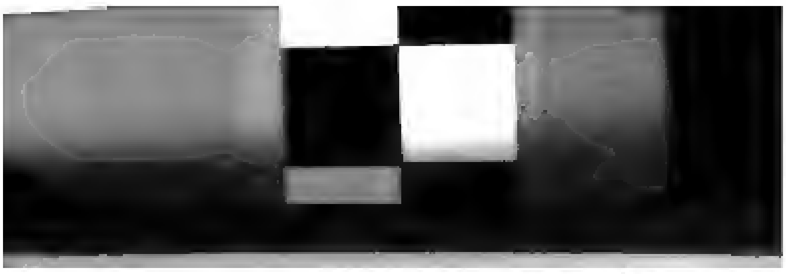


A COLLEGE TEXT-BOOK

OF

PHYSICS

BY

ARTHUR L. KIMBALL, PH. D.

PROFESSOR OF PHYSICS IN AMHERST COLLEGE

SECOND EDITION, REVISED

NEW YORK
HENRY HOLT AND COMPANY
1917

THE MAPLE PRESS YORK PA

PREFACE

In offering this work to my fellow teachers, a word of explanation is due.

The book was undertaken some years ago when the writer felt the want of a text-book adapted to the needs of students taking the general first year course in college. As the work has slowly progressed several text-books of very similar aim have appeared, and it must be admitted that the call is not so imperative now as formerly; and yet it is hoped that the treatment here presented may meet some still existing demand and so justify its existence.

What may be called the physical rather than the mathematical method has been preferred in giving definitions and explanations, because it is believed that the ideas presented are more easily grasped and more tenaciously held when the mind forms for itself a sort of picture of the conditions, instead of merely associating them with the symbols of a formula.

There are many minds that do not easily grasp mathematical reasoning even of a simple sort; and it is often the case also that a student who may be able to follow an algebraic deduction step by step has very little idea of the significance of the whole when he reaches the end. Algebra is not his native tongue and it takes considerable time and experience for him to learn to think in it. While all will agree that for the more advanced study of physics, mathematics is quite indispensable, many will grant that in a general course, which is to furnish to most of those taking it all that they will ever know of physics as a science, the ideas and reasonings should be presented as directly as possible and in the most simple and familiar terms.

This then has been the central aim in the preparation of this book: to give the student clear and distinct conceptions of the fundamental ideas and phenomena of physics, and to aid him in thinking through the relations between them, to the end that he may gain some understanding of the underlying unity of the subject; and to carry out this aim in such a manner that students may not be repelled by any unnecessary prominence of symbolic methods, and yet that the treatment may have all the exactness and

precision in statement and deduction which the subject demands. This is a large ambition and I cannot hope to have been wholly successful, but I shall be grateful if my attempt is found in any degree to have subserved its purpose.

My grateful acknowledgements are due to Dr. G. S. Fulcher of the University of Wisconsin, who has read nearly all the manuscript with great care, and to whom I am indebted for important suggestions, and to my colleague Professor J. O. Thompson whose criticism at all stages of the work and painstaking correction of the proof has been most helpful.

AMHERST, A. L. K.
March, 1911.

PREFACE TO REVISED EDITION

Recent advances in physical science having made it necessary to rewrite some paragraphs of the earlier edition, especially those relating to X-rays and the electron theory of matter, advantage has been taken of the opportunity to make a few additional changes which class-room experience has shown to be desirable. Certain paragraphs relating to force and motion, which had been introduced before the section on statics, are now placed among the introductory paragraphs to kinetics, where they fall in line with the logical development of the subject. The electromagnetic units, volt, ampere and ohm, are defined and introduced earlier than before. The sections on wireless telegraphy have been made more complete and wireless telephony is touched upon. A section also has been added treating of the [illegible] photometer. At the end of the volume a short discussion of Carnot's cycle and the thermodynamic basis of the absolute scale of temperature has been introduced as an appendix [illegible] proof is given of Newton's wave formula. Quite a number of [illegible] problems have been added, but the old problems have been [illegible] to serve their purpose well and are for the most part retained.

The author gratefully acknowledges his indebtedness to Dr. G. S. Fulcher, and to Professors W. E. McElfresh and [illegible] Miller, for valuable suggestions and criticisms.

AMHERST, MASS. A. L. K.
July, 1917.

CONTENTS

INTRODUCTION

MECHANICS

RADIOACTIVITY

LIGHT

INTRODUCTION

Aim and Method of Physical Sciences

1. Physical Sciences.—The study of nature includes two great divisions, biological and physical sciences. The former includes those that involve the complex phenomena of life, while the latter are concerned with the investigation of the fundamental phenomena of matter. Physics and Chemistry are the fundamental physical sciences and form the basis upon which Astronomy, Geology and Meteorology rest in investigating their special realms in the world of nature. Formerly Physics was called Natural Philosophy, in distinction from Natural History which described the world of plants and animals.

Physics deals with the properties and phenomena of inanimate matter as affected by forces, and is expecially concerned with the properties common to all kinds of matter and those changes of form and state which matter undergoes without being changed in kind, as well as such general phenomena as sound, heat, electricity and magnetism. Chemistry is distinguished from Physics in that it is chiefly concerned with the phenomena that result when different kinds of matter are brought together and enter into combination. It deals largely with the qualities in which one kind of matter differs from another. There are, however, many points where these sciences merge into each other, and the domain of *physical chemistry* lies largely in this borderland.

2. The Aim of Physical Science.—It is the aim of physical science so to systematise our knowledge of the material world that all its phenomena shall be seen as special instances under a few far-reaching and more inclusive generalizations called laws. And when a given phenomenon is analyzed in this way into separate parts or phases each of which is but a special case under some general law, the phenomenon is said to be explained.

In seeking an explanation we determine the *causes* of the

phenomenon in question; that is, the *essential* circumstances or those circumstances without which the given event does not occur; and then we seek to determine the effect of each of these circumstances separately, and exhibit, if possible, each such effect as a special instance under some general law.

For example, the complex motion of a ball struck by a bat is found to be dependent on the motion given to it by the blow of the bat, on the presence of the earth, and on air resistance.

We first try and determine how a body moves when set free in the presence of the earth without any initial blow or impulse and in a vacuum. We find in this way an unvarying rule of motion that applies to all bodies of whatever size or shape, and we call it the law of falling bodies. That part of the motion of the ball which depends only on the nearness of the earth is but a special instance under this law. Now, making allowance for the motion due to the earth, we seek to determine that part of the motion due to the initial blow, and here again we find that the actual motion seems to be exactly according to a general rule which is found to hold whenever an impulsive force acts on a mass. And finally we investigate the effect of air resistance, determining how it affects a body at rest and how it modifies the motion of a body moving through it, and here again certain general rules are found which apply not only to the special case under consideration, but to all cases of bodies moving through air. When the effects of all three circumstances are taken into account, the motion is found to be exactly accounted for, and is then said to be explained.

Leverrier and Adams, in analyzing the motion of the planet Uranus, found that after taking account of all the known circumstances, such as the attractions of the sun and other planets upon it, there still remained a part of its motion which was not accounted for, and assuming it to be due to an unknown planet they computed its position and mass, and thus the planet Neptune was discovered.

But in analyzing our problem we may go deeper and show that the motion of the ball near the earth is such as would result from a force urging the two bodies together, and we may then discover that it is merely a special instance of the law that all bodies are influenced by forces urging them together or, in other words, that

all bodies attract each other. When we can show also that the forces between the air and the moving body are due to the motion given to the air and so are simply particular exhibitions of the general rule which holds whenever matter is set in motion, we feel that a still higher degree of understanding is reached.

By such a process all the complex facts of nature are assigned their places in an orderly system. But a limit is soon reached beyond which the mind cannot go, because thinking is conditioned by experience, and even in its profoundest theories and speculations the mind must employ those conceptions which it has obtained from the world about it.

3. Experiment.—Physics is an experimental science, its generalizations rest solely upon experiment, and although reasoning upon established facts has often led to the discovery of new truths of great importance, the final appeal must always be to experiment. If the deduction is thus disproved, it appears either that the reasoning was wrong or that there are certain elements entering into the problem that were neglected. In seeking for the causes of such discrepancies new truths have often been discovered.

An experiment is a combination of circumstances brought about for the purpose of testing the truth of some deduction or for the discovery of new effects. The usual course of an experimental inquiry is to modify the circumstances one by one, noting the corresponding effect until the influence of each is thoroughly understood.

4. Necessary Assumptions.—In every experimental science it is assumed that the same causes always produce the same effects and that the position of the event as a whole in either time or space only affects the absolute time and position of the result, provided there is no change in the *relative* time or space relations of the various circumstances involved. For example, if all the other circumstances are the same, a stone will fall in exactly the same manner next week as it does to-day, or if the solar system be changing its place in space no change in the manner of the stone's falling will take place from that cause alone.

Experience up to this time has justified these assumptions, and without them progress in physical science would be impossible.

Fundamental Conceptions

5. Force.—Our ideas of force are derived primarily from muscular effort. It requires an effort to lift a weight, to throw a ball, or to compress a spring. The upward pull upon a weight at any instant while it is being lifted is a force acting on it, and the downward tendency of the weight which the pull opposes is also a force.

Anything that serves to accomplish what would require muscular exertion to bring about exerts a force. Thus a support exerts a force on the weight which rests upon it, and the weight exerts an equal and opposite force on the support, compressing it. A bat exerts a force against a ball in giving it motion, a clock spring exerts a force against the stop that prevents it from unwinding.

6. Matter.—Through our muscular sense and sense of touch we are made conscious of bodies around us which resist compression, and may, therefore, be said to occupy space. Such bodies are said to be *material substances* or made of *matter*.

Every object that we know of possesses weight; that is, it requires some muscular effort to support it, or if it is hung on a spring the spring is stretched. What we call its weight is a force urging it toward the ground, and as the weight of two quarts of water is twice that of one quart we are led to think of the weight of a body as a proper measure of the quantity of matter which it contains.

But besides weight all bodies have inertia; that is, to produce a definite change per second in the motion of a body a certain force is required.

If a given body is isolated from other portions of matter, it may be heated or cooled or bent or twisted or compressed into small volume or allowed to expand into a large one, but in all these changes its weight and its inertia remain unchanged.

It will be seen later that if the body is taken from one place on the earth to another its *weight* also may change, so that **the only general property of a given portion of matter that cannot be changed is its inertia.**

It is this property, therefore, by which quantities of matter are defined, and two bodies which have equal inertias are said to have equal *masses* or to contain equal quantities of matter.

› actual comparison of two masses, however, is usually by weighing, since under the ordinary circumstances of ing, bodies which have equal weights have equal inertias.

Conservation of Matter.—The mass of a given portion of r as measured by its inertia cannot be changed by any s known to man. Not only may a piece of wood be bent visted or compressed without changing its mass, but it may rned in the fire, and chemistry shows that if the ashes and s and gases that have come from it are collected and separated from the gases of the air with which they may have united, be found that the united mass of the ash and the gases and s is the same as the mass of the original piece of wood.

s principle is known as the *conservation of matter*, and is ished by innumerable experiments, both physical and cal.

States of Matter.—Different kinds of matter differ greatly power of preserving their shape. Some, such as steel or r, offer very great resistance to any attempt to change their . Such bodies are said to be *rigid* or *solid* bodies. Others, ater or air, have no permanent shape, but flow under the of the weakest forces and take the shapes of the vessels ning them; they are called *fluids*. There are no substances re either perfectly rigid or that are perfect fluids, for the rigid bodies may be distorted, and those substances that nost freely offer some resistance to change of form.

ome cases it is difficult to say whether a substance is to be led as solid or fluid.

ids are again divided into *liquids* and *gases*. *Liquids* are fluids that can have a free surface and do not change much ıme under great changes in pressure. A mass of liquid has ly definite bulk though no permanent shape. Water is an le of a liquid.

s, on the other hand, are fluids that do not have a free sur- ut completely fill the containing vessel, however much it e enlarged.

ıass of gas may be regarded as having neither permanent nor size, since both of these are entirely determined by the which contains it. Air is a familiar example of a gas.

1 yard = 91.43835 cm.
1 foot = 30.47945 cm.

13. Unit of Mass.—The unit of mass in the C. G. S. system is the **gram**, or one-thousandth part of the standard kilogram, which is a mass of platinum kept at Paris and known as the *Kilogramme des Archives*. The standard kilogram was intended to represent the exact mass of a cubic decimeter of distilled water at its greatest density or at the temperature 4°C.

The gram is, therefore, equal to the mass of a cubic centimeter of pure water at 4°C. This relation between the cubic centimeter and the gram is exceedingly convenient, for it enables us to determine the volume of an irregular vessel from the weight of water which it can contain. But it is not a direct relation like that between the unit of length and unit of volume. Aside from convenience, there is no reason why a cubic centimeter of copper or mercury or of anything else might not have been taken as the unit of mass.

Since two masses may be compared with a far higher degree of accuracy than that with which the weight of a cubic centimeter of water can be determined, the **Kilogramme des Archives is the real standard on which all metric weights are based.**

14. Unit of Time.—Intervals of time are always compared by the motions of bodies. Two intervals of time are *defined* as equal when a body, moving under exactly the same circumstances in both cases, moves as far in the one time as in the other. The heavenly bodies have in their motions always furnished measures of time. One of the simplest natural units of time is the period of rotation of the earth, which is the interval of time between two succesive meridian passages of the same star. This is known as the *sidereal* day, and time reckoned in this way is called sidereal time. By considering the possible effect of tidal friction in retarding the earth's motion, Adams concludes that the period of rotation of the earth has not changed by more than one-thirtieth of a second in 3000 years.

The ordinary day is determined not by the rising and setting of the stars, but by the motion of the sun. When the sun is on the meridian it is said to be *solar* or *apparent* noon. The interval of time between two successive apparent noons is called the appar-

ent or solar day. It is this time which is indicated by the sun dial. By means of clocks, which are machines constructed to run with great uniformity, one solar day may be compared with another, and it is thus found that they are not of equal length. The average length of the solar days in a year is known as the *mean solar day*.

The ordinary standard time used in everyday life is mean solar time.

The unit of time in the C. G. S. system is the mean solar second or the 86400th part of a mean solar day.

MECHANICS

I. GENERAL PRINCIPLES

15. Definitions.—*Mechanics* treats of the motions of masses and of the effect of forces in causing or modifying those motions. It includes those cases where forces cause relative motions of the the different parts of an elastic body causing it to change its shape or size, as when a gas is compressed or a spring bent. Such changes in size or shape of different portions of a body are called *strains*. Bodies which do not suffer strain when acted on by forces are said to be *rigid*.

All known bodies yield more or less to distorting or compressing forces, but when considering the motion of a body as a whole, all bodies in which the strains are small may be regarded as practically rigid. Thus we may treat the motion of a grindstone or of a shell from a rifled gun as though these bodies were rigid, though we know that they are slightly strained by the forces acting.

Mechanics is usually subdivided into kinematics and dynamics.

Kinematics treats of the characteristics of different kinds of motion, and of the modes of strain in elastic bodies without reference to the forces involved.

Dynamics treats of the effect of forces in causing or modifying the motions of masses and in producing strains in elastic bodies.

It is usual to treat *dynamics* under the heads *statics* and *kinetics*.

Statics is that part of dynamics which deals with bodies in equilibrium or when the several forces that may be involved are so related as to balance or neutralize each other, so far as giving motion to the body as a whole is concerned.

Kinetics is that part of dynamics which treats of the effect of forces in changing the motions of bodies.

Ideas and Definitions of Kinematics

16. Motion Relative.—When a body is changing its position it is said to be in motion. There is no way of fixing the position

of a body except by its distance from surrounding objects. When it is said, therefore, that a body has moved, it is always meant that there has been a change in its position with reference to some other objects regarded as fixed, or in other words, there has been *relative* motion. Thus we know only relative motion, and when we speak of an object as at rest we usually mean with reference to that part of the earth's surface in our vicinity.

17. Displacement.—The distance in a straight line from one position of the body to another is called its displacement from the first position. To completely describe any displacement, its amount and direction must both be given.

If an extended rigid body is displaced, as when a book is moved on a table, it may be moved in such a way that its edges will remain parallel to their original directions, in which case the displacements of all points in the body will be the same both in amount and direction. The motion is said to be one of simple *translation* without *rotation*. But in general when a rigid body is moved there is rotation as well as translation, so that to bring it into the second position from the first we may first imagine it to be translated till some point in the object is brought into its second position. Then by a rotation about a suitable axis through that point the whole body may be brought into the second position.

18. Vectors and Their Representation.—All quantities which involve the idea of direction as well as amount are said to be *vector quantities* or *vectors*. Such are displacements, velocities, forces, etc. While quantities having magnitude only, without any reference to direction, are known as *scalar* quantities. Volume, density, mass, and energy are scalar magnitudes. A vector quantity is represented by a straight line which indicates by its position the direction of the vector, and by its length the magnitude of the vector, the length being measured in any convenient units, provided the same scale is used throughout any one diagram or construction.

It must be remembered, however, that a vector represented by a line AB is not the same as that represented by BA, one is the opposite of the other, or $AB = -BA$. This will be evident if AB represent a displacement from A to B. A displacement BA will exactly undo what the other accomplished, and bring the

body back to its starting point. The straight line representing a vector is, therefore, commonly represented with an arrow-head indicating its positive direction.

19. Composition of Displacements.—If a man in a railway car were to go directly across from one side to the other, say from A to B (Fig. 1), then the line AB will represent both in amount and direction his displacement considered only with respect to the car. But if the car is in motion and in the meantime has advanced through the distance AC, the man will evidently come to D instead of to B. The displacement of the car with reference to the earth is AC and the displacement of the man relative to the earth is AD. This is called the *resultant* displacement of the man, of which AB and AC are the components.

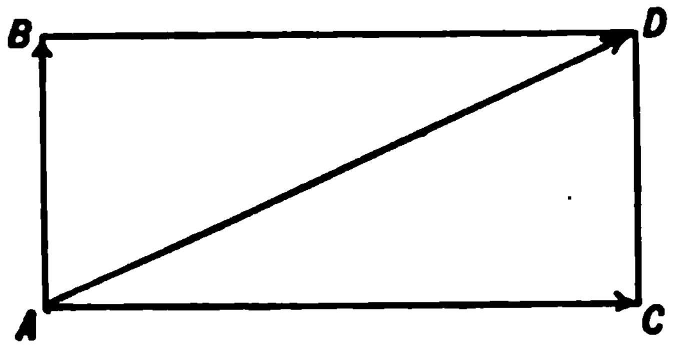


FIG. 1.

Another way of stating this is that the man received simultaneously two displacements AB and AC, for if he had not been displaced in the direction AC he would have gone to B, while if he had not had the displacement AB he would have been carried to C.

From the above it is evident that the resultant of any number of simultaneous displacements may be found just as if they had been taken successively.

For example, let it be required to find the resultant of four displacements represented in amount and direction by the vectors A, B, C, D. If A were the only displacement, the body would be brought from O to a, but B is also a component displacement, therefore draw B' equal and parallel to B, and the result of the two displacements will be represented by the distance $O\,b$. Then in like manner draw C' and D' equal and parallel, respectively, to C and D, and it is clear that the result of the four

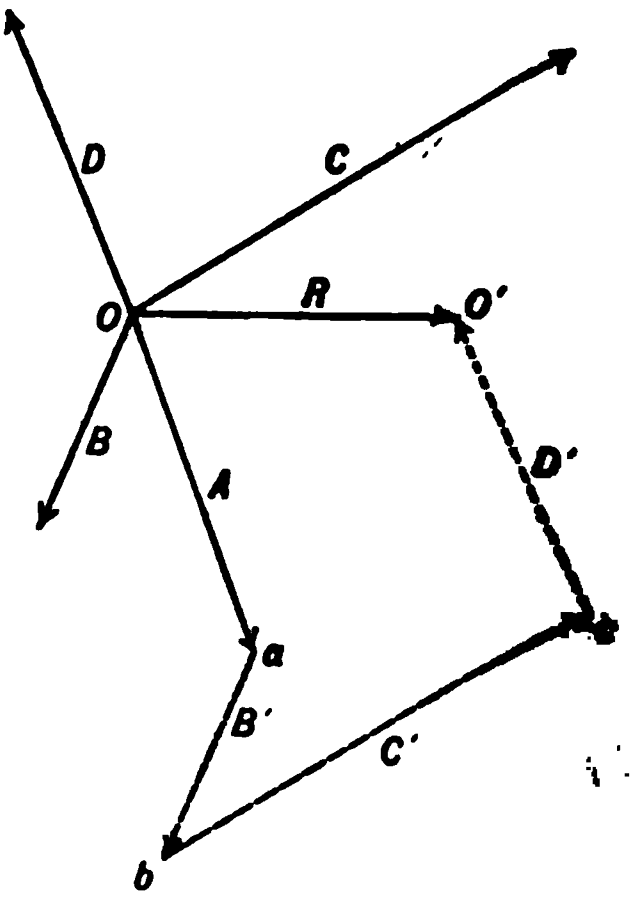


FIG. 2.

displacements on a body originally at O would be to transfer it to O'. Therefore the *resultant* of the four displacements is the single displacement R, and this is so whether the component displacements occur simultaneously or successively. The particular order in which the several components are taken is quite immaterial.

This construction by which the resultant is found is called the diagram of displacements, it is perfectly general and applies whether the components are in the same plane or not.

20. Composition of Vectors.—The above construction is a particular instance of the addition or composition of vectors. By a precisely similar process the resultant of any set of vectors may be obtained whether they represent forces, velocities, momenta, or any other quantities having direction as well as magnitude.

21. Resolution of Displacements.—There is only one resultant displacement that can be found when the components are given, in whatever order they may be taken. If it is required, however, to *resolve* a given displacement into its components, there are an infinite number of ways in which it may be done.

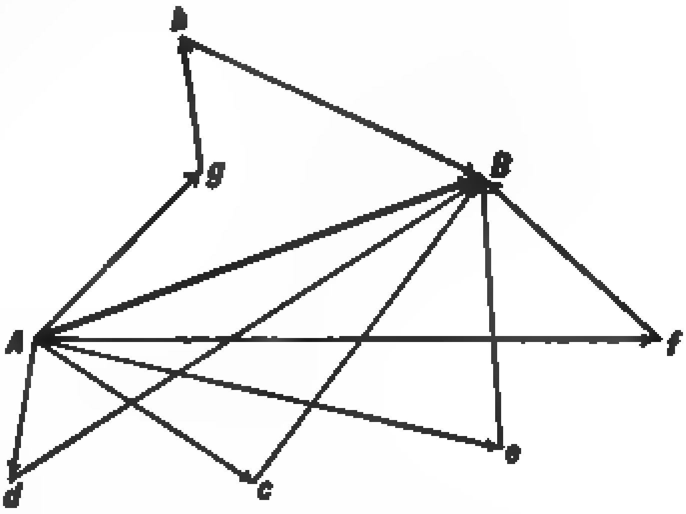


Fig. 3.

For example, the displacement AB (Fig. 3) may be regarded as having Ac and cB as its components, or Ad and dB or Ae and eB, or it may be considered the resultant of the three displacements Ag, gh, hB. Or if any broken line whatever be taken starting at A and terminating at B, AB will evidently be the resultant of the displacements which are represented in amount and direction by the several parts of the broken line.

22. Resolving of Vectors.—What has been just said of the resolving of a displacement into components is equally true of the resolving of any other vectors whatever into component vectors, and applies to the resolution of velocities, forces, etc.

23. Velocity.—The velocity of a body is the rate at which it passes over distance in time. It is a vector quantity, its direction being as important as its amount. The term *speed* is famil-

ilary used to express the *amount* of velocity without reference to its direction. Two bodies may be moving with the same speed, but if they are not going in the same direction their velocities are different.

This is the strict use of the word velocity; it is often somewhat loosely used to express merely the speed of motion.

24. Constant Velocity.—When a body moves in a straight line always passing over equal distances in equal times it is said to have constant or uniform velocity. It is evident that the motion must be in a straight line, otherwise the *direction* of the velocity would not be constant.

In this case of motion if the length of any part of the path be divided by the time taken for the body to traverse that portion, the result is what is called the rate of *motion*, or the distance passed over per unit time, and is the same whatever part of the path may be chosen. It is this quantity which is the *speed* or the amount of the velocity.

Thus when a train is moving with constant velocity, the number of miles run in a given time divided by that time expressed in hours, is the speed in miles per hour.

25. Variable Velocity.—When either the rate or direction of motion of a particle is changing, it is said to be moving with variable velocity. Thus the velocity is varying in case of a falling body which constantly gains in speed or in case of a railway train rounding a curve where the direction of motion is changing.

Fig. 4.

To understand what is meant by the *speed* of motion at a particular point when the velocity is constantly changing we may consider a short portion *b c* (Fig. 4) of the path of the body having at its middle the point *a* at which the speed is to be determined. Divide the length of *b c* by the time taken by the body in traversing it. The result will be what may be called the *average* speed over that part of the path. If, now, the part chosen is taken smaller and smaller, always having the given point at its center, the average velocities thus found will approximate more and more nearly to the true velocity at the given point, and that value which these successive approximations continually approach as a *limit*, as the distance *b c* approaches zero, is the speed of motion at

the point a. At each instant a body has a certain speed, but it may not be constant even for the shortest interval of time that can be conceived.

So also with regard to the *direction* of motion. If the body moves in a curved path, its direction of motion at any point is the direction of the tangent to the curve at that point, and as the direction of the tangent constantly changes as we pass along the curve, so the direction of the velocity in such a case may be different at one point from what it is at a neighboring one, however near together the two points may be.

26. Composition of Velocities.—If a body has at any instant several component velocities, the resultant velocity may be found by the vector diagram as in the case of the composition of displacements.

For instance, suppose a ball is thrown in a moving railway car, it is required to find the velocity of the ball with reference to the earth. Let AB represent the velocity of the railway car, say 50 ft. per second, and let AC be the velocity of the ball as thrown obliquely across the car with a velocity of, say, 40 ft. per second. Then, laying off the vectors AB and BD with the proper relative direction and length, the resultant velocity is represented by the vector AD, which is found by measurement (using the same scale as in laying off AB and BD) to be 76 ft. per second, and this is the resultant speed of the ball relative to the earth. If the angle between AB and AC is given, the side AD of the triangle ABD may be calculated by trigonometry, using the formula

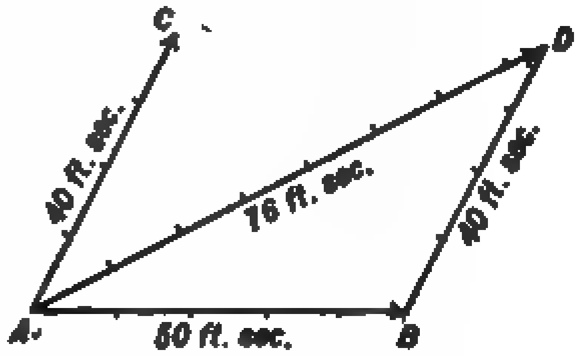


Fig. 5.

$$AD^2 = AB^2 + AC^2 + 2AB.\,AC.\,\cos CAB.$$

27. Resolution of Velocities.—Any given velocity may also be resolved into component velocities. For instance, suppose a man is rowing a boat with a velocity of 10 ft. per second in a direction making an angle of 30° with the straight shore of a lake, and it is required to determine how fast he is moving along the shore and how fast he is moving out into the lake. Let AB (Fig. 6) represent a velocity of 10 ft. per second. Draw AC paral-

lel with the shore and making an angle of 30° with AB. Draw CB perpendicular to AC. Then AC and CB will represent two velocities, one parallel to the shore and one at right angles to it, whose resultant is AB. Therefore the boat may be regarded as having a velocity AC parallel with the shore and a velocity CB at right angles to it, and the amounts of these may be found by measurement, using the same scale as in laying off AB. Or we may calculate them by trigonometry, for

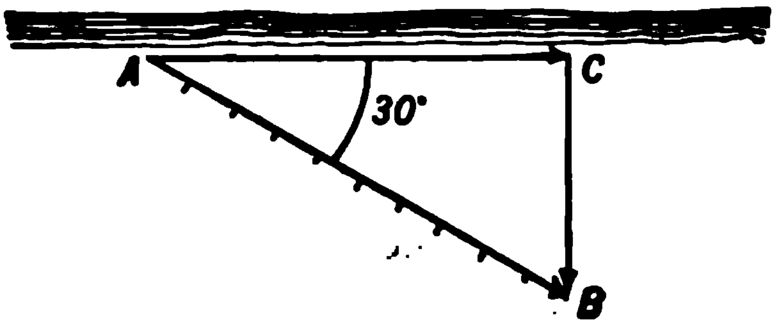


Fig. 6.

$$\begin{aligned} AC &= AB.\cos 30^\circ \quad \therefore \quad & AC &= 8.66 \text{ ft. per second} \\ CB &= AB.\sin 30^\circ & CB &= 5 \quad\ \text{ft. per second} \end{aligned}$$

It is frequently necessary in practice to resolve a velocity or other vector into two components which are mutually at right angles as in the case just discussed, and so this case, while one of the simplest, is one of much importance.

28. Acceleration.—When the velocity of a body changes either in amount or direction the motion is said to be *accelerated*, and

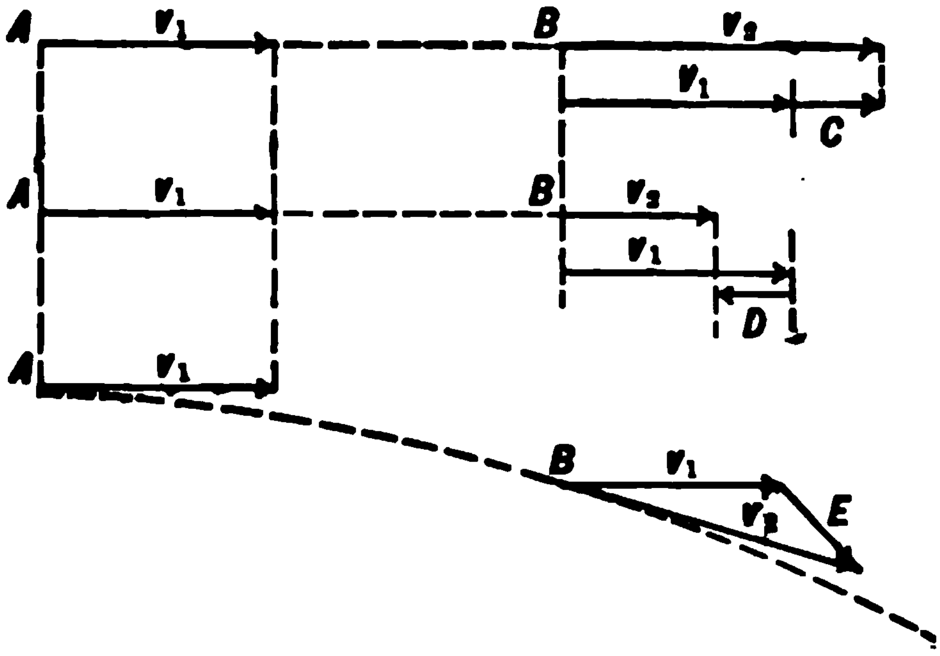


Fig. 7.

the change in velocity per unit time, or the time rate of change of the velocity, is called the *rate of acceleration* or simply the *acceleration*.

Change in velocity may always be thought of as due to the body receiving an additional component velocity which is compounded

with the original velocity, the velocity after the change being the resultant of the two.

For example, in the upper diagram of figure 7 a body moving in a straight line is represented as having a velocity v_1 at A and a greater velocity v_2 at B. The gain in velocity is represented by the vector C which must be added to v_1 to give v_2. The average rate of acceleration between A and B is therefore found by dividing C, the increase in velocity, by the time taken by the body in passing from A to B. In the second diagram v_2 is less than v_1, and so the change in velocity is represented by the arrow D and is negative or opposite to the original motion. In the third case figured the motion is along a curve and the velocity at B is not in the same direction as the velocity at A, but a velocity represented by E if compounded with v_1 will give v_2 as the resultant. The velocity E, is therefore the change in velocity between A and B, and dividing it by the time during which the change has taken place or the time of motion from A to B, the *average* rate of acceleration between A and B is found.

29. Acceleration in Rectilinear Motion.—If the motion is in a straight line the velocity changes only in amount and not in direction, and the acceleration is calculated by dividing the change in speed during a given interval of time by the time interval. Or, expressing it in a formula,

$$a = \frac{v - u}{t}$$

where u represents the velocity at the beginning of the interval of time t while v represents the velocity at its end. This formula gives in general the average rate of acceleration during the interval of time t, but if the acceleration is constant it gives the actual rate.

Thus if a ball with a velocity of 50 ft. per second has, after one-half second, a velocity of 40 ft. per second in the same direction, the average rate of the acceleration during the interval is

$$\frac{40 - 50}{\frac{1}{2}} = -20,$$

the negative sign indicating that the acceleration is opposite in direction to the original velocity and therefore the velocity is decreasing.

[redacted]

30. Composition and Resolution of Accelerations.—When a moving body has several different accelerations, as when a man in a railway car starts to walk in the aisle while the speed of the train is changing or while it is rounding a curve, the several accelerations may be compounded and their resultant found just as with other vectors. So also an acceleration may be resolved into two or more components.

Problems

1. A man walks ½ mile in 10 minutes. What is his average velocity in feet per second?
2. A train has a velocity of 30 miles per hour; what is its velocity in feet per second?
3. A bicycle rider is traveling north at the rate of 10 miles per hour. If the wind is blowing from the east at the rate of 6 miles per hour, what is its apparent direction and velocity to the rider? Show the direction by a diagram.
4. A man rows a boat at the rate of 4 miles per hour, making an angle of 30° with the straight shore of the lake. How fast is he moving away from the shore?
5. Draw a diagram to scale showing the direction in which a man must row across a river in order to reach a point directly opposite, if he rows 3 miles per hour while the speed of the current is 2 miles per hour.
6. If the river in the last problem is ¼ mile broad, how long does it take to cross it as described, and what is the velocity of the boat relative to the shore?
7. A ball rolling down an incline has a velocity of 60 cm. per sec. at a certain instant, and 11 seconds later it has attained a velocity of 181 cm. per sec. Find its acceleration.
8. A body having an initial velocity of 60 ft. per sec. has an acceleration −32 ft. per sec. per sec. Find its velocity at the end of 1, 2, and 3 seconds.
9. A railroad train having a velocity of 40 miles per hour is brought to rest in 1 minute. Find the acceleration in feet per second per second.

First Principles of Dynamics

31. First Law of Motion.—When a ball on a table starts to roll, experience convinces us either that the table is not level or that some external force has caused the motion. On the other hand, when we see a ball that has been set rolling on a level table, gradually losing its speed, we are equally satisfied that there is some force resisting its motion. For it is found in such a case

that if the table is made smoother and if air resistance is gotten rid of, the ball loses speed much more slowly than before. We are thus satisfied that if there were no force resisting its motion the speed of the ball would remain unchanged.

This conviction, arrived at through experience, was clearly enunciated by Sir Isaac Newton in the first of his celebrated Laws of Motion, published in his *Principia*, in 1686.

First Law of Motion.—Every body continues in its state of rest, or of moving with constant velocity in a straight line, unless acted upon by some external force.

32. Discussion of the First Law of Motion.—The first law asserts that force is not required to *keep* a body in motion, but simply to *change* its state of motion. After a railroad train has attained a constant speed the entire force of the locomotive is spent in overcoming the various resistances that oppose the motion, such as friction of wheels and bearings and air resistance. But for these the train would maintain its speed without aid from the locomotive.

Therefore, when any object is observed to be at rest or moving with constant speed in a straight line, we conclude either that no external force acts upon the body, or that whatever forces act are so related as to neutralize or balance each other.

Since we measure equal times by the equal angles through which the earth has moved, the law that freely moving bodies move through equal distances in equal times may seem simply a consequence of the mode of defining equal times and without any physical significance. But the statement of the law really asserts the physical fact that *in case of any two bodies whatever, unacted on by external forces, while one body moves through successive equal distances, the distances traversed simultaneously by the other body are also equal among themselves.*

That this is true whatever the nature of the bodies concerned is a fact of nature that rests on experience, and cannot be regarded as known *a priori.*

33. Inertia.—The property, common to all kinds of matter, that no material body can have its state of rest or motion changed without the action of some force, is known as *inertia.* The amount of force required to produce a given change in the motion of a body depends both on the body and on the suddenness of the change to be produced.

Any force however small can give as great a velocity as may be

desired to any mass however great, provided it acts for a long enough time.

If a weight rests on a sheet of paper on a table it may be drawn along by means of the paper, for there is friction between the two and it requires a certain force to slip the one over the other. *If, therefore, we do not attempt to accelerate the weight too rapidly,* the friction will move the weight along with the paper. But if we attempt to start the weight suddenly, or change its velocity suddenly while moving, the paper will at once slip from under it, for the force required to produce the sudden change of motion is greater than the friction between the two.

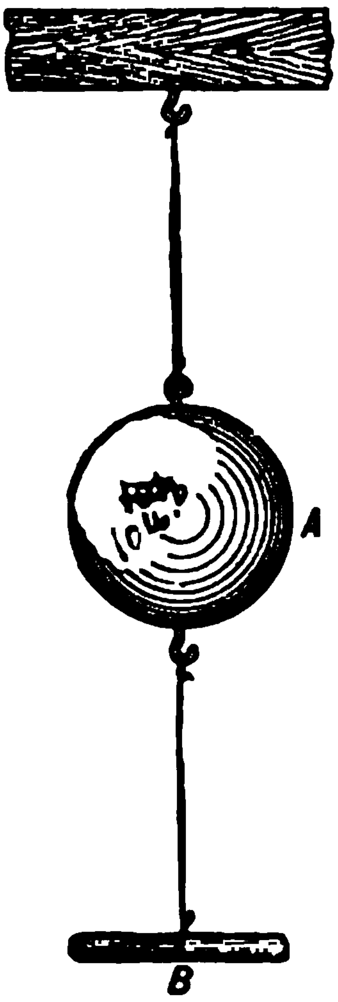


Fig. 8.—Inertia.

Again if a 10-lb. weight (Fig. 8) is hung by a cord from a fixed support and if it is drawn steadily downward by a piece of the same cord attached to it underneath, the cord will break *above* the weight, for the force exerted by the lower cord upon the weight will cause it to move downward, straining the upper cord with the combined force due both to the weight and the pull. But a sudden pull will break the cord *below* the weight. For in consequence of its inertia the weight cannot be set in motion as suddenly as the cord is pulled without the exertion of a greater force than the cord is able to bear, so that the cord breaks even before the weight has moved downward enough to strain the upper cord to the breaking point.

The complete statement of the effect of a force upon the motion of a body is embodied in Newton's Second Law of Motion, and will be discussed when we take up the study of Kinetics (§93).

34. Measure of Mass.—The inertias of bodies may be compared quantitatively by the amounts of force required to accelerate them at the same rate, and when they are thus compared it is found that *the inertia of a given portion of matter is always the same and cannot be increased or diminished by any known process.* Consequently the inertia of a body is the sum of the inertias of its several parts, the inertia of two quarts of water is twice that of one quart whether t bined or separate.

It is for this reason that the quantity of matter in a body, or its *mass* as it is called, is measured by its inertia as compared with that of some standard piece of matter taken as the unit of mass.

Therefore, **masses are said to be equal which acquire equal velocities when acted on by equal forces for the same length of time.** For example, suppose two masses are drawn side by side over a frictionless surface by two spring balances at such a rate that each balance is kept constantly stretched, say to the 4-oz. point, so that they exert equal forces; then, if the masses after starting together keep pace with each other, they are acquiring velocity at the same rate and consequently are equal.

Of course such an experiment serves chiefly to illustrate what is meant by saying that equal masses have equal inertias, for it would be impossible to directly compare masses in this way with any degree of accuracy.

The actual comparison of masses is accomplished with great accuracy by *weighing;* for it is found that masses which have equal inertias have also equal weights, provided they are weighed in a vacuum at the same point on the earth. (§102.)

35. Measure of Force.—A force may be measured in three ways:

1. *By the weight that it can support.* This is the gravitation method.

2. *By its power to strain an elastic body*, as in the ordinary spring balance.

3. *By its power to give motion to a mass.* This is the dynamical method.

The first method is very convenient and forms the basis of most measurements of force in engineering and ordinary life, but it has the disadvantage that the force required to support a pound weight varies from place to place on the earth.

The second method is convenient for comparing forces, but the elastic properties of one substance differ from those of another, besides being dependent on temperature and physical condition, so that a standard force could not be preserved or accurately defined by this method.

The third method is difficult to apply except indirectly, but furnishes a unit of force which depends only on the inertia of matter and is, therefore, absolutely invariable and well suited to be a standard force.

36. Equal Forces.—Two forces are said to be *equal* when the velocity of a given mass is increased at the same rate per second by one force as by the other. When two such forces act in opposite directions on a given mass they neutralize or balance each other so far as any effect on the *motion* of the mass is concerned. Thus when a cord is stretched horizontally between two springs, the forces exerted by the springs are equal and opposite so long as the cord remains at rest or moves with uniform velocity.

37. Stress.—When a weight is supported by a uniform cord, every part of the cord is stretched, and if the weight of the cord itself is so small that it may be neglected, the stretch of every inch of it is the same whether it is near the upper or lower end and whatever may be the total length. The section AB (Fig. 9) is pulled up by the cord above A and is pulled down by the cord below B and is, therefore, stretched until the contractile force of its own elasticity balances the external stretching force. In this way every portion of the cord is subject from without to an external stretching force, and it exerts in opposition to this an internal contractile force and is said to be *in a state of stress*. In this case the stress is called *tension*, and every portion is subject to forces which tend to elongate it. When a weight is supported on a vertical rod or column the whole support is in another state of stress called *pressure* or *compression*, for the forces that act on any part of the rod tend to shorten it. Besides *tension* and *pressure* there is a third kind of stress, called *shearing stress*, which tends to distort or force out of shape the parts of a body. This is the stress in a rod that is being twisted. But the further discussion of this matter must be left until the elasticity of bodies is considered. (§234.)

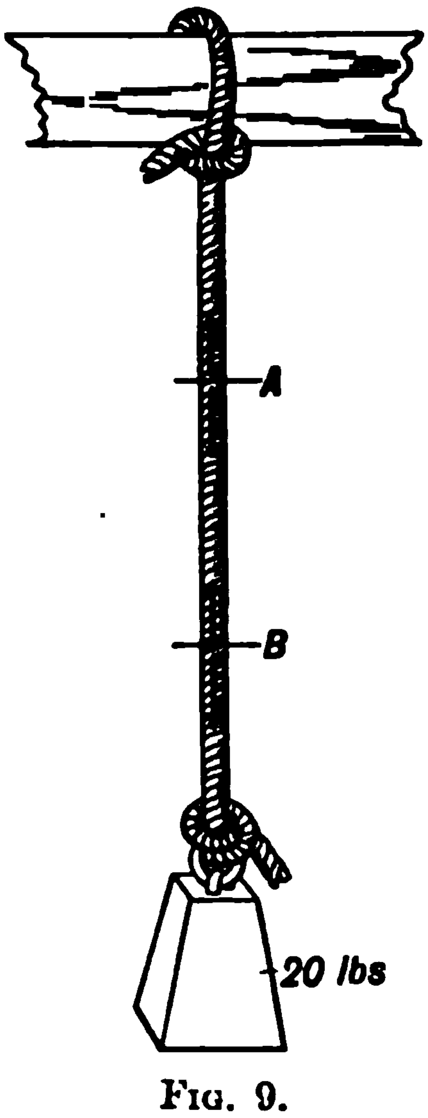


Fig. 9.

It is believed that *all forces are transmitted by stresses.* Even the attraction between a magnet and a piece of iron is explained by stress in the ether around them, and the gravitation attraction which exists between all masses is also supposed to

due in some unexplained way to a stress in the surrounding edium.

38. Action and Reaction.—Every stress has a double aspect. hus when a weight rests on a table the force between the two ay be regarded as a pressure down on the table or an upward sh against the weight. When a cord is supporting a weight, at ery cross section in the cord there is a downward pull on the rd above the section, and an upward pull on the cord below d these two are exactly equal. When a magnet attracts a ece of iron the force may be regarded as drawing the iron ward the magnet or the magnet toward the iron.

These two aspects of a stress are known as the action and action; they are exactly equal and opposite. This fundamental ct was stated by Newton as the Third Law of Motion.

Third Law of Motion.—To every action there is an equal and posite reaction.

39. Discussion of Third Law of Motion.—When a weight rests on a table it is pushed up with a force equal to that which it erts upon the table. The table, therefore, presses a heavy ight upward with more force than it exerts on a small weight. e only limit of the power of the table to react is its strength. is instructive to consider what happens when a weight heavy ough to crush the table is placed upon it. As it is lowered upon table it presses more and more until the limit of the table's wer of resistance is reached, when in breaking down it begins move away from the weight at such a rate that the reaction ich it exerts is at every instant exactly equal to the pressure to ich it is subjected by the weight. For if a body moves away t enough from another which is pressing upon it the pressure ay be diminished to any extent.

When a ball is struck by a bat the force upon the bat at every stant while they are in contact is the same as that which the bat erts upon the ball.

40. Composition and Resolution of Forces.—When several rces act simultaneously on a particle **the single resultant force ay be found by the diagram of vectors** (§ 19) just as in case of celerations; for the several component forces each cause a rresponding component of acceleration, and the resultant eleration corresponds to the resultant force.

So also any force may be considered as the resultant of two or more component forces, and these components may be found just as the components of an acceleration are found.

41. A Special Case.—Suppose the vector AB, five units long, represents a force of five pounds, acting obliquely on a block of wood resting on a table, and it is required to find how much force is pressing the block against the table and how much is urging it along its surface. The vector AB may be resolved as just explained into the two components AC and CB, where AC represents the force pressing the block against the surface and CB represents the force pushing it along the surface. The amount of these components may be determined either by direct measurement, using the same scale as in laying off AB, or they may be calculated as follows: If F is the amount of the force AB and if a is the angle between AB and the table top, then

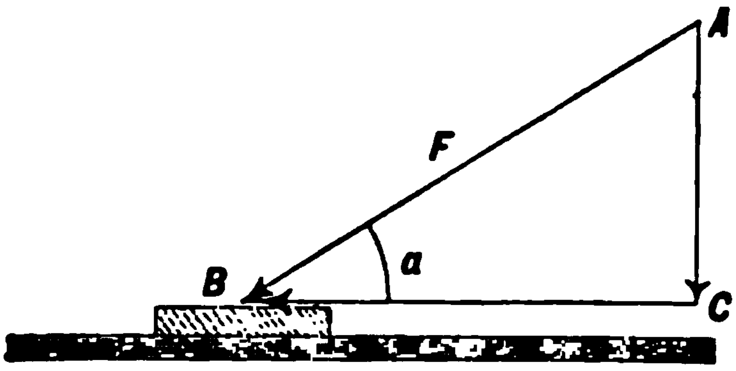


Fig. 10.—Oblique force on block.

$$AC = F. \sin a$$
$$CB = F. \cos a$$

II. STATICS

Equilibrium of a Particle

42. Equilibrium.—Before taking up the study of the motions of bodies as determined by forces, we shall consider some cases in which the various forces concerned are so related as to balance each other, so far as the motion of the body on which they act is concerned.

A body is said to be in equilibrium when any forces which act on it are so related that the body is not accelerated. Thus a body at rest is in equilibrium, also a body moving with constant velocity in a straight line, also a body turning with constant speed of rotation about an axis through its center of mass, as in case of a well-balanced wheel, also when such a wheel is not only turning with constant speed, but moving along with constant speed. Thus a wheel rolling in a straight line along a level su

face or a wheeled vehicle like a car on a straight level track is in equilibrium if moving with constant speed.*

We shall first consider the equilibrium of a *particle* or a body so small that the forces acting on it may all be considered as acting at one point. Afterward the conditions of equilibrium of an extended rigid body will be taken up.

Whether a body is to be treated as a particle or not depends on circumstances. For instance, in astronomy the sun and planets are treated as particles when their shapes and distribution of mass do not affect the question considered.

43. Equilibrium of a Particle.—A particle is in equilibrium when the resultant of the forces acting on it is zero. Evidently in this case the diagram of the forces must be a closed triangle or polygon.

For let the forces acting in a given case be $a\,b\,c$ (Fig. 11), then if we draw the diagram of forces as in the lower part of the figure and if the vectors $a\,b\,c$ form a closed triangle, as shown, the resultant is zero and the particle is in equilibrium.

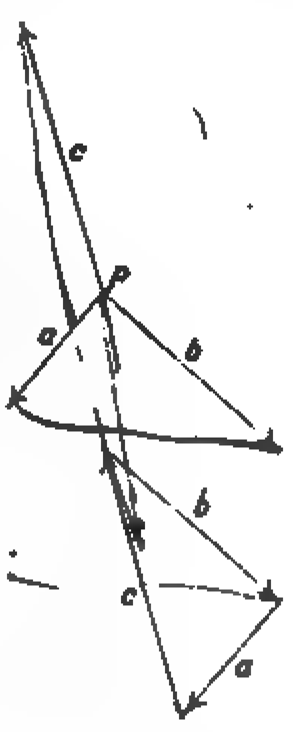


FIG. 11.—Three forces in equilibrium

So also in case of any number of forces in equilibrium, **the diagram of forces, formed by drawing successively the several vectors representing the forces, must be a closed polygon;** that is, the last vector drawn must terminate at the starting point.

An example of four forces in equilibrium is shown in figure 12, the several forces $a\,b\,c\,d$ forming a closed polygon.

It is interesting also to observe that if we resolve each of these forces into two components, one directed toward the top or bottom of the page and the other sidewise, as, for example, b is resolved into b' and b'', c into c' and c'', etc., we find that the components a' b' directed from left to right exactly balance the components c' d' directed from right to left, so also the upward components a'' and d'' are together equal to the sum of the downward components b'' and c''.

In the above diagram the four forces have for convenience all been represented in one plane. This restriction is not necessary,

* When moving as just described, a wheel *considered as a whole* is in equilibrium, but its parts are not in equilibrium, for they move in circles and are therefore accelerated (§28).

the same construction is the test of equilibrium in whatever directions the four forces may act.

1/5/26

44. Illustrations.—If three cords joined at P suspend weights of 3, 4, and 5 lbs. respectively, those supporting the 3-lb. and 4-lb. weights passing

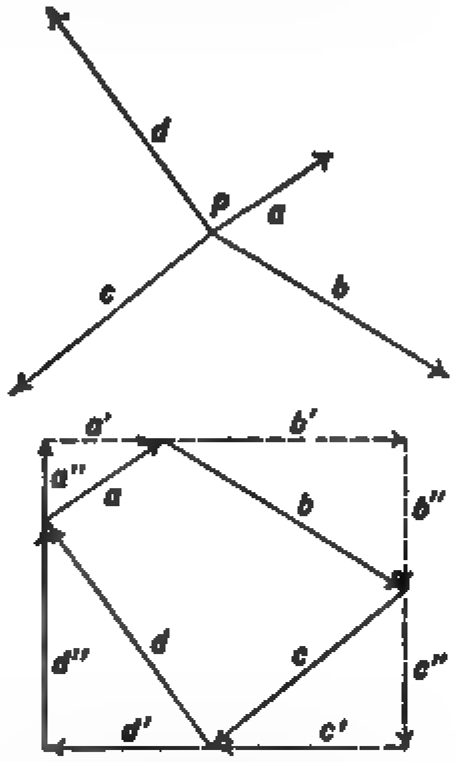


FIG. 12.—Four forces in equilibrium.

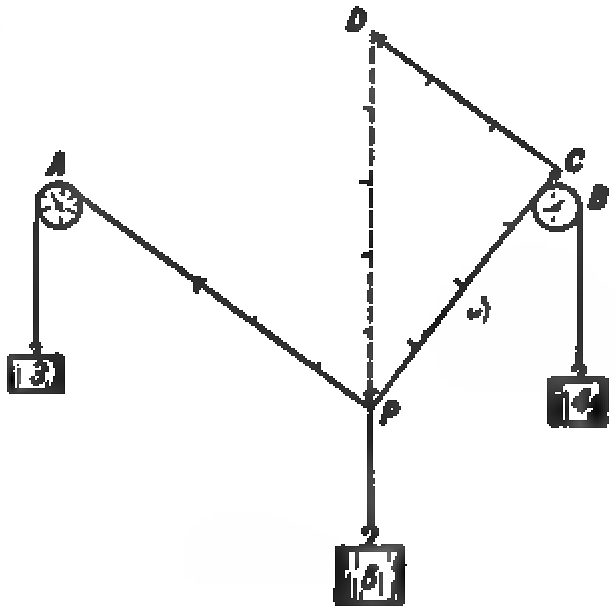


FIG. 13.

over frictionless pulleys as shown in figure 13, then the point of junction P will assume a definite position to which it will return if pushed aside and the cords PA and PB will be at right angles to each other.

For the point P is in equilibrium under the three forces 3, 4, 5, and therefore the force diagram must be the closed triangle PCD, the three

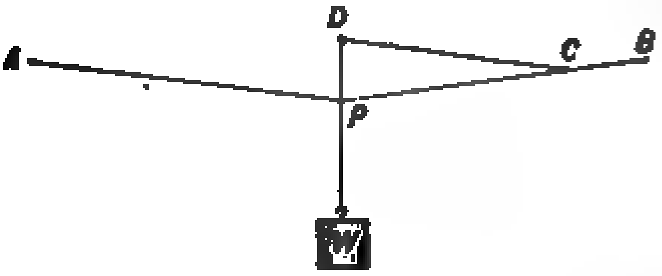


FIG. 14.

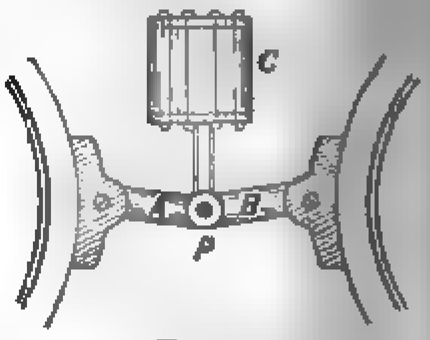


FIG. 15.

sides of which are in the ratio of 3 : 4 : 5. But such a triangle is right-angled and therefore the force 4 and force 3 must be at right angles to each other.

Suppose a cord is fastened at A and B and is then stretched by a weight W hung at P (Fig. 14). As before, the diagram of forces is PCD, where

PC represents the stress on the cord between P and B while CD represents the stress on AP, and DP represents the weight W. Evidently the more nearly AP and PB are to being in a straight line the larger will CD and PC be in comparison with the force W which is represented by DP. So that a comparatively small pull down at P, if APB is nearly straight, may produce a force great enough to break the cord between A and B. Thus the stress brought to bear on hammock ropes may be much greater than the weight of the person supported if it is hung with insufficient sag.

The student may easily determine under what conditions the stresses on AP and PB will be each equal to the weight W.

The jointed device used in hand printing presses, and shown in figure 15 as applied to the brakes on a locomotive, illustrates the same principle. Here when compressed air is admitted to the cylinder C the piston is forced upward, thus straightening the two connecting pieces A and B, thereby forcing the two brake shoes against the wheels with a force which is greater the more nearly the connecting pieces are pulled into a straight line.

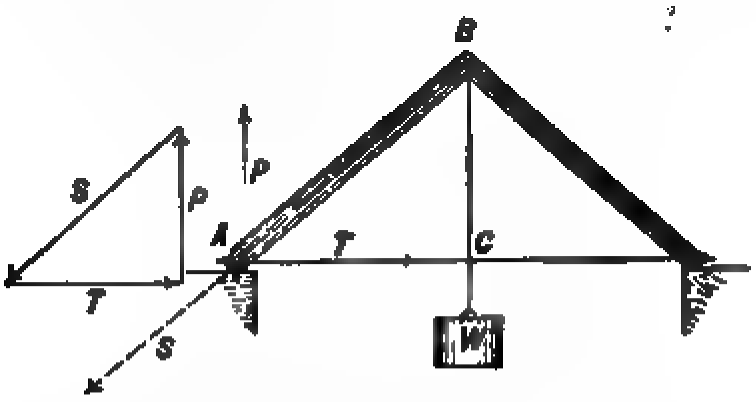


FIG. 16.—Bridge truss.

45. Bridge Stresses.—Let it be required to find the stresses, tensions or pressures, in case of the various parts of the truss which is shown in figure 16, supporting at its center the weight W.

Consider what forces are acting on the end of the truss at A. It will be shown later (§54) that in such a case half the total weight will be borne by one abutment and half by the other. The end of the truss at A therefore presses down on the abutment with a force equal to $\frac{W}{2}$, if we neglect the weight of the truss itself. Now A is in equilibrium under the three forces represented by the arrows, P indicating the upward pressure of the abutment, S representing the oblique downward thrust of the strut AB, while T represents the inward pull of the tie rod AC; therefore the diagram of these forces must be a triangle as shown above. But this triangle is similar to the triangle ACB, for the sides are respectively parallel, and so the forces P, S, and T are in the same proportion as the sides BC, AB, AC, and since the pressure P is equal to $\frac{W}{2}$, the other forces are at once known by proportion when the sides of the triangle ABC are known.

In the somewhat more complicated case shown in figure 17 where the total

weight of 10 tons is supported between the two abutments the upward pressure P will equal 5 tons. The stresses on AB and AC may be found as in the preceding case, but to find the stress on the rod BC we must make a diagram of the forces under which the point B is in equilibrium as shown in the diagram.

46. Crane Problem.—A weight of 100 lb. is suspended from a crane of dimensions shown in the figure. It is required to find the tension on the tie rod AB and the compression on the strut BC.

The point B is in equilibrium under three forces, the downward weight W = 100 lb., the pressure P of the strut which acts outward, and the tension T of the tie rod which acts in the direction BA. The diagram of forces must therefore be a triangle with sides parallel to the directions of the three forces as shown in the figure.

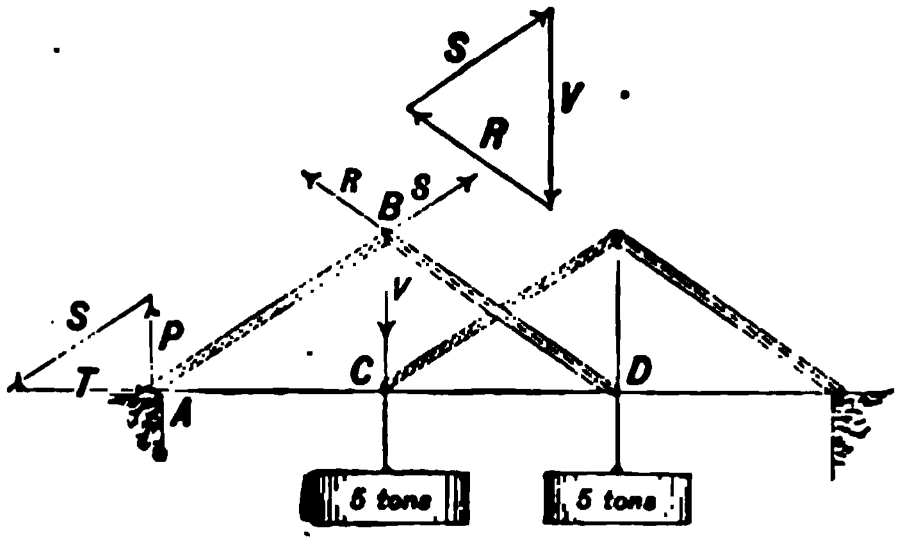


FIG. 17.

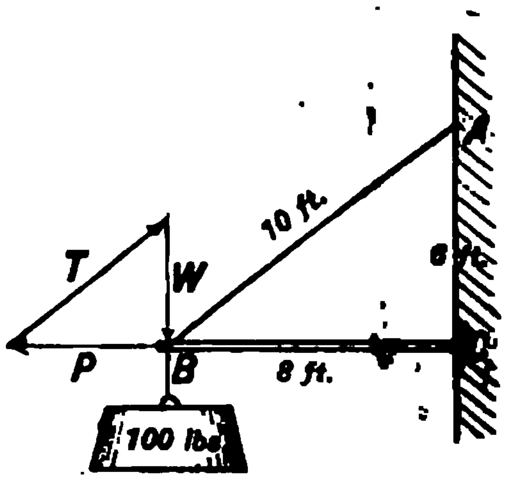


FIG. 18.—Crane.

Problems

1. A force of 300 grams and a force of 400 grams act at right angles to each other on the same point. Find the single force to which they are equivalent and also its direction, by a diagram.
2. Four forces of 3, 4, 5, and 6 lbs. act on the same point in directions east, northeast, north, and northwest, respectively. Construct the force diagram and find by measurement the amount of the resultant and the angle which it makes with the north line.
3. Three cords fastened together at a point free to move, have tensions 60, 70, and 80 grams respectively. Construct the force diagram and find by measurement the angles between the cords when at rest.
4. Two forces of 10 lbs. each act upon a single point in such a way that they are equivalent to a single force of 10 lbs. Find the angle between their lines of action.
5. How much force must be exerted at an angle of 45° to the top of a tabl to push along a weight when the frictional resistance to be overcome i a force of 2 kgms.?
6. When a force of 20 lbs. is required to draw along a sled by a rope makin an angle of 30° with the ground, find the force moving the sled forwar and the force diminishing its pressure on the ground.

7. A weight of 2 lbs. hung from a nail by a cord 30 in. long is pushed aside by a horizontal force of 1 lb. How far will it be moved away from the vertical line through its point of support?

8. A 32-gram weight is hung by a cord 60 cm. long from a point on a vertical wall. How far will it be pushed out from the wall by a force of 24 grams acting perpendicular to the wall?

9. Make a diagram showing the angle between the two ropes of a hammock when the tension on each rope is twice the weight of the person in the hammock.

10. A rope supporting a weight of 180 lbs. at its middle point is hung between two hooks which are on the same level and 18 ft. apart. If the middle point sags 3 ft. below the level of the hooks, find the force on each hook.

11. In case of a crane, like figure 18, in which the horizontal strut is 5 ft. long and the vertical distance AC is 3 ft., find the tension on the oblique tie rod and the pressure on the strut when a weight of 270 lbs. is supported.

12. Suppose the wall in figure 18 overhangs so that A is 6 ft. vertically above a point 1 ft. to the left of C on the bar BC, which is horizontal and 9 ft. long. Find the stresses on AB and BC when a weight of 240 lbs. is suspended at B.

Equilibrium of Rigid Body

47. Equilibrium of a Rigid Body.—A rigid body is in equilibrium when its velocity of translation is not changing in any direction and when its velocity of rotation is not changing about any axis.

Or, in other words, **a rigid body is in equilibrium when it has no acceleration either of translation or rotation.**

48. Condition for Translational Equilibrium.—In order that there may be no *translational* acceleration, the relation between the forces acting on the body must be exactly the same as that required for the equilibrium of a particle. For if there is to be no acceleration in any direction the resultant force in any one direction must be zero, and this is evidently the case *when the diagram of forces is a closed polygon.*

49. Case of Two Forces.—The relation which must hold between the forces in order that there may be no *rotational* acceleration may be most easily reached through the study of some simple cases of equilibrium. That a body may be in equilibrium under two forces it is necessary that the two forces P and Q (Fig. 19) should be equal and opposite in order to satisfy the condition of no translational acceleration as just shown. In order

that there may be no tendency of the forces to rotate the body it is clearly necessary that they shall act *in the same straight line,* as shown in the figure. The only effect of the forces applied at A and B in the figure is to compress the body between these points.

50. Resultant of Two Oblique Forces in the Same Plane.—Let P and Q represent two forces acting at A and B upon an extended body, and let their lines of action when produced intersect at C. A force equal and opposite to P if applied at C will exactly balance P, as shown in the preceding paragraph, and a force equal and opposite to Q, also applied at C, will balance Q, therefore a single force R equal and opposite to the resultant of P and Q as found by the triangle of forces, will, if applied at C, exactly balance both P and Q and produce equilibrium. Evidently the force R may be applied to the body at any point in its line of

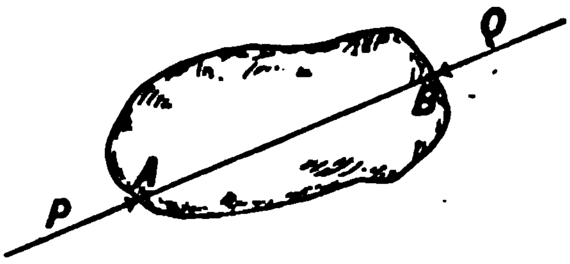


Fig. 19.

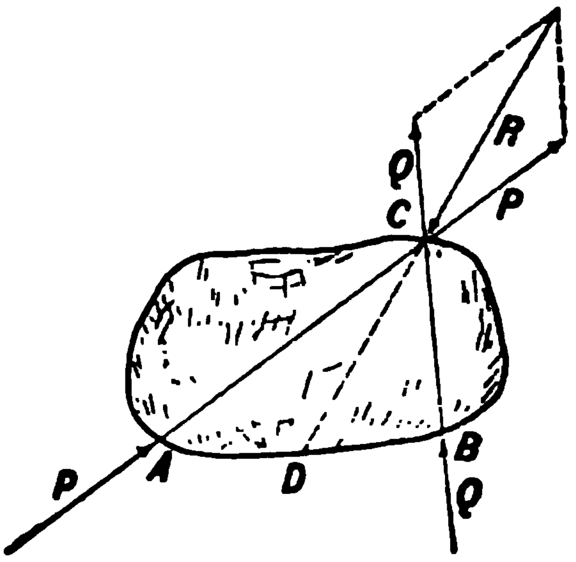


Fig. 20.

action CD. The resultant of P and Q is, therefore, a force equal and opposite to R and acting along the line CD.

51. Moment of Force.—In the case of equilibrium just discussed we may imagine the force R to be produced by pressure against a *pivot* fixed somewhere in the line CD, say at E (Fig. 21).

The forces P and Q then balance each other, so far as causing rotation about the axis at E is concerned. Two forces so related to any axis are said to have equal and opposite *moments* with respect to that axis.

Common experience shows us that the farther the line of action of a force is from the pivot or axis, the greater will be its ability to rotate the body about that axis. Thus in opening a heavy gate we take hold of it as far as possible from its hinges and pull at right angles to the gate. A pull in line with the hinges would hav

no effect to turn it whatever. The moment of a force to turn a body about an axis depends, therefore, both on the amount of the force and the distance of its line of action from the axis.

Let x be the perpendicular distance from E to the line of action of the force P, and let y be the perpendicular distance from E to the line of action of Q. We shall now show that in this case, where the moments of P and Q about the axis E balance each other, $Px = Qy$.

Construct the parallelogram $CFGH$ having $CF = P$ and $CH = Q$. Draw FE and HE; then the area of the triangle CFE is equal to $\frac{1}{2}Px$ for P is its base and x is its altitude. So also the area of CHE is equal to $\frac{1}{2}Qy$. But the two triangles CFE and CHE have equal areas, for they have a common base CE and equal altitudes HI and FK; therefore, $Px = Qy$.

It has thus been shown that when the axis E lies on the line CG the two forces P and Q have equal and opposite moments about it and also in that case $Px = Qy$. *These products Px and Qy may, therefore, be taken as representing the abilities of the forces to produce rotation about the given axis*, and are, therefore, used to measure the moments of the forces.

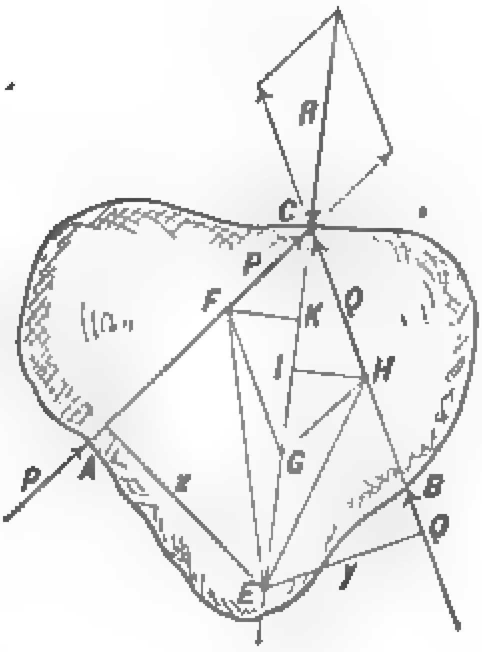


Fig. 21.

The moment of a force with reference to a given axis is its ability to produce rotation about that axis, and is measured by the product of the force by the perpendicular distance from the axis to the line of action of the force.

52. Second Condition of Equilibrium.—The second condition of equilibrium for an extended rigid body is that the various forces must be so related that there is no rotational acceleration about *any* axis in the body.

Since the ability of a force to produce rotation is measured by its moment, **this condition is satisfied when the sum of the moments of the forces tending to produce clockwise rotation about any axis whatever is equal to the sum of the counter-clockwise moments about that axis.**

53. Forces in One Plane.—When the forces acting on a body all lie in one plane, such as the plane of the paper in figure 22, they can have no tendency to rotate the body except about an axis at right angles to that plane.

In this case if the diagram of forces is a closed polygon showing that there is no translational acceleration, and if the clockwise and counter-clockwise moments are equal about some one axis at right angles to the plane of the forces, then the body is in equilibrium and the resultant moment of the forces is also zero about any other axis that may be chosen in the body.

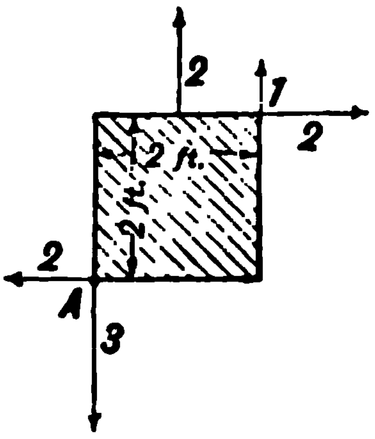


Fig. 22.

For example, in case of a board 2 ft. square, with forces applied to it as shown in figure 22, the diagram of forces is a closed figure, as the student may easily verify. The forces, therefore, balance so far as *translation* is concerned.

Now calculate the moments of the forces about an axis perpendicular to the plane of the forces, say through the point A. Designating clockwise moments *plus* and counter-clockwise *minus*, and taking the forces in order beginning at the top, we have the moments

$$\left.\begin{array}{r} -2 \times 1 = -2 \\ -1 \times 2 = -2 \\ +2 \times 2 = +4 \\ 3 \times 0 = 0 \\ 2 \times 0 = 0 \end{array}\right\} \quad \text{Sum of the moments} = 0.$$

Therefore the board is in equilibrium under these forces and consequently the sum of their moments will be zero if reckoned for any axis whatever.

Compute in this way the moments about an axis through the center of the board and show that their sum is zero.

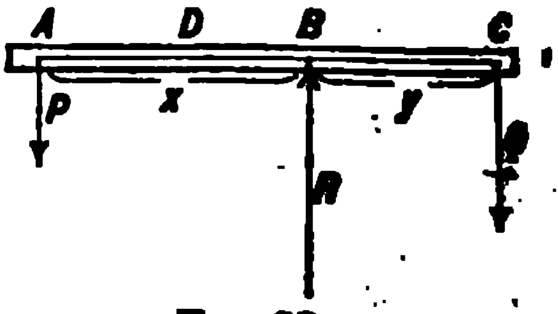


Fig. 23.

54. Three Parallel Forces.—When a bar is in equilibrium under three parallel forces, as in figure 23, to satisfy the condition of no translational acceleration the *up* forces must be equal to the *down forces*, or $P + Q = R$. While to satisfy the second condition, that the moments of the forces shall balance, we have $Px = Qy$, for these are the moments about point B, and R has zero moment about that point. Or we may take moments about A and find $Rx = Q(x+y)$. If moments are taken about any point other than A, B, or C, there will be thr

moments to reckon. If, for example, the point D is taken as the axis the clockwise moment of Q must be equal to the sum of the counter-clockwise moments of P and R.

55. Parallel Forces in General.—Any case of equilibrium with parallel forces may be discussed in a similar way, two conditions being met, namely, the sum of the forces acting in any one direction must be equal to the sum of the forces in the opposite direction, and the sum of the clockwise moments about any axis must be equal to the sum of the counter-clockwise moments.

56. Illustration.—A certain bar having no weight is acted on by four forces as shown in figure 24, forces of 4 lbs. and 2 lbs. acting upward and 3 lbs. and 5 lbs. acting downward, and it is required to find the single force necessary to produce equilibrium and the point on the bar where it must be applied. Since the total upward force is 6 while the downward force is 8, the required force F must be an upward force 2 to satisfy the *first condition* of no translational acceleration.

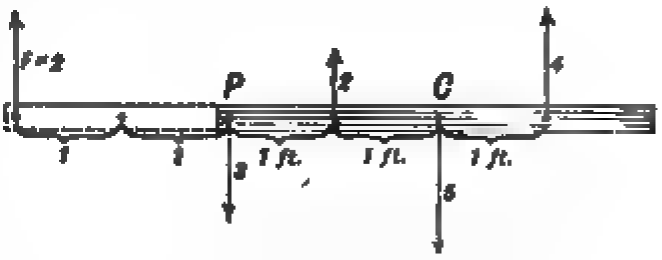


FIG. 24.

This force must be applied at such a point on the bar as to satisfy the *second condition*, and make the clockwise moments balance the counter-clockwise moments about any axis. Take an axis through P, for instance. The moments about P are

$$\left.\begin{array}{r} 3 \times 0 = 0 \\ -2 \times 1 = -2 \\ +5 \times 2 = +10 \\ -4 \times 3 = -12 \end{array}\right\} \quad \text{Sum} = -4 \text{ counter-clockwise.}$$

therefore, to produce equilibrium the applied force 2 must produce a clockwise moment 4. Since it must also act upward, it must be applied at a distance 2 to the *left* of P, and consequently the bar must be extended 2 feet in that direction.

Any point whatever on the bar might have been taken as the origin of moments, and the reader should show that the same conclusion is reached taking moments about some point such as C.

57. Couple and Torque.—If in the case just treated the upward force 4 is changed to 6, we have a case that calls for special consideration. The upward forces are exactly equal to the downward forces and yet the bar is not in equilibrium, for taking moments about P we find that the clockwise moment is 10 while the counter-clockwise moment is $2 + 18 = 20$. Here, then, there is *a com-*

[redacted]

bination of forces that does not tend to produce translation, but simply rotation. Such a combination is known as a *couple,* and its moment is commonly known as a *torque.* It cannot be balanced by any single force, for any force applied either upward or downward would cause translation. **A couple can be balanced only by another couple having an equal and opposite moment, or torque.**

The simplest case of a couple is when two equal parallel forces act in opposite directions not in the same straight line. For instance, the forces FF, figure 26, constitute a clockwise couple the moment of which is Fx where x is the distance between the lines of action of the forces. **The moment of a couple about any axis is the same as about any parallel axis.** For, take an axis perpendicular to the paper and through P at a distance y from the nearer force, then the moments are Fy counter-clockwise and $F(x+y)$ clockwise, hence subtracting we have Fx clockwise, as the resultant of the two.

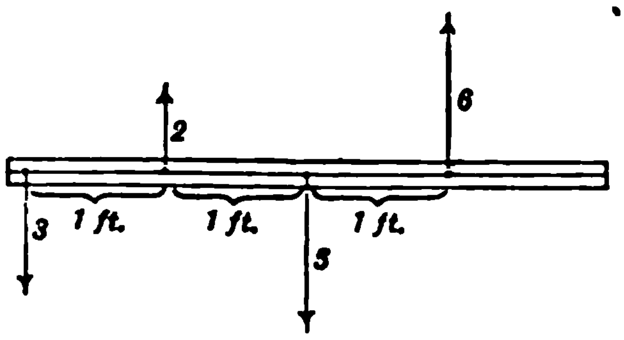


Fig. 25.

The moment of such a couple about an axis perpendicular to the plane in which the two forces lie is, therefore, measured by the product of the amount of either force by the perpendicular distance between their lines of action.

To produce equilibrium, then, in the case under consideration a couple having a clockwise moment 10 must be applied to the bar, and it may be applied at any point we choose. The following figure illustrates different modes of producing equilibrium in this case.

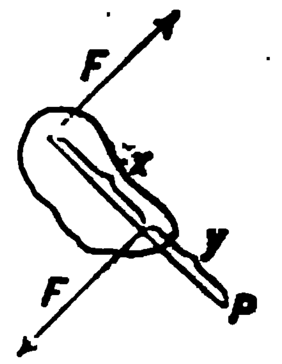


Fig. 26.—Couple.

In every case of equilibrium the forces acting may be resolved into a number of balancing couples.

58. Center of Gravity.—The weight of a mass is the force with which it is drawn toward the earth. All parts of a body have weight and so the weights of the several parts into which a body may be conceived to be divided constitute a system of parallel forces acting downward toward the earth. The resultant of this system of forces is a single force equal to their sum and is the total weight of the body. It may be proved that there is a

certain point in the body through which the resultant force due to weight always acts whatever may be the position of the body. This point is called the *center of gravity* of the body.

In all problems that involve the weight of a body we may ignore the fact that the weight is distributed throughout the body, and treat it as a single force applied at the center of gravity.

59. Proof of Center of Gravity.—Let M and m be the masses of two parts of a body and let the line joining them be inclined as shown in figure 28. Since the weights of masses are proportional to the masses themselves (§38), the single upward force necessary to balance the weights of the two masses must be applied in the

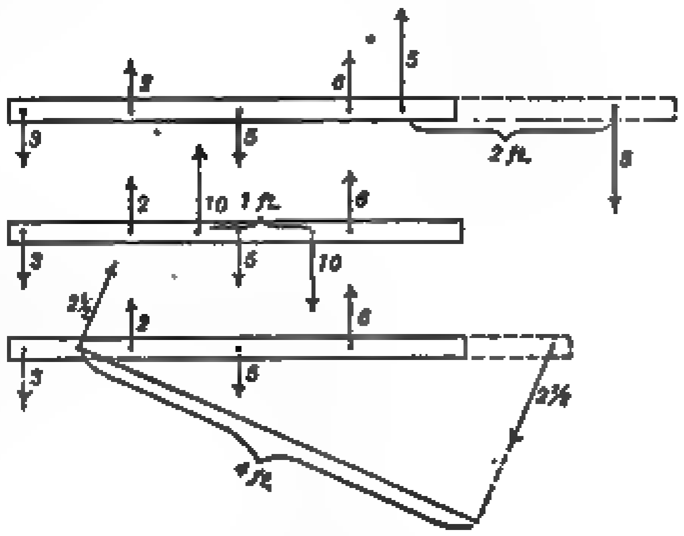


FIG. 27.

vertical line AB, so situated that $Mx = my$. But AB intersects at P the line joining the two masses, dividing it into the two segments a and b which, by similar triangles, are in the same ratio as x and y, and consequently $a : b :: M : m$; and since this ratio does not depend on the inclination of the line joining M and m, it follows that the balancing force must pass through the point P whatever the inclination may be. P is, therefore, the center of gravity of M and m. Now conceive the masses M and m concentrated at P and find similarly a point P' through which the resultant weight of $(M+m)$ and of another mass m' must always pass. Continue in this manner until account has been taken of all the masses into which we may conceive the body to have been subdivided. The point through which the final resultant passes is the center of gravity of the body.

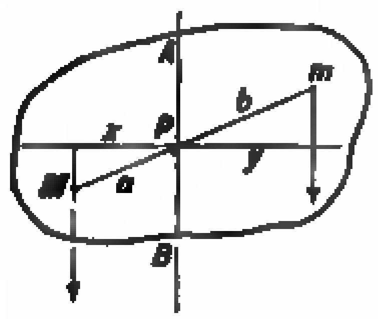


FIG. 28.

60. Center of Mass and of Inertia.—The center of gravity as has just been explained is determined by the distribution of mass in a body or system of bodies. It has certain remarkable

[redacted]

properties quite independent of weight, and is therefore also called the *center of mass* or *center of inertia* of the body or system.

For example, a freely rotating body like a spinning projectile will always rotate about an axis through its center of mass.

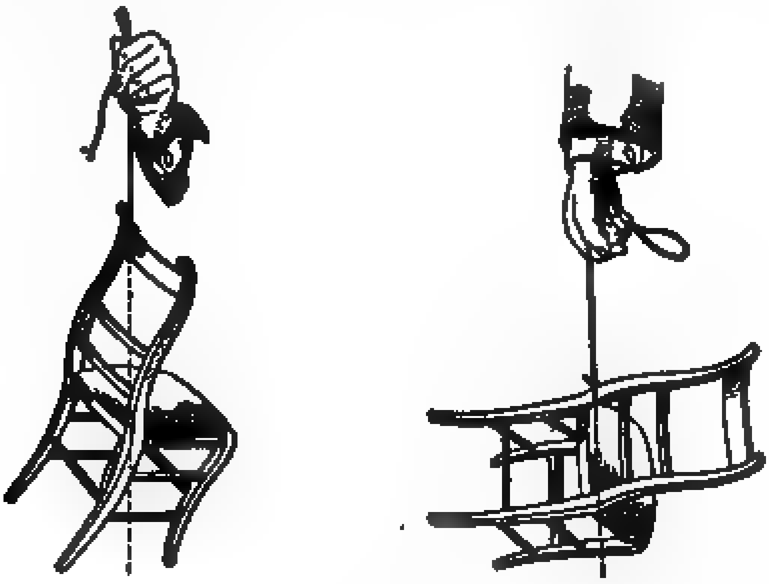


FIG. 29.

61. Position of Center of Gravity.—When a body is hung by a cord or balanced on a point the center of gravity must be in the vertical line passing through the point of support. For two equilibrating forces must act in the same straight line. If, therefore, a body is hung first from one point and then from another the intersection of the two lines thus determined marks the position of the center of gravity, as shown in the figure, where it is seen to be a point outside the actual substance of the chair.

The center of gravity of a uniform bar is at its center; in a uniform thin plate, square, rectangular, or in the form of a parallelogram, it is at the intersection of the diagonals. In case of any homogeneous symmetrical body it lies in the plane of symmetry. Thus it is at the center of a sphere, circle, or circular disc and at the center of a cube.

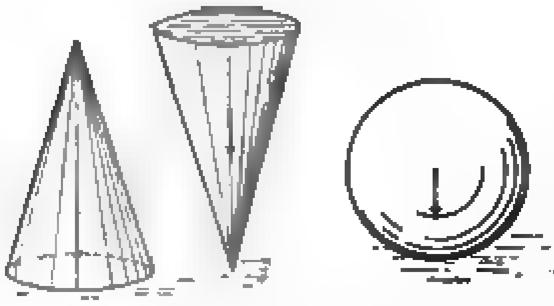


FIG. 30.

62. Equilibrium under Gra ity—That a body may be equilibrium under gravity, it must be supported by a force equ to its own weight and acting upward through its center of gravi Thus the two cones and sphere shown in the figure as resting o

level table are in equilibrium, the upward force being supplied by the reaction of the table. But the first cone is said to be in *stable* equilibrium, because if slightly tipped it will fall back to its original position. The second cone is said to be in *unstable* equilibrium because if disturbed it will fall away from its original position, while the sphere is said to be in *neutral* equilibrium because it remains in equilibrium when displaced. *It will be observed that in the first case the center of gravity of the cone is raised when it is tipped; in the second it is lowered, and in case of the sphere it is neither raised nor lowered.* The weight of a body being considered as acting at its center of gravity will always cause that point to move *down* or toward the earth unless opposed by some other force. In case of a loaded wagon on a hillside the vertical line through its center of gravity may remain between the wheels if the center of gravity is low, when if it were high the line of action of the weight might fall outside the wheel base causing the load to overturn.

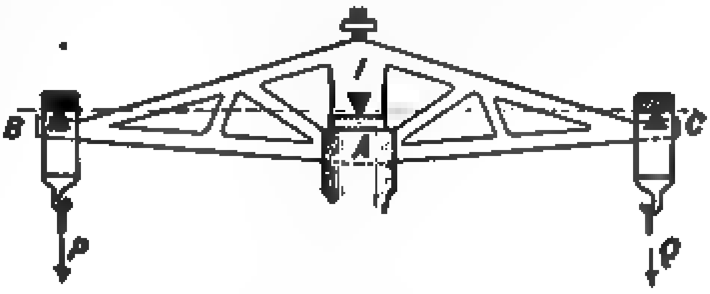


Fig. 31.

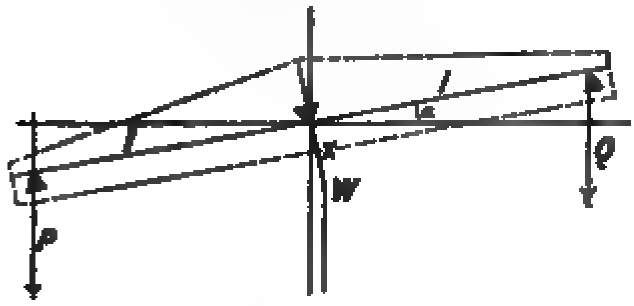


Fig. 32.

63. Balances.—The beam of a balance rests on a sharp steel "knife edge" A, while the pans are hung on the knife edges B and C. These three knife edges are rigidly fixed in the beam and should be parallel, and in the same plane, and the arm AC should be equal to the arm AB. (Fig. 31.)

Now if A, B, and C are all in the same straight line, and if the weight P is greater than the weight Q, the balance will tip entirely over unless some counteracting force is available. This is found in the weight of the balance beam itself which is so adjusted that its center of gravity does not lie exactly on the edge A, but slightly below it, say a distance x. Then when P is greater than Q the balance beam inclines and its center of gravity is displaced until the restoring moment due to the weight of the beam W, acting down through its center of gravity just balances the deflecting moment due to the difference between P and Q. The deflection is therefore very nearly proportional to $P - Q$, and the greater the deflection for a given difference between P and Q, the greater the *sensitiveness* of the balance is said to be.

64. Double Weighing.—If the arms of a balance are not of equal length, the true weight of a body may be obtained by the *method of substitution*, in which the body is placed in one pan of the balance and some convenient counterpoise, such as sand or shot, which will exactly balance it, placed in the other pan. The body is then removed and weights substituted for it until equilibrium is again reached, the substituted weights are then equal to the weight of the body.

Or the method of *double weighing* may be employed. Let r be the length of the right arm of the balance and l the length of the left arm, then if a body whose real weight is P is placed in the right pan and balanced by weights W in the left pan, we have by the equality of moments

$$Pr = Wl.$$

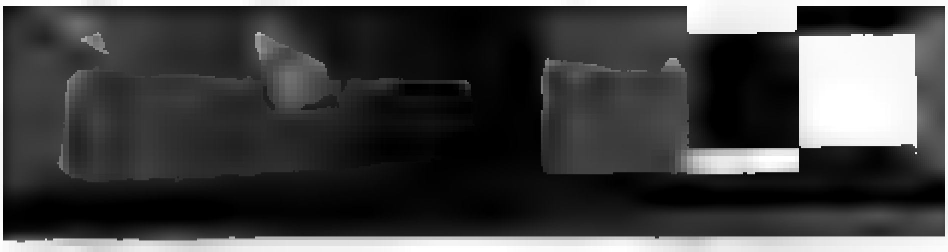
Then interchanging body and weights, it is found that a weight W' is required to balance it when it is in the left pan, which gives

$$Pl = W'r.$$

Multiplying the two equations together we have $P^2lr = WW'lr$ or $P^2 = WW'$; therefore $P = \sqrt{WW'}$, or if W is very nearly equal to W', $P = \frac{W + W'}{2}$.

Problems

1. A wooden bar 5 ft. long and weighing 2 lbs., the ends of which are supported by spring balances, has a 10-lb. weight hung on it 2 ft. from one end. Find the force exerted on each balance.
2. A beam 20 ft. long is carried by three men, one at one end and the other two supporting it between them on a cross-bar at such a point that each man carries an equal weight. Find where the cross-bar must be placed.
3. A man standing on a uniform beam, 16 ft. long and weighing 120 lbs., at a point 1 ft. from its end causes it to just balance as it lies horizontally across a support 4 ft. from that end. What is the man's weight?
4. At what point on a pole must a weight of 52 lbs. be hung so that a boy at one end may carry $\frac{4}{9}$ as much as the man at the other end, and how much does each carry? Neglect the weight of the pole.
5. If the pole in problem 4 is uniform and weighs 10 lbs., where must a 50-lb. weight be hung so that the man may carry twice as much weight as the boy?
6. Find the center of gravity of a uniform bar weighing 6 lbs. and having a 2-lb. weight on one end and a 7-lb. weight on the other.
7. Forces 2 and 4 acting upward are applied to a horizontal bar at 2 ft. and 4 ft. from the left-hand end, respectively, also forces 3 and 1 acting downward are applied at 1 ft. and 5 ft. from the same end. Find amount and point of application of a single force producing equilibrium.
8. How produce equilibrium in problem 7 when an additional force 2 acts downward at a point 3 ft. from the left-hand end of the bar.



9. A board 2 ft. square is acted on by five forces applied at the same points as shown in figure 22, but the forces instead of being 2, 1, 2, 3, 2, beginning at the top, are 4, 2, 4, 5, 3, respectively. Find the direction and amount of the force needed to produce equilibrium and how far from the center of the board its line of action must lie.

10. A ladder standing 6 ft. from a smooth vertical wall rests against it at a point 30 ft. from the ground. If the ladder weighs 60 lbs. and its center of gravity is ⅓ of its length from the bottom, find the force with which it presses against the wall, also the amount and direction of its force against the ground; that is, its vertical and horizontal components. *Note.*—The force between ladder and wall must be perpendicular to the latter if there is no friction between them.

11. When a man weighing 150 lbs. is halfway up the ladder in problem 10, find the pressure of the ladder against the wall and also the two components of its force against the ground.

12. When is the ladder in problem 11 more liable to slip, when a man is near the top or bottom, and why?

Work and Energy.

65. Work.—A man digging a ditch is said to work, so also a team of horses drawing a load, and a carpenter supporting temporarily the end of a beam may also, in ordinary speech, be said to be working; for in each case a useful end is secured by the exertion of force.

But there is a difference between these cases. In the first two there is motion and a permanent change is effected; while in the third case the beam is not moved but simply supported, and any prop would have served as well as the carpenter.

In physics the term work is restricted to such cases as the first two where motion results from the action of force, and **the amount of work is measured by the product of the force by the distance through which the body moves along the line of action of the force.** Thus when in digging a ditch a ton of earth is thrown to an average height of 6 ft., the work done is $2000 \times 6 = 12{,}000$ foot-pounds.

If the motion of the body is not in line with the resultant force, **then in estimating work only that component of the motion which is in the direction of the force is to be taken into account.** For instance, in raising a barrel into a wagon *the work done is the same whether the barrel is lifted directly from the ground or rolled up an inclined plane.* For the weight of a body is a force

that acts vertically downward, consequently **in estimating work done against weight, only the vertical distance through which the body is moved is to be considered.**

When a body yields to a force work is said to be done *by the force* or *upon the body;* but when a moving body is retarded by some resisting force, work is then said to be done *by the body* or *against the force.*

The work done in raising a weight or compressing a spring is the same whether done in a second or in an hour. The time required to do the work determines the *rate of working,* but has nothing to do with the amount of work.

It is remarkable that although *force* and *distance* are both vector quantities, *work,* which is their product, *is not a vector quantity.* It has nothing to do with *direction,* and consequently to get the total work done upon a body by several different forces, the work of each may be reckoned separately and then the sum taken.

Motion is essential to work. A great weight may rest on a support, but no work is done in supporting the weight though a great force is exerted.

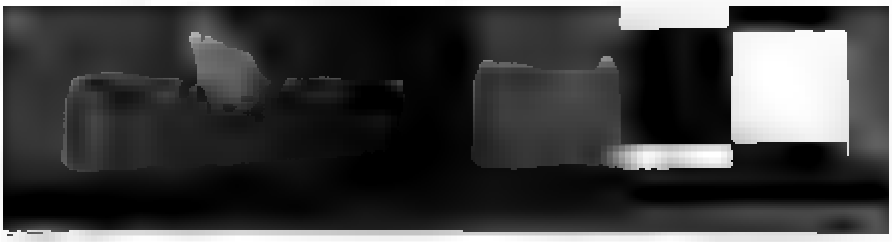
66. Rate of Working. Horse-power.—A given amount of work may be done either in a short time or a long time, and in commercial operations the *rate of working,* or the work done per second or per hour, is an important consideration. Thus in case of an engine we wish to know how much work it can do in a given time, and its rate of working is known as its *power.*

Power may be measured by the number of grams weight that can be raised one centimeter per second, or by the number of pounds that can be raised one foot per second; but the unit of power introduced by James Watt and commonly used in engineering practice is the *horse-power* (written H.P.).

One horse-power = 550 foot-pounds per second, or 33,000 foot-pounds per minute.

That is, a 10 H.P. engine can raise 330 lbs. through a height of 100 ft. in one-tenth of a minute, or 3300 lbs. through a he of 10 feet in the same time.

67. Energy.—The importance of the idea of work lies in fact that a body upon which work is done acquires the capacity to do an equal amount of work in returning to its ori



The capacity to do work is called energy. Thus work is when a spring is bent, and the spring acquires energy which asured by the work that it can do as it unbends. Also a weight raised 100 ft. above the earth has had 1000 ft.-f work expended in raising it, and it has gained the power that same amount of work in returning to its original on.

e energy of a bent spring resides in the spring itself in virtue internal stresses; but in case of the raised weight the energy gs not to the weight alone, but to the system of two bodies, rth and the weight, which are separated in opposition to the or attraction between them.

Kinds of Energy.—In both illustrations given above the y depends on the relative positions of bodies or parts of s between which there exist stresses. There is another form rgy which depends not upon stress, but upon the motion of r.

pose the raised weight is set free and allowed to fall with ng to resist it, the force of the earth's attraction is exerted the mass as it falls and consequently work is done and energy ded, but in this case the work is all spent in giving velocity e falling mass. When the weight reaches the bottom it st all its advantage of position, but it still has power to do in virtue of its motion, and experiment shows that the work do before coming to rest is exactly equal to the work that one upon it in giving it motion. The mass, therefore, still s the energy that it had in the raised position, but it is now *of motion.*

energy which a body or system of bodies has in virtue of es is called potential energy.

energy which a body has in consequence of the velocity of ss is called its kinetic energy.

Illustration.—If a mass is hung so that it can freely swing as a um, when it has been raised to the position A (Fig. 33) it has been through the vertical distance h from B to D, and, therefore, has more ial energy at A than at B by the work done in raising it from B to allowed to fall freely it will reach the bottom, moving with suf-velocity to carry it up to C on the same level as A. At the bottom s has energy of motion or kinetic energy. It has entirely lost the

advantage of position which it had at A, the work done in raising it to A being now wholly transformed into energy of motion.

But as the mass rises from B toward C it loses velocity for it is doing work and using up the store of kinetic energy that it received in falling, changing it again to potential energy. The pendulum has thus a constant store of energy which changes back and forth from one form to the other, the sum of the two being always constant, except as energy is gradually lost through friction and air resistance.

70. Work against Friction.—There is one case, however, in which the work done upon a body does not *seem* to increase its energy or power to do work. When a weight is pushed from one point to another on a level table force has to be exerted to overcome friction. The weight, however, remains at the same level above the earth and has no more power to do work in the new position than before it was moved. The work expended seems to be quite lost.

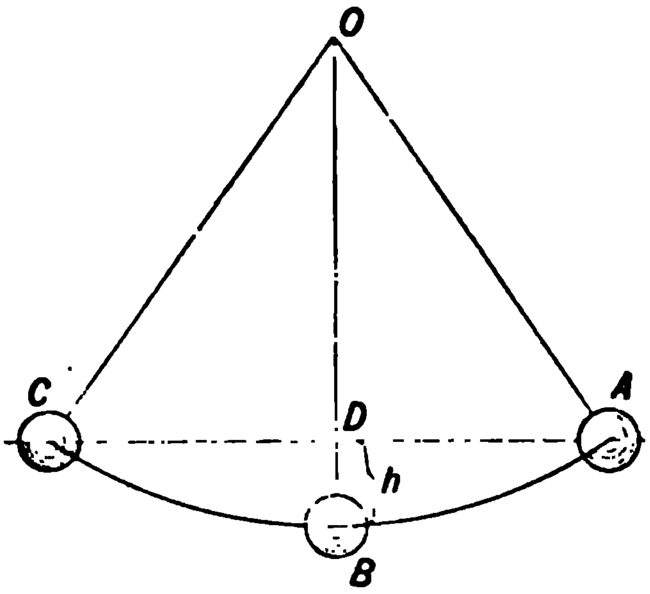


Fig. 33.

But investigation has shown (§405 *et seq.*) that whenever work is done against friction *heat* is developed in amount exactly proportional to the work done; and also that when work is obtained from a heat engine a precisely corresponding amount of heat disappears. It is, therefore, concluded that **the work which seems to be lost in friction is not really lost or annihilated, but is transformed into heat as into another form of energy.**

When, therefore, a pendulum comes to rest in consequence of friction (at its point of support, or between it and the air through which it swings) the original energy of the pendulum is not lost but transformed into heat.

71. Forms of Energy.—From the results of innumerable experiments physicists have concluded that not only is heat a form of energy, but sound, light, and all electrical and magnetic actions are manifestations of energy, and require energy to be expended in causing them, just in proportion as they are capable of doing mechanical work or developing heat.

The different manifestations of energy may be summarized follows:

Manifestations of Energy

Energy of masses	Masses in motion—kinetic.	
	Elastic bodies in a state of stress	potential.
	Gravitation, energy of attracting masses	
	Sound, both kinetic and potential.	
Energy of molecules and atoms	Heat.	
	Molecular and atomic energy	
	Chemical action.	
Energy of ether	Electric and magnetic phenomena.	
	Light and radiation.	

When the energies involved in all these varied phenomena are studied it is found that one form of energy may be transformed into another, and that again into a third, but in every change the *amount* of the energy as measured by its power of doing work or of developing heat is unchanged.

72. Conservation of Energy.—The recognition of these varied forms of energy and careful measurements of the transformations from one form to another have led to the enunciation of a great principle or law known as the *Conservation of Energy*, which may be thus stated:

In any system of bodies which neither receives energy from without nor gives up any, the total amount of energy is unchanged whatever actions or changes may take place within that system, whether the energy manifests itself in mechanical forms, in sound, heat, light, electric, or magnetic effects, or in chemical action or molecular or atomic changes.

In most cases the tracing of all the changes is a difficult matter. For example, a cannon ball receives energy from the work done by the powder gases as they expand forcing the ball from the gun. As it travels it is resisted by the air, losing kinetic energy exactly equivalent to the heat energy developed by friction in the air. On striking the target, sound waves carry off a small part of the energy, there may also be a flash of light which also takes away some energy, and the rest will be found in the form of heat developed in the target and in the ball itself and also in the form of kinetic energy in the fragments which may be thrown off. The principle of the conservation of energy asserts that if we add together all the energy that is derived from the motion of the ball the sum will be eactly equal to the amount of work which was required to give it its motion.

This law is the most important and extensive generalization of the science of physics, and much of the progress of modern physics is due to its recognition. Every experiment in which the quantities of energy can be accurately determined is a test and confirmation of its truth, and no principle of physics is better established.

In consequence of this law, the determination of the energy involved in any action assumes new importance and is an essential part of the study of every physical phenomenon.

73. Availability of Energy.—The presence of friction and analogous forms of resistance everywhere in nature causes a constant transformation of various forms of energy into heat, in which stage it is conducted from one body to another and gradually becomes uniformly diffused so that although the energy still exists it is no longer available for the purpose of obtaining other forms of energy that may be desired. There is thus a constant degradation of energy going on throughout the universe, more available forms being constantly frittered away into heat.

Friction

74. Friction.—When one body slides over another the motion is resisted by a force which is called *friction*. It is always a resistance, acting against the motion, and depends on the character of the surfaces in contact and on the force pressing them together.

It is a force of the greatest importance in daily life. If it were not for friction, nails and screws and knots would not hold, ropes could not be made, nor could we even walk across a floor. On the other hand, we would gladly be rid of friction in machines, for it is the cause of a large proportion of energy being lost in heat.

Friction appears to be due to the interlocking of minute roughnesses on the surfaces, together with the clinging together or adhesion of the points of closest contact. It is, therefore, diminished by polishing the surfaces, which diminishes the roughness and also makes the points of contact broader so that the film of air or oil is more effective in preventing adhesion.

When two surfaces have been resting in contact the friction at sta is greater than after the motion has been established. It seems prob

that this may be due to the closer contact due to the film of air or oil being squeezed out by the continued pressure.

Friction also resists the rolling of one body on another, though *rolling friction* in case of two given surfaces is much less than *sliding friction*. Rolling friction when surfaces are well polished appears to be due both to cohesion and to a slight deformation both of the surface and of the roller at the point of contact; for the surface is compressed as it passes under the roller, and though it may spring back again *it does not exert quite as much force in recovering as it opposed to the deformation.*

75. Laws of Friction.—Let the block P be drawn along by the weight F, which is not sufficient to start it in motion, but will keep it moving with constant velocity when once started. The weight F is then equal to the force of friction, for it just balances it, neutralizing the resistance to the motion.

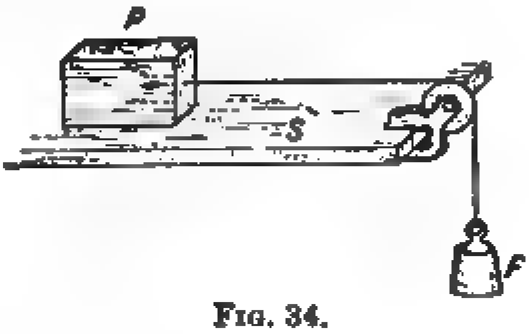


Fig. 34.

It is found in this way that **the friction between two given surfaces is proportional to the force pressing them together.** If the block P weighs 5 lbs. and if an additional weight of 5 lbs. is placed on the block, the force of friction is doubled.

It is also found that **the force of friction, within wide limits, is independent of the area of the surface of contact.** For instance, the friction of the block P is almost the same whether it slides on a narrow or a broad side, provided they are equally smooth.

The velocity with which one surface slides over the other makes little difference, the friction being appreciably the same for all moderate speeds; but the resistance to starting, or *static* friction, is greater than the friction after the motion is established.

It is evident, however, that these laws do not hold without limit. For if one surface is very small, as in case of a point resting on a plane surface, or if the pressure is so great that one body presses into the other, then one cannot move on the other without tearing or injuring the surface, and the law no longer holds.

76. Coefficient of Friction.—It follows from the first law of friction that **the force of friction divided by the force pressing the surfaces together is a constant, this constant is called the**

coefficient of friction of the surfaces concerned; **it is a fraction which when multiplied by the force pressing two surfaces together gives the force of friction to be overcome.**

Thus if the coefficient of friction in case of iron wheels on iron rails is 0.004, then, if the wheels weigh 1000 lbs., a force of 4 lbs. will be required to overcome the friction.

When an engineer wishes to know how much force will be required in moving a house to cause it to slide on its ways, he has only to multiply the coefficient of friction for the soaped beams on which the house rests by the weight of the house itself.

Some Coefficients of Friction

Sliding Friction

Oak upon oak, fibers parallel,	without lubricant	0.42
	rubbed with dry soap	0.16
Oak upon oak, fibers crossed without lubricant		0.29
Iron on bronze	without lubricant	0.25
	thoroughly lubricated, may be as small as	0.06

Rolling Friction

Cast-iron wheels on rails	0.004

77. Limiting Angle of Repose.—The angle at which a surface may be inclined before a body resting on it begins to slip down is determined by the coefficient of friction between the surfaces. Thus let a weight W rest on a surface inclined at an angle a. The earth attracts the weight with a force W which acts vertically downward. We may resolve this force into two components, one P which is perpendicular to the inclined surface and represents the pressure of the weight against the surface, and another F which is parallel to the surface and represents the force urging the weight down along the slope. The force of friction between the weight and the inclined plane is equal to the product kP, where k is the coefficient of friction. If the friction is less than F the weight will slide with increasing speed down the incline, while if it is greater than F the weight will remain at rest.

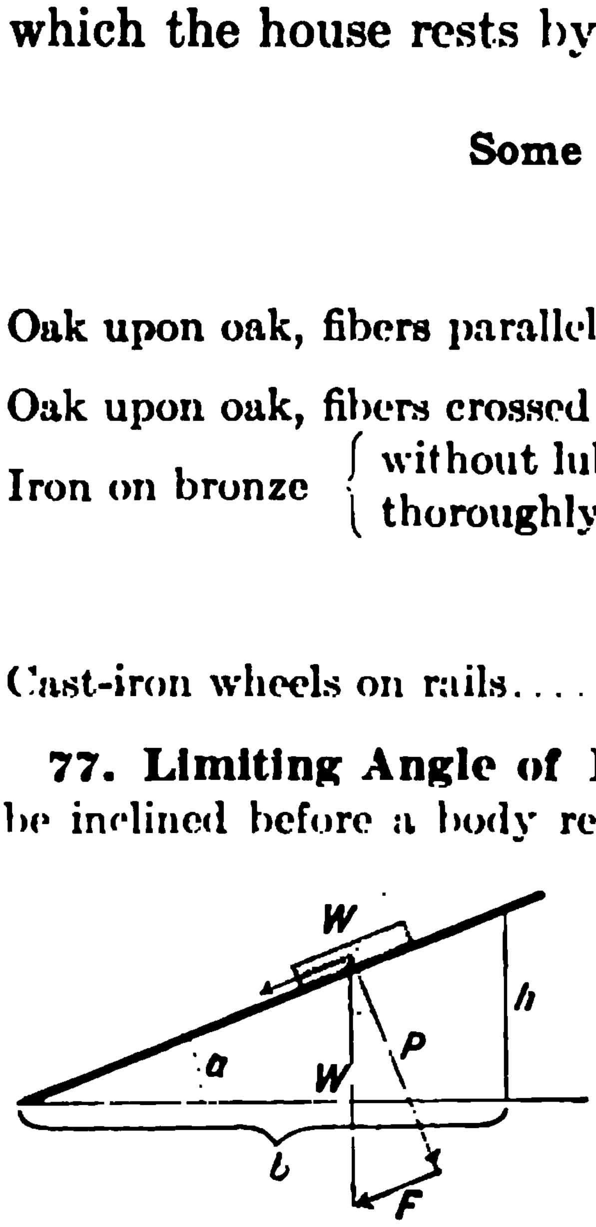


Fig. 35.

It will be noticed that P is made smaller by increasing the slope of the incline, and since k remains constant, the force of friction is less the grea the slope, and is zero when the slope is vertical.

At one particular angle a, which may be called the limiting angle of rep the force of friction balances the force F, and we have $kP = F$. At that an the weight does not start to slide of itself but *if started, slides down* 1

constant speed. In this case $k = \frac{F}{P}$, and by similar triangles $\frac{F}{P} = \frac{h}{b}$, therefore $k = \frac{h}{b}$ or $k = \tan a$.

Hence by finding the limiting angle of repose in a given case the coefficient of friction is at once determined.

78. Means of Diminishing Friction.—To make friction small the surfaces should be very hard and of fine even polish. Where there is much wear it is customary to make one of the bearing surfaces of a harder material than the other. Thus the crank pins on steam engines are made of polished steel and turn in brass boxes, the friction between the brass and steel being less than it would be between two parts of steel.

Rolling friction is very much less than sliding friction, therefore, wheels are used on carriages, etc. It depends to some extent on the diameter of the wheels, being less when the diameter is greater. But even when wheels are used there is sliding friction in the hubs. The resistance to the motion of the vehicle due to this sliding friction is diminished by making the axles of small diameter, but the length of the axle in the hub of the wheel or the length of its bearing surface does not affect the frictional resistance.

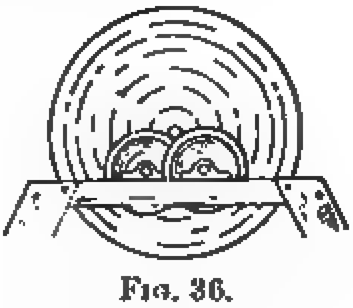


Fig. 36.

To avoid the friction due to the sliding between wheel and axle, ball bearings are used; but even in these bearings there is some sliding friction where adjoining balls rub against each other.

In some cases the axle is made to rest on the rims of two smaller wheels which are called friction wheels (Fig. 36). This is a common practice in mounting grindstones.

Friction is greatly diminished by the use of *lubricants*, of which those most in use are oil, grease, soap, and black lead. The substances used as lubricants cover or wet the surfaces so that the rubbing takes place between layers of these substances instead of between the original surfaces. When the bearing surfaces are subjected to great pressure, as in heavy machinery, a thick oil is used that is not driven out by the pressure; in very light machinery, as in clocks and watches, a very thin oil is used. If oil were used on wooden bearings it would only increase the friction, for it would soak into and swell the wood; dry soap or paraffin may be used as a lubricant for wood surfaces.

Machines

79. Machines.—*Machines are devices by which the amount or mode of application of a force is changed for the sake of gaining some practical advantage.* Simple machines, known also as the mechanical powers, are the rope and pulley, lever, wheel and axle, inclined plane, and screw. All afford interesting cases of forces in equilibrium; but they may also be discussed from the point of view of the conservation of energy, for the work done on a machine must be equal to the work done by it if there is no loss of energy in friction.

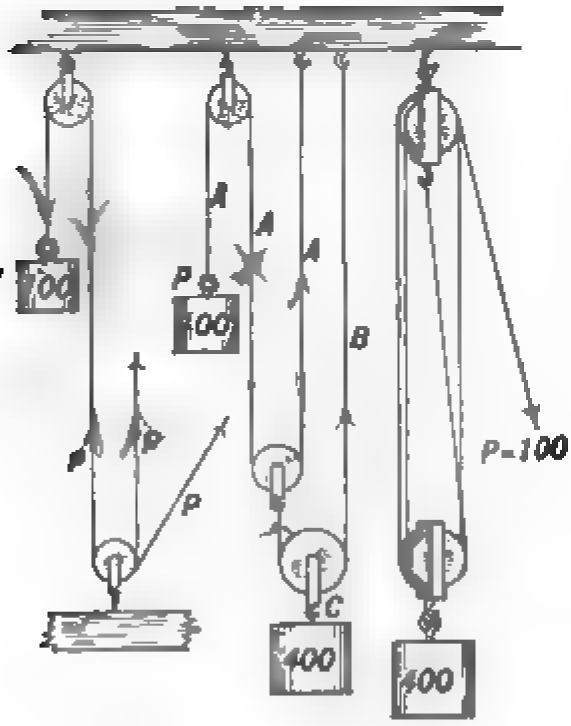


Fig. 37.

The ratio of the force exerted by a machine to the force applied is called its mechanical advantage.

80. Rope and Pulley.—In all tackles where ropes are used the tension or force is the same at every point in a continuous rope, whether it passes over pulleys or not, if there is no friction.

Let us apply this principle to a few cases. In case 1 (Fig. 37) there is only one rope and the 100-lb. weight is supported by it, therefore all parts of the rope are under a tension of 100 lbs., and that force must be exerted at P in whatever direction the pull may be made. In case 2 the 100-lb. weight is supported by the rope A, all parts of this rope are therefore under that tension; but B is attached to a pulley which is drawn up by *two* parts of A. Since the pulley is in equilibrium, it follows that the upward pull of the two parts of A must be equal to the downward pull of B together with the weight of the pulley. If we neglect the latter the tension on B must be 200 lbs. and similarly that on C must be equal to twice that on B. Hence, neglecting friction and the weight of the pulleys, a weight of 400 lbs. on C will balance a weight of 100 lbs. on A.

In case 3 there is one continuous rope which is fastened at the top and passes over two sheaves in each pulley, the lower pulley is therefore sustained by four parts of one rope, hence when a weight of 400 lbs. is support by the lower pulley the tension on the rope is 100 lbs.

The mechanical advantage in the first case is 1, while in the second and third cases it is 4.

81. Principle of Work Applied.—From the conservation of energy it is clear that the work done by a machine must be equal to the work done upon it, provided there is no friction and the energy stored in the machine is not changed. In illustration of this principle consider the various tackles of the preceding paragraph, and let x represent the distance that W is raised in a given case while the end of the rope at P is pulled through a distance y. Then the work done by the machine when W is raised is Wx, and the work spent in raising the weight is Py, and therefore $Wx = Py$.

In the first case $x = y$, therefore $P = W$.

In the second case $x = \frac{1}{4}y$, therefore $\frac{1}{4}W = P$.

In the third case also $x = \frac{1}{4}y$, therefore $\frac{1}{4}W = P$.

It should be noted that in the last two cases if the weights of the pulleys are taken account of we cannot say that $Wx = Py$, for some of the work done is spent in raising the movable pulleys. Thus, in case 2, if each pulley weighs w, we have

$$Py = w\frac{y}{2} + (w + W)\frac{y}{4} \quad \text{or} \quad P = \tfrac{1}{2}w + \tfrac{1}{4}(w + W).$$

82. Lever.—In the lever a rigid bar resting on a point of support, or *fulcrum*, is used to exert a great force near the fulcrum when a smaller force is exerted at the end of the longer arm of the lever. A crowbar as used in moving a stone, a hammer in drawing a nail, are examples of levers. Levers are sometimes divided into three classes depending on the relation between the position of the fulcrum and the points where the weight is raised and the force applied, as shown in the figure, where P represents the force applied to support the weight W, and F is the fulcrum.

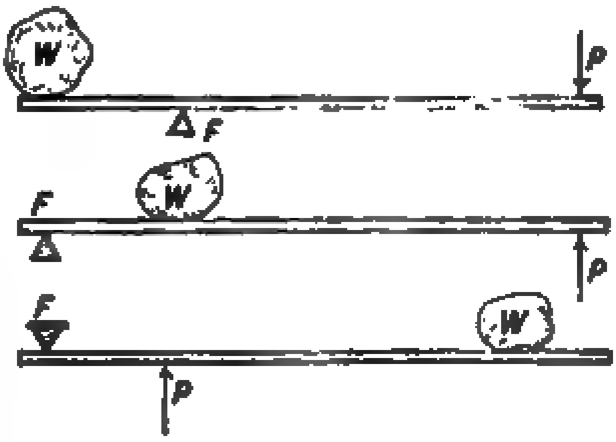


Fig. 38.—Classes of levers.

The upper lever in the figure belongs to the *first class;* the next to the *second class;* and the lowest to the *third class.*

The distance from P to the fulcrum is called the power arm and that from W to the fulcrum is called the weight arm, and the

principle of moments tells us that if these distances are measured perpendicular to the lines of action of the forces P and W, then *the product of P by the power arm is equal to the product of W by the weight arm.* In other words, the moments of the two forces about the fulcrum as axis must be equal and opposite.

The pressure F against the fulcrum, since the three forces P, W, and F must be in equilibrium, is represented by the vector necessary to form a triangle with P and W. Of course if P and W are parallel, F must be either their sum or difference, depending on circumstances.

83. Crank and Axle.—In case of the crank and axle, shown in figure 39, the relation between the weight and the force applied at the crank to support it, is at once obtained from the principle that the moments of F and W about the axis must be equal, since the only motion that the system can have is one of rotation. Hence if R is the length of the crank arm and r the radius of the axle or drum on which the rope supporting W is wound, we have $FR = Wr$ in case the force F acts at right angles to R.

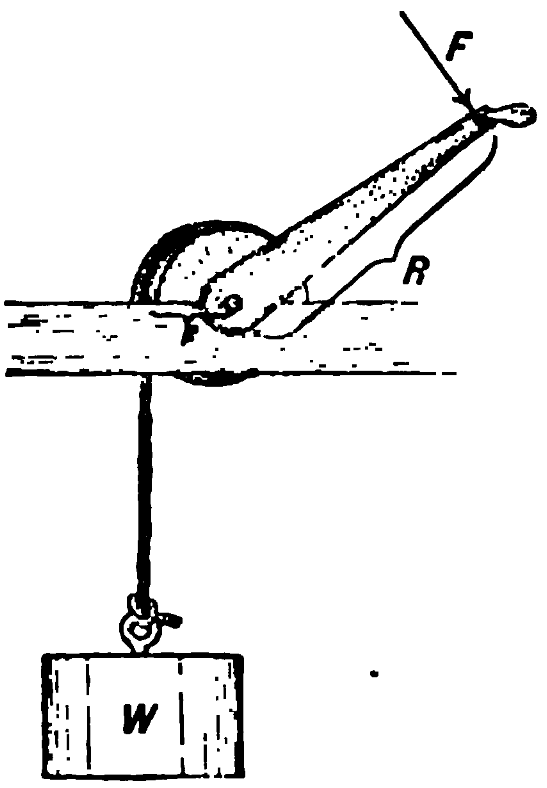


FIG. 39.

Here, again, we may apply the principle of work, for in one revolution of the crank the weight W is raised a distance equal to the circumference of the drum or $2\pi r$, while the balancing force F acts through a distance $2\pi R$. We have, therefore, in case of equilibrium

$$W\ 2\pi r = F\ 2\pi R \quad \text{or} \quad Wr = FR.$$

84. Inclined Plane.—Barrels or casks are sometimes rolled up inclined planes and thus raised where they could not be directly lifted. The advantage of the inclined plane may be understood from figure 40, where W represents a weight resting on the inclined plane having length l, height h, and base b. The attraction of the earth is a force vertically downward on W, but it may be resolved as is shown into the components N at right angles to the inclined plane and F parallel to it.

The component N is balanced by the pressure of the plane,

while the component F represents the force that must be balanced by the push P necessary to support the weight on the plane. From the similarity of the two triangles it is clear that W, N, and F are proportional to l, b, and h, respectively. That is $F : W :: h : l$, or in words, **the force required to support the weight on the inclined plane is to the whole weight as the height of the plane is to its length.**

The same conclusion may also be reached by the principle of work, for if the weight is pushed up the plane the supporting force P acts through the length l, while the weight W is only raised against the earth's attraction through a distance h. Hence $Pl = Wh$.

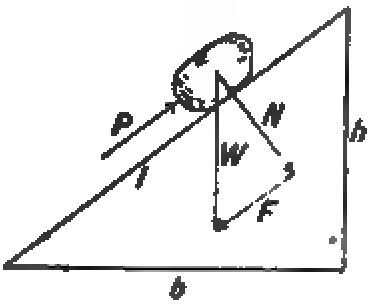


Fig. 40.

If the force P, instead of acting parallel to the *length* of the inclined plane, were parallel to its base we should resolve the weight W into components N and F as in figure 41, where F is parallel to the base. Then

$$F = P = W\frac{h}{b}.$$

85. Screw.—The screw as used in the ordinary letter press may cause enormous pressures by the application of a very moderate force to the lever arm. In one complete revolution of the screw it advances the distance between consecutive threads measured parallel to the axis. This distance is called the *pitch* of the screw.

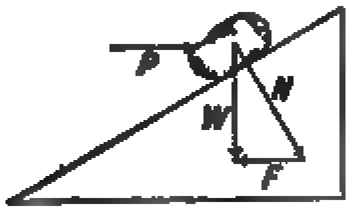


Fig. 41.

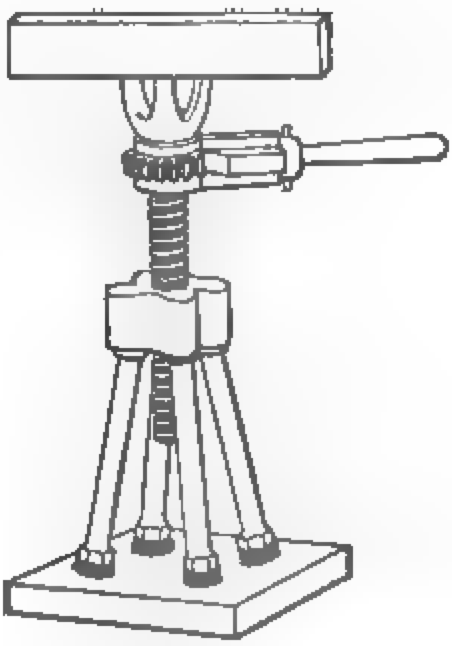


Fig. 42.

The mechanical advantage of the screw may be determined by considering the thread as a sort of inclined plane wrapped around the axis, but we may deduce it more conveniently from the principle of work; for if the force P operating the screw acts at right angles to the end of a lever arm of length R, in one revolution of

the screw the force P acts through a distance $2\pi R$, while the screw advances through a distance h equal to the pitch of the screw. Hence if W is the force exerted by the screw we have by the principle of work

$$2\pi RP = Wh$$

or

$$\frac{W}{P} = \frac{2\pi R}{h}.$$

86. Chinese Capstan and Differential Pulley.—In the Chinese capstan a drum or axle having two parts of somewhat different diame-

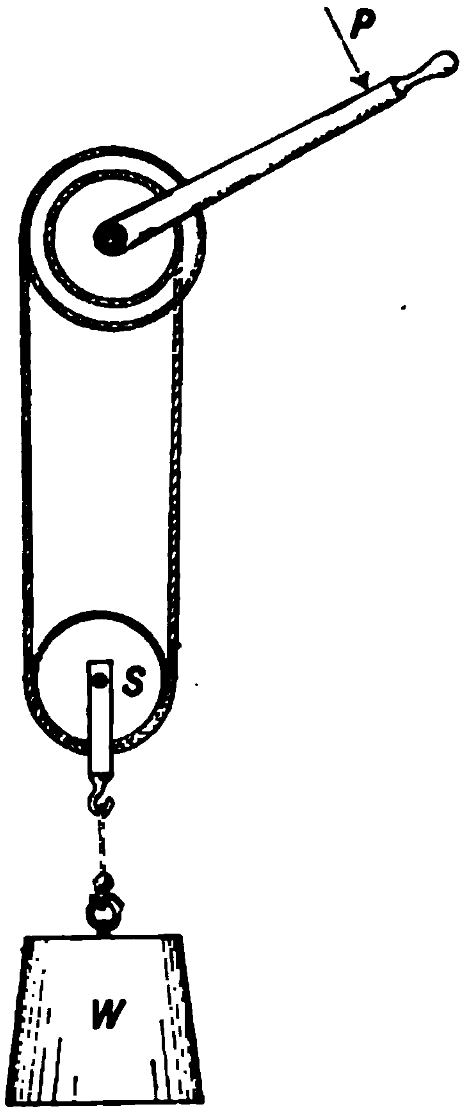


FIG. 43.

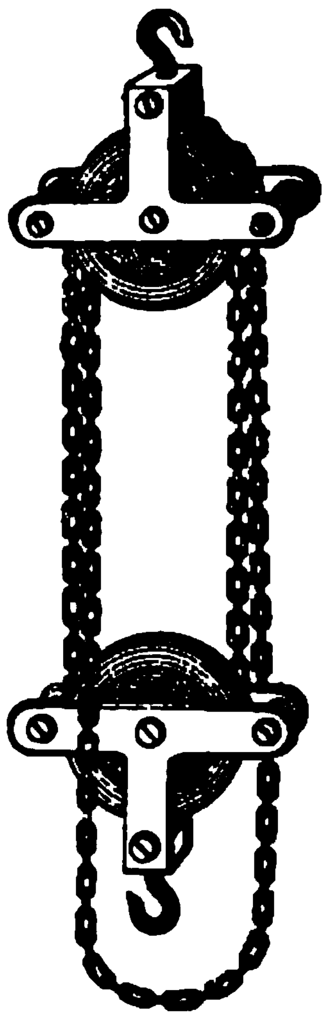


FIG. 44.—Differential pulley.

ters is operated by lever arms or capstan bars, so that one end of a rope is wound up on the drum of larger diameter while the other end unwinds from the smaller drum. The rope passes around a pulley S which is attached to the anchor or other weight to be raised. The force W is divided between the two parts of the rope pulling on S, so that the rope is under a tension $\frac{W}{2}$. If r and R are the radii of the small and large drums, respectively, the moments of the forces exerted by the rope on the drum are $\frac{W}{2}r$ and $\frac{W}{2}R$ and the difference between these two moments must be balanced by the moment of the force P acting on the end of the capstan bar of length l. Hence we have in case of equili

$$Pl = \frac{W}{2}(R - r).$$

The advantage of such an arrangement is evidently the same as if one ıd of the rope were fixed and the other, after passing around S, were wound p on an axle whose radius was $R - r$. But such an axle being of small ıameter would not have the strength of the larger axle with two drums.

The differential pulley is a similar device used for raising heavy weights. 'here is an upper pulley having a single sheave with two grooves of different ıameters like the two drums of the Chinese capstan. An endless chain asses over one groove in the upper pulley then around a pulley attached to he weight to be raised, and then around the second groove of the upper or xed pulley. The grooves of the upper pulley have notches to receive the hain so that it cannot slip, and the chain is passed over it in such a way that is wound up on one groove at the same time that it unwinds from the other. f the difference in diameters of the two grooves in the upper sheave is small, small pull on the chain may suffice to support a large weight.

Problems

1. A 180-lb. barrel is rolled up an inclined plane 12 ft. long to a platform 4 ft. above the ground. How much force must be exerted along the plane and how much work is done? Find also the force and work when the plane is 20 ft. long, the height being the same.
2. Find the force which the barrel exerts against the plane in both the cases specified in the first problem.
3. How much force parallel to the plane is required to support a weight of 39 kgms. on a frictionless inclined plane 13 meters long and 5 meters high? Also find the force with which the weight presses against the plane.
4. If the coefficient of friction between weight and plane in the last question is 0.20, find the force of friction and how much force must be exerted parallel to the plane in drawing the weight up, also in lowering it.
5. When the coefficient of friction between a weight and the inclined plane on which it rests is 0.30, find the ratio of its height to length when the plane is so steep that when the weight is started it slides down without acceleration.
6. A certain jack-screw has a screw 2 in. in diameter with three threads to the inch, and is operated by a lever arm 2 ft. long. What weight can be raised by a force of 48 lbs. applied at right angles to the end of the lever arm, neglecting friction?
7. When the coefficient of friction of the oiled surfaces of the jack-screw described in problem 6 is 0.06 and when a weight of 5 tons is raised, find the force required at the end of the lever arm to overcome friction, and the additional force required to raise the weight.
8. In problem 7, find the ratio of the work required to raise the weight 1 ft. without friction, to the actual work with friction, and thus determine

the efficiency of the screw. Would the efficiency be the same if one-half as large a weight were being raised?

9. Find the tension on a bicycle chain when the pedal is pressed down with a force of 120 lbs.; the crank arm being 6 in. long and the sprocket wheel 8 in. in diameter.

10. If a force of 40 lbs. must be exerted on the arm of a windlass in raising a weight of 120 lbs. while a force of only 20 lbs. is required in lowering the same, find the force expended in overcoming friction, and the efficiency of the windlass, and what per cent. of the work done is lost in friction.

11. How much force must be exerted on the crank of a windlass to raise a weight of 180 lbs., if the crank arm is 20 in. long and the drum on which the rope is wound is 8 in. in diameter.

12. Find the direction and amount of the force on the bearings of the windlass in the previous question, first, when the crank is in a horizontal position and being pressed down; second, when the crank arm is vertical.

13. A man weighing 150 lbs. raises himself in a sling by means of a rope passing over a movable pulley attached to the sling and a fixed pulley overhead. With how much force must he pull? Show also how to obtain your result by the principle of work.

14. A man weighing 180 lbs. runs up 24 steps, each 7 in. high, in 8 seconds. How much work does he do and what horse-power does he expend?

15. A donkey-engine is required to raise by means of a tackle a 2-ton weight to a height of 100 ft. in $\frac{1}{2}$ minute. What horse-power is required if the efficiency of the tackle is 70 per cent.

16. When 1 H.P. is expended by a horse in pulling a load at the rate of 6 miles per hour, find the force with which the horse pulls the load.

17. What load can two horses draw along a level road at the rate of 3 miles an hour if they spend 2 H.P. in pulling the load, when the coefficient of friction of wagon on road is $\frac{1}{10}$. **Ans. 2500 lbs.**

18. A locomotive drawing a train along a level track at 30 miles per hour expends 75 H.P., find the total air and frictional resistance overcome. **Ans. 937.5 lbs.**

19. A locomotive draws a 300-ton train along a level track at the rate of 20 miles per hour; while working at the same rate it draws it up a $\frac{1}{4}$ per cent. grade at 15 miles per hour; what horse-power is expended, supposing the frictional and air resistances the same in both cases, and what is the resistance in pounds. Ans. Resistance = 4500 lbs.; H.P. = 240.

III. KINETICS OF A PARTICLE

Rectilinear Motion of a Mass

87. Introductory.—Up to this point we have studied especially cases of *equilibrium*, where the forces acting are balanced

that there is no acceleration. We must now examine in some detail the various forms of motion where forces are involved in such a way as to cause acceleration.

This part of mechanics, as Mach says, "is a wholly modern science. All that the Greeks achieved in mechanics belongs to the realm of *statics*. *Dynamics* was first founded by Galileo."

Before 1638, when Galileo first published the results of his experiments, so little progress had been made in this direction that it was currently held that heavy bodies fell faster than light ones.

In studying the effect of force in giving motion to matter, the simplest case to examine is where a definite portion of matter is acted on by a constant force. This is the case with falling bodies; for while a body is falling freely it is being urged downward by a constant force which we call its weight. Therefore, Galileo carefully studied the motion of falling bodies, and of bodies rolling down inclined planes, and showed that in each of these cases the motion was with *constant acceleration*. As pendulum clocks had not been invented at that time, he made use of a simple water clock to measure short intervals of time in his experiments. This consisted of a large vessel of water having a jet closed by the finger, from which water was allowed to escape during the time interval to be measured. Thus the weight of water escaping while a body rolled down an inclined plane served to measure the time of descent.

These experiments also showed that when a plane was inclined at such an angle that the force parallel to the plane required to keep a body from sliding down was one-half the weight of the body, then its acceleration in sliding down was one-half its acceleration when falling vertically. That is, *the acceleration was proportional to the force causing the motion.*

38. Atwood's Machine.—A convenient device for studying the effect of forces in giving motion to masses is the apparatus known as Atwood's machine (Fig. 45). Two equal weights A and B are hung over a very light carefully balanced wheel mounted so that it shall run with as little friction as possible. An additional weight or rider w, having two projecting arms, is laid on top of the weight A, which is supported so that it can be liberated at any instant. When the weight A is freed it moves down accelerated by the rider w, until it reaches the ring C which picks off the rider w and allows A to pass freely through. After passing the

ring C there is no longer any accelerating force, since the rider is removed, and the weight A continues to move with the velocity which the rider had given to it.

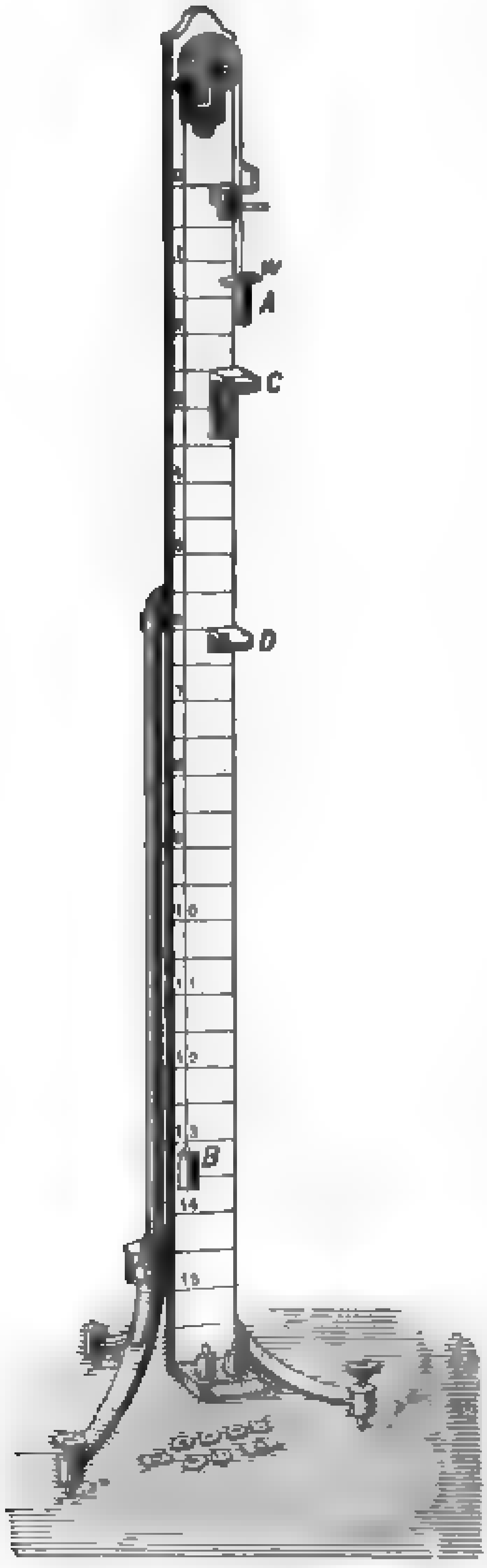


Fig. 45.—Atwood's machine.

Thus if the ring C is so adjusted that A passes through it exactly 2 seconds after being liberated, and if D is so placed that A moves from C to D in the next second, then if C and D are found to be 30 cm. apart, we conclude that A acquired a velocity of 30 cm. per second by a force which acted steadily for 2 seconds. If the same force is now allowed to act for 1 second, a velocity of only 15 cm./sec. will be acquired. By varying the weight of the rider or using instead of A and B a pair of weights, having double the mass, the following conclusions may be established:

(a) The motion is with constant acceleration.

(b) The acceleration is proportional to the weight of the rider so long as the total mass $A+B+w$ is constant.

(c) If the mass of the moving system is doubled, a given rider will cause only half as great acceleration as before.

89. General Principle.—The effect of a force in giving motion to a body, as brought out in the experiments just described, may be thought of as due to a general principle which may be thus stated: **the effect of a force in changing the motion of a mass is not in any way affected by the state of rest or motion of the mass which is acted upon.**

For instance, while a force is acting on a mass and increasing its velocity, suppose a second and equal force to act in the same direction upon the same mass. The second force being equal to the first will produce just as great an increase in velocity per second as

is being produced by the first; and since both effects take place simultaneously and without interference, the total change in velocity will be twice that which would have been produced by the original force. It follows that **the change in velocity per second when a force acts on a body is proportional to the amount of the force.**

And the change in velocity of a body when acted on by a force is also *proportional to the length of time during which the force acts*, for suppose a mass has acquired velocity by a force acting upon it for 1 second, if the force now acts for another second it will increase the velocity of the mass as much more in the same direction, since the effect of a force is in no way conditioned by the state of rest or motion of the body upon which it acts.

90. Impulse.—The change in velocity which a given mass experiences is proportioned therefore both to the amount of the force and to the time during which it acts. A large force acting for a short time may produce the same change in the velocity of a mass as a small force acting for a longer time.

A billiard ball may be made to roll as fast by pushing it as by striking it with the cue; the force in the second case is very much greater than in the first, but is exerted during an exceedingly short time; the impulse in both cases must be the same.

The product of the amount of a force by the time during which it acts is called the *impulse*.

91. Force and Motion.—Again, suppose two equal masses moving side by side are acted on by equal forces in the same direction, they will both gain in velocity equally and will accordingly continue to move side by side, and their motion will evidently not be affected in any way if the two masses are connected forming a single large mass.*

From this consideration we see that if a force gives a certain acceleration to a given mass then twice the force will be required to give the same acceleration to a mass twice as great, etc. Or, in order that different masses may all have the same change in velocity per second, the forces acting on them must be proportional to the masses.

But if the mass is doubled without any corresponding change in

* This cannot be regarded as known *a priori* for it results from the experimental fact that the inertia of one body is not affected by its proximity to another.

the force which acts upon it, the gain in velocity will be only half as great as before, for the motion in that case will be the same as if the original mass were acted on by half the original force.

92. Momentum.—A given impulse may produce a great change in the velocity of a small mass, or a proportionally small change in the velocity of a greater mass; therefore, to measure the effect of an impulse, a quantity is employed which is proportional both to the mass and velocity of the moving body; this is called its *momentum.*

The momentum of a body is the product of the amount of its mass by the amount of its velocity, and is a directed or vector quantity.

93. Three Laws of Motion.—The relations between forces, masses and motion, were first clearly enunciated in the form of three laws of motion, by Sir Isaac Newton in his celebrated *Principia*, published in 1686. Two of these laws have been already discussed (§§31,38), but are here repeated in order that all three may be presented together.

First Law.—Every body continues in its state of rest or of moving with constant velocity in a straight line, unless acted upon by some external force.

Second Law.— Change of momentum is proportional to the force and to the time during which it acts, and is in the same direction as the force.

Third Law.—To every action there is an equal and opposite reaction.

94. Discussion of Second Law.—This law may be also expressed in the formula

$$mv - mu \propto Ft$$

where F is a force acting on a mass m for a time t, and u is the velocity at the beginning of the time interval t, while v is its velocity at the end of that time. Thus mv is the momentum after the force has acted, while mu is the original momentum of the mass. The gain in momentum is, therefore, $mv - mu$, and according to the law this is proportional to the force F and to the time t jointly, or to their product Ft.

The above formula may be written:

$$Ft = k(mv - mu) \quad \text{or} \quad F = km\left(\frac{v-u}{t}\right)$$

here k is a constant, the value of which depends on the particular nits which are employed in measuring the various quantities oncerned.

In the above equations F represents the *average* value of the rce during the time t in which the velocity of the mass has anged from u to v; but when t is exceedingly short $\frac{v-u}{t}$ pproaches as its limit the actual rate of acceleration at the given stant, while F is the corresponding force at that same instant, nd we may write,

$$F = kma \tag{1}$$

hat is, **the acceleration of a body is proportional to the force cting upon it and inversely proportional to its mass.**

This may be called the fundamental formula of dynamics as it a direct expression of the second law of motion, is absolutely eneral, and enables us to determine the forces acting in any case here the mass and motion of a body are known, since the acceleration is determined from the motion.

Thus it follows that if the force acting on a mass is constant the ass moves with constant acceleration, while if the force varies e acceleration varies in the same proportion.

95. Dyne and Poundal.—In dealing with cases of equilibrium e have used the ordinary gravitation measures of force, the eight of a pound or gram, but in studying the accelerating ffect of forces it will be found more convenient to use as the unit force which will make the constant k equal to unity in the above xpression, so that we may write simply

$$F = ma$$

Defined in this way, unit force is one which will give unit cceleration to unit mass, or unit force acting for unit time on nit mass will change its velocity by unity.

When the centimeter gram and second are the fundamental nits as in the C. G. S. system, the unit force is called the **dyne,** rom the Greek word for force. **It is a force which, acting on a ass of one gram for one second, will change its velocity by one entimeter per second.**

[illegible] find the force in dynes in a given case of motion it is

only necessary to multiply the mass in grams by its rate of acceleration measured in centimeters per second per second.

Thus a force of 100 dynes acting on a mass of 10 grams will give it an acceleration 10, or in one second will give it an increase in velocity of 10 cms. per second.

A unit of force similarly based on the foot, pound, and second as units of length, mass, and time, respectively, is sometimes used and is called the **poundal**, it is **the force which acting on a mass of one pound will increase its velocity one foot per second for every second that it acts.**

The dyne and poundal have the advantage of being absolutely definite units of force, and *do not vary from point to point on the earth as the weight of a gram or pound varies.*

96. Unit of Work or Energy.—The unit of work on the C. G. S. system of units where the force is measured in *dynes* and the distance in centimeters is known as the **erg** (from the Greek word for work). **It is the work done when a body moves one centimeter in the direction in which it is urged by a force of one dyne.**

The corresponding unit of work or energy on the foot-pound-second system is the **foot-poundal**, and **is the work done when a body moves one foot in the direction in which it is urged by a force of one poundal.**

97. Motion in a Straight Line with Constant Velocity.—When a body moves in a straight line with constant velocity the acceleration is zero and therefore the force must be zero according to the formula $F = ma$.

The moving mass is, therefore, in equilibrium. This is the case considered in Newton's first law of motion.

A railway train while running at constant speed is in a state of equilibrium. The force of the locomotive urging it on is exactly balanced by the resistance of the air and friction of the wheels. So when a bucket is drawn up out of a well with constant speed it is in equilibrium and the upward pull on the rope is exactly equal to the weight of the bucket of water.

98. Motion in a Straight Line with Constant Acceleration.—When a body moves in a straight line with a velocity which is increasing or diminishing at a constant rate, it has a constant acceleration in the direction of the motion in one case and opposite to it in the other.

When the acceleration a is constant, the change in velocity of the moving body in t seconds is at. And if the velocity at the beginning of the time t is u, and that at the end of the time is v, then

$$v = u + at \quad \text{when the speed is increasing;}$$
$$v = u - at \quad \text{when the speed is decreasing} \qquad (1)$$

The space passed over in t seconds will be found by multiplying the *average velocity* during the interval by the time t. But since the acceleration is constant the velocity increases uniformly with the time, and therefore the average velocity is the arithmetical mean of the initial and final velocities, or $\frac{v+u}{2}$. The space traversed in time t may, therefore, be expressed by the formula

$$s = \frac{v+u}{2}t \qquad (2)$$

Substituting for v its value

$$v = u \pm at,$$

we find

$$s = ut \pm \tfrac{1}{2} at^2 \qquad (3)$$

But equation (1) may be put in the form

$$a = \frac{v-u}{t},$$

and this multiplied by (2) gives

$$2as = v^2 - u^2 \qquad (4)$$

By the use of these formulas (1 to 4) any two of the quantities u, v, a, t, s may be determined when the other three are given.

The student should thoroughly memorize these formulas and exercise himself in applying them to simple problems, such as those on page 70.

39. Force Causing Rectilinear Motion with Constant Acceleration.—The kind of motion just discussed is produced whenever a mass is acted on by a constant force in one direction; for in such a case the acceleration is constant and given by the relation

$$a = \frac{F}{m}.$$

Thus when a car is drawn along a track by a stretched spring

which is kept constantly at the same tension, the motion is with constant acceleration. So also a falling body has this kind of motion, for it is constantly urged downward by its own weight, which is a nearly constant force. When a body slides down an inclined plane, the force urging it down along the plane is the same at one point as at another, and, therefore, in this case also the acceleration is constant.

100. Falling Bodies.—Freely falling bodies are the most familiar examples of bodies moving with constant acceleration. For a body near the surface of the earth is attracted or urged downward with a certain constant force which we call its weight, and when it is set free so that its weight is the only force acting, it falls with constantly accelerated motion. In ordinary experience, however, where bodies fall through air, the resistance of the air is another force which modifies the motion. If the resistance in a given case were constant, the body would still fall with constant acceleration, but the air resistance increases greatly with the velocity of the falling body, so that in case of a light body, as the speed increases the air resistance may become equal and opposite to its weight, and when that is the case it falls without acceleration. This is the case with scraps of paper and rain drops.

Strictly speaking, even the weight of a body is not constant as it falls, but increases as it approaches the surface of the earth. The weight of a kilogram one mile above the earth's surface is less by $\frac{1}{2}$ a gram than at sea level, and at the ceiling of a room 3 meters high a kilogram weighs about one milligram less than at the floor. This variation of force with height causes a corresponding increase in the acceleration of a falling body as it approaches the earth's surface; but this is so small, however, that except in case of great heights it may be neglected.

101. Acceleration of Gravity.—The early philosophers speculated as to *why* bodies fell; Galileo was the first to carefully determine *how* bodies fell. He also showed, contrary to the universally accepted opinion of his day, that except for air resistance all falling bodies are equally accelerated. A large stone or a small one, an iron cannon ball, a lump of lead, or block of wood when dropped from the top of a tower reach the ground in the same time. If a feather, scraps of paper, and some bits of metal or lead shot

are placed in a long tube (Fig. 46) from which the air is exhausted, on quickly inverting the tube all reach the bottom at the same instant. Hence the rate of increase in velocity, or *acceleration, is constant* at any given place on the earth *for all kinds and sizes of bodies.*

This constant acceleration is called the *acceleration of gravity* at the given place, it is usually represented by the symbol g and is measured most accurately by pendulum experiments.

The value of g at the sea level for the latitude of New York is 980.2 cm./sec.^{2}, or 32.16 ft./sec.^{2} The table on page 108 gives also the values at some other places.

The formulas for falling bodies are, therefore, obtained from those of §98 by making the acceleration equal to g. Thus

$$v = u + gt$$
$$s = ut + \tfrac{1}{2} gt^2$$
$$2gs = v^2 - u^2.$$

When a body is simply dropped, with no initial velocity, u is zero, and we have

$$v = gt$$
$$s = \tfrac{1}{2} g t^2$$
$$2gs = v^2.$$

In approximate calculations and in working problems for practice, g may be taken as 980 cm./sec.^{2} or 32. ft./sec.^{2}

102. Mass Proportional to Weight—Galileo's discovery that all kinds and sizes of bodies when dropped to the earth at the same place are accelerated at the same rate except for air resistance, leads to an important conclusion. For when two bodies are equally accelerated their masses must be proportional to the accelerating forces (§91), which forces, in the case under consideration, are the weights of the bodies; therefore *the masses of bodies are proportional to their weights, if weighed at the same place.*

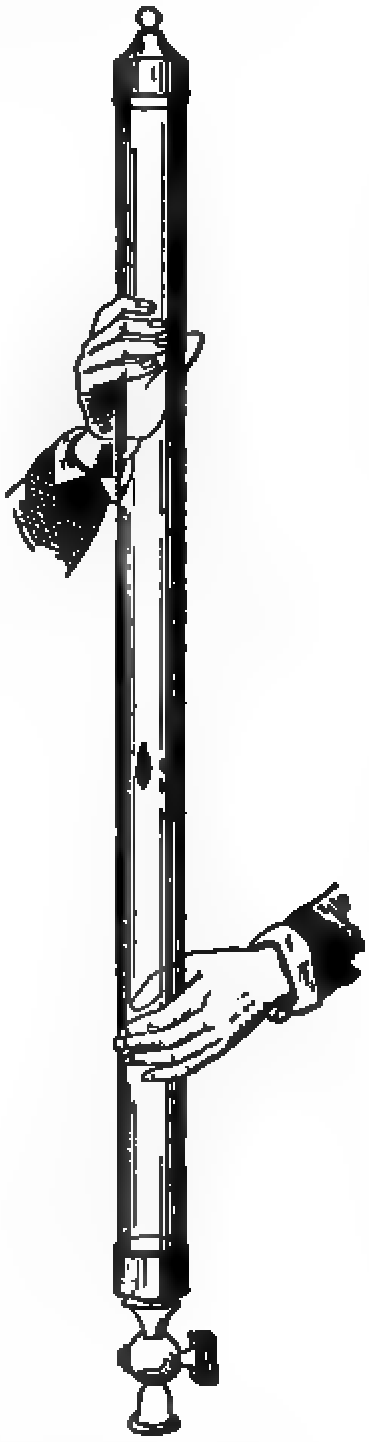


Fig. 46.—Fall in vacuo.

103. Relation between Dyne and Gram.—The force urging downward a freely falling mass m is expressed by the formula

$$F = mg$$

the force being in *dynes* if C. G. S. units are used. Suppose $m = 1$ gram and $g = 980$, then $F = 980$ dynes; but the force with which a mass of one gram is attracted toward the earth is called the weight of one gram therefore the weight of one gram = 980 dynes, or the force which we have called a dyne is slightly more than the weight of a milligram at the earth's surface.

The student may show similarly that one poundal is about equal to the weight of a half-ounce; that is, *one pound weight at New York = 32.16 poundals.*

104. Gravitation Units of Force.—The weight of a gram or pound is often a convenient unit of force; indeed, engineers in English speaking countries almost always measure forces in pounds; for though the weight of a pound varies from place to place on the earth, its weight at some selected spot may be taken as standard.

For example *the standard force of a pound* may be defined as the weight of a pound mass at New York where the acceleration of gravity is 32.16 $ft./sec.^2$, and in that case it will be equal to 32.16 poundals. So also *the standard force of a gram* might be defined as the weight of a gram at a point where $g = 980$ $cm./sec.^2$, in which case it is equal to exactly 980 dynes.

If these *gravitation units of force* are used the constant k in formula (1) §94 is no longer unity, but we have

$$(F, \text{ in pounds}) = \frac{1}{32.16}(m, \text{ in pounds}) \times (a, \text{ in ft./sec.}^2)$$

or

$$(F, \text{ in grams}) = \frac{1}{980}(m, \text{ in grams}) \times (a, \text{ in cm./sec.}^2)$$

But most of the formulas in this book are based on the relation $F = ma$; it will therefore be best for the student in working problems to use consistently either the centimeter-gram-second system with the force in *dynes*, or the foot-pound-second system with the force in *poundals*, changing, when required, dynes or poun into grams or pounds weight by dividing by 980 or 32 as the

may be; 32 being used instead of 32.16, as the value of g in ft./sec.^{2}, for convenience in numerical work.

105. Atwood's Machine Problem.—Suppose two masses, one 40 and the other 50 grams, are connected by a cord running over a light frictionless pulley as in Atwood's machine, and suppose for simplicity that the mass of the cord and of the pulley may be neglected. It is required to find the acceleration and the tension on the cord.

In this case the whole mass $40 + 50$ moves together and the resultant force which gives it motion is the weight of $50 - 40 = 10$ grams, or $10g$ dynes.

Since force = mass × acceleration

we have $10g = 90 \times a$, therefore $a = \frac{1}{9}g$, hence the acceleration is one-ninth that of a freely falling body. 10 = 90a a = $\frac{1}{9}$

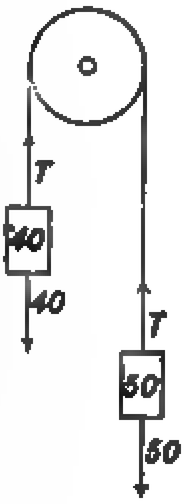


FIG. 47.

This result may also be reached by considering that a force of 10 grams acting on a mass of 10 grams gives it an acceleration g, and therefore if that same force act on a mass 9 times as great it will give it an acceleration $\frac{1}{9}g$.

To find the tension on the cord consider the forces acting on the mass 40. It is urged downward by its own weight, 40 grams, and upward by the tension of the cord, which we may call T grams. It moves upward with an acceleration $\frac{1}{9}g$, as has been shown, hence the resultant force must be upward and equal to $(T-40)$ grams or $(T-40)g$ dynes, and we have, since $F = ma$,

$$(T - 40)g = 40 \times \tfrac{1}{9}g$$

whence

$$T = 44\tfrac{4}{9} \text{ grams' weight.}$$

106. Motion on an Inclined Plane.—When a mass M rests on an inclined plane, the force due to gravity, or its weight, may be resolved into two components, as shown in figure 48, one N perpendicular to the plane and the other F parallel to it. If M is the mass in grams, its weight in dynes is Mg. And from the similarity of the two triangles, we have Mg, N, and F respectively proportional to the sides of the large triangle formed by l, b, and h.

That is, $F:Mg::h:l$ or $F = Mg\frac{h}{l}$ dynes. Thus the force

F causing the motion is constant, and is the same fractional part of the whole weight of the body as the height of the inclined plane is of its length. The acceleration is therefore constant.

Since $F = Ma$, we have $a = g\frac{h}{l}$ or $a = g \sin c$.

To find the velocity which the body acquires in sliding the length of the plane l, we have only to use the formula (4) of §98.

$$2as = v^2 - u^2.$$

The body starts from rest, hence $u = o$ and $s = l$ in this case, therefore

$$2g\frac{h}{l}\,l = v^2 \quad \text{or} \quad v^2 = 2gh;$$

but this is precisely the velocity which a freely falling body will gain in falling through a vertical distance h, and there is nothing

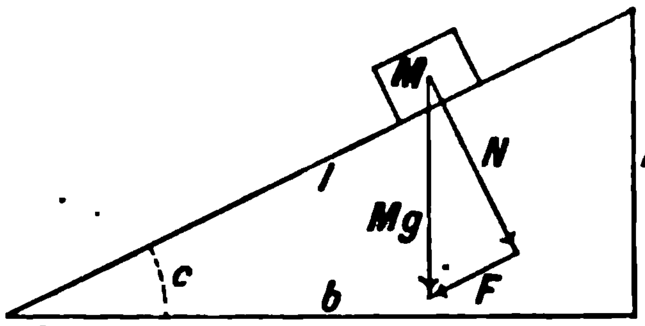


FIG. 48.

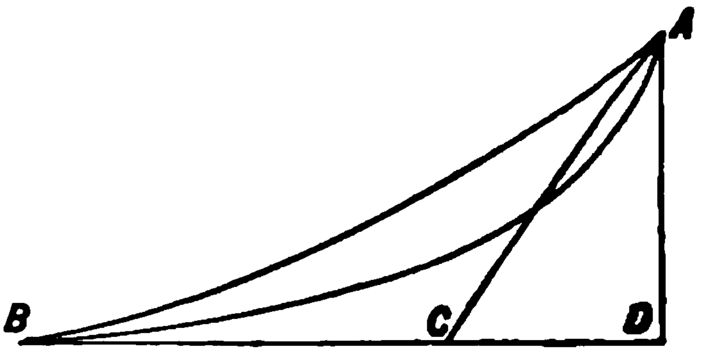


FIG. 49.

in the result which depends on the slope of the plane, **therefore the velocity gained by a body in sliding down a frictionless inclined plane of any slope whatever is the same as that gained by a body in falling freely the same vertical distance.**

Since the *velocity* does not depend on the slope of the plane, it will be the same at B (Fig. 49) for any smooth, frictionless curve down which it may slide from A, and it will be the same at B as at C or D.

The *time of descent*, however, from A to B depends on the curve and may be proved to be a minimum when A and B are joined by the arc of a cycloid.

107. Kinetic Energy.—We will now calculate the effect of a certain amount of work in giving motion to a mass m. Suppose a force of F dynes acts on m in the direction of its motion while it is moving through a space of s centimeters; the work done is by definition, Fs dyne-centimeters or *ergs*. But while the const

force F acts there is a constant acceleration a and the equations of §98 therefore apply to the motion, and we have

$$2as = v^2 - u^2,$$

also

$$F = ma.$$

Multiplying these equations together we obtain

$$Fs = \tfrac{1}{2}mv^2 - \tfrac{1}{2}mu^2 \qquad (1)$$

The change from $\frac{1}{2}mu^2$ to $\frac{1}{2}mv^2$ therefore expresses the amount of work required to change the velocity of the mass m from u to v. Starting from *rest*, the energy required to give it velocity v is $\frac{1}{2}mv^2$; this is also the measure of the work that the body can do before coming to rest again, therefore **the quantity $\frac{1}{2}mv^2$ is the measure of the kinetic energy or energy of motion possessed by a mass m moving with velocity v.**

$$\tfrac{1}{2}mv^2 \begin{cases} = \text{kinetic energy in } \underline{\textit{ergs}} \text{ when} \begin{cases} m \text{ is in grams,} \\ v \text{ is in cms. per sec.} \end{cases} \\ = \text{kinetic energy in } \textit{foot-poundals} \text{ when} \begin{cases} m \text{ is in pounds} \\ v \text{ is in ft. per sec.} \end{cases} \end{cases}$$

108. Velocity at Foot of Inclined Plane.—The principle of the conservation of energy may be applied to motion on an inclined plane and leads at once to the conclusion previously stated, §106, that the velocity of a body at the foot of an inclined plane depends only on its height and is independent of the slope.

For the work done in lifting the body from the bottom of the plane to the top depends only on the height of the plane, since the work is done only against gravity and serves to increase the potential energy of the body. In sliding down the plane, if no work is done against friction, all the potential energy gained will be transformed into kinetic energy, so that when it reaches the bottom its kinetic energy will be equal to the work that was done in lifting it. The kinetic energy, of the body and consequently its velocity will therefore be independent of the slope of the plane.

The work done in lifting the mass m the height of the plane h is mgh, for mg is the weight of the mass expressed in dynes. The kinetic energy of the mass at the bottom is $\frac{1}{2}mv^2$, hence

$$mgh = \tfrac{1}{2}mv^2 \quad \text{and} \quad v^2 = 2gh.$$

109. Kinetic Energy and Momentum Compared.—Kinetic energy and momentum are both quantities that depend on the mass and velocity of the moving body, but while kinetic energy is expressed by $\frac{1}{2}mv^2$ and measures the *work* done on the body in giving it motion, momentum, expressed by mv, measures the *impulse* given to it, or the product of the force by the *time* during which it was acting on the body, for the second law of motion (§94) gives the relation

$$Ft = mv - mu,$$

or change in momentum is equal to the impulse when the force is measured in the appropriate unit.

Hence if a force acts upon a body through a certain *distance* and it is required to find the change in velocity of the body, the formula for kinetic energy must be used,

$$Fs = \frac{1}{2}mv^2 - \frac{1}{2}mu^2;$$

while if the *time* of action of the force is given, the change in velocity is found from the equation of momentum,

$$Ft = mv - mu.$$

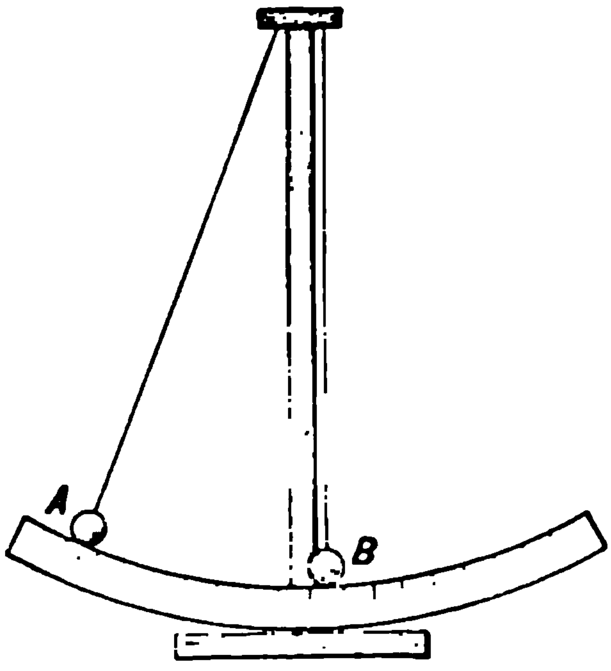
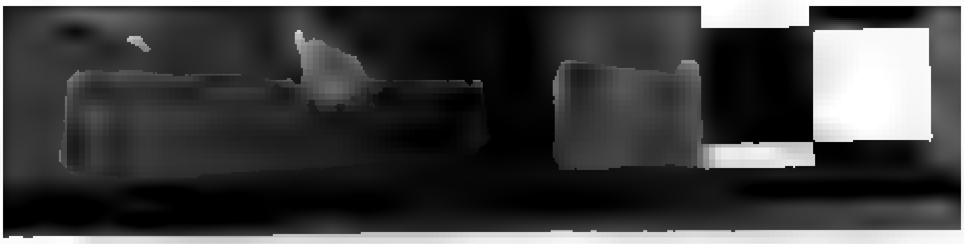


Fig. 50.

110.—Impact.—When one freely moving body strikes against another there is said to be *impact.*

When the ball B is at rest and A is allowed to swing against it, if the bodies are inelastic like two balls of lead or putty, they will keep together after impact, the forward momentum of the combined mass being equal to the momentum of A before impact. If the two balls are perfectly elastic or resilient and of equal masses, like two ivory billiard balls, A will come to rest giving up its whole momentum to B, which will, therefore, swing out just as far as A has fallen. If the masses are elastic but not equal, then A may continue forward or have its motion reversed at the instant of impact depending on whether B is the less or greater mass.

In all cases of impact, whether the masses are elastic or in-



elastic, the total momentum of the two bodies is not changed by the impact. That is, if one body *loses* forward momentum the other *gains* an exactly equal forward momentum.

Stated algebraically,

$$Av + Bu = AV + BU$$

where A and B are the two masses, respectively, while v and u are their velocities before impact, and V and U are their velocities after impact.

This law is easily seen to be a direct consequence of the laws of motion. For at each instant during impact the forward pressure of A upon B is equal to the backward pressure of B against A, as expressed in the statement that action and reaction are equal and opposite. Hence the total forward *impulse* given to B is equal to the backward impulse sustained by A, and by Newton's second law the change in the momentum of A must be equal and opposite to the change in the momentum of B, consequently the sum of the momenta of the two is not changed.

If the two are inelastic they move together after impact with a common velocity x, whence

$$Av + Bu = (A + B)x.$$

In case of elastic bodies there is a certain instant during the impact when the compression is a maximum and the two bodies are neither approaching nor receding from each other. At that instant they are moving with the same velocity x which they would have acquired if quite inelastic. But suppose they are perfectly resilient and the pressure between them at any instant as they spring apart is exactly equal to what it was during the corresponding instant of compression. The total backward impulse given to A will then be twice what it would have been if the bodies had been inelastic, hence the total change in velocity of A will be twice as great as $v - x$, or $2(v - x)$, and its final velocity V will be

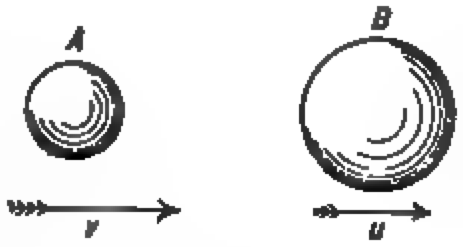


Fig. 51.

$$V = v - 2(v - x) = v + 2(x - v),$$

so also

$$U = u + 2(x - u).$$

If the above expressions are written

$$V = v + \mu(x - v)$$
$$U = u + \mu(x - u)$$

the coefficient μ will serve to indicate the degree of resiliency. If $\mu = 1$ the bodies are quite inelastic for $V = x$ and $U = x$, but if $\mu = 2$, the resiliency is perfect.

Problems

1. A ball is thrown vertically upward with a velocity of 64 ft. per sec.; how soon will it reach the ground again and how high will it rise, and what will be its velocity when half-way up?
2. A falling body has a velocity 200 cm./sec.; how far will it drop before its velocity becomes 10,000 cm./sec.? Take $g = 980$.
3. A weight thrown forward on ice with velocity 60 ft. per sec. is resisted by a constant force, and after 5 seconds has half its original velocity; how far has it gone in that time?
4. Find the acceleration in the previous problem, also how far the weight will go before coming to rest.
5. A mass of 10 gms. is acted on by a constant force which changes its velocity from 100 to 500 cm. per sec. in 5 seconds. Find the acceleration and amount of the force.
6. What steady forward pull must be exerted by a locomotive in starting a 200-ton train to give it a velocity of 20 miles per hour in 5 minutes, neglecting friction. Find force in poundals and then in pounds.
7. A weight of 10 lbs. is thrown forward on ice with a velocity 50 ft. per sec.; if the coefficient of friction between it and the ice is 0.10, how far will it go and in how many seconds will it stop.
8. A 300-lb. mass is lowered by a rope with uniform velocity. What is the tension on the rope? If it is lowered with a constant acceleration of 10 ft. per sec. per sec. what is the tension? What if it is lowered with acceleration g?
9. An elevator weighing 2000 lbs. is pulled upward with a force of 3000 lbs. What is its acceleration, and how long will it take to gain an upward velocity of 2 ft. per sec.?
10. A mine bucket weighing 2000 lbs. and being lowered with a velocity of 3 ft. per sec. is stopped in a distance of 1 ft. What is the average force on the supporting cable while stopping?
11. If a man weighing 75 kgms. is in an elevator which is going up with constant velocity, how much force does he exert on its floor? What if the elevator has an upward acceleration of 3 meter/sec.^{2}?
12. What is the least acceleration with which a man weighing 150 can slide down a fire-escape rope which can only sustain a weight of 100 lbs.? And what velocity will he have after sliding 50 ft.?

13. A 30-gm. weight is drawn up by a 70-gm. weight by means of a cord over a frictionless pulley. Find the acceleration (taking $g = 980$) and also the tension on the cord. How far will the weights move in 3 seconds from the start?

14. A 38-lb. weight resting on a level, frictionless table is drawn along by a 4-lb. weight by means of a cord over a frictionless pulley. Find the acceleration and also the tension on the cord.

15. If in the previous problem the friction between the weight and table is a force of 2 lbs. find acceleration and tension as before.

16. How many foot-poundals of work are required to give a 500-lb. shell a velocity of 2000 ft. per sec? Find the work also in foot-pounds. If this work is done by the powder gas in a gun 25 ft. long, find the average force in pounds against the shell as it is discharged.

17. How much energy in foot-pounds must be expended in giving a 300-ton train a velocity of 30 miles an hour? If the locomotive works at the rate of 100 H.P., how long will it take to bring the train up to speed?

18. A 3-kgm. hammer with a velocity of 5 meters per sec. drives a nail 4 cm. into a plank. Find the average resistance in dynes and grams and how much weight resting on the nail would be required to force it into the wood.

19. A bullet weighing 1 oz. and having a velocity of 1000 ft./sec. is fired through a plank 3 in. thick which resists it with a force of 800 lbs. With what velocity will it come out, and how many such planks could it pierce?

20. A bullet weighing 1 oz. is shot into a suspended block of wood weighing 18 lbs. 11 oz. and gives it a velocity of 6 ft. per sec. What is the combined momentum of block and bullet after impact? What was the momentum of the bullet before impact? Thence find velocity of bullet before impact.

21. What was the kinetic energy of the bullet in problem 20 before impact? What is the kinetic energy of the block containing bullet after impact? How much energy in foot-pounds was expended by the bullet in penetrating into the block? What proportional part of the original energy of the bullet remains as energy of motion after impact?

22. A bullet weighing 15 gms. is shot into a suspended block of wood weighing 2985 gms. and gives it a velocity of 200 cm. per sec.; find the velocity of the bullet.

23. How high above its original level will the suspended block, in the last question, swing in consequence of the velocity given to it?

24. If all the energy of a 640-lb. shell having a velocity of 2000 ft. sec. could be spent in raising a 10,000-ton battle ship, how high would it lift it?

25. A bullet weighing 10 gms. has a velocity of 600 meters per sec. and penetrates 30 cms. into a pine log. What is the force in kilograms with which the bullet is resisted, and how far would it penetrate if it had half the original velocity?

26. A monkey clings to one end of a rope passing over a frictionless pulley, and is balanced by an exactly equal weight on the other end of the rope.

Explain what will happen to the counterpoise if the monkey climbs 10 ft. up the rope and then suddenly stops. The mass of the rope and wheel are to be neglected.

27. A cord passes over two fixed pulleys and hangs down vertically between them supporting a movable pulley which with attached weight weighs 5 lbs. A 3-lb. weight is hung on one end of the cord and a 4-lb. weight on the other end. Find the accelerations of all three weights and the tension on the cord.

Note.—First find a simple relation between the accelerations of the three masses from the fact that the cord is inextensible.

Motion of a Particle in Curved Path

111. Motion of a Projectile.—When a body near the surface of the earth is thrown in any direction, such as AB, it is subject to the steady force of the earth's attraction vertically downward, and, therefore, it has constantly the downward acceleration of gravity g. The initial impulse, however, gave it a forward velocity V in the direction AB, in which direction it would have continued to move with constant velocity if no force had acted on it. The actual path in which it moves may then be regarded as the resultant of motion with constant velocity V in the direction AB combined with a motion downward with constant acceleration g. Thus after a time t the body will have traveled a distance $AC = Vt$ in the direction AB, but it will also have fallen from C to D a distance $s = \frac{1}{2}gt^2$.

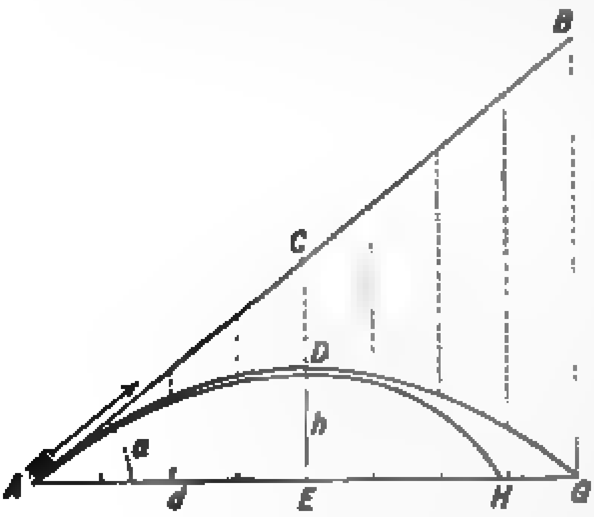


Fig. 52.—Curve of projectiles and jets.

If a is the angle of elevation of AB above the horizontal, and if d is the distance AE which the projectile has advanced in a horizontal direction and h is its height, we have

$$d = Vt \cos a$$
$$h = Vt \sin a - \frac{1}{2}gt^2.$$

The path traversed may be shown to be parabola with its axis vertical and passing through the highest point of the path. The highest point is half-way between the point of projection and the point G where the projectile again reaches the earth. The distance AG is called the range [illegible] maximum when the angle a is 45°.

These results are easily deduced from the above equations, but it must be borne in mind that the influence of air resistance has been neglected. This force in rapidly moving bodies, like bullets, may be very great and changes the form of the trajectory to something like that shown by the second curve from A to H. In consequence of this the maximum range in gunnery is found at a much smaller elevation.

The form of the path of a projectile or ball is beautifully shown by a water jet, for each particle in the jet is a freely falling body.

112. Curved Pitching.—If a ball when thrown forward is rapidly rotated the resistance of the air causes it to swerve from the path that it would otherwise take. This is seen in the curving of a pitched ball and in the drifting of projectiles from rifled guns. It results from the viscosity of air in consequence of which the rotating ball drags air in on one side and flings it out on the other as it advances.

Suppose, for example, that a ball is spinning about an axis perpendicular to the paper as shown by the curved arrow in figure 53, while it is moving forward in the direction of the straight arrow *cd*; the rotation of the ball drags air in from the side at *a* and carries it around toward the front of the ball at *c*, giving it a greater forward momentum than is given to the air between *c* and *b* where the surface of the ball is spinning backward.

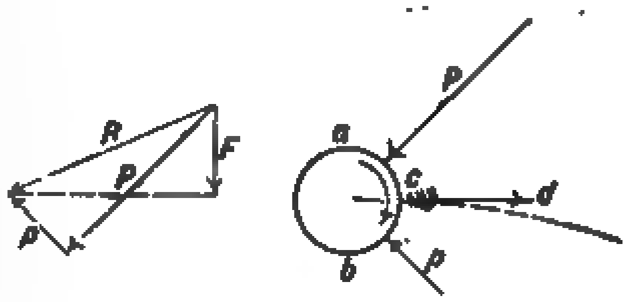


Fig. 53.—Curving of pitched ball.

The force against the ball is, therefore, greater between *a* and *c* than it is between *b* and *c*. Let P and p represent these pressures against the ball. Their resultant as shown in the diagram of forces is the oblique force R which in part resists the forward motion of the ball, but also has a component represented by F which is at right angles to the path of the ball and causes it to swerve to one side in the direction of the dotted line.

As the force F acts *constantly* it causes the ball to move sidewise with constantly *accelerated* motion, and, therefore, the curving rapidly increases as the ball advances.

113. Motion Around the Earth.—Suppose it were possible to shoot a cannon ball in a horizontal direction from the top of some high mountain on the earth with a velocity so great that

while it advanced a mile it would drop just enough to follow the curvature of the earth. Then, if there were no air resistance, the ball would continue around the earth and return to its original point of projection with undiminished velocity and would, therefore, continue to circulate forever around the earth as a satellite.

For suppose A (Fig. 54) is the point from which the projectile is shot in the direction AB. As it advances it drops away from the line AB; but the earth's surface also drops away from AB in consequence of its curvature, by about 8 in., in the first mile. If, therefore, the cannon ball has a velocity which will carry it a mile in the same time that it will drop 8 in., it will, on reaching the end of the mile, say at C, be just as high above the earth as at the start and be moving in the direction CD, tangent at C. As the ball moves forward the force due to the earth's attraction is always at right angles to the direction of motion, and hence the speed of the ball is neither increased nor diminished and all the conditions of the motion remain constant.

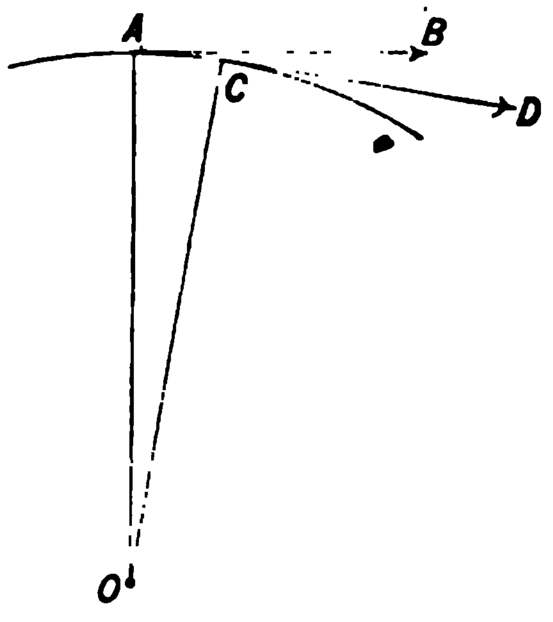


FIG. 54.

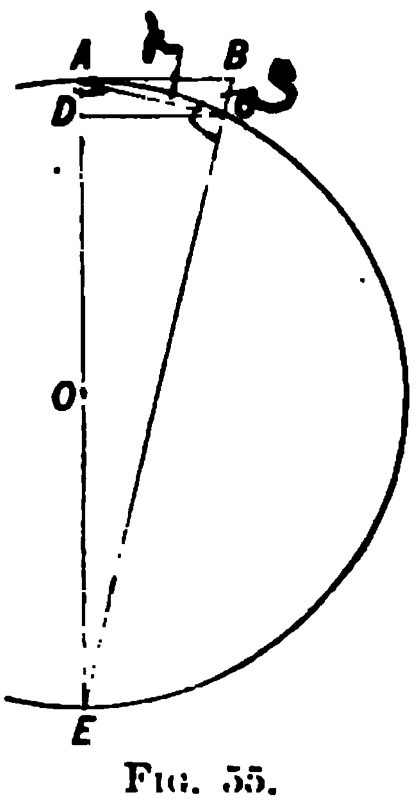


FIG. 55.

The time required for a body to drop 8 in. toward the earth is found from the formula (§101)

$$s = \tfrac{1}{2}gt^2.$$

Since $s = 8$ in, or 0.66 ft., and taking $g = 32$ ft./sec.², we find $t = 0.20$ sec.; hence our cannon ball must have a velocity of a mile in 0.2 sec. or 5 miles per second.

The complete calculation may be made thus. Let the ball drop a distance $BC = s$ (Fig. 55) in going forward the distance $AB = d$. If R is the radius of the earth the relation between s and d may be found from the similarity of the triangles ACE and ADC from which we find

$$AD : AC :: AC : AE$$

or

$$s : d :: d : 2R$$

and

$$s = \frac{d^2}{2R}, \tag{1}$$

but s is the distance fallen with constant acceleration g in t seconds, therefore

$$s = \tfrac{1}{2} g t^2,$$

and d is the distance which the ball, moving with constant velocity v, advances in t seconds; that is $d = vt$.

Substituting in (1) we have

$$\tfrac{1}{2} g t^2 = \frac{v^2 t^2}{2R}$$

whence

$$g = \frac{v^2}{R}.$$

Taking $g = 32$ ft./sec.^{2} and $R = 5280 \times 4000$ ft.,

$$v^2 = 32 \times 5280 \times 4000$$

and we obtain

$$v = 26{,}000;\ \text{ft./sec.} = 4.92 \text{ miles per sec.}$$

114. Motion in Any Circle with Constant Speed.—The case just discussed is in no way different from *any* case when a mass moves in a circle with constant speed. To cause it to constantly change its direction of motion there must be a force constantly acting at right angles to the direction of motion, and if the speed does not change there can be no force at all in the direction of motion. The relation between the velocity in the circle, the

radius of curvature of the path, and the acceleration are given as above in the formula $a = \frac{v^2}{r}$.

It will be interesting to derive this relation in another way, from the simple conception of acceleration as the change in velocity per second.

As a particle moves from A to B in the circle (Fig. 56) its velocity changes only in *direction* from v_1 to v_2. But this change in velocity is equivalent to compounding with the original velocity v_1, another velocity represented by the vector f. This vector therefore represents the change in velocity between A and B and when divided by t, the time taken by the body in moving from A to B, gives the *average rate of acceleration between* A and B or $a = \frac{f}{t}$.

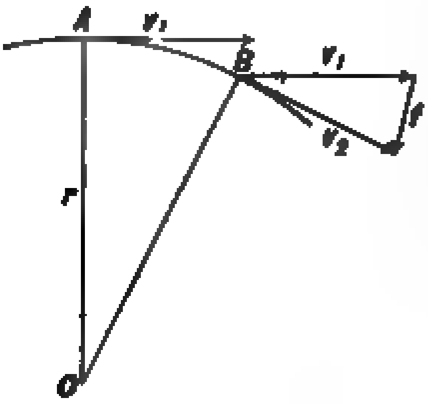


Fig. 56.

Let d represent the distance AB, and since the sector OAB is almost exactly similar to the triangle formed by v_1, v_2, and f we have the proportion $r : d :: v : f$ where v is the *amount* of v_1 or v_2. But the distance $d = vt$ and $a = \frac{f}{t}$ or $f = at$ hence substituting

$$r : vt :: v : at$$

from which

$$a = \frac{v^2}{r};$$

and this relation is *exact* and not approximate, for as B approaches A, the triangle and sector approach exact similarity as a limit and the *average* acceleration between A and B approaches the actual acceleration at A. It will be noticed also that f is parallel to a line bisecting the angle AOB, hence as B approaches A the direction of f approaches the direction AO as a limit. We conclude therefore that **the acceleration at any point of the circle is directed toward the center and is equal to $\frac{v^2}{r}$.**

115. Acceleration in any Curved Path.—Since
tion just found depends only on the instantaneo

various quantities involved, it applies to any curved path ıtever. The acceleration at any point in a curved path may 'esolved into two components, one along the curve or tangent t and the other at right angles to it. The component *along* curve is the rate of change of speed of the moving body, le the component at right angles to the path, is $a = \frac{v^2}{r}$ where the speed at the given point r is the radius of curvature of path at that point.

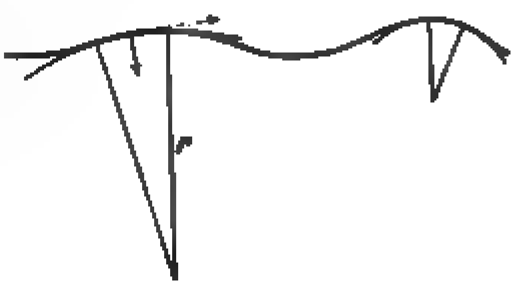


FIG. 57.

16. Force in Circular Motion.—enever a mass is accelerated it :ted on by a force determined by relation $F = ma$, hence when a ıs m moves in a circle with con-ıt velocity it is acted on by a force $F = m\frac{v^2}{r}$ directed toward center of the circle; or, more generally, whenever a mass is /ing in a curved path it is subject at any point to a force $m\frac{v^2}{r}$ directed toward the center of curvature, r being the radius of curvature of the path and v the velocity of the mass m at the given point.

FIG. 58.

117. Centripetal and Centrifugal Force.—The force by which a mass is constrained to move in a curved path is, as has just been shown, always directed toward the center of curvature of the path, and is therefore called the *centripetal* force. The reaction aginst this force by the moving body is called *centrifugal* force. Both are different aspects of the same stress, and are of course equal and opposite. For example, when a weight is rled in a circle by a cord, it is held in the circle by the tension he cord which supplies the *centripetal* force, while the reaction outward pull of the weight against the cord is called the *rifugal* force.

Vhen the string breaks *both* forces instantly disappear, *there is endency for the weight to fly outward*, it simply keeps moving he tangential direction in which it was moving when freed.

118. Other Expressions for Centripetal Force.—If a mass moves in a circle and makes n complete revolutions per second, since the distance traveled in one revolution is $2\pi r$, in n revolutions it will be $2\pi rn$ and hence $v = 2\pi rn$, and the centripetal force $F = \frac{mv^2}{r}$ becomes

$$F = m4\pi^2n^2r \tag{1}$$

Or we may express the velocity in terms of the time required to make one complete revolution, which may be called the *period* P. In that case

$$v = \frac{2\pi r}{P}$$

and

$$F = m\frac{4\pi^2r}{P^2} \tag{2}$$

Or if ω represents the angular velocity of the rotating body, or the arc in radians traversed per second, we have $\omega r = v$ (§130) and therefore

$$F = m\omega^2r \tag{3}$$

119. Illustrations.—When a railway train rounds a curve it is kept in the curve by the pressure of the rail against the flanges of the wheels. The weight of the train is balanced by the upward pressure of the track, represented at A, figure 59, while the centripetal force exerted by the rails is represented by B; the resultant force R is therefore inclined and the track is tilted toward the center so that the pressure may be equal on both rails.

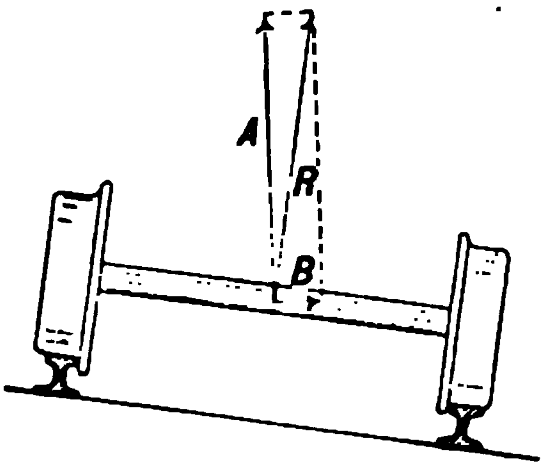


FIG. 59.

When a fly wheel rotates, it is under great tension due to the centrifugal force of the heavy rim, and great destruction may result from the bursting of such a wheel. A grindstone also may burst if driven at too high a speed.

A glass of water held in a sling may be swung in a vertical circle without spilling the water. For the acceleration downward due to gravity when it is at the top of the circle may not be enough to hold it in the circular path, consequently the bottom,

of the glass must exert an additional force upon the enclosed water, so that even at the top of its path the water presses against the bottom of the glass. In this case the tension on the cord when the mass is at the top of the circle is lessened by the weight of the body, while at the bottom the tension is increased by the same amount. The tension on the cord will therefore be

$$F = \frac{mv^2}{r} - mg \qquad \text{at the top.}$$

$$F = \frac{mv^2}{r} + mg \qquad \text{at the bottom.}$$

If a cylinder of wood having a length three or four times its diameter is suspended from the axle of a whirling machine by a short cord attached to one end, on setting it in rotation it will soon be disturbed from its initial position (Fig. 60) and will rotate in the oblique position as shown, in consequence of the centrifugal force of the two ends of the cylinder balancing the force toward the axis due to the cord and weight of the cylinder. With rapid rotation the cylinder comes into a horizontal position rotating now about an axis at right angles to its length.

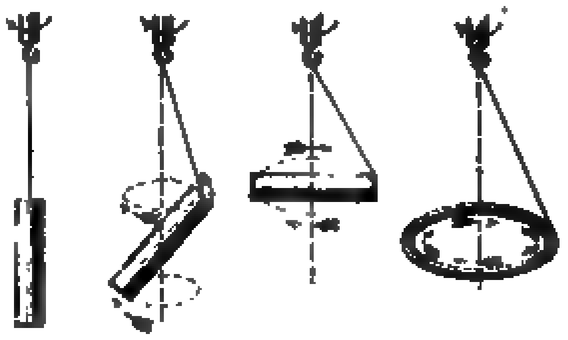


FIG. 60.

So a ring suspended by a cord from a rotating point of su port will tip up into the horizontal position and rotate like a wh about a vertical axis through its center. And even a chain w similarly suspended will first widen out into a loop in conseq of centrifugal force and then take the form of a horizonta rotating as though it were rigid; the required centripetal in these cases being supplied by the tension on the chain or In all these cases it will be observed that equilibrium is atta when the mass is on the whole as far as possible from the axis revolution.

120. Conical Pendulum.—Suppose a mass m, as in an old-fashioned steam engine governor, is swung aroung in a circle with uniform speed, it will swing out from the axis and come into equilibrium at a certain angle depending on the speed of rotation.

Evidently in order that there may be equilibrium the mass must be acted on by a force F directed toward the axis and just sufficient to hold it in the circle. But if the mass, or conical pendulum as it may be called, makes one revolution in P seconds, then the centripetal force is

$$F = \frac{m\,4\pi^2 r}{P^2},$$

and this force is the resultant of the tension on the cord T and the weight of the mass, which is mg dynes. Constructing the diagram of forces as in the figure, it is clear that $F : mg :: r : h$; therefore,

$$F = \frac{mgr}{h}.$$

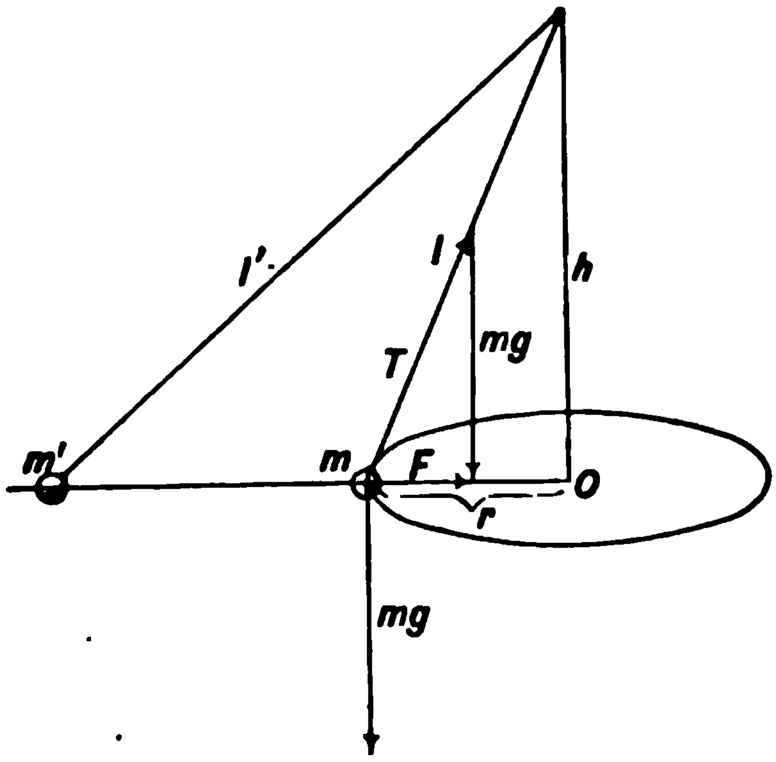


Fig. 61.

We have, therefore

$$\frac{mgr}{h} = \frac{m\,4\pi^2 r}{P^2}$$

whence

$$P = 2\pi\sqrt{\frac{h}{g}}.$$

Consequently if we have two masses m and m' hung by cords of different lengths, they will have the same period of tation if the height h is the same in both cases.

he more rapid the rotation the higher the mass rises, and in team-engine governor the rising of the weights operates a hrough which steam is cut off and the speed of the engine ied.

Problems

ream of water from a horizontal nozzle falls 3 ft. below the level ne nozzle in a distance of 20 ft., measured horizontally. Find the city of the escaping jet.

jet of water is directed upward at an angle of 45° to the vertical, and trikes the ground at a distance of 64 ft. from the nozzle. Find the time taken by a water particle in passing from nozzle to ground, and the velocity of the jet.

---

3. How much weight can a cord sustain by which a mass of 100 gms. can be whirled in a circle of 1 meter radius making 2 turns per s. neglecting the effect of gravity in the circular motion?

4. A stone weighing 1 lb. is whirled by a string in a circle 6 ft. in diameter. The string breaks and the stone flies off with a velocity of 30 ft. per sec. Find the strain on the string when it broke.
5. A mass of 50 gms. has a velocity of 750 cms. per sec. in a circle of radius 60 cms. Find the acceleration in amount and direction and centripetal force in dynes. Also find angular velocity.
6. A 100-ton locomotive rounds a curve at a uniform speed of 40 miles per hour. Find the acceleration if the radius of curvature of the track is 1000 ft. Also find the horizontal force exerted against the rails.
7. In case of the last problem, how much higher must the outer rail be than the inner, in order that the resultant force due both to the weight of the locomotive and its centrifugal force, may be perpendicular to the road bed?
8. A mass of 1 lb. is whirled in a circle of 2 ft. radius on a smooth level table, being held in the circle by a cord which passes without friction through a hole in the center of the table and supports a 2-lb. weight. Find the angular velocity and revolutions per sec. of the 1-lb. mass necessary to support the weight.
9. A 200-gram mass is whirled in a vertical circle of radius 80 cms. with a uniform angular velocity 8 radians per sec. Find the period of revolution and the acceleration. Also what is the tension on the cord in grams when the mass is at the top of the circle and what when it is at the bottom?
10. A weight of 2 lbs. is whirled in a vertical circle. If its velocity is 100 cms. per sec. at the top of the circle, what will be its velocity at the bottom, the gain being due to the acceleration of gravity as it falls, just as in an inclined plane (see §106)? Radius 80 cms.
11. A 10-lb. mass is hung as a pendulum by a cord 4 ft. long. How *high* must it swing in order that the tension on the cord at the lowest point of its swing may be double the tension when hanging at rest?
12. In case of "looping the loop," how high above the level of the top of the circle must the car start that it may just have speed enough to keep to the circle, neglecting friction? Circle 30 ft. in diameter.
13. Find the angular velocity and period of a conical pendulum hung by a cord 1 meter long and swinging around in a horizontal circle of 60 cms. radius.

Vibratory Motion

131. Simple Harmonic Motion.—If a body moving with constant speed in a circular path is observed from a distant point in the plane of the circle, it appears to oscillate back and forward in a straight line.

The kind of vibratory or oscillatory motion that the particle appears to have in this case is known as simple harmonic motion,

[redacted]

it may be defined as **the projection upon a straight line of uniform motion in a circle.**

There are other kinds of vibratory motion that are *not* simple harmonic, such, for example, as the particle would appear to have in the above instance if it moved around the circle in any manner whatever *except* with constant speed. Simple harmonic vibration is, therefore, one particular mode of oscillation; but it is by far the most important, for it is the most common of all, and all other modes of vibration may be expressed as the resultant of a sum of simple harmonic vibrations, as was shown by the French mathematician Fourier.

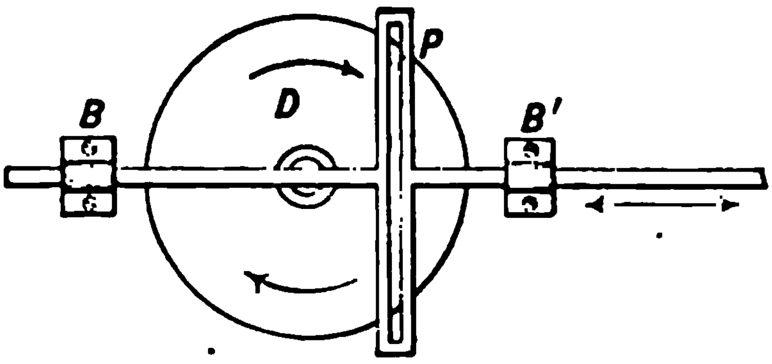


FIG. 62.

A simple mechanical device illustrating this kind of motion is shown in figure 62. A pin P projects from the face of a rotating disc D and fits in a slot in a cross head which is attached to rods that can slide back and forth in the bearings BB'. When the disc rotates with uniform speed every point in the rods and cross head will move back and forth with simple harmonic motion.

The *amplitude* of the vibration is the distance that the vibrating body moves on each side away from its central or mean position.

122. Velocity in Simple Harmonic Motion.—Let a particle A move around the circle (Fig. 63) with constant speed, and let another particle B move back and forth along a diameter DC in such a way that the line joining A and B is always perpendicular to DC. Then B oscillates with simple harmonic motion. Let v_0 represent the velocity of A. It may be resolved into two components, as shown in the diagram, one at right angles to the direction in which B moves and the other parallel with B's motion. Since B always keeps abreast of A, the velocity of B at any point must be equal to that component of A's velocity which is parallel to DC, namely to the

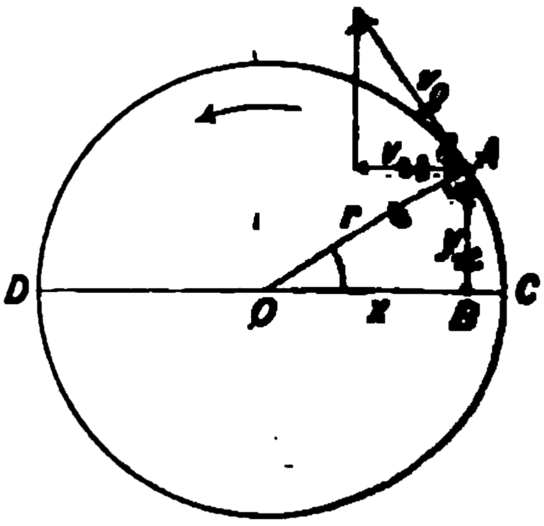


FIG. 63.

component v. Letting r represent the radius of the circle and y the distance AB, we have by similar triangles

$$r : y = v_o : v$$

whence

$$v = \frac{y}{r} v_o \quad \text{or} \quad v = v_o \sin e$$

where e is the angle AOC.

The velocity of B is, therefore, zero at the ends of its path at C and D, for there $y = 0$. While at the center $y = r$ and the velocity of B is equal to v_o, its maximum value.

The complete period of an oscillation of B is evidently the same as the time in which A goes completely around the circle. Let P represent this period, and the velocity of A is

$$v_o = \frac{2\pi r}{P},$$

which also expresses the velocity of B at its middle point.

123. Acceleration in Simple Harmonic Motion.—Since A and B have exactly the same motion in the direction DC, the acceleration of B must be the same as that component of the acceleration of A which is parallel to DC. The acceleration a_o of A, moving with uniform speed in a circle, is directed toward the center of the circle and is equal to $\frac{4\pi^2 r}{P^2}$ (§118).

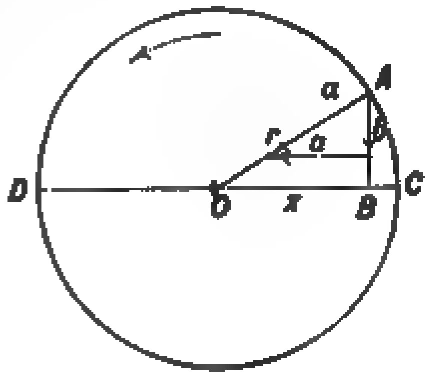


Fig. 64.

Resolving the acceleration a_o into two components and letting a represent the component parallel to DC and b that perpendicular to it, we have by similar triangles,

$$a : a_o = x : r$$

and, therefore,

$$a = \frac{a_o}{r} x$$

or since

$$a_o = \frac{4\pi^2}{P^2} r$$

$$a = \frac{4\pi^2}{P^2} x.$$

The acceleration of B is, therefore, proportional to its distance from the center, it is greatest when $x = r$ and is zero when B is at the center. It will also be noticed that the acceleration of B is always directed *toward* the center; that is, B is always losing velocity as it moves away from the center and gaining velocity as it moves toward the center, and consequently its velocity is greatest at the center as we have already seen.

124. Force in Simple Harmonic Motion.—The fundamental dynamical equation $F = ma$ enables us to express at once the force in simple harmonic motion. When the mass of the oscillating particle is m and its complete period of oscillation is P, the acceleration at the instant when the particle is a distance x from its central position has just been shown to be

$$a = \frac{4\pi^2 x}{P^2}.$$

The force at that instant is, therefore,

$$F = m\frac{4\pi^2 x}{P^2} \tag{1}$$

and is directed always toward the center, or equilibrium position.

Therefore when a mass in equilibrium is so situated that if displaced it is always urged back toward its equilibrium position by a force which is proportional to the displacement, it will, on being displaced and then set free, oscillate with simple harmonic motion about its position of equilibrium as a center.

Now the force required to cause a *small* strain in almost any elastic body is proportional to the amount that the body is strained, whether the body is bent or stretched or twisted (Hooke's Law, §237), hence when such bodies are strained and then let go they oscillate to and fro in simple harmonic motion, as in case of the small vibrations of a tuning fork.

125. Problem.—Let a mass of 1 kgm. be supported by a steel spring of such stiffness that an additional weight of 100 grams will stretch it just 1 cm. It is required to find the period of oscillation of the weight if disturbed, neglecting the mass of the spring.

If the kilogram weight is pushed up or pulled down as it hangs on the spring, it will move through a distance which is proportional to the force used, a force of 100 gms. being required to displace it 1 cm. To produce a

displacement of x cms. the force required is 100 x gms. or 100 xg dynes. But from equation (1) above we have, since $m = 1000$

$$F = 1000 \frac{4\pi^2 x}{P^2}$$

but

$$F = 100\ gx.$$

Therefore, $\quad 100\ g = 1000 \dfrac{4\pi^2}{P^2}$ and $P^2 = \dfrac{4000.\pi^2}{100.g}$
which gives $P = 0.634$ sec.

126. Simple Harmonic Motion Isochronous.—It will be noticed that the expression $P^2 = \dfrac{4\pi^2 xm}{F}$ does not contain r and is, therefore, independent of the amplitude of the vibration, so that it does not make any difference in the period of vibration whether the amplitude is large or small, provided the ratio $\dfrac{x}{F}$ is constant, in which case the motion is truly simple harmonic.

When vibrations have this property they are said to be *isochronous.*

127. Energy of a Vibrating Mass.—The energy of an oscillating mass is all potential at the ends of its vibration, but in the middle where the velocity is greatest it is all kinetic and so may readily be computed. For we have seen, §122, that the maximum velocity of the vibrating body is $v_0 = \dfrac{2\pi r}{P}$, and since kinetic energy $= \frac{1}{2}mV^2$ we find, kinetic energy at middle or total energy $= \dfrac{2m\pi^2 r^2}{P^2}$ where m is the mass of the vibrating particle, P is its period of vibration, and r is the amplitude of its motion.

128. Simple Pendulum.—A mass suspended from a fixed point so that it can swing freely in a circular arc about the fixed point as a center, is called a pendulum. As a simple or ideal case we may suppose the whole mass of the pendulum to be concentrated at the point B, the mass of the suspending cord or wire being so small as to be neglected. The forces acting on the mass m are its weight mg and the tension T of the suspending cord. The weight mg may be resolved into two components, one in line with the cord and opposing its tension and one at right angles to the suspending cord and in the direction in which the mass m moves. It is this latter component F (Fig. 65) which gives it motion along

the circle. Since the diagram of forces is a triangle similar to BCO we have

$$F : mg = BC : BO;$$

but

$$BO = l,$$

and if the angle a through which the pendulum swings to and fro is small, BC is very nearly indeed equal to the arc BA, the length of which may be represented by x.

Then *approximately*

$$F : mg = x : l$$

and

$$F = \frac{mgx}{l} \qquad . \qquad . \qquad (1)$$

Therefore the force F urging m along the arc toward A is proportional to the displacement x measured along the arc. But with such a law of force there is simple harmonic vibration (§124) and the relation of force to period of vibration is expressed in the formula

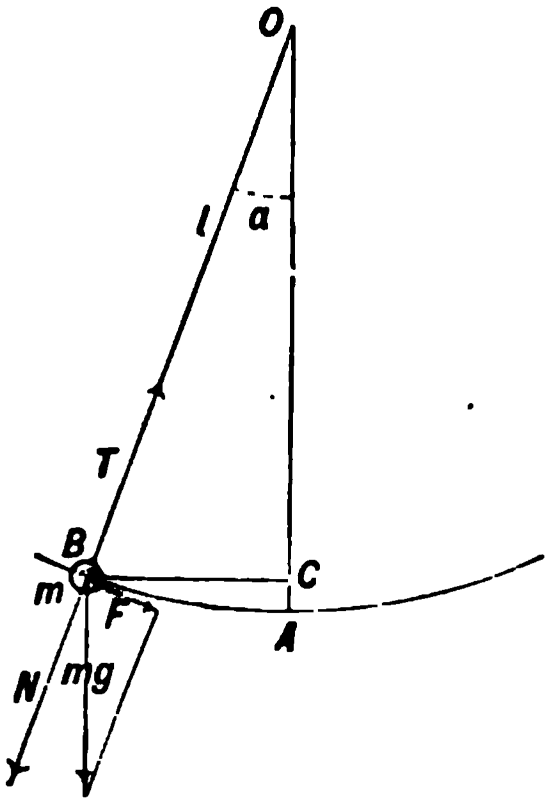


Fig. 65.

$$F = m\frac{4\pi^2 x}{P^2} \qquad (2)$$

Equating (1) and (2) we have

$$\frac{g}{l} = \frac{4\pi^2}{P^2}$$

from which

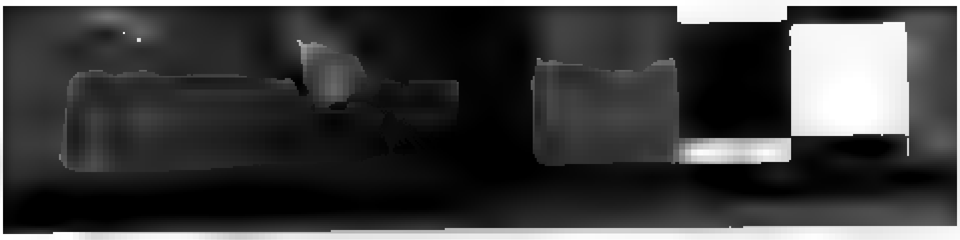
$$P = 2\pi\sqrt{\frac{l}{g}}$$

The period of vibration therefore depends only on the length of the pendulum and the acceleration of gravity at the place where it is swung, and is independent of its mass and of the length of the arc, provided the arc is so small that the approximation made above is justified.

The effect of the length of arc upon the period is shown by the following more exact formula,

$$P = 2\pi\sqrt{\frac{l}{g}}\left(1 + \frac{a^2}{16}\right),$$

where a is the arc AB measured in radians. Thus if a pendulum 1 meter



ong swings 5 cm. on each side of the lowest point, the arc $a = \frac{1}{20}$ and $\frac{a^2}{16} = \frac{1}{6400}$, so that the period is greater by one part in 6400 than if the arc had een infinitely small.

129. Pendulum Clocks.—The pendulum affords a valuable neans of regulating the motion of a clock, since when it swings hrough a small arc its oscillations are nearly *isochronous; i.e.*, ts period of oscillation is nearly independent of the amplitude of its swing.

When an ordinary clock driven by a spring is just wound up t gives a greater impulse to the pendulum through the escapenent than when it is nearly run down, and even in clocks driven oy weights the friction is not always constant and the swing of he pendulum will vary accordingly.

It must be remembered also that the pendulum of a clock is not *free*, but the little backward and forward impulses which it eceives from the escapement hasten somewhat its motion. To ecure regularity of motion, therefore, the pendulum should be heavy, so that its natural period will be only slightly affected by the pushes of the escapement.

In the finest astromonical clocks what is known as a *gravity escapement* s used, in which the pendulum does not receive any impulse directly from the spring or weight that drives the clock, but its motion is kept up by a mall weighted lever which is set free just as the pendulum reaches the end of its swing, and in falling gives a slight push to the latter.

Between successive impulses the lever is raised and set in position by the action of the clockwork.

Full details as to some forms of gravity escapement will be found in the article *Clocks* in the Encyclopedia Britannica.

Problems

1. Show that the motion of the piston of a steam engine when the crank is turning with uniform velocity is not simple harmonic. At which end of the piston's motion is the acceleration greatest and why?
2. Assuming that the motion of the piston is simple harmonic, find its velocity in the middle of its stroke when the crank is 8 in. long and makes 200 revolutions per minute. Also find acceleration at middle and end of its stroke.
3. If the piston and connecting rod weigh 100 lbs. in the last problem, find the maximum force against the crank pin due to their inertia alone, neglecting the effect of steam pressure.

4. A mass of 4 lbs. is made to oscillate to and fro by a spring at the rate of 2 vibrations per sec. Find the force on the mass when it is 2 in. from its middle position.
5. A pendulum 1 meter long swings 10 cms. on each side of its lowest point; find the direction and amount of the acceleration at the ends of its swing and at middle.
6. How long must a pendulum be to beat seconds at a place where $g = 980$. If made 1 mm. too long will it gain or lose and how much per day?
7. A clock having a pendulum which beats seconds where g is 980, is taken to another place where $g = 981$; will it gain or lose, and how much in one day?
8. Each prong of a tuning fork making 100 complete vibrations per second vibrates to and fro through a distance of 1.5 mm. Find the velocity of the prong in the middle of its swing.
9. A 400-gm. weight when hung on a long and light helical spring stretches it 30 cms. What will be its period of oscillation if drawn down a little and then set free? Take $g = 980$ and neglect mass of spring.

Ans. 1.099 sec.

IV. ROTATION OF RIGID BODIES

Motion of a Rigid Body

130. Translation and Rotation.—If a rigid body moves in such a way that any straight line joining two points in the body remains parallel to itself as it moves along, the motion is said to be a *translation* without rotation. A book slid about on a table keeping one edge always parallel to one edge of the table is a case of pure translation. If the edge of the book changes its direction there is said to be *rotation.*

Any motion of a rigid body may be considered as made up of the motion of its center of mass combined with rotation about an axis through that center.

Motion of the Center of Mass.—It may be proved that when any *external* forces act on a rigid body the center of mass of the body moves just as though the whole mass of the body were concentrated at that point and all the forces were applied directly to it, and it makes no difference at what points on the body the forces may be applied.

When a top spins on a smooth frictionless table its center of gravity remains at rest, for the external forces acting on the top are its *weight* due to the attraction between it and the [illegible]

the upward *pressure of the table on its point.* These two forces are equal and opposite and consequently the center of gravity has no translational acceleration even when the top is inclined as in figure 76 (p. 102).

When a stick of wood is hurled through the air its center of mass moves in a simple parabolic curve (§111) just as a particle would move, except as affected by air resistance.

Besides this translational force which depends only upon the amounts and directions of the several forces and so is found by the simple force polygon, *there is usually also a couple which causes the body to rotate about an axis through its center of mass.* This couple depends not only on the amounts and directions of the forces, but also upon their points of application to the body.

131. Angular Velocity.—When any line in a body is at rest while other points in the body move in circles about that fixed line or *axis*, the motion is called *rotation.* In case of a rigid body, like a wheel, all parts whether near the axis or far from it must rotate through equal angles in the same time. The rate at which the body is turning at any instant, measured in *radians per second,* is known as its *angular velocity* and is represented by ω (the Greek letter *omega*).

Since the length of a radian of arc at a distance r from the axis is equal to r, we have

$$v = \omega r$$

where v is the linear velocity of a particle at a distance r from the axis.

Example.—If a wheel of radius 15 cms. is making 3 revolutions per sec., its angular velocity is $3 \times 2\pi$ radians per sec., and the linear velocity of a point on the rim is $3 \times 2\pi \times 15$ cms. per sec.

132. Angular Acceleration.—When the angular velocity of a body is changing, the rate of change per second is known as its *angular acceleration,* and may be represented by A. If the angular velocity ω_1 changes to ω_2 in t seconds, then

$$A = \frac{\omega_2 - \omega_1}{t}$$

or

$$\omega_2 = \omega_1 + At$$

where A is the average rate of acceleration during the time t.

The direction of the axis of rotation may change, and this also constitutes an *angular acceleration*, even though the speed of rotation about the axis may remain constant. This is illustrated by the motion of a spinning top when its axis is inclined, for the axis swings around in a circle keeping a constant inclination to the vertical.

133. Vector Representation of Angular Velocity.—The angular velocity of a body may be represented by a vector or arrow drawn along the axis of rotation and having a length proportional to the amount of the angular velocity, and pointing in the direction that a person must look along the axis to see the body rotating in a clockwise direction. For example, if the rotating disc shown in figure 66 has an angular velocity 10, it will be represented by a vector 10 units long drawn in the direction of the arrow.

134. Change in Direction of Angular Velocity.—If the axis of rotation is changing in direction the angular velocity at one

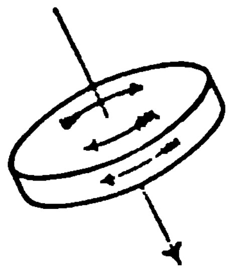


Fig. 66.

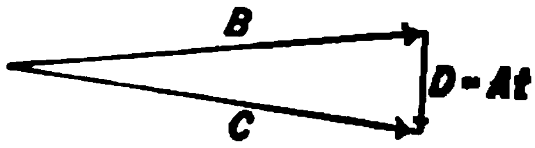


Fig. 67.

instant might be represented by the vector B and a short time later by C (Fig. 67). The change in angular velocity would then be represented by the vector D, for this combined with B gives C according to the composition of vectors. If t is the time during which the change has taken place, then $D = At$ where A is the *angular acceleration*. An angular acceleration of this character is found in the motion of a top. (§147.)

135. Rotation with Constant Acceleration.—The equations for rotation with constant acceleration are exactly analogous to those for simple translation (§98), as may be seen thus:

Translation in Straight Line	*Rotation about a Fixed Axis*
s = displacement in time t.	α = angle through which body turns in time t.
v_1 = velocity at *beginning of* interval t.	ω_1 = angular velocity at *beginning* of interval t.

v_2 = velocity at *end* of interval t. | ω_2 = angular velocity at *end* of interval t.

a = acceleration. | A = angular acceleration.

$$a = \frac{v_2 - v_1}{t}. \qquad A = \frac{\omega_2 - \omega_1}{t}. \quad (1)$$

$$s = \left(\frac{v_2 + v_1}{2}\right)t \text{ or } s = v_1 t + \frac{at^2}{2}. \qquad \alpha = \left(\frac{\omega_2 + \omega_1}{2}\right)t \text{ or } \alpha = \omega_1 t + \frac{A}{2}t^2 \quad (2)$$

$$2as = v_2^2 - v_1^2. \qquad 2A\alpha = \omega_2^2 - \omega_1^2 \quad (3)$$

Problems

1. If a wheel revolves 1800 times per minute, what is its angular velocity; and if it is 6 in. in diameter what is the linear velocity of a point on its periphery?
2. What is the linear velocity of a point 1 ft. from the axis of a wheel making 2.5 turns per sec.? Also the velocity of a point 1.4 ft. from axis? What is the angular velocity of each?
3. Find angular velocity of a wheel in which a point 6 in. from the axis has a velocity of 4 ft. per sec.
4. A locomotive rounds a curve having a radius of 800 ft. at 15 miles per hour; what is its angular velocity?
5. A wheel is given a speed of 100 revolutions per min. in 2 minutes; what is its angular acceleration in radians per sec. per sec.?
6. How many revolutions will a fly wheel make in 20 seconds, while its angular velocity is changing from 3 to 10 radians per sec., if the acceleration is constant?
7. A body rotates about an axis with constant angular acceleration 8 radians per sec. per sec.; how many turns will it have made in 10 seconds from the start?
8. How many revolutions will a body make starting from rest with angular acceleration 4 radians per sec. per sec. before it will be revolving at the rate of 20 turns per sec.?

Kinetics of Rotation about a Fixed Axis

136. Angular Acceleration Caused by Torque.—Suppose the bar shown in figure 68 is acted on by a force F at a distance d from the axis; it is required to find how rapidly the speed of rotation of the bar about the axis will increase in consequence of the moment of force, or torque Fd.

Imagine the bar divided into little masses m_1, m_2, m_3, etc., and suppose the effect of the force F is to cause an *angular acceleration* A in the rotation of the bar; that is, its angular velocity is increased at the rate of A radians per sec. per sec. The *linear acceleration* of the mass m_1 at distance r_1 from the axis will then

be r_1A, and consequently the force acting on m_1 must be m_1r_1A and may be represented by f_1. This force f_1 is due to F and is transmitted to m_1 by the rigidity of the bar. So also m_2 must be acted on by a force $f_2 = m_2r_2A$ since it has the acceleration r_2A. And similarly every one of the masses m_1, m_2, m_3, etc., into which the bar is divided is acted on by the force needed to give it its acceleration, as indicated by the small arrows in the figure.

Now, if a force equal and opposite to f_1 is applied to m_1, and a force equal and opposite to f_2 is applied to m_2, and so on, applying to each of the little masses a force just such as to counteract its acceleration, it is clear that there will be no acceleration and the bar will be in equilibrium. That is, a system of forces equal and opposite to f_1, f_2, etc., will just balance the turning moment of the force F about the axis O. Consequently the sum of the moments of f_1, f_2, etc., about O must be equal to the moment of F about that axis. Thus,

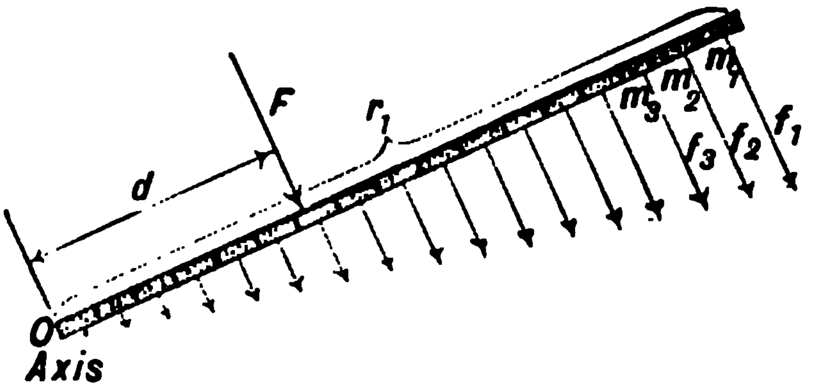


Fig. 68.

$$Fd = f_1r_1 + f_2r_2 + f_3r_3 +, \text{etc.}$$

But it has been shown that

$$f_1 = m_1r_1A, \quad f_2 = m_2r_2A, \quad \text{etc.}$$

Therefore

$$Fd = m_1r_1^2A + m_2r_2^2A + m_3r_3^2A +, \text{etc.},$$

or

$$Fd = A\,(m_1r_1^2 + m_2r_2^2 + m_3r_3^2 +, \text{etc.}).$$

The quantity in the parenthesis, which depends only on the mass of the body and its distribution with reference to the given axis is called the **moment of inertia** of the body about that axis, and may be represented by the symbol I.

The *torque* or sum of the moments of whatever forces may be acting to rotate the body around the given axis may be represented by L, and we have then,

$$L = IA \quad \text{or} \quad A = \frac{L}{I} \tag{1}$$

That is, **the angular acceleration caused by a given torque is**

equal to the torque divided by the moments of inertia of the body about the given axis.

Notice the analogy to the formula $F = ma$, moment of force or torque corresponds to force, moment of inertia corresponds to mass, and angular acceleration corresponds to linear acceleration.

The effect of torque in causing angular acceleration may be illustrated by the apparatus shown in the figure. A light bar carrying two masses M and M' is mounted on a horizontal axis perpendicular to the bar and is set in motion by a weight W hung from a cord wrapped around a drum on the axis. When the masses M and M' are in the position shown, the bar gains angular velocity slowly, for the farther the masses are from the axis, the greater the moment of inertia of the rotating system. When the masses are close to the axis the moment of inertia is smaller and the bar gains angular velocity very much more rapidly than before.

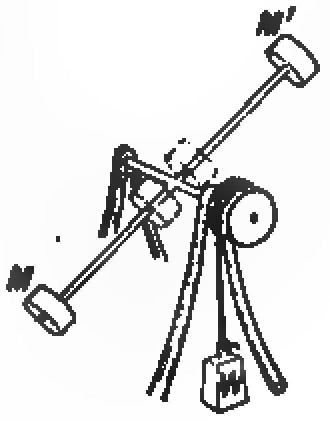


FIG. 69.

The calculation of moments of inertia will be discussed in paragraphs 139 to 141.

137. Angular Momentum.—The formula of the last paragraph

$$L = IA$$

may be put in the form

$$L = I\frac{\omega_2 - \omega_1}{t} \qquad \text{see (§135)}$$

or

$$Lt = I\omega_2 - I\omega_1 \qquad (1)$$

which is exactly analogous to

$$Ft = mv_2 - mv_1 \qquad (§94)$$

The product of the moment of inertia by the angular velocity about an axis is known as the **angular momentum** of the rotating body about that axis, and equation (1) above, states that the **change in the angular momentum of a body about any axis is equal to the moment of force or torque about that axis multiplied by the time during which it acts.**

When the axis of torque is *perpendicular to the axis of rotation* of the body its only effect is to change the *direction* of the axis

of rotation, but the *amount* of the angular momentum remains unchanged. This is illustrated by the top (§147).

138. Kinetic Energy of a Rotating Body.—When all parts of a body have the same velocity the kinetic energy of the body as we have already seen is $\frac{1}{2}Mv^2$ where M is the mass of the body and v its velocity. But in case of a rotating rigid body the velocity of any part depends on its distance from the axis. In this case we may imagine the whole mass to be divided into small portions, and calculate the kinetic energy of each of these portions separately and then add them together to find the total energy of rotation.

The body represented in figure 70 is supposed to rotate about an axis perpendicular to the paper. Imagine the whole body cut up into little rods parallel to the axis whose ends are seen as the reticulation in the diagram. Let the mass of one of these rods be m, its distance from the axis r, and its velocity due to the rotation of the body v. Then its kinetic energy is $\frac{1}{2}mv^2$.

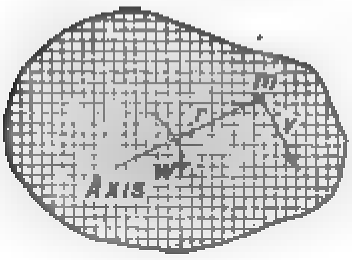


Fig. 70.

But if ω is the *angular velocity* of the body, ωr will be the linear velocity of a mass at a distance r from the axis.

Thus,

$$\omega r = v \quad \text{and} \quad \tfrac{1}{2}mv^2 = \tfrac{1}{2}m\omega^2r^2.$$

Now, let m_1 represent the mass of another of the rods into which the body has been imagined divided and r_1 its distance from the axis, then its kinetic energy is $\frac{1}{2}m_1\omega^2r_1^2$, and so the total kinetic energy of the body is

$$K.\,E. = \tfrac{1}{2}m\omega^2r^2 + \tfrac{1}{2}m_1\omega^2r_1^2 +, \text{etc.},$$

there being one term for each part into which the body is conceived to be divided. Or we may write

$$K.\,E. = \tfrac{1}{2}\omega^2(mr^2 + m_1r_1^2 +, \text{etc.}),$$

since the angular velocity of every part of the body is the same. But the quantity in parenthesis is the moment of inertia I of the bar about the axis, therefore

$$K.\,E. = \tfrac{1}{2}I\omega^2.$$

Notice again the analogy between this expression and the formula for kinetic energy of translation $\frac{1}{2}Mv^2$.

Moment of inertia corresponds to mass.
Angular velocity corresponds to linear velocity.

139. Moment of Inertia of a Rod.—The method of computing moments of inertia may be illustrated by the case of a straight uniform rod with the axis at one end. Let l be the length and M the mass of the rod. Conceive it to be divided into n equal parts, each part having a mass m. Then the length of each part will be $\frac{l}{n}$, and if the distance of any part from the axis is taken as the distance of its farther end, the distances of the successive parts are $\frac{l}{n}$, $\frac{2l}{n}$, $\frac{3l}{n}$, etc., and the moment of inertia is, therefore,

$$I = m\frac{l^2}{n^2} + m\frac{2^2l^2}{n^2} + m\frac{3^2l^2}{n^2} + \text{etc., or } I = \frac{ml^2}{n^2}(1^2 + 2^2 + 3^2 + \ldots n^2).$$

Now, it may be shown that the larger n is taken the more closely does the sum in the parenthesis approach the value $\frac{n^3}{3}$, and accordingly if the rod is supposed to be divided into an infinite number of parts,

$$I = \frac{ml^2}{n^2}\cdot\frac{n^3}{3} = \frac{Ml^2}{3} \text{ since } mn = M.$$

The moment of inertia of the bar is, therefore, the same as though its mass were concentrated at a distance k from the axis, where $k^2 = \frac{l^2}{3}$.

The distance k is known as the *radius of gyration* of the rod about the given axis.

140. Formulas for Moment of Inertia.—In case of bodies of simple figure and having the mass uniformly distributed throughout the volume the moments of inertia may be calculated by the methods of calculus. But in more complicated cases they must be determined by experiment.

The following formulas are given for reference:

Thin rod, of mass M and length l, having a transverse axis at one end

$$I = \frac{Ml^2}{3}.$$

Thin rod, of length l, having a transverse axis through the center,

$$I = \frac{Ml^2}{12}.$$

Rectangular block, of width a and length b and of any thickness whatever, about an axis through the center perpendicular to a and b,

$$I = M\left(\frac{a^2 + b^2}{12}\right).$$

Circular disc or cylinder, of any length and of radius r, about an axis through the center and perpendicular to the circular section of the disc or cylinder,

$$I = \frac{Mr^2}{2}.$$

Circular cylinder, of length l and radius r, about a transverse axis through its center perpendicular to its length,

$$I = M\left(\frac{r^2}{4} + \frac{l^2}{12}\right).$$

Sphere, of radius r about an axis through its center,

$$I = M\frac{2r^2}{5}.$$

141. Moment of Inertia about a Parallel Axis.—If the moment of inertia of a body is known about an axis through its center of mass, it may readily be calculated about any parallel axis. For if I_0 is the moment of inertia about the axis through its center of mass and if M is the mass of the body, then the moment of inertia about a parallel axis at a distance h from the center of mass of the body is $I = I_0 + Mh^2$; that is, the moment of inertia I about any axis is equal to the moment of inertia which the whole mass would have about that axis if it were concentrated at the center of mass of the body, added to the moment of inertia of the body about the parallel axis through its center of mass.

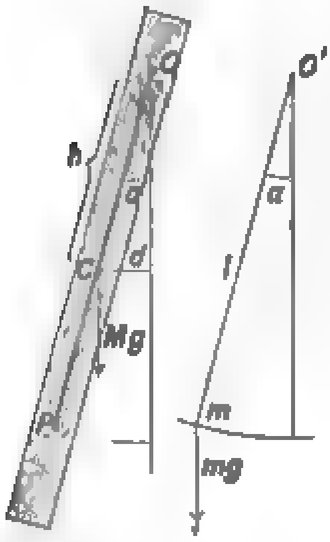


FIG. 71.

142. The Compound Pendulum.—In discussing the *simple pendulum* it was assumed that the oscillating mass was so small that it might be consdered as concentrated at a point, and the mass of the suspending system was entirely neglected.

A pendulum which has distributed mass and so does not satisfy either of the above simple conditions is said to be a *compound* or *physical pendulum*. All actual pendulums belong to this class.

Let it be required to find the length of a simple pendulum having the same period of oscillation as a given physical pendulum. Suppose the pendulum to have mass M and let its axis of suspension O be a distance h above its center of gravity C (Fig. 71). Then, when a line joining O and C makes an angle a with the vertical, the pendulum may be considered as acted upon by a force Mg acting downward through its center of gravity and producing a moment of force about the axis O equal to Mgd or $Mgh \sin a$. If I is the moment of inertia of the pendulum about O, we have by equation (1) §126,

$$Mgh \sin a = IA$$

therefore,

$$A = \frac{Mgh \sin a}{I}.$$

But in case of a simple pendulum of length l the moment of the force mg about the axis O' is $mgl \sin a$ and the moment of inertia of m about O' is ml^2; therefore, $mgl \sin a = ml^2A'$ and the angular acceleration is,

$$A' = \frac{g \sin a}{l}.$$

If the two pendulums are to have the same period of vibration their angular accelerations A and A' must be equal when both pendulums make equal angles with the vertical; that is,

$$\frac{Mgh \sin a}{I} = \frac{g}{l} \sin a$$

and, therefore,

$$l = \frac{I}{Mh}.$$

The length of the equivalent simple pendulum calculated from the above formula will always be greater than h, since the moment of inertia I of the pendulum is always greater than if the whole mass were concentrated at its center of gravity (see §141); that is, I is greater than Mh^2 and, consequently, l is greater than h.

The point P in line with O and C and at a distance l from O is called *the center of oscillation*. Each portion of the mass of the pendulum between P and O is constrained to swing slower than it would if it were free to oscillate by itself about O as a center, while all portions of the pendulum below P have to swing more quickly than if they were free. The mass between P and O, therefore, tends to quicken the motion of the pendulum while the mass below P tends to retard it, while the mass situated at P is neither hastened nor retarded, but swings exactly as it would if freely suspended from O.

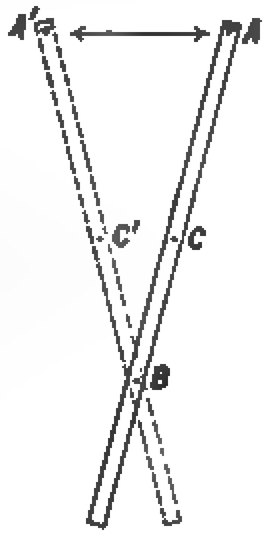


Fig. 72.

143. Center of Percussion.—If a rod or pendulum is suspended from an axis A (Fig. 72) and if that axis is given a sudden sidewise impulse or if it is moved rapidly back and forth from side to side, the inertia of the rod will cause it to move as though a certain point B was fixed and the rod turned about that point as axis.

This instantaneous center of the motion is not the center of gravity C, but is the center of oscillation corresponding to the axis of suspension at A. A marble placed on a little shelf at B is scarcely disturbed by the sudden to-and-fro movements of the axis A, while at any other point it would be instantly thrown off.

On the other hand, when the pendulum suspended from the axis A is hanging at rest, if a sudden sidewise impulse is given to the bar at B, as when it is struck a blow at that point, no sidewise impulse is communicated to A in consequence, but the bar simply tends to turn about A as an axis. For this reason the point B is also called the *center of percussion* corresponding to the axis A.

In case of a baseball bat the blow is given to the ball with the least jar to the hands when the ball is struck at the center of percussion of the bat corresponding to an axis at the point where it is grasped.

Problems

1. A cylinder weighing 30 kgms. and having a diameter of 1 meter is mounted on an axis and set rotating by a pull of 2 kgms. on a cord wound on an axle 10 cms. in radius. Find the acceleration produced and the speed of rotation 3 sec. from the time of starting. The moment of inertia of a cylinder about its axis is $M\frac{r^2}{2}$ (§140) or

$$I = \frac{30 \times 1000 \times 50^2}{2} = 37{,}500{,}000 \text{ grm. cm.}^2$$

The force acting is 2 kgms. or 2000 gms. or 2000×980 dynes and the moment of the force is $2000 \times 980 \times 10$ dyne-cm.
Substitute in the formula $L = IA$

$$2000 \times 980 \times 10 = 37{,}500{,}000A \quad \therefore \quad A = 0.523.$$

Hence the system will gain in 1 second an angular velocity of a little more than half a radian per sec.
In 3 seconds it will acquire an angular velocity $\omega = 3A = 1.569$; that, is, it will be turning at the rate of about 1 revolution in 4 seconds, since $\omega = \frac{2\pi}{P}$ where P is the period of revolution.

2. What is the kinetic energy of a wheel which has a moment of inertia 20 lb. ft.^{2} and is rotating at the rate of two turns per sec.?
3. If a 5-lb. weight is raised by means of a rope wound on the axle of the wheel in problem 2, how high will it be raised before the wheel comes to rest?
4. A uniform rod 40 cms. long and weighing 200 gms. can rotate about a transverse axis through its middle point. How many ergs of work will be required to make it revolve at the rate of three turns per sec.?
5. Suppose the rod in question 4 is set in rotation by means of a 200-gm. weight attached to a cord wrapped around a cylindrical axle 4 cms. in diameter. How far will the weight have descended in giving a speed of rotation of 3 revolutions per sec.
Note.—First solve neglecting the kinetic energy acquired by the 200-gm. weight as it sinks. Then obtain the more exact solution, taking account of this energy.
6. The fly wheel of an engine weighs 1200 lbs., the bulk of the weight being in the rim of the wheel at a distance of about 3 ft. from the axis. What is approximately its moment of inertia and how many ft.-lbs. of work must be done by the engine to set it rotating 3 times per sec.?
7. How much energy will be given out by the fly wheel in problem 6 in slowing down from 3 to 2.5 revolutions per sec.

8. A uniform bar 3 ft. long swings as a pendulum about an axis at one end. Show that the equivalent simple pendulum is 2 ft. long.
9. A uniform spherical steel ball 6 cms. in diameter is hung as a pendulum by a steel wire so that the center of the ball is just 100 cms. below the axis of suspension. Find how far the center of oscillation is below the center of the ball and what is the length of the equivalent simple pendulum, neglecting the mass of the suspending wire.
10. A rectangular bar of steel $1 \times 1 \times 12$ cm. and weighing 90 gms., when suspended in a horizontal position by a wire attached to its middle point, is set oscillating about a vertical axis through its center and makes 4 complete vibrations in 10 sec. Find the moment of force or torque due to the twist in the wire when the bar is at right angles to its equilibrium position.
11. Find the period of oscillation of a solid metal sphere 6 cms. in diameter and weighing 800 gms. when hung by the same wire as the bar in problem 10 and set oscillating about a vertical axis through its center.

Some Cases of Motion with Partly Free Axis

144. Foucault's Pendulum Experiment.—It occurred to the French physicist Foucault that since a pendulum undisturbed by external forces must persist in its original direction of vibration, if one were swung at the north pole by some suspension which could not transmit torsion, its direction of vibration would remain constant while the earth turned around under it, so that to an observer moving with the earth the pendulum would seem to change its direction of vibration at the rate of 15° per hour.

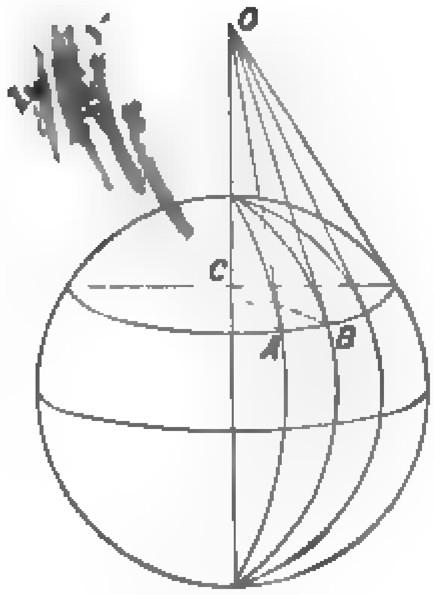


Fig. 73.

At the equator the direction of the meridian remains parallel to itself as the earth rotates, and consequently the plane of vibration of the pendulum would remain unchanged.

At any intermediate latitude the tangents to the meridians at two points differing in longitude by 15°, such as A and B (Fig. 73), will meet the axis at O, and the angle AOB measures the change in direction of the meridian per hour. Consequently a Foucault pendulum in that latitude will shift in one hour through an angle equal to AOB. This interesting experiment was car-

ried out by Foucault in 1851. He used as pendulum a massive ball of copper, hung by a wire more than 50 meters long, from the dome of the Pantheon in Paris.

145. Conservation of Angular Momentum.—In any body or system of bodies **the total angular momentum of the system cannot be changed by any internal forces:** for suppose A and B (Fig. 74) are two parts of the system which act on each other, since action and reaction are equal and opposite the force on A is equal and opposite to the force on B; and since the distance from the axis to the line of action of the forces is the same for both, the *moments* of the *forces* about the axis will be equal and opposite, so that in the same time they will give equal and opposite angular momenta to the system, and consequently the total angular momentum will not be changed.

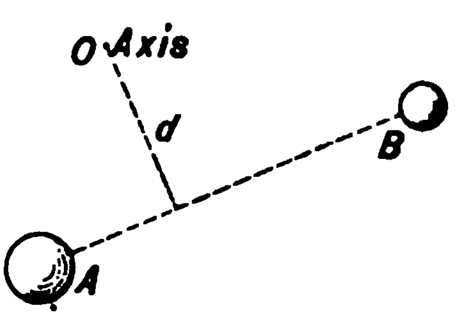


Fig. 74.

For example, in the solar system the planets have not only angular momenta about their own axes, but also angular momenta about the common center of gravity of the system. These angular momenta may be represented as vectors and their resultant found from the vector diagram, and neither the direction nor amount of this resultant is changed by any internal forces, such as the attraction of one planet for another or any possible collisions between them.

146. Angular Momentum of Projectiles.—A body having angular momentum tends to keep the direction of its axis of revolution constant, and the greater the angular momentum the harder it is to disturb the direction of the rotation; that is, the slower its axis of revolution will change in direction under any given torque.

So the spin of the rifle bullet or shell from a rifled gun causes it to keep pointing in a nearly constant direction as it flies through the air in spite of the tendency of a long bullet to turn sidewise in consequence of air resistance.

147. Motion of a Top.—When a rotating body is acted on by forces which tend to turn it about an axis perpendicular to its axis of rotation the effect is to change the *direction* of the axis of rotation without producing any change in the *amount* of the angular momentum about that axis; precisely as when a force acts on

a body at right angles to the direction of its linear motion (§114) it changes the *direction*, but not the *speed* of the motion.

The motion of a top affords an excellent illustration of this principle. The top in figure 75 is represented as spinning in the direction indicated by the arrow, but in the inclined position shown it is subject to a downward force W due to its own weight acting through its center of gravity G, and the upward pressure of the floor against the point of the top at A. These two forces are equal and constitute a couple which tends to turn the top about an axis DA perpendicular to its axis of revolution. The effect of the couple is to cause a steady change in the direction of the axis of revolution, the upper end of the top moving around in the circle $EFLH$. This change in the direction of the axis of the top may be called its *precessional* motion.

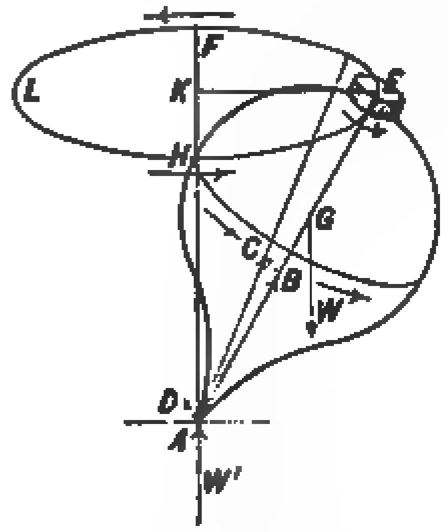


FIG. 75.—Top with point fixed.

The precession of the top may be explained as follows: let the vector AB represent in amount and direction the angular momentum of the top about its axis, the vector being drawn so that the top is seen to revolve clockwise by an observer looking along the vector AB in the direction in which it points. Similarly the vector AD may represent the angular momentum which would be given to the top in a very small interval of time t by the couple consisting of the forces W and W'. The resultant of the two vectors AD and AB is the vector AC, showing that the resultant angular momentum will have AC as its axis, and the axis of the top will accordingly move through the angle BAC in the time t. And as the vector DA is always at right angles to the plane EKA, the top will move at right angles to this plane, and therefore its upper end E will describe a circle about the vertical axis AK.

In the case just discussed the friction of the floor is supposed to be sufficient to keep the point of the top fixed at A. But when the top spins on a *frictionless* level surface it remains at a constant inclination and its precessional motion is

about a vertical axis through its *center of gravity*, as shown in figure 76.

How it is possible for a top to rise to a vertical position as it spins was first explained by Lord Kelvin. It depends on the fact that the peg of the top is rounded and the friction between it and the floor causes it to roll around in a circle; and when this rolling of the peg on the floor urges the top around faster than the regular precessional motion, it causes the inclination of the top to gradually diminish until it stands vertical, and "goes to sleep." On a perfectly frictionless surface a top could not rise in this way.

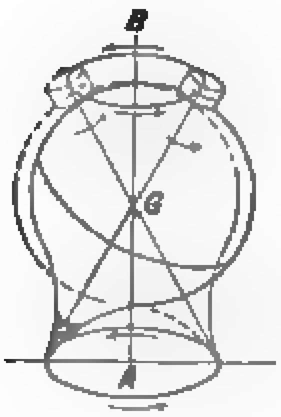


Fig. 76.

148. Gyroscope.—In the gyroscope shown in figure 77 a wheel with heavy rim is mounted in two pivoted rings so that the axis of rotation of the wheel may be inclined at any angle and the whole may also turn freely about a vertical axis. When the wheel is in rapid rotation a sharp blow given with the hand to one of the rings as if to change the direction of the axis of rotation, will cause the wheel to vibrate as though it were held in its position by stiff springs.

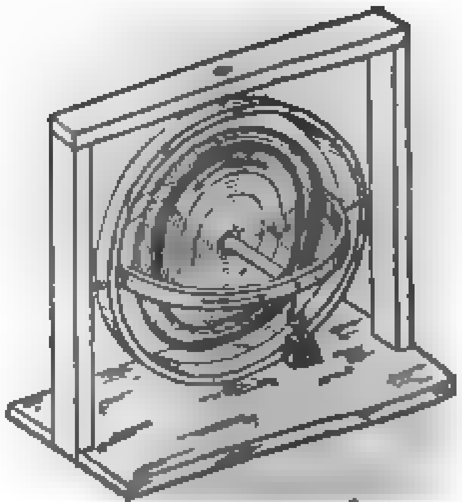


Fig. 77.

When a small weight is hung on near one end of the axis of rotation, the wheel, instead of tipping down, rotates slowly around the vertical axis as indicated by the arrow; if the weight is hung from the other end of the axis this precessional motion is reversed. A bicycle wheel serves admirably as a gyroscope.

References

A. Gray: "Gyrostats and Gyrostatic Action," Smithsonian Reports, 1914, p. 193.

"The Sperry Stabilizer for Aeroplanes," *Scientific American*, Aug. 8, 1914.

"Gyro-compass," *Scientific American Supplement*, v. 72, p. 200, 1911.

John Perry: *Spinning Tops.*

H. Crabtree: *Spinning Tops and Gyroscopic Motion.*

Worthington: *Dynamics of Rotation.*

149. Precession of the Equinoxes.—The earth itself illustrates the precessional motion of the gyroscope. It is a rotating body with eas-

mous angular momentum. But as it is not a sphere and its axis is not perpendicular to the plane of its orbit, the attraction of the sun on the bulging equatorial belt tends to turn it over and make its axis perpendicular to the ecliptic. The effect of this rotational force is a slow precessional motion of the axis of the earth, just as in the gyroscope. The axis remains inclined 23½° to the pole of the ecliptic, but describes a circle about that pole in a period of about 25,800 years.

If we take the pole of the ecliptic as a center and describe a circle of 23½° radius it will pass through the present pole star and will mark the path which is being described by the polar axis of the earth. In about 13,000 years the bright star Vega in the constellation of the Lyre will be very nearly at the pole.

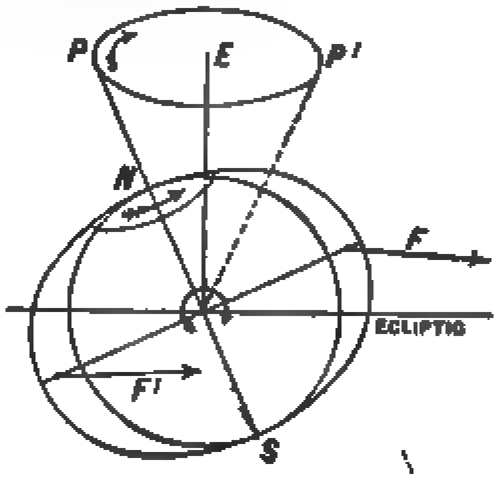


FIG. 78.

V. UNIVERSAL GRAVITATION

150. Kepler's Laws.—The German astronomer Kepler in the year 1609, having made a careful study of the observations made by Tycho Brahé, came to the conclusion that the orbits of the planets were not circular as had been supposed, but elliptical, and announced his discovery in the following laws:

1. *The orbits of the planets are ellipses having the sun at one focus.*

2. *The area swept over per hour by the radius joining sun and planet is the same in all parts of the planet's orbit.* Hence the planet moves faster in its orbit when near the sun than when farther away.

After nine years more of persistent search for some relation between the periodic times of the planets and their distances from the sun, he discovered and announced his third law:

3. *The squares of the periodic times of the planets are proportional to the cubes of their mean distances from the sun.*

151. Newton's Principia.—In 1686 Sir Isaac Newton published his great work, the *Principia,* in which he clearly enunciated the fundamental principles of mechanics and applied them to a great variety of important problems. In this work he showed from the laws of mechanics that if the planets moved about the sun in ellipses in the manner described in the first two laws of

Kepler, then each planet as it moves in its orbit must be subject to a force which is directed toward the sun, and varies inversely as the square of the distance between them.

152. Universal Gravitation.—From the above result Newton concluded that probably all masses, great and small, attract each other with a force proportional to their masses and inversely proportional to the square of the distance between them.

According to this law, the attractive force between any two masses m and M is expressed by the formula

$$F = \frac{mM}{r^2} C$$

where r is the distance between the centers of the masses if they are spherical. The quantity C is an absolute constant for all kinds of matter and depends only on the units in which force, mass, and distance are measured. It is called the gravitation constant and is equal to the force with which two unit masses attract each other when placed unit distance apart.

153. Moon's Motions Connected with Fall of Apple.—Newton conceived that the weight of a body near the surface of the earth is due to this gravitation attraction between the earth and the body, and that an apple drops toward the earth in accordance with the same gravitation law which determines the motion of the moon in its orbit.

To test this point let us, following Newton, find the acceleration which the apple would have if it were dropped toward the earth when as far off as the moon, and compare this acceleration with that which the moon is known to have.

According to the law of gravitation (§152), the earth attracts a body at its surface with 3600 times the force that it would if the body were 60 times as far from its center, or at the distance of the moon. Consequently the acceleration toward the earth of a body at the distance of the moon should be $\frac{1}{3600}$ of the acceleration of gravity at the earth's surfaces.

But the acceleration of the moon toward the earth may be computed from the formula

$$a = \frac{v^2}{r} \quad \text{or} \quad a = \frac{4\pi^2 R}{P^2} \qquad (§114)$$

where R is the radius of its orbit (240,000 miles) in feet and P is its period of orbital revolution (27.322 days) in seconds.

Substituting, we have

$$a = \frac{4\pi^2 \times 240{,}000 \times 5280}{(2{,}360{,}620)^2} = 0.008974 \text{ ft./sec.}^2$$

which is $\frac{1}{3600}$ of 32.30 ft./sec.,^{2} while the acceleration of gravity at the pole, where it is not affected by the earth's rotation is 32.26 ft./sec.^{2} The two results therefore agree as exactly as could be expected with the data used.

We conclude, then, that the motion of the moon and the fall of an apple or stone are both according to the same law of gravitation.

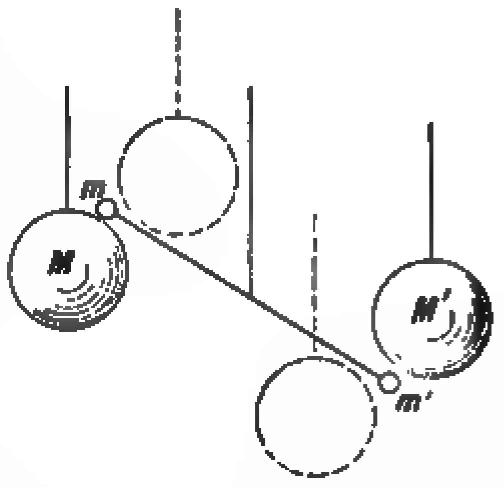


Fig. 79.

154. Determination of the Gravitation Constant.—To determine the constant of gravitation the force of attraction between two known masses must actually be measured. The extreme minuteness of this attraction between small masses makes the exact determination of its value very difficult.

It was first accomplished by Cavendish in 1798, using a form of apparatus indicated in figure 79. Two small spherical balls m and m' were mounted on the ends of a light crossbar which was suspended by a fine silver wire at its center. Two large spherical balls of lead M and M' weighing 158 kilograms apiece were suspended one near m and the other near m' but on opposite sides so that their attractions tended to turn the bar in the same direction. To protect the suspended bar from being disturbed by air currents it was entirely enclosed in a narrow box, its deflections being observed by a telescope through a glass window.

Having observed the deflection of the bar when the large masses were in the positions shown, the masses were moved into the dotted positions where their attractions produced a deflection of the bar in the opposite direction. From these observations, combined with a measurement of the force required to turn the suspended bar through a given angle, the force of attraction between the masses was determined.

In the year 1889 C. V. Boys, who had discovered the remarkable elastic properties of fine quartz fibers, devised an apparatus similar in principle to that of Cavendish, but much more compact, in which the small suspended masses were hung by a quartz fiber so fine that larger deflections and greater accuracy of measurement were attained.

According to Boys' determination, $C = 6.6576 \times 10^{-8}$ in C. G. S. units. That is, the attraction between two masses of one gram each concentrated at two points a centimeter apart, or of two spherical masses of one gram each with a distance of one centimeter between centers, is 0.000,000,066,6 dyne.

Two kilogram masses 10 cms. between centers attract with a force of 0.000666 dyne, or about seven ten-millionths of a gram weight.

The constant of gravitation has also been reckoned by estimating the mass contained in an isolated mountain and then measuring its deflecting effect on a plumb-line near its base.

155. Mass of the Earth.—When the gravitation constant is known the mass of the earth itself may readily be determined. For consider the earth as attracting a gram mass at its surface. The force of attraction is g dynes or approximately 980, and from the law of gravitation

$$F = \frac{mM}{r^2} C$$

Take M = mass of the earth, $F = 980$, $m = 1$, r = radius of earth in centimeters, and $C = 6.66 \times 10^{-8}$.

All of these quantities are known except M, which may be calculated. In this way the mean density of the earth is found to be 5.527, a result which is especially interesting as the average density of the *surface* materials of the earth is only about 2.5.

156. Mass of a Planet.—So also the mass may be found of any planet having a satellite whose distance and period of orbital revolution about the planet can be observed. For the attraction between the planet and satellite is expressed by $\frac{mM}{r^2}C$, while the centripetal force in case of a satellite of mass m and period P and moving in a circle of radius r, is $\frac{4\pi^2 m}{P^2} r$, and since it is the attraction which holds the satellite in its orbit we have

$$\frac{mM}{r^2} C = \frac{4\pi^2 m}{P^2} r.$$

In the equation the mass of the satellite m cancels, and as all the other quantities except M are known, the mass of the planet may be computed.

157. Significance of Kepler's Third Law.—Let M represent the mass of the sun, E the mass of the earth, r the mean distance between them, and P the period of the earth's revolution about the sun. Then, as in the last paragraph

$$\frac{ME}{r^2}C = \frac{4\pi^2 Er}{P^2} \quad \text{or} \quad \frac{MC}{4\pi^2} = \frac{r^3}{P^2} \tag{1}$$

So also if J is the mass of some other planet, such as Jupiter, and if r_1 and P_1 represent its distance from the sun and period of revolution in its orbit, respectively, we have

$$\frac{MJC}{r_1^2} = \frac{4\pi^2 Jr_1}{P_1^2} \quad \text{or} \quad \frac{MC}{4\pi^2} = \frac{r_1^3}{P_1^2} \tag{2}$$

If the constant of gravitation C has the same value in case of the sun and earth as it has in case of the sun and Jupiter, then

$$\frac{r^3}{P^2} = \frac{r_1^3}{P_1^2}$$

which is precisely what Kepler's third law asserts to be true throughout the solar system. It is concluded, therefore, that the same gravitation constant holds everywhere throughout the solar system and probably throughout the material universe.

158. Variation of Gravity on Earth.—The force of gravity is not the same everywhere on the earth's surface. There are three circumstances which determine this variation, namely, the fact that the earth is not a sphere, its rotation, and the height above sea level of the given station.

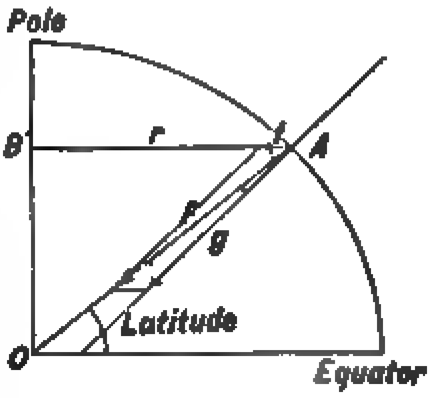


Fig. 80.

The earth is approximately an oblate spheroid having its polar radius less than its equatorial by 13.2 miles or 21.2 kilometers and in consequence of this the value of g at the poles is greater than at the equator by 1.6 cm./sec.^{2}, due to this cause alone. But there is another circumstance which still further reduces the value of g at the equator. The *rotation of the earth* affects both the direction and amount of the acceleration g. For the resultant attraction F of the earth on a gram of matter situated at A (Fig. 80) is directed toward the center O, but this resultant attraction serves both to supply the centripetal force f, which holds the mass on the earth as it rotates, and also the

force which we call its weight which gives it acceleration g when dropped. The centripetal acceleration f is directed perpendicular to the polar axis and is equal to $\frac{4\pi^2}{P^2}r$, where P is the period of rotation of the earth and r is the distance AB.

The distance $AB = R \cos l$ where R is the radius of the earth and l the latitude of A. Evidently then, f is a maximum at the equator and has zero value at the poles. Since F is the resultant of f and g, and is directed toward the center of the earth, it is clear from the diagram that g cannot be directed toward the earth's center except at the poles or equator. The direction of g is the direction in which a plumb-line will hang or a body will fall at A. Also a liquid surface, as the surface of the ocean, must be at right angles to g (see §172).

At latitude 45° the plumb-line points away from the center of the earth about 6.9 miles.

At the equator the centrifugal force of a mass of one gram is 3.36 dynes. Hence the acceleration of gravity is less at the equator than at the poles by 3.36 cm./sec.^{2} on this score alone.

The height of a place above sea level also affects the value of g, as it must diminish with the increase in distance from the center of the earth. If h represents the height in centimeters or in feet, the corresponding change in g is $(0.000003)h$.

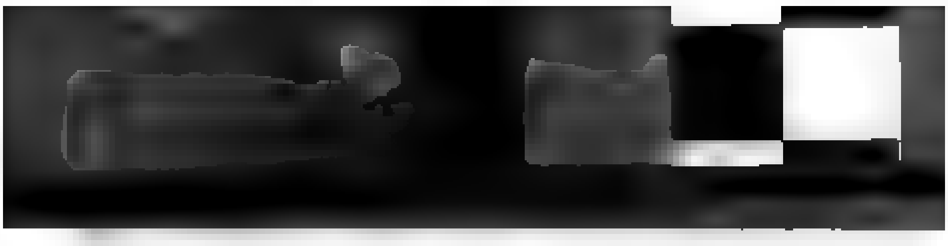
Though on account of the irregular shape and distribution of the earth's mass the exact value of g at any place can be determined only by pendulum experiments, an approximate value may be calculated for any place on earth by the following formula due to Clairaut:

$$g = 980.6056 - 2.5028 \cos 2\lambda - 0.000003h.$$

where λ represents the latitude of the place and h its height above sea level.

Some Values of g at Sea Level

Place	Cm./sec.^{2}	Ft./sec.^{2}	Place	Cm./sec.^{2}	Ft./sec.^{2}
Pole	983.1	32.25	New York	980.2	32.16
London	981.2	32.19	Washington	980.0	32.15
Paris	980.9	32.18	Equator	978.1	32.09



MECHANICS OF LIQUIDS AND GASES

PART I.—FLUIDS AT REST

Pressure in Liquids and Gases

159. Fluids.—Certain substances, such as air, water, glycerin, etc., are characterised by great mobility, changing their shapes and flowing under the smallest forces. They are known as *fluids.*

Fluids are divided into two classes, liquids and gases.

Liquids change but slightly in volume when subjected to great pressure and may have a free surface.

Gases are far more compressible than liquids and fill all parts of the containing vessel. Water is a type of liquid, and air of gas.

160. Density.—The mass of any substance contained in unit volume is known as its density. In the C. G. S. system of units density is expressed in grams per cubic centimeter, while in the foot-pound-second system it is expressed in pounds per cubic foot.

Thus the density of water is 1.0 on the first system, while it is 62.5 on the latter system.

A table showing the densities of some substances will be found on page 148.

161. Viscosity.—Fluids differ greatly in mobility. If a dish of water is tilted, the flow is so rapid that it gives rise to waves that surge to and fro, while in case of glycerin or syrup the flow is slow and the liquid only gradually settles to the new level. This difference in mobility is due to *viscosity* or internal friction (§245). Substances like pitch or tar are very viscous, while water, alcohol, and ether are but slightly so.

A *perfect* fluid is one that has no viscosity and is an ideal. All known fluids, even gases, have some viscosity.

162. Force in Fluid at Rest.—The force exerted by a fluid at rest against any surface is perpendicular to that surface. Otherwise, owing to the mobility of the fluid, flow must take

place along the surface, which of course cannot be in a liquid at rest.

This law is true of all fluids, even those which are very viscous, *after they have settled into equilibrium.*

163. Pressure.—Let a very small flat surface be imagined at some point in a fluid. The fluid on one side of that surface exerts a force perpendicular to the surface against the fluid on the opposite side. This force is proportional to the surface, and the force per unit surface is called the pressure.

In C. G. S. units pressure is measured in dynes per square centimeter; it may also be measured in grams per square centimeter, pounds per square inch, etc.

164. Hydrostatic Pressure.—**At any point in a fluid at rest the pressure is the same in every direction.** This is a direct consequence of the mobility of fluids, for a little sphere of liquid at the given point could not be in equilibrium if the pressure against its surface were not the same in every direction.

165. Pressures on Same Level.—**In a liquid at rest the pressure is the same at all points on the same level.**—For a horizontal cylindrical column of liquid reaching from A to B is in equilibrium under the pressure of the surrounding liquid. The pressure against its sides is perpendicular to the line AB, and therefore has no influence to move the column toward A or B. And since it is level it has no tendency to slide toward A or B by reason of its weight. The force against the end at A must therefore be balanced by the force against the end at B. These forces are due to the pressures at A and B, and since the ends have equal areas the pressure at A must be equal to the pressure at B.

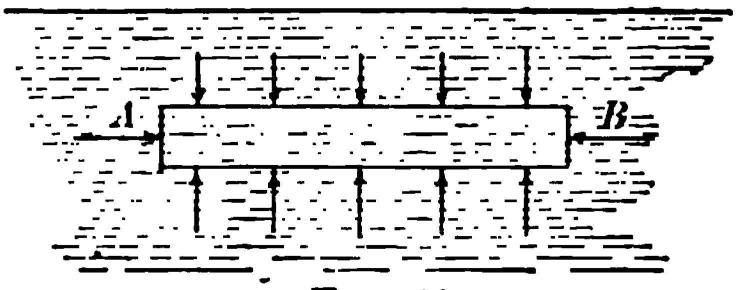


Fig. 81.

166. Pressures at Different Depths.—**The difference in pressure between two points at different levels in a mass of fluid at rest under gravity, is equal to the weight of a column of the fluid of unit cross section reaching vertically from one level to the other.** For a vertical cylindrical column of the fluid of unit cross section reaching from B to C is in equilibrium under the pressure of the surrounding fluid. The pressure against the sides

of the vertical column is horizontal and has no power to support its weight, consequently the upward force at C must balance the weight of the column in addition to the downward force at B. Hence, since the force against the end of a unit column is equal to the pressure, the pressure at C is greater than the pressure at B by the weight of the column of fluid of unit cross section reaching from B to C.

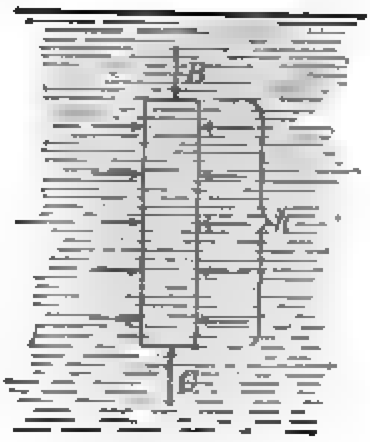


Fig. 82.

If h is the height of the column in centimeters and d is the weight of one cubic centimeter of the fluid in grams, then hd is the weight of the column and is thus the difference in pressure between B and C in grams per sq.cm. The difference in pressure expressed in *dynes* per sq.cm. is hdg where g is the acceleration of gravity in cm./sec.[2] The total pressure at a point h centimeters below the surface, is therefore as follows:

Pressure in *grams* per sq.cm. $= hd +$ pressure on surface in *grams* per sq.cm.

Pressure in *dynes* per sq.cm. $= hdg +$ pressure on surface in *dynes* per sq.cm.

Note as to Units.—In calculating pressure by the use of the formula hd, it must be remembered that if the pressure is to be found in pounds per square inch, then h must be expressed in inches and d is the weight of one cubic inch of the liquid in pounds. The student is advised, however, to compute directly the weight of a column of the substance of unit cross section without thinking of any formula.

In gases the density is so small that the pressure is practically the same everywhere throughout a *small* volume.

Pascal's Principle.—Pressure is transmitted equally in all directions throughout a mass of fluid at rest, or if the pressure at any point is increased, it is increased everywhere throughout the fluid mass by the same amount.

167. Hydraulic or Hydrostatic Press.—An important mechanical device known as the *hydraulic press* is a good illustration of the application of the laws of fluid pressure. It was first constructed by Bramah in 1796, and is sometimes known as Bramah's press.

It consists of a strong cylinder in which works a cylindrical piston or ram of larger diameter. A collar of oiled leather or copper surrounds the piston in such a way that the greater the pressure of the liquid filling the cylinder, the more closely does the collar fit the piston. By means of a small pump, oil or water is

forced into the large cylinder, a check-valve preventing its return. In consequence of the law of pressure just enunciated, whatever pressure is communicated to the liquid by the pump will be exerted everwhere equally against the walls of the containing cylinders. So that if the large piston has 100 times the area of the other it will exert a force 100 times as great as that applied to the pump piston.

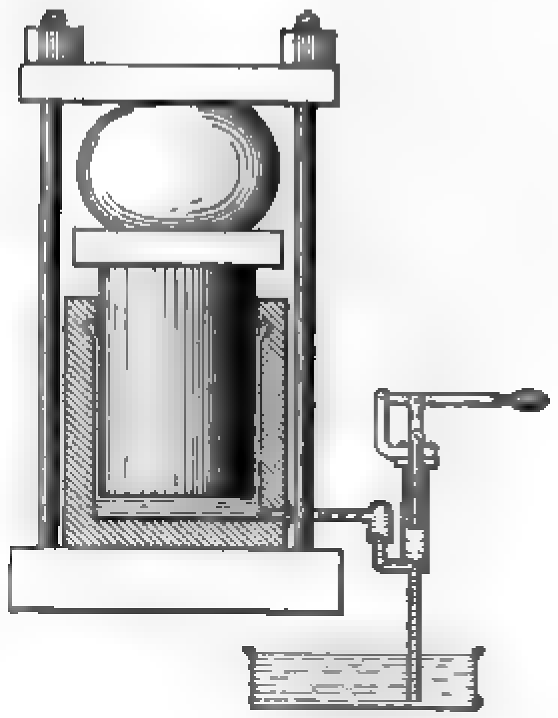


Fig. 83.—Hydrostatic press.

Hydraulic jacks act on this principle: they contain a reservoir of oil which may be pumped into the main cylinder, thus forcing up the ram; opening a small stop-cock permits the flow of oil back to the reservoir. Oil is used as it keeps the machine lubricated and does not freeze.

It is to be observed that when the liquid in the hydraulic press is incompressible as much work is done by the large piston as is expended upon the smaller one.

168. Pressure Independent of Shape of Vessel.—It has been shown that the pressure at any point in a liquid under gravity depends only on the depth of the point below the surface, on the density of the liquid, and on the pressure on its surface.

The total force exerted against the bottom of a vessel by the pressure of the liquid which it contains is the product of the pressure at the bottom by its area, and may therefore be very different from the actual weight of liquid which the vessel contains; and when a vessel is filled with water to a given height the force against its bottom is the same whether the upper part of the vessel is flaring, cylindrical, or narrow. The reasonableness of this result will be evident from the following considerations.

In the case of the vessel with flaring sides we may think of a cylindrical column resting on the bottom and pressed upon by the surrounding water as shown in the figure (Fig. 84). This pressure is necessarily perpendicular to the surface of the cylindrical column and, therefore, can have no effect in either supporting it or

[redacted]

pressing it down. The whole weight of the cylindrical column is, therefore, supported by the bottom plate. In case of the vessel which is narrow at the top, the liquid exerts a downward force on the bottom greater than its weight because the sides of the vessel press the liquid down. Just as a man in a box may brace himself

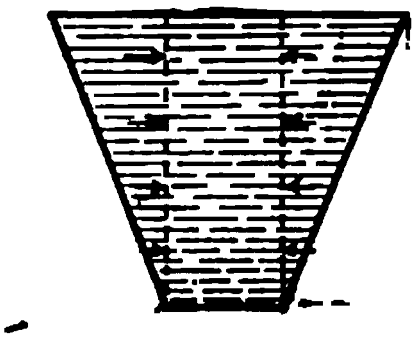


FIG. 84.

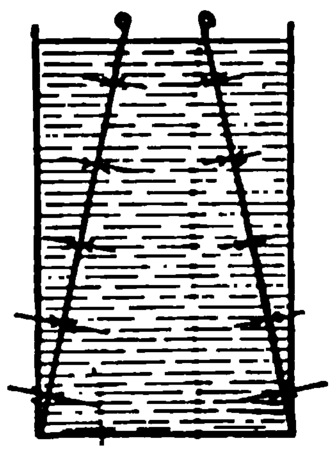


FIG. 85.

against the top and press against the bottom with a force far greater than his own weight.

This fact that the force exerted on the bottom of a vessel may be greater than the weight of all the liquid in the vessel has been called the *hydrostatic paradox*.

Pascal succeeded in bursting a strong cask by the pressure produced by a column of water in a narrow pipe 40 ft. high.

169. Center of Pressure.—The *center of pressure* of a surface is the point of application of the resultant force due to the pressure against the surface. The pressure is so distributed that the surface will just balance if supported at that point.

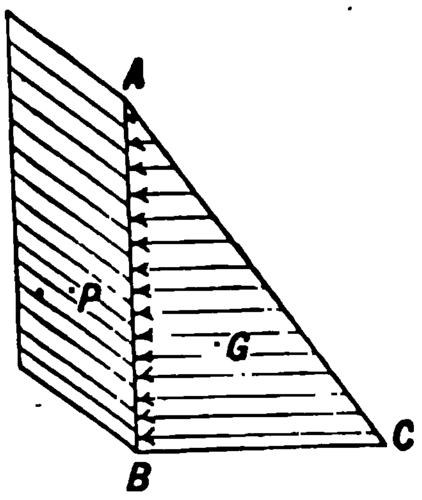


FIG. 86.

In case of a tank having rectangular sides and filled with water, the center of pressure on a side will evidently be nearer the bottom than the top, because the pressure increases with the depth. Suppose the side to be divided into narrow horizontal strips of equal widths, the force exerted on each strip by the liquid pressure may be represented by an arrow as in the diagram, and it is clear that each of these forces will be proportional to the depth, since the force on any strip is the product of the area of the strip by the pressure at that depth. By the methods employed in finding the resultant of parallel forces it may be shown that the center of pressure in this case is at P, $\frac{1}{3}$ of the total depth from the bottom.

It is not difficult to see that the center of pressure P must be on the same level as the center of gravity G of the triangle ABC formed by the lines representing the forces against the equal horizontal strips.

If a cylindrical water tank were to be bound by a single hoop, this should be situated $\frac{1}{3}$ the height of the tank from the bottom. The hoops on water tanks are placed closer together at the bottom than at the top for the same reason.

Liquid Surfaces

170. Free Surface of a Liquid.—When a liquid is at rest or in equilibrium the force which a surface particle exerts against the adjoining liquid must be perpendicular to the free surface at that point, otherwise the particle would move along the surface. This force depends upon gravity, on the attraction of neighboring particles, and on the atmospheric pressure on the surface, and also upon any acceleration which the particle may have.

171. Level Surface.—When a liquid is at rest on the earth, all parts of the surface which are not too near the walls of the containing vessel are at right angles to the direction of gravity or to the direction in which a plumb-line points. Such a surface is called *level.* A level surface is not a *flat* surface, but has the same curvature as the earth. In a pond 1 mile in diameter the center is 2 in. higher than a plane passing through the edges.

The force is not necessarily the same at all points of a level surface. This is well illustrated in case of the earth, for the force of gravity at sea level near the poles is decidedly greater than at the equator.

172. Surface of a Rotating Liquid.—When a vessel containing a liquid is rotated by a whirling machine, the liquid by virtue of its viscosity soon comes into equilibrium,[1] and turns at the same rate as the vessel. If the speed is slow the upper surface of the liquid is slightly concave, at greater speed it will become deeply hollowed, but it always has the form of a paraboloid of revolution. Here a little mass m exerts against the adjoining liquid a downward force mg due to gravity, and an outward centrifugal force[2] equal to $m\omega^2 r$. The components q due to gravity (Fig. 87) are the same at all points of the surface, while the centrifugal components l_1, l_2, l_3 increase in proportion to the distance of the particle from the axis of rotation. The resultant

[1] That is, it is in equilibrium considered as a whole, though the individual particles move in circles and are therefore accelerated.

[2] The pressure of the adjoining parts against any little liquid mass supplies the *centripetal* force urging it toward the axis as it rotates. Its outward reaction against that pressure is the *centrifugal* force.

forces a_1, a_2, a_3 will therefore be differently inclined, and the surface must be of such a curve as to be at right angles to them. It will be noted that the resultant force is greater at points higher up on the surface, so that a surface particle near the top presses against the surrounding liquid with far more force than it would if at the bottom of the curve.

The oblate form of the earth is similarly explained. A unit mass at the earth's surface exerts a downward force a toward the center of the earth due to attraction, and also a centrifugal force c due to rotation. The latter component is zero at the poles and reaches a maximum at the equator and is always at right angles to the polar axis. The resultant downward force g is, therefore,

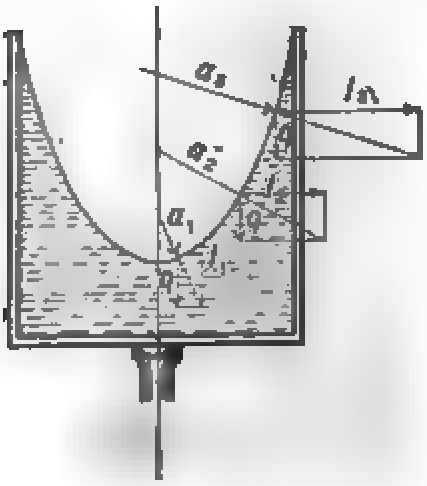


FIG. 87.—Surface of rotating liquid.

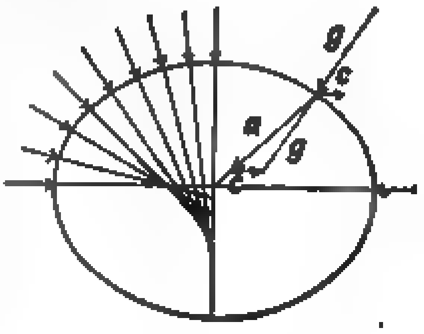


FIG. 88.

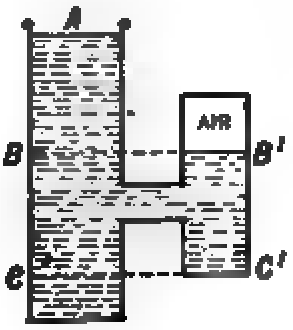


FIG. 89.

directed exactly toward the center only at the poles and at the equator, and the surface of the ocean when calm must be everywhere perpendicular to g.

173. Surface in Connected Vessels.—In a continuous mass of one kind of liquid all points on the same level must be at the same pressure, even though they may be in separate branches of the containing vessel. Thus the pressure at B (Fig. 89) is the same as at B', and that at C is the same as at C'. It is clear that the enclosed air is under greater pressure than that of the atmosphere at A.

When communicating parts of a vessel of liquid are open to the air the free surfaces must lie all on the same level because all are at the same pressure.

174. Case of Two Liquids.—If a bent tube containing mercury, as shown in the figure, have some other liquid, as water or oil,

poured into the longer arm, the mercury will be pressed down on that side and raised on the other. Since all below A is one continuous liquid, the pressure at A must be the same as at A' on the same level, hence the column of mercury BA' must produce the same pressure as the column of liquid CA.

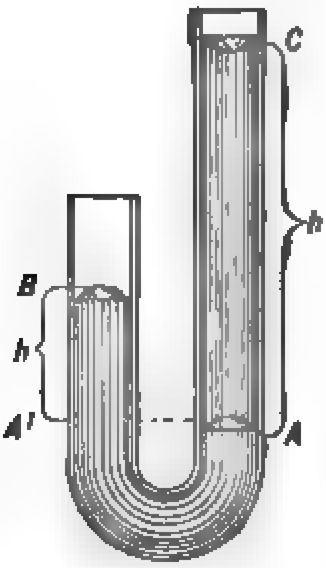


FIG. 90.

Letting h and h' represent the heights of the two columns of liquid and d and d' their densities, then, since the pressures of the two columns must be equal,

$$hd = h'd'.$$

175. Spirit-level.—The ordinary spirit-level consists of a glass tube hermetically sealed, nearly filled with alcohol or ether, a bubble of air or vapor being left. The tube is bent slightly, forming the arc of a large circle, and the bubble always rests in equilibrium at the highest point.

A level is said to be *sensitive* when a small inclination will cause a large motion of the bubble. In a sensitive level the curvature of the tube is very slight, and the bubble is usually large, otherwise it would be sluggish in its movements. For fine levels the tube is carefully ground on the inside so as to have a uniform curvature.


FIG. 91.—Spirit level.

Problems

1. Find the pressure 3.50 meters below the surface in a pond of water; in grams per sq. cm. and in dynes per sq. cm.
2. Find the pressure in pounds per sq. in. 30 ft. below the surface of a pond, taking the weight of 1 cu. ft. of water as 62.5 lbs.
3. A piston 1 ft. in diameter carries a weight which together with that of the piston amounts to 200 lbs. How high a column of water will be required to produce enough pressure under the piston to support the weight.
4. What is the pressure 1 mile below the surface of the ocean, in pounds per sq. in., taking the relative density of sea water as 1.03.
5. Find the difference between the pressure at the bottom of a vessel 75 cms. deep filled with water, and the pressure when the vessel is full of mercury. Density of mercury = 13.6.

6. A jar has a square cross section 5 cms. each way and is 30 cms. deep. It is half-full of mercury and half-full of water; find the pressure halfway down and also at the bottom, also the total force due to pressure against the bottom,
7. Find the total force against one side due to pressure in the preceding problem.
8. If a cubical tank 4 ft. each way is level full of water, find the pressure in pounds per sq. in. on bottom. Also the total force against one side in lbs. weight. Where is the center of pressure on the bottom? Where the center of pressure on one side?
9. Oil of density 0.7 is poured into one branch of a U-tube which contains enough mercury to keep the bend full. When the column of oil is 39 cm. high, how much higher will it stand than the mercury in the other branch?
10. When the atmospheric pressure is just 1,000,000 dynes per sq. cm., how far below the surface of a pond of water will the total pressure be just twice as much as at the surface?
11. In a pail of water spinning about a vertical axis through its center the surface of the water is hollowed so that at a point 10 cms. from the axis the surface is inclined 45°. Find the number of revolutions per sec. which the pail is making.

Buoyancy and Floating Bodies

176. Buoyant Force of a Fluid.—Suppose that a mass of wood or iron is immersed in a liquid and it is required to find the force exerted upon it by the surrounding liquid. Imagine the given substance removed and its place filled by the liquid, and conceive of this portion as separated from the surrounding liquid by an imaginary surface *ABC* of the same shape as the original body. The liquid is in equilibrium, and since the mass enclosed in the surface *ABC* is urged down by its own weight, this weight must be exactly balanced by the force due to the pressure of the surrounding liquid on the surface *ABC*. Hence the resultant force due to pressure on the surface is an upward force equal and opposite to the weight of the enclosed mass of liquid, and since the whole weight of the enclosed mass acts down through its center of gravity *G*, the *center of pressure* must also be at the same point.

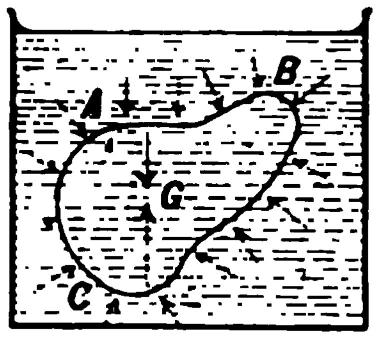


FIG. 92.

Now, neither the amount nor direction of the pressure will

be changed at any point of the surface *ABC* if it is filled with wood or iron instead of the liquid. Therefore **when any object is wholly or partially immersed in a liquid it is buoyed up by a force equal to the weight of the displaced liquid, and the center of pressure is where the center of gravity of the submerged portion would be if it were homogeneous.**

There is nothing in the above reasoning which restricts this conclusion to *liquids*, it may therefore be stated as a general law of *fluids* and is known as **Archimedes' principle**, from its discoverer.

177. Experimental Illustration.—A brass cylinder which exactly fits into and fills a cup is suspended together with the cup from one pan of a balance and exactly counterpoised by weights. A vessel of water is raised under the cylinder until it is quite immersed, and the weights will now greatly overbalance the cup and cylinder; but if the cup is just filled with water the balance is restored.

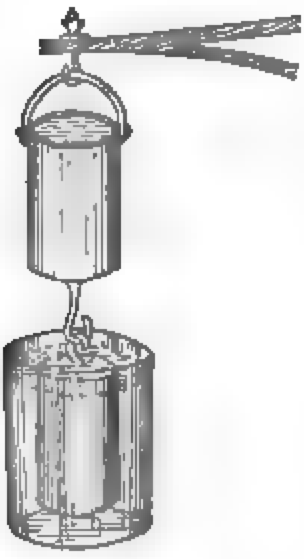


Fig. 93.

178. Buoyancy at Great Depths.—Since buoyant force depends on the weight of the liquid displaced and not directly on the pressure, it makes no difference whether the immersed body is 1 in. or 100 ft. below the surface of the liquid except for the compression due to increased pressure. If the immersed body is more compressible than the surrounding liquid it will displace less liquid where the pressure is great than at the surface and so will be less buoyed up at great depths. If it is less compressible than the liquid, it will be more buoyed up at great depths than when near the surface.

The heavy iron shot used in deep sea soundings is buoyed up slightly more at great depths than at the surface because water is more compressible than iron.

179. Cartesian Diver.—The Cartesian diver is a small bulb of glass open at the bottom and containing just enough air to cause it to float in a jar full of water. A sheet of rubber is tied firmly over the mouth of the jar, and by pressing on the rubber the pressure in the liquid is increased and the air in the bulb compressed into smaller volume. The bulb with the contained

air may thus be made to displace less than its own weight of water and will then sink to the bottom, but rises again when the pressure is relieved and the air expands.

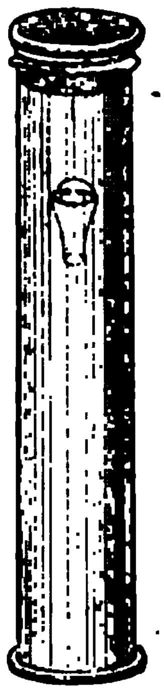


Fig. 94.

180. Equilibrium of Floating Bodies.—A floating body may be considered as acted on by two forces: its own weight acting down through its center of gravity and a buoyant force equal to the weight of the displaced liquid acting up through the center of pressure. It can be in equilibrium only when these two forces are equal and opposite. The conditions for equilibrium may then be thus stated:

1. **The weight of the displaced liquid must be equal to the weight of the floating body.**

2. **The center of gravity of the floating body must be in the same vertical line as the center of pressure.**

The displacement of a ship is the weight of water which it displaces, and is therefore the total weight of the ship and equipment.

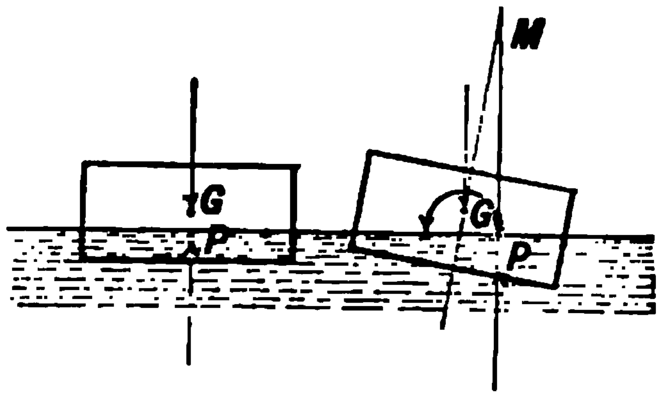


Fig. 95.

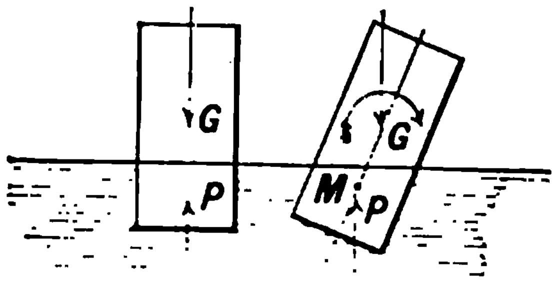


Fig. 96.

181. Stability of Equilibrium.—If, when a floating body is slightly inclined from its position of equilibrium, the couple resulting from its own weight and the buoyant force of the liquid tends to turn it back into its original position, the equilibrium is said to be *stable*. In figure 95, G is the center of gravity and P the center of pressure of the floating block. When it is tipped slightly P is displaced to one side in such a way that the combined action of the forces through G and P tends to turn the body in the direction of the arrow, bringing it back into its original state of equilibrium, which is therefore *stable*. In figure 96 is shown a state of equilibrium such that when the body is slightly displaced the couple acts to increase the displacement and to turn the body away from its original position. In this case the equilibium is *unstable*.

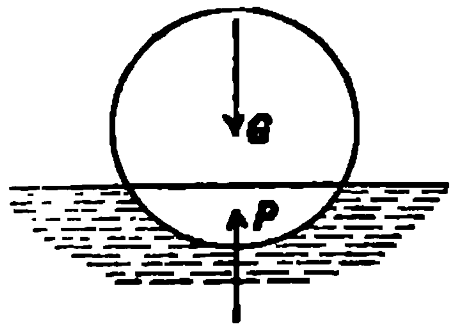


Fig. 97.

A floating homogeneous sphere may be turned in any way and the center of

pressure P will always be directly under the center of gravity, and the equilibrium will remain undisturbed. Here the equilibrium is *neutral.*

SPECIFIC GRAVITY AND ITS MEASUREMENT

182. Specific Gravity.—The relative density of a substance as compared with some standard substance is known as its *specific gravity.* Solids and liquids are usually compared with water as a standard, while gases are often referred to air or hydrogen.

The specific gravity of a substance referred to water is found by dividing the weight of the given substance by the weight of an equal volume of pure water at the temperature of 4°C.

The specific gravity of a substance is a *ratio* and is therefore the same whatever system of units is employed.

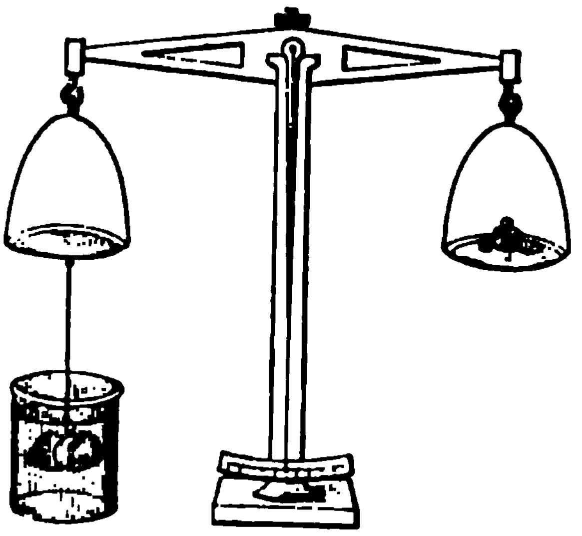


FIG. 98.

Since 1 c.c. of pure water at 4°C. has a mass of 1 gram, the density of a substance in *grams* per cubic centimeter is equal to its specific gravity referred to water.

183. Specific Gravities by Balance.—The substance, of which the specific gravity is to be determined, is suspended by a fine fiber from one arm of a balance and weighed, first in air and then when immersed in water. The second weighing will be less than the first by the weight of the water displaced by the substance. The difference between the two weighings will then give the weight of a mass of water of the same volume as the substance, and therefore if the weight in air is divided by the difference between the weights in air and water the *specific gravity* is obtained.

184. Mohr's Balance.—A convenient balance for determining the specific gravity of liquids is that shown in figure 99. A glass bulb weighted so as to sink in liquids is hung from one arm of a balance and exactly counterpoised by the weight P on the other arm. The glass bulb is hung in the liquid to be examined and the buoyant force of the liquid balanced by riders hung on the balance arm. From the weight and position of the riders the specific gravity of the liquid is obtained directly without calculation; for the several riders are so adjusted that each has one-tenth the weight of the

next larger, and the position of each on the balance arm gives the figure for the corresponding decimal place in the result.

185. Hydrometers of Constant Weight.—These instruments are usually made of glass and consist of a rather long light bulb having a slender stem above and a weighted bulb below so that the instrument floats in a vertical position in the liquid whose density is to be determined. By means of a scale on the stem the specific gravity of the liquid may be read directly from the point on the scale to which the instrument sinks.

In such a case the weight of the whole hydrometer must be equal to the weight of the displaced liquid, so that if v is the

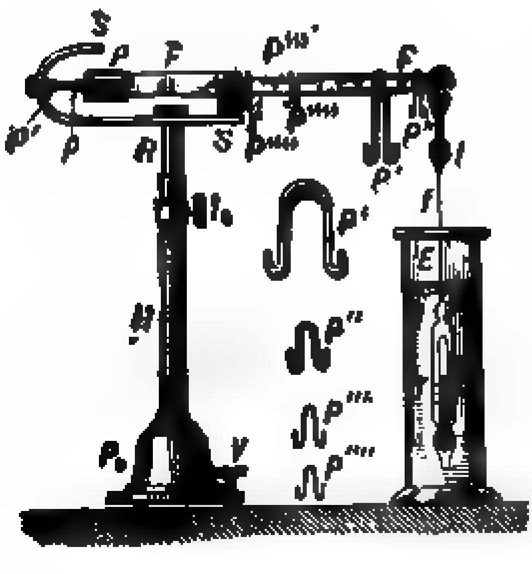


Fig. 99.

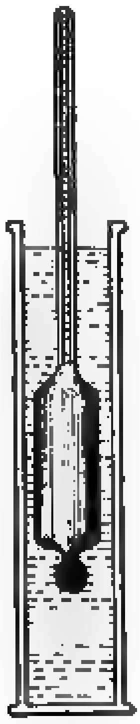


Fig. 100.

volume of the hydrometer below the mark to which it sinks in a given liquid and if d is the weight of unit volume of the liquid, then $W = vd$ where W is the weight of the hydrometer.

The specific gravity scale of a hydrometer is not a scale of equal parts, corresponding divisions being farther apart at the upper end of the stem than at the lower. The Beaumé scale is an arbitrary scale of equal parts in which hydrometers are often graduated.

Hydrometers are made for liquids lighter than water and also for liquids heavier than water.

by a suction pump. Torricelli believed that this was because water was raised in such a pump by the pressure of the atmosphere. He concluded that as mercury was 13.6 times as dense as water the atmospheric pressure would be able to support a column of mercury only $1/13.6$ times as high, or about 30 in. in length, and to test it tried the following experiment.

A tube nearly 3 ft. long and closed at one end was filled with mercury and then the open end being closed with the finger to prevent the escape of mercury the tube was inverted and placed with its open end below the surface of mercury in a dish, after which the finger was withdrawn. The mercury at once sank in the tube till it stood at a height of about 30 in. or 76 cms. above the level in the dish. The space above the mercury in the tube was a vacuum except for the presence of mercury vapor.

Fig. 102.

As 1 c.c. of mercury weighs 13.6 grams., the atmospheric pressure able to support a column 76 cms. high must be $76 \times 13.6 = 1033.6$ gms. per sq. cm., and would, therefore, sustain a column of water 1033.6 cms. high, or 33.9 ft.

Pascal, reasoning that if the pressure of the atmosphere was due to its weight the pressure should be less on top of a mountain than at its base, caused the experiment to be tried and established the fact.

190. Magdeburg Hemispheres.—Otto von Guericke, of Magdeburg, shortly after he had invented the air pump, demonstrated the pressure of the atmosphere by means of two hemispherical cups of copper carefully fitted together to form a spherical vessel about 2 ft. in diameter. When the air was exhausted from the vessel two teams of horses were unable to pull the cups apart.

The force with which the cups are pressed together in such a case is found by multiplying the area of the circular opening of the cups by the difference between the air pressure on the inside and outside.

191. Barometer.—Instruments for the measurement of the atmospheric pressure are known as barometers. The best barometers usually employ a column of mercury, as in Torricelli's experiment.

A form much used is the Fortin barometer, the reservoir of which is shown in the figure. The tube containing the mercury is sheathed with brass to protect it from injury, the height of the column being read through an opening by means of a vernier which slides on a scale graduated on the brass sheath. As the mercury sinks in the barometer tube it flows out into the vessel at the bottom and raises the level there, it is therefore necessary to provide some means of adjusting the height of the mercury in the lower vessel. This is accomplished by the screw C, on turning which the flexible leather bottom of the vessel is raised or lowered until the surface of the mercury exactly touches the ivory point a, which is the zero point from which the scale is graduated. As the lower vessel is not air-tight, the external air pressure is freely transmitted to the surface of the mercury. The greatest care is taken in filling such a barometer that no air is left clinging to its sides, the mercury being usually heated and even boiled in the tube.

FIG. 103.—Fortin barometer.

192. Capillary Correction.—The upper surface of the mercury column in a barometer tube is rounded upward in a *meniscus*, higher at the center than at the edges, and the height of the barometer is measured to the highest point of this curved meniscus.

The effect of the curvature is to make the column stand slightly *lower* than if the surface was flat. Hence to obtain the true height a small correction, called the *capillary correction*, which depends on the curvature of the surface, must be added to the apparent height.

In a standard barometer the tube should be so large (2 cms. in diameter) that there is no curvature at the center of the surface, in which case there is no capillary correction.

Capillary Correction Millimeters

Capillary Depression	1.4	0.8	0.5	0.3	0.2 mm.
Internal Diameter of Tube	4.0	6.0	8.0	10.0	12.0 mm.

193. Temperature Correction.—It must be remembered that the scale by which the height of a barometer is read is correct at only one temperature, and also that the density of the mercury itself varies with the temperature; in order, therefore, that barometer readings may be definite, what is known as the *reduced reading* is always given; this is the height at which it would stand if the mercury had the density which it has at 0°C.

Effect of Gravity.—It might be supposed that if the reduced heights of the barometers at two places were the same that the atmospheric pressures at those places would be equal, but this is not necessarily so. The pressure in grams per square centimeter would be the same, but the weight of a gram depends on the force of gravity. Near the equator a gram weighs 978 dynes, while near the poles it weighs over 983 dynes. If the reduced height of the barometer in centimeters be multiplied by the density of mercury at 0°C. and the product by the acceleration of gravity at the given place, the pressure recorded by the barometer will then be determined in dynes per square centimeter, which is absolutely definite.

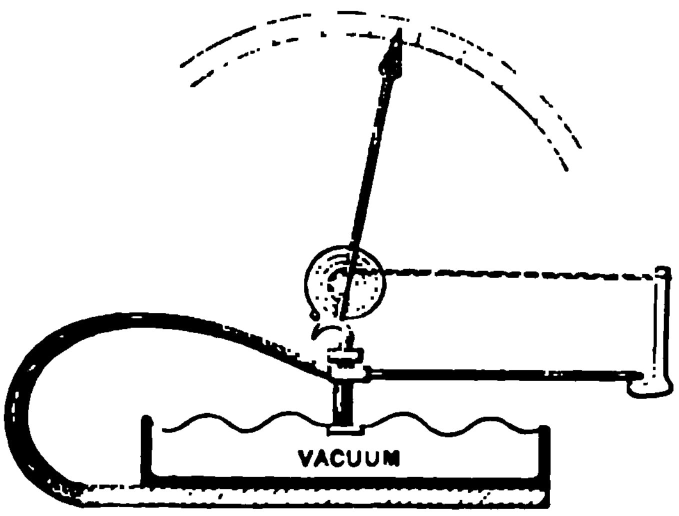


FIG. 104.—Diagram of mechanism of aneroid barometer.

194. Aneroid Barometer.—An exceedingly convenient and portable form of barometer is known as the *aneroid* (from the Greek, meaning *without liquid*). A disc-shaped metal box, like a small blacking box, is provided with a top made of thin metal corrugated so as to be extremely flexible. The air is exhausted from the box and it is permanently sealed, the top being supported by a stout steel spring which prevents it from collapsing. As the atmospheric pressure increases the spring yields a little and its point moves downward, acting by means of levers and a delicate chain to give a greatly increased motion to the pointer which moves over a graduated dial. A hair-spring serves to take up the slack of the chain. Such an instrument may be

made as compact and portable as a watch. It is subject to change, however, and needs to be compared with a mercurial barometer from time to time. Also the elasticity of the spring varies with the temperature.

195. Standard Atmospheric Pressure.—It is customary in stating the densities of gases to give them at what is called atmospheric pressure. This *standard atmospheric pressure*, sometimes called a *pressure of one atmosphere*, is the pressure of a column of mercury 76 cms. high at 0°C.

When the acceleration of gravity has the value that it has at Paris (980.94) this pressure is 1,013,600 dynes per square centimeter.

At London its value is 1,013,800 dynes per square centimeter.

196. Buoyancy.—The law of buoyancy, known as Archimedes' principle, that bodies immersed in a fluid are buoyed up with a force equal to the weight of the displaced fluid, holds for gases as well as for liquids. This may be easily illustrated by the apparatus shown in the figure. A hollow globe is balanced by a solid mass of lead or brass hung from the other arm of the balance. When the globe is closed and the whole is placed under the bell jar of an air pump, it is observed that as the air is exhausted from the receiver the globe settles down; when air is readmitted, however, the globe is again balanced by the weight. The globe with its greater volume displaces a greater volume of air than the weight, and by the law of buoyancy it must be buoyed up with a greater force.

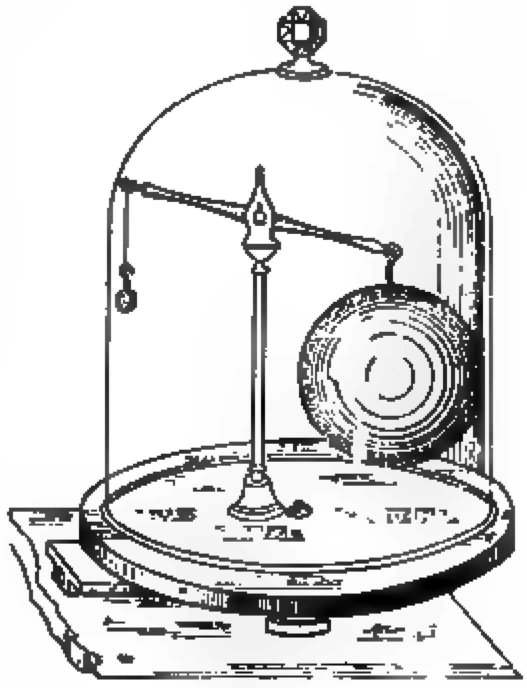


FIG. 105.

If a solid mass of brass is being weighed, using brass weights, the buoyant force of the air on both sides of the balance will be the same. But if the density of the weights is greater than that of the body weighed, the apparent weight of the body will be less than its true weight. When the apparent weight of a body is w, its true weight W may be found by the formula,

$$W = w + w\delta\left(\frac{1}{d} - \frac{1}{d_1}\right),$$

where δ is the density of air, d the average density of the object being weighed, and d_1 the density of the weights used.

197. Balloons.—Balloons ascend in consequence of the buoyancy of the surrounding atmosphere. The gas within the envelope simply supplies the pressure to keep the balloon distended; in so far as it has weight it is a disadvantage. To find the supporting power of a balloon we must determine the weight of the balloon itself together with the enclosed gas and subtract this from the weight of an equal volume of atmospheric air. The difference is the portative force of the balloon.

As the balloon rises the pressure of the atmosphere decreases and the gas in the interior expands and completely fills the balloon, and then as it expands still farther the excess escapes through an opening at the bottom.

Expansion of Gases

198. Expansion of Gases.—When a vessel containing gas is enlarged the gas expands, keeping the vessel full however great its volume may become, and at the same time the pressure of the gas diminishes.

If a small thin rubber bag containing a little air is closed and placed under the bell jar of an air pump, and the air exhausted from the space around the bag, the latter will be distended by the expansion of the enclosed air as the pressure upon it diminishes.

199. Boyle's Law.—The exact way in which the pressure of a gas changes when its volume is varied was first investigated by the English physicist Robert Boyle in 1662 and by Mariotte in France in 1679.

The form of apparatus used by Boyle is illustrated in figure 106. The short arm of the tube is closed and contains a mass of air separated from the outer air by the mercury in the bend of the tube. The enclosed air is at the same pressure as the outer air since the mercury stands at the same level in each branch. Mercury is now poured into the long arm of the tube until the

enclosed air is compressed to one-half its original volume, as shown in figure 107. The height of the mercury in the long branch above that in the closed branch is then found to be just equal to the height of the barometric column. That is, the enclosed air is under a pressure of *two* atmospheres, one due to

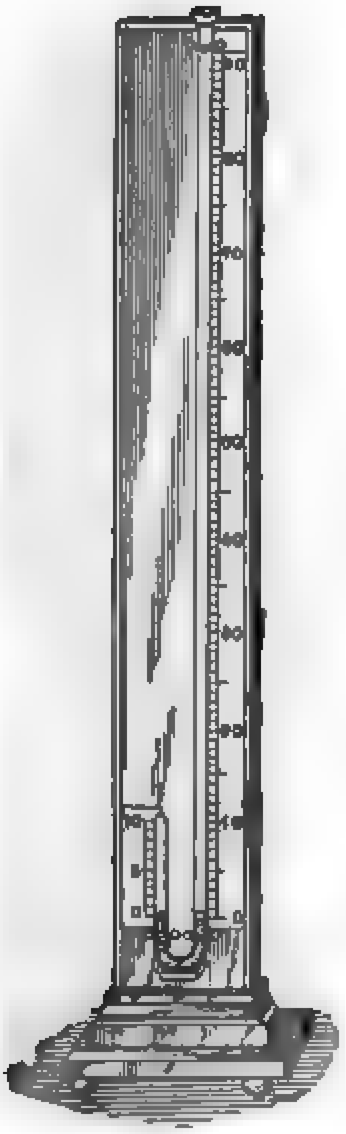


FIG. 106.

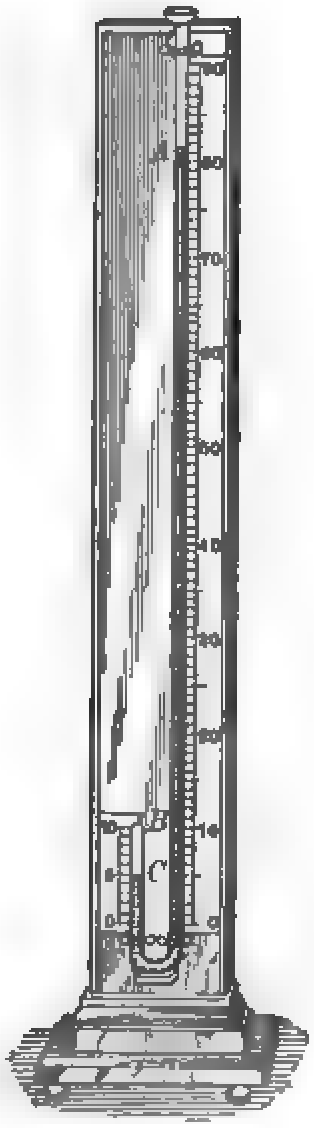


FIG. 107.

the external air pressure and the other due to the height of the mercury column.

If more mercury is added the air is still further compressed, and when the total pressure is three atmospheres, the mercury column having twice the barometric height, the air is found to be compressed to one-third of its original volume.

The law of compressibility of air, which is also found to be *approximately* true for all the more perfect gases may then be stated thus:

Boyle's Law.—When the volume of a mass of gas is changed, keeping the temperature constant, the pressure varies inversely as the volume; or the product of the pressure by the volume remains constant.

That is, if a mass of gas has a volume v at a pressure p and if the volume is changed to v' while the temperature is kept constant, the pressure will become p' such that

$$pv = p'v' = \text{constant} \qquad (1)$$

This constant is evidently proportional to the mass of gas used, for if the pressure is kept constant we must take twice the original volume in order to get double the mass of gas. We may, therefore, express Boyle's law by the equation,

$$pv = mk$$

or

$$\frac{pv}{m} = k \qquad (2)$$

where k is a constant which depends only on the kind of gas and its temperature.

Thus if we have a mass of gas m having pressure p and volume v, and another mass m' of *the same gas at the same temperature,* but with pressure p' and volume v', we have by (2)

$$\frac{pv}{m} = \frac{p'v'}{m'} \qquad (3)$$

Letting d represent the density of the gas, since $d = \frac{m}{v}$, we have from formula (3)

$$\frac{p}{d} = \frac{p'}{d'} = k; \qquad (4)$$

FIG. 108.

that is, **the density of a gas is directly proportional to its pressure when the temperature is constant.** This is directly shown by Boyle's experiment, for with doubled pressure the volume is diminished to one-half and the density is consequently doubled.

To study the relation between pressure and volume for pressures less than one atmosphere, Mariotte used the apparatus shown in figure 108.

A long tube of glass closed at the upper end and plunged in a deep bath of mercury contains a small mass of air or other gas. The volume of the air or gas is given by graduations on the tube while its pressure is found by subtracting the height of the mercury column *CD* from the barometric height which measures the pressure of the external air. The volume and pressure are varied by raising or lowering the tube in the bath.

200. Variations from Boyle's Law.—Boyle's law is not *exactly* true in case of any actual gas.

The following table will indicate the degree of departure from the law, with increasing pressures, of some common gases:

Volume	Pressure in meters of mercury			
	Air	Nitrogen	CO_2	Hydrogen
1	1.0000	1.0000	1.0000	1.0000
½	1.9978	1.9986	1.9824	2.0011
¼	3.9874	3.9919	3.8973	4.0068
⅛	7.9456	7.9641	7.5193	8.0339
1/10	9.9161	9.9435	9.2262	10.0560
1/20	19.7198	19.7885	16.7054	20.2687

It will be noted that air and nitrogen are slightly more compressible than if they followed Boyle's law exactly, while hydrogen is rather less compressible; the departures from the law are, however, less than 1 per cent. up to 10 atmospheres' pressure. Carbon dioxid shows marked increase in compressibility as the pressure increases and it approaches its point of condensation.

The French physicist Amagat has made an exhaustive study of the compressibilities of gases at different temperatures and up to pressures as great as 3000 atmospheres. His results show that as pressure is increased the product pv slightly diminishes at first, but when the pressure exceeds a certain amount, which depends on the gas and its temperature, the product pv steadily increases up to the highest pressures used.

The Dutch physicist Van der Waals has shown that the formula

$$\left(p + \frac{a}{v^2}\right)\left(v - b\right) = \text{constant},$$

in which a and b are small constants depending on the kind of gas, expresses quite exactly the relation of pressure to volume in gases at constant temperature for a far wider range of pressures than the simple formula of Boyle.

201. Measurement of Heights by Barometer.—The difference in pressure at two different heights in the atmosphere is equal to the weight of the unit column of air reaching from one level to the other. If the average density of the air between the two levels were known then the height could easily be ascertained by dividing the difference in pressure by the average weight of unit volume of the air.

Let H represent the height in centimeters, P and p the two pressures measured in grams per square centimeter, and d the average density in grams per cubic centimeter, then

$$Hd = P - p \quad \text{and} \quad H = \frac{P - p}{d} \tag{1}$$

As the average density of the air between the two levels depends on pressure, temperature, and moisture, it is clear that the chief difficulty lies in determining this quantity.

An *approximate* result may be obtained by assuming that the average pressure between the two levels is $\frac{P+p}{2}$. Then, if d_0 is the density of the air at standard atmospheric pressure p_0, and at the average temperature between the two stations, we have by Boyle's law

$$\frac{p_0}{d_0} = \frac{\frac{P+p}{2}}{d};$$

therefore

$$d = \frac{d_0}{p_0} \cdot \frac{P+p}{2},$$

and by (1)

$$H = \frac{2\,p_0}{d_0} \cdot \frac{P-p}{P+p}.$$

If we take the average temperature at 15°C. and neglect moisture, we find $d_0 = 0.00122$ and $p_0 = 76 \times 13.6 = 1033.6$, hence

Approximate height

$$H = \frac{2 \times 1033.6}{0.00122}\,\frac{P-p}{P+p} = 1{,}694{,}000 \cdot \frac{P-p}{P+p} \text{ cms.}$$

or

$$H = 55{,}600\,\frac{P-p}{P+p}\text{ ft.}$$

Since the final expression involves the *ratio* of $P - p$ to $P + p$, the pressures may be measured in any units whatever, centimeters of mercury or inches of mercury or whatever unit is most convenient.

Pumps and Pressure Gauges

202. Air Pump.—Air pumps were first made by Otto von Guericke, of Magdeburg, in 1650. For rapid exhaustion when a vacuum of 0.1 mm. of mercury is sufficient, a very convenient pump is Gaede's rotary air pump, shown in figure 109, in which the cylinder *A* mounted close to one side of a somewhat larger cylindrical cavity, is rapidly rotated by an electric motor and sweeps out the air from the crescent shaped space by means of two sliding vanes *ss*, which are carried in slots in *A* and are pressed against the walls of the cavity by means of springs. In this way air is drawn in at *C* and forced out at *D* finally escaping at *J*.

For higher exhaustion, pumps are used in which oil or mercury prevents leakage. In figure 110 the cylinder of the Geryk pump is

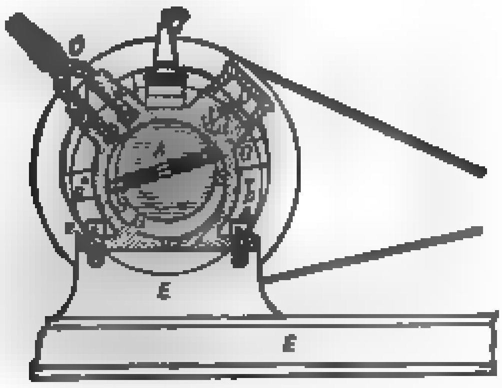


Fig. 109.—Gaede rotary pump.

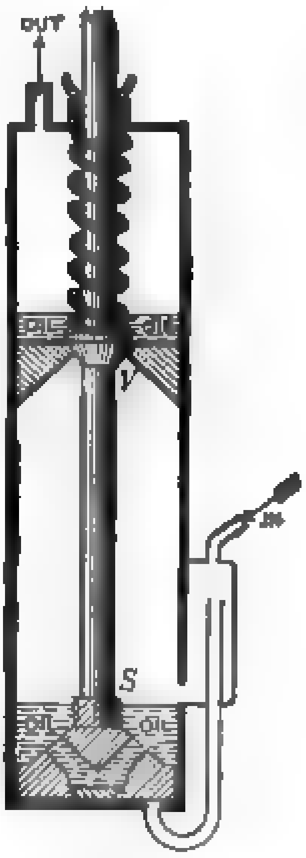


Fig. 110.—Cylinder of Geryk air pump.

shown in which a deep layer of oil covers the piston and valves so that no leakage of air back through the pump is possible. When the piston is raised the air above it is forced out through the valve *V* which is finally lifted by the shoulder *S* when he piston reaches the top, permitting the last bubbles of air to escape through the oil into the upper chamber, while at the same time oil flows down through the valve, filling the small space above the piston. In this way the air in the cylinder is *completely* expelled in each stroke.

Oil pumps for high exhaustions should never be operated without

a drying tube to absorb all water vapor from the air before it reaches the pump, as moisture absorbed in the oil prevents the securing of a high vacuum.

A most effective pump of this type is one devised by Gaede in which three cylinders, connected in series and mounted one above the other, form a single

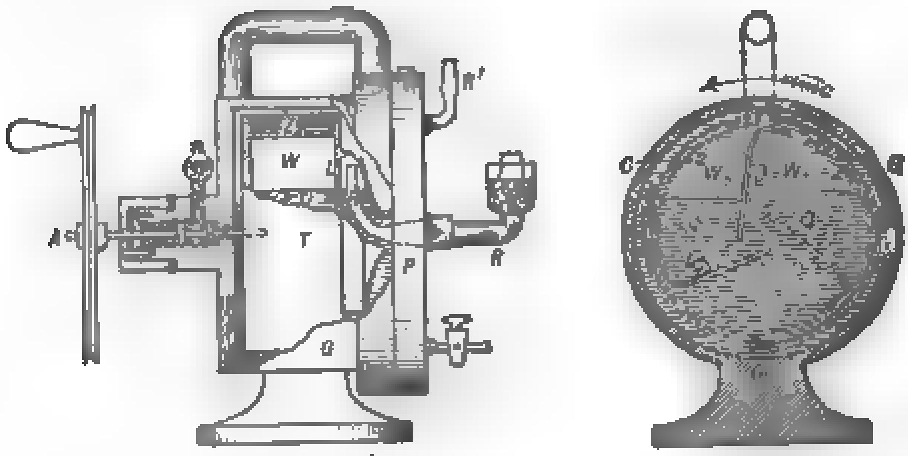


Fig. 111.—Rotary mercury pump.

long cylinder and are operated with one piston rod. Air is drawn in at the bottom and forced successively through the three cylinders and escapes at the top. Only a small amount of oil is used and the presence of water vapor does not interfere with the action as it does in most oil pumps.

203. Rotary Mercury Pump.—One of the most perfect pumps for high exhaustion is a rotary pump, also devised by Gaede, in which a peculiar spiral-shaped drum of porcelain T (Fig. 111) is rotated in a cylindrical case rather more than half full of mercury. As the spiral drum rotates in the direction of the arrow, the space W, inside the spiral and above the level of the mercury, enlarges and air is drawn in through the opening L which is connected by the curved tube R with the vessel to be exhausted. But as the motion continues L passes below the surface of the mercury into such a position as L_2 and the air that has been drawn into the spiral is caught in the space W_2 whence as the drum rotates it is driven out by the mercury through the narrow space between the turns of the spiral and escapes into the space surrounding the drum, from which it is removed by an auxiliary pump connected at R'.

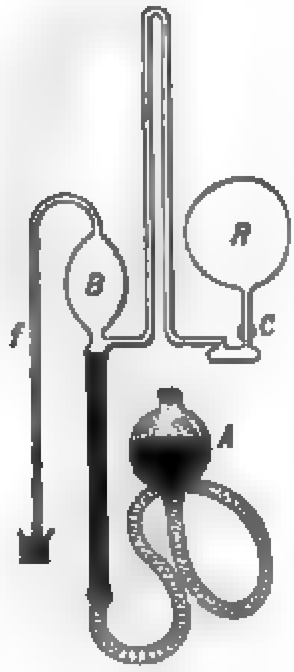


Fig. 112.—Geissler Toepler air pump.

This pump will not act unless a vacuum of a few millimeters of mercury is maintained in the space around the drum, and for this purpose the rotary pump described in the last section is very well suited, both pumps being conveniently driven by the same electric motor.

204. Mercury Air Pumps.—A simple form of air pump with which high vacua may be obtained is shown in Fig. 112. The vessel R to be

exhausted is connected with B. On raising A which is an open vessel containing mercury, the mercury rises into B driving out the air through the narrow tube f which dips into a cup containing a little mercury. On lowering A the mercury sinks out of B, but air cannot re-enter through f because the mercury rises in that tube balancing the external pressure. The air in R expands filling B again and is removed as before. The rarefaction may thus be pushed as far as desired by alternately raising and lowering A.

205. MacLeod Gauge.—For measuring the very low residual pressures in the vacua produced by air pumps, a device, shown in Fig. 113, and known as the *MacLeod gauge** is employed.

It is connected by the tube C with the exhausted vessel, so that the pressure in the bulb A is the residual pressure to be determined. The bulb D is raised causing the mercury to rise into A and C and compressing the air in A into the upper part of the narrow tube B. Suppose the air is thus compressed into one-thousandth part of the original volume $A + B$, the pressure in B will then be 1000 times the original pressure, while the pressure in the tube C is unchanged. The difference between the mercury levels in B and C will then measure the difference between the pressures, which in the case supposed is 999 times the pressure in C, so that 1 mm. difference in level corresponds to an original pressure of only 0.001 mm. of mercury.

FIG. 113.—McLeod gauge.

206. High Vacua.—In obtaining the highest vacua chemical means also are employed. Sir Humphrey Davy was the first to use this method. Having put into the vessel to be exhausted some caustic potash and then filled it with carbonic acid gas, he pumped out the gas as far as possible, and, having sealed the vessel, left the residual gas to be absorbed by the caustic potash, and thus obtained a very good vacuum. Or the tube, in which some copper filings are introduced, may be filled with oxygen and when exhausted, sealed and heated, the oxygen combining with the copper leaves a high vacuum. By these means vacua higher than a millionth of an atmosphere may be obtained.

These high exhaustions are called by courtesy *vacua*, as they are the nearest approaches to an absolute vacuum that physicists have been able to make by the most refined methods known to science; and yet there is reason to believe that in every cubic inch of such a vacuum there are 400 million million molecules of gas. To form

* Pronounced MacLeod.

some idea of the vastness of this number, we may consider that if through the side of a little glass bulb of 1 cu. in. capacity, exhausted to this extreme degree, a minute hole were to be made through which a million molecules should enter in every second, it would take 10 years for the pressure in the bulb to be doubled.

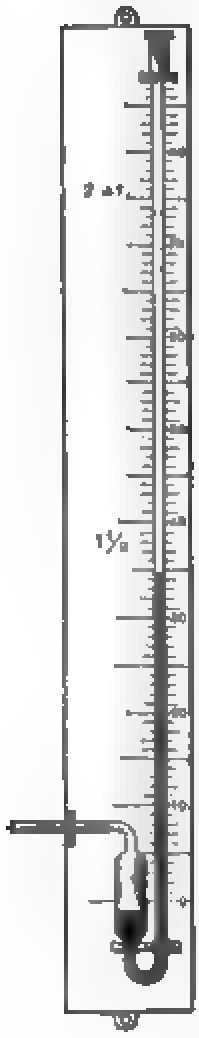


Fig. 114.—Open manometer.

The highest vacua are now conveniently obtained by enclosing in a bulb connected with the exhausted tube some fragments of cocoanut or box-wood charcoal, which when cooled to the temperature of liquid air absorbs powerfully the residual gas.

207. Pressure Gauges.—One of the simplest forms of pressure gauge is the *open manometer*. It consists of a bent tube containing mercury, one arm being open to the air and the other connected with the vessel in which the pressure is to be measured. The difference between the pressure in the vessel and that of the atmosphere is measured by the height of one end of the mercury column above the other. If the difference in pressure to be measured is very small, it is often best to use water or even kerosene oil instead of mercury on account of their small densities.

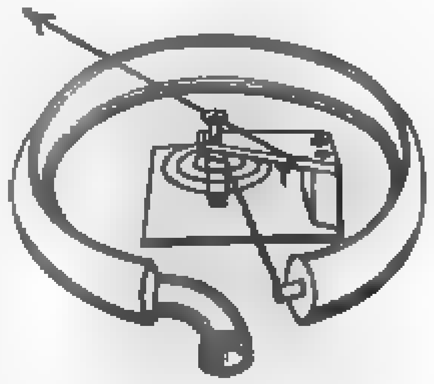


Fig. 115.—Bourdon spring gauge.

208. Bourdon Spring Gauge.—A device commonly used in steam gauges is the *Bourdon spring*, so called from its inventor. It consists of a tube of brass of elliptical section, bent into a nearly complete ring, the flatter sides of the tube forming the inner and outer sides of the ring. One end of the tube is closed and into the other the fluid under pressure is admitted by a pipe. This end of the tube is firmly fixed, while the closed end is free though connected with a pointer by levers and rack work or by a fine chain wrapped around a small spindle (Fig. 115) by which the motion is greatly amplified.

Suppose the pressure to increase, the flattened tube will spring a little and become more nearly circular in cross section,

and in so doing it will slightly unbend as if to straighten out, causing the pointer to move over the scale. When this device is employed as a steam gauge the pipe leading to it is usually bent downward so that it fills with condensed water, preventing the hot steam from reaching the gauge.

209. Common Suction Pump.—In this pump there are two valves opening upward, one in the piston and one at the bottom of the cylinder. As the piston is raised, its valve being shut, the atmospheric pressure forces water from the cistern to rise through the pipe and follow the piston, the lower valve opening and permitting this flow. As the piston descends the lower valve closes, preventing return to the cistern, and the valve in the piston opens allowing the water to pass through. Such a pump cannot raise water from a level more than about 34 ft. below the piston.

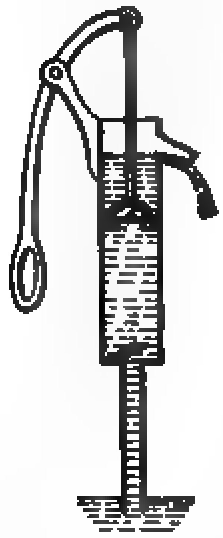


Fig. 116.—Lift pump.

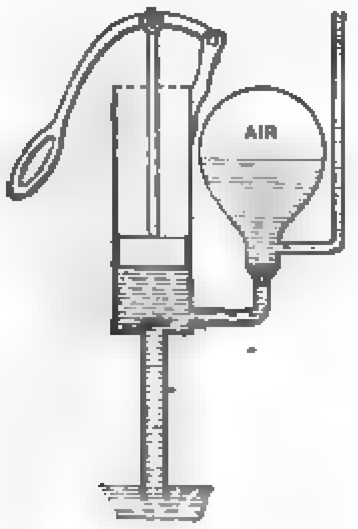


Fig. 117.—Force pump.

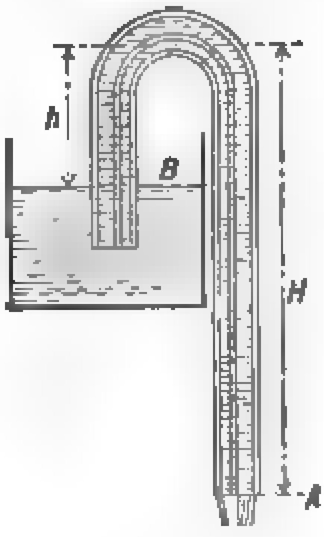


Fig. 118.—Siphon.

210. Force Pump.—Water may be raised, however, to any desired height by the use of the force pump. In this pump the water is drawn into the cylinder as in the suction pump, but the downward stroke of the solid piston forces the liquid in the cylinder out through the side tube into the rising pipe, which may be extended to any height. A valve in the side tube prevents flowing back, and an air chamber is provided which acts as a spring, the air yielding to sudden movements of the piston, which the water column on account of its great inertia could not do.

211. Siphon.—If a bent tube is filled with a liquid and one end is introduced into a vessel of the liquid while the other end is open and held at a lower level than the surface, the liquid

will escape through the tube. Such an arrangement, known as a siphon, is represented in figure 118.

The upper surface of the liquid in the vessel and also the open end of the syphon are subject to the atmospheric pressure, but this is partly balanced on the short side by the column of liquid of height h, while on the other it is opposed by the longer column H. The pressure which is effective in causing the flow is, therefore, that of a column of liquid of height $H - h$. From this it appears that the velocity of liquid through a siphon would be the same as from an opening directly into the vessel at the level of the outer end of the siphon, if it were not for the loss due to friction in the pipe.

Clearly the liquid can only rise in the siphon to a height where it can be supported by the atmospheric pressure; water, therefore, cannot be lifted by a siphon more than 34 ft. above its level and mercury not more than 30 in.

Problems

1. How high would the atmosphere have to be to cause the barometer to stand 76 cm. high, if its density was the same throughout as at the earth's surface, taking this density as 0.0012 gms. per c.c.
2. How much higher will a barometer stand at the base of a mountain than at a station 1000 meters higher; taking the average density of air between the stations as 0.0012.
3. The air chamber of a force pump contains at the start 600 cu. in. of air at pressure 75 cms. of mercury. What volume will the air occupy while water is being forced to a height of 150 ft. above the pump?
4. How deep must a pond be that an air bubble on reaching the surface may have twice the volume that it had at the bottom? Suppose the barometric pressure at the surface to be 75 cm. of mercury.
5. How deep must a pond be when a bubble having a volume of 12 c.c. at the bottom has a volume of 30 c.c. as it reaches the surface. Barometer reading 75 at the surface.
6. A barometer on top of a tower stands at 75.20, at the bottom it stands at 75.40. How high is the tower if the average density of the air between the top and bottom is 0.0012 gms. per c.c.?
7. A barometer having a little air in the top of the tube stands at 72; but if the level of the mercury is raised so that the air space is half as great as before, it stands at 70. What is the correct barometric height.
8. If the tube in the apparatus shown in figure 108 contains 100 c.c. of air, and the mercury stands in the tube 15 cm. above the level in the outer vessel, while the barometer stands at 75, find what would be the volume of the enclosed air if it were at atmospheric pressure, also what will the

volume of the enclosed air become when the tube is raised sufficiently to make the mercury stand 20 cm. high inside the tube?

9. A glass bottle containing 100 c.c. of air floats at the surface of a pond with its open mouth downward. The bottle weighs 130 grm. and the density of the glass is 2.6. If the barometric pressure is 75 cm. of mercury, how deep below the surface must the bottle be pushed that it may just float in equilibrium, neither tending to rise nor sink? Neglect the weight of the enclosed air. Will the equilibrium be stable or unstable and why?

10. What force must be exerted on the piston of a force pump 3 in. in diameter to raise water 100 ft.?

PART II.—FLUIDS IN MOTION

212. Steady Flow.—When a fluid is in motion if the pressure, velocity and direction of flow remain unchanged at every point in a certain region, the motion there is said to be steady. A line drawn in the fluid so that at every point it is in the direction of the flow at that point, is called a *stream line.*

213. Continuity.—In case of steady flow as much fluid must flow into any region as flows out of it in the same time.

Let the figure represent either an open channel or a pipe conveying water. The total volume of water crossing the section of A per second will be vs cu. ft. per second if the velocity is v ft. per second and the cross section of the stream at that point is s sq. ft.

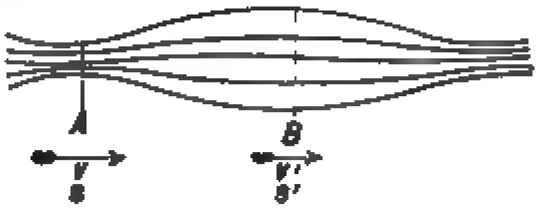


Fig. 119.

If d represents the density at A, or the number of pounds mass per cubic foot, then vsd is the mass of water crossing A per second and similarly $v's'd'$ is the corresponding mass crossing B in the same time, and therefore $vsd = v's'd'$. This equation holds for the steady flow of any fluid whether gas or liquid. But for liquids since the density does not appreciably change during the flow, we may take $d = d'$ and so

$$vs = v's'$$

or the velocity is inversely as the cross section of the stream. If at a narrow place in a stream the velocity is not correspondingly great, we may be sure that the stream is deep at that point. The extremely small cross section of a stream at the edge of a dam is due its great velocity at that point.

214. Momentum of Liquid Stream.—When a liquid is in motion each moving particle has momentum and kinetic energy. When a jet escapes through an opening in the side of a vessel the pressure which gives the jet its forward momentum acts at the same time as a reaction pressing the vessel in the opposite direction. If the orifice is free to move backward it will do so, as in case of the device known as Barker's mill shown in the figure. In case of the end of a hose the rush of water around a curve will by its centrifugal force tend to straighten the hose. If the end is free it will very probably swing over too far, in consequence of its inertia, when it will be flung back again, thus thrashing to and fro.

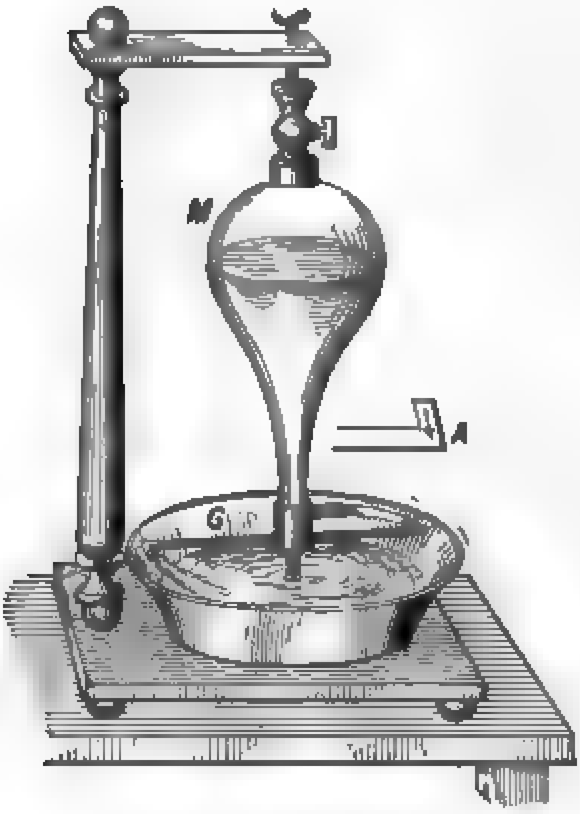


FIG. 120.—Barker's mill.

215. Turbine Water Wheels.—The centrifugal force of a stream as it moves by curved guides is made use of as a means of obtaining power in turbine water wheels. Such a wheel is shown in section in the diagram. The water flows inward toward the wheel through the fixed guides, which cause it to enter in the proper direction, and then driving the wheel forward and sweeping by the wheel guides *BB*, it escapes at the center of the wheel. The guides *AA* may be made adjustable so as to regulate the flow of water. The entering water from the flume is conducted to the turbine by a pipe which is kept constantly full, thus giving the advantage of its pressure. The turbine may be set at the

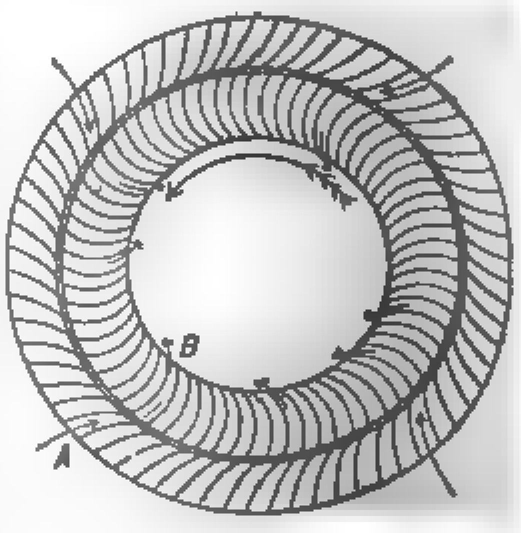


FIG. 121.—Turbine water wheel diagram.

lowest level so that the water escapes directly into the tail race, or it may be set higher if the water escaping from the wheel enters a closed draft pipe which leads down to the tail water.

The sinking of the water in this draft pipe produces a suction which increases the efficiency of the wheel. In the great 5000 H. P. turbines in use at Niagara the water enters the wheel from below in such a way that the weight of the wheel and shaft are almost exactly balanced by the upward pressure of the water, making the friction in the bearings extremely small.

216. Efficiency of Water Wheels.—When water flows from one level down to another it loses potential energy. That proportion of the potential energy lost by the water which is transformed into useful work in a water wheel is called its efficiency. It is clear that to be efficient a wheel must as far as possible let the water down from the higher to the lower level without dashing, and the water escaping at the bottom should have little velocity, its energy having been expended in useful work.

217. Various Water Wheels.—The old-fashioned overshot wheel, taking water from the upper level and lowering it to the bottom of the fall, uses the whole energy of the fall, but its size and weight cause great frictional loss.

Where a small supply of water at high pressure is available, some form of jet wheel is often best. Here the wheel is driven at high speed by the force of a jet escaping against cups set around the periphery of the wheel.

218. Hydraulic Ram.—The hydraulic ram is an appliance by which a small quantity of water may be raised a considerable height by using a small fall in a stream. The water is conducted to the ram through a straight, smooth, inclined pipe offering little resistance to the flow. At *C* is a valve opening *downward* through which the water at first escapes; but as its speed increases, it catches the valve in its rush and shuts it. This sudden stoppage of the stream causes a great pressure at this end of the pipe in consequence of the forward momentum of the stream, and the valve *d* which opens *upward* is forced open and some water driven into the pipe *e*. The valve *d* then closes and prevents any return of water from *e*. But with the sudden stoppage of the stream the valve *C* if properly weighted rebounds and opens

again, the stream again escapes at C with increasing velocity until the valve is again caught and closed, when water is again driven through the valve d by the hammer-like blow of the column of water in A. The action is thus kept up indefinitely, water being gradually forced up the pipe e until it may reach many times the height through which the stream falls. The air

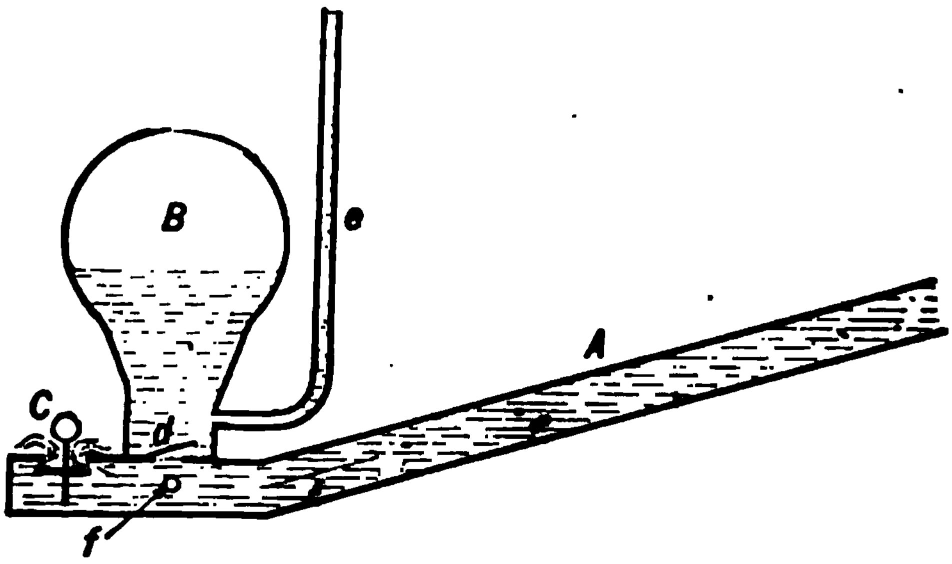


FIG. 122.—Hydraulic ram.

chamber B is essential to the action of the ram as it presents an elastic cushion with but little inertia, enabling the valve d to yield instantly. At f there is a minute opening, the *air sniff*, through which, in the recoil of the water, air is drawn in, maintaining the supply in the air chamber. *If a hydraulic ram were perfectly efficient*, it would raise one-tenth of the amount of water flowing into it through ten times the height of the fall or one-half the water twice the height of the fall. But in practice the efficiency of a good ram is about 50 per cent.

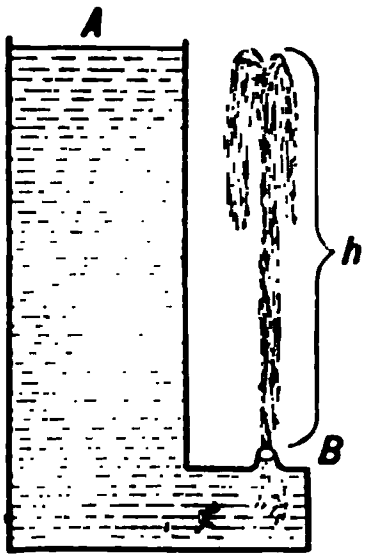


FIG. 123.

Rams are now made in which the supply pipe is as much as 4 ft. in diameter. In these rams the valve which arrests the flow is moved by a piston operated by water from a small branch of the main pipe.

219. Velocity of a Jet.—While a liquid is escaping from a vessel through an opening which is small compared with the upper surface of the liquid, no *change* takes place within the vessel except the gradual lowering of the surface or disappearance of liquid from the top, while a corresponding mass appears outside in the escaping jet. If no energy is lost in friction or

viscosity, the energy of a mass escaping at B must be the same as the energy of an equal mass at A. But since the potential energy at B due to gravity is less than at A the kinetic energy at B must be correspondingly greater; that is, it must be great enough to cause the escaping mass to rise from B to A when the jet is directed upward.

If h is the height of A above B we have—

The difference between the potential energy of a mass m at A and $B = mgh$ ergs.

The kinetic energy of mass m escaping at B with velocity $v = \frac{1}{2}mv^2$ ergs.

Therefore

$$mgh = \tfrac{1}{2}mv^2$$

and

$$v^2 = 2gh \tag{1}$$

This velocity is the same as that which a freely falling body would acquire in a distance h, a conclusion known as *Torricelli's theorem.*

Torricelli's Theorem.—The velocity of an escaping jet is equal to the velocity which a body will acquire in falling from the level of the upper surface to that of the opening.

The density of the liquid and direction of the jet do not affect its velocity.

When the pressure alone is known, the height of the escaping liquid required to produce the given pressure may be calculated and then used in the above formula. Thus the pressure on the level of B is $p = hdg$ in *dynes;* using this to eliminate h from equation (1) we obtain

$$v^2 = \frac{2p}{d} \tag{2}$$

220. Vena Contracta.—The liquid as it approaches the opening moves in from all sides along ***stream lines*** like those shown in the diagram. Liquid coming from each side has a certain momentum toward the axis of the jet, hence the jet narrows and does not become cylindrical until just after it has left the orifice. A short cylindrical neck of the size of the opening is found to increase the

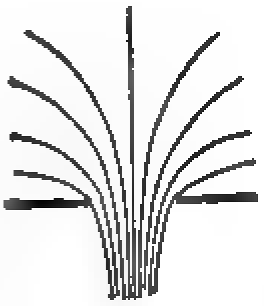


Fig. 124.

quantity escaping per second, and if the neck is somewhat flared out the flow is still greater.

221. Efflux of Gases.—The velocity with which a gas escapes through a small opening when the difference between the pressures on the two sides of the opening is p, is also determined by Torricelli's theorem.

$$v = \sqrt{\frac{2p}{d}}.$$

Since for a given pressure the velocity of efflux is inversely proportional to the square root of the density of the gas, the densities of gases may be compared by observing the times in which measured quantities escape through a small opening.

222. Energy Due to Pressure.—**When a liquid is forced into a vessel against pressure, the work done is equal to the product of the pressure by the volume of the liquid which is introduced.** This expenditure of work is not wasted in friction, but exists as energy in the mass, ready to be transformed into energy of motion if an opening allows the mass to escape. The amount of this energy, since the volume of the mass m, equals $\frac{m}{d}$ is

$$E = \frac{pm}{d}.$$

223. Energy Equation.—Consider a small mass of liquid at a in the vessel shown in the diagram; it is in equilibrium, and may be moved without offering any resistance from a up to the surface. Clearly there is no change in its total potential energy as it is moved from one part of the vessel to another. At the top its gravitation potential energy referred to the earth is a maximum, but then it has no energy due to pressure, while at a its gravitation energy is less but its pressure energy is correspondingly greater. If h represents the height of the mass m above some fixed plane, say the surface of the earth, its gravitation potential energy referred to that plane is mgh. We have seen that its pressure energy is $\frac{mp}{d}$; and if the mass is in motion it will have kinetic energy $\frac{1}{2}mv^2$, and its total energy may be written

$$\frac{mp}{d} + mgh + \tfrac{1}{2}mv^2 = \text{energy of mass } m.$$

If the stream is flowing in conduits or channels without doing work, the energy of the mass will remain constant except as it is wasted in internal or external friction. The fact that in steady irrotational motion of a frictionless fluid, the expression $\frac{mp}{d} + mgh + \frac{1}{2}mv^2$ remains constant for a little mass m as it moves along, is known as *Bernoulli's Principle*.

As an illustration of the above equation, conceive the vessel A in the figure to be kept filled to a constant level while the liquid is flowing out freely through the pipe D, and follow the changes in the mass m. As it moves

downward h grows less and so its gravitation energy diminishes while the pressure energy increases, the kinetic energy being scarcely changed; but as it approaches the opening B its velocity increases and consequently more of its energy is kinetic and less due to pressure than at the same level farther in the vessel, the pressure at B must then be less than at b. When it reaches C its velocity will be less, and consequently the pressure there will be greater than at B. Throughout D the cross section, and consequently the velocity, is constant, and since it is all at the same level the pressure must be constant except as influenced by friction in the pipe.

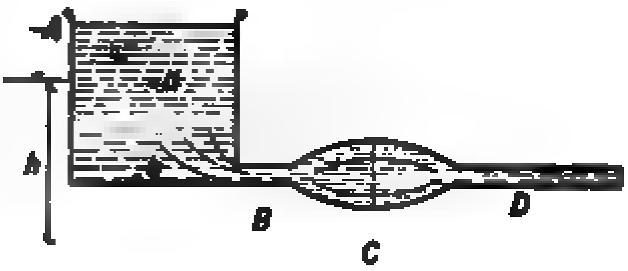


Fig. 125.

224. Friction in Pipes.—When water escapes from a reservoir through a horizontal pipe of uniform section, as ab in figure 126, the velocity will be the same at all points in the pipe, and *if there is no friction* the pressure will be constant throughout the length of the pipe and equal to the atmospheric pressure at the end b. In that case the water will not rise in any of the gauge tubes shown. In practice, however, there is always some friction in a pipe, and, therefore, a constant expenditure of energy. But the energy equation is

$$E = \frac{mp}{d} + mgh + \tfrac{1}{2}mv^2,$$

and if the pipe is level and cylindrical h and v cannot change, consequently if there is any decrease in E there must be an equal decrease in the first term $\frac{mp}{d}$, and therefore, a *fall in pressure*. If the friction is uniform throughout the pipe the pressure will decrease uniformly, becoming equal to the atmospheric pressure at the opening b. The height h (see figure) which determines the pressure at b when the opening is stopped up so that there is no flow, is called the *pressure head*. When b is open the head required to produce the observed velocity of escape reckoned from the law $v = \sqrt{2gh}$, is called the *velocity head*. In the above case it is h', and the remaining head $(h - h')$ is spent in overcoming friction.

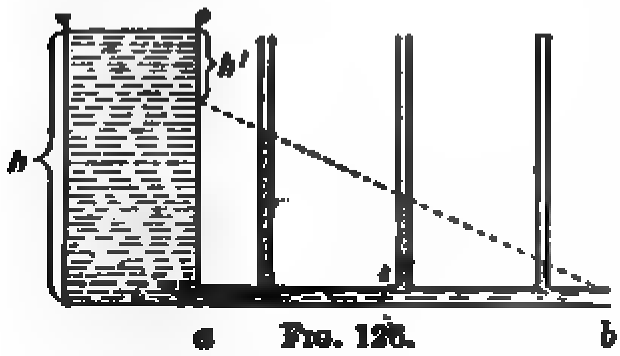


Fig. 126.

The loss of pressure when water is flowing in pipes is a fact that has to be constantly taken into account in practice. The friction and consequent loss of pressure, increase with the velocity of flow.

225. Pressure Varies with Velocity.—The fact just demonstrated that in a horizontal pipe of variable section the pressure

will be greatest when the cross section is the greatest and velocity least is so interesting and important that it merits a brief examination from another point of view.

Consider a little mass of liquid at A (Fig. 127) where its velocity is clearly diminishing. It is under pressure on all sides, but since its velocity is diminishing the pressure backward on its forward side must be greater than the pressure on its left which urges it forward. When the mass reaches B, however, its velocity is increasing, hence the pressure behind it which urges it forward must be greater than the pressure in front which is opposite to its motion; the point of slowest motion must therefore be a point of maximum pressure.

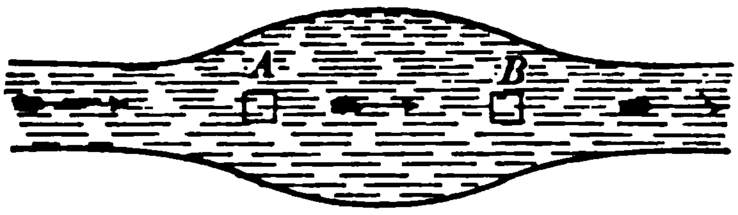


Fig. 127.

226. Aspirating Pumps.—The principle just established is made use of in aspirators for exhausting air. Such an instrument is shown in figure 128. It provides a narrow channel through which water flows with great velocity, the stream widening out and moving slower before it reaches the atmospheric pressure at the open end. The pressure at the narrowest point must then be very much less than that of the atmosphere, and air is accordingly drawn in through the side tube C and carried out at B by the rush of water.

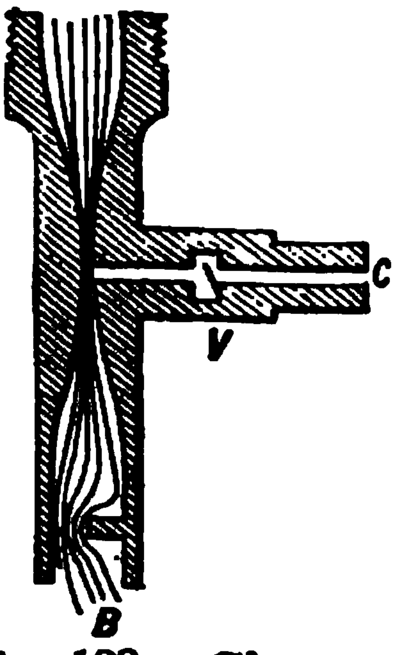


Fig. 128.—Chapman exhaust pump.

227. Ball on Jet, etc.—A jet of water or even of air may support in stable equilibrium a light ball. The explanation is that a slight shifting of the ball, say to the right, would cause the main stream to rush on the left, the velocity of flow would be greatest there, and therefore, the pressure less than on the right, and so the ball would be pressed back again.

A card with a pin through it and laid over the open end of a spool cannot be blown off by blowing through the spool because the velocity of the air stream as it spreads out under the card is least at the outer edge where it comes to the atmospheric pressure, the pressure nearer the center where the velocity is greater

will, therefore, be less than that of the atmosphere and the card will accordingly be pressed against the end of the spool.

Similarly a ball will be held in a cup by a jet escaping around it, as in the ball nozzle used for fire hose.

If a stream of air is directed between two sheets of paper they are drawn together. So also leaves and other light objects are drawn *toward* a moving train as it passes.

Problems

1. Find the velocity of the stream of water in a pipe having a cross section of 3 sq. in. and discharging 450 cu. ft. of water per hour.
2. How much water will escape per minute from a 2-in. hole in the side of a water tower 50 ft. high.
3. With what velocity will water spurt out of a hole in a boiler in which the pressure is 80 lb. to the sq. in., in addition to atmospheric pressure?
4. The stream of water below a certain dam has a cross section of 10 sq. ft. and a velocity of 5 ft. per sec. Find the horse-power available if the dam is 15 ft. high.
5. What horse-power would be obtained from a 20-ft. fall by a turbine wheel of 80 per cent. efficiency, when the flow is 300 cu. ft. per minute?
6. If the water escaping from a turbine water wheel which uses the water from a 10-ft. fall has a velocity of 6 ft. per sec., what is the greatest possible efficiency of the wheel?
7. If the efficiency of a hydraulic ram is 50 per cent., how much water per day will it raise to a tank at a height of 100 ft. above the ram, when the supply pipe has a fall of 8 ft. and discharges 1 gallon per minute?
8. While water is flowing with a velocity of 2.2 ft. per sec. in a pipe 1 in. in diameter the pressure drops off from 70 to 10 lbs. per sq. in. in a length of 500 ft. Find the energy in foot-pounds spent in overcoming friction per cu. ft. of water.
9. Find the horse-power spent in friction in the 500 ft. length of pipe specified in problem 8.
10. Derive formulas (1) and (2) of §219 for the velocity of an escaping jet, from the energy equation of §223.
11. A horizontal water pipe of 1 sq. in. cross section widens out to 3 sq. in. in section. If the velocity is 5 ft. per sec. in the narrower pipe and the pressure 5 lbs. to the sq. in., what will be the pressure in the adjoining part of the wider pipe? Ans. 5.14 lbs. per sq. in.

The pressure given is *gauge pressure*, or the excess above that of the atmosphere. The total or absolute pressure is 5 + 14.7 = 19.7 lbs. per sq. in.

12. The nozzle of a fire hose has an opening 2 in. in diameter, while the pipe just back of it is 3 in. in diameter. Find the pressure just back of the nozzle when it can throw a jet 60 ft. vertically upward. Ans. 20.9 lbs. per sq. in.

Note.—At the opening of the nozzle the pressure is that of the atmosphere, or 14.7 lbs. per sq. in. absolute, while the velocity is found from the height to which the water is thrown. Use energy equation of §223.

PROPERTIES OF MATTER

AND ITS

INTERNAL FORCES

STRUCTURE

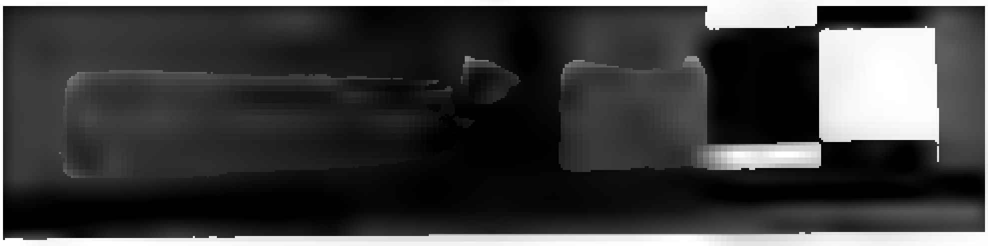
228. Density.—On comparing a block of wood or aluminum with an equal weight of lead or gold, it is clear that substances differ greatly in the quantity of matter concentrated in a given volume. *The mass of any substance contained in unit volume is known as its density.*

Densities of some Substances in Grams per Cubic Centimeter

Solids		*Liquids*		*Gases at 0°C. and 1 Atm.*	
Aluminum . .	2.7	Mercury	13.596	Air	0.001293
Iron	7.2–7.8	Sea water	1.026	Oxygen . .	0.001430
Tin	7.3	Water at 4°C.	1.00	Nitrogen.	0.001256
Copper	8.8	Alcohol	0.8	Hydrogen	0.00008988
Lead	11.4	Ether	0.72		
Gold	19.3				
Silver	10.5				
Platinum . . .	20.5–22.0				
Brass	8.3–8.6				
Glass	2.5–3.5				
Oak wood . . .	0.84				

229. Molecular Forces.—When a lead bullet is divided by a clean cut, if the two halves are pressed together they will cling with considerable force. This is an imperfect exhibition because of poor contact of the force which originally held the two parts together. This force is known as cohesion, or when the attraction is between different substances it is known as adhesion. A drop of water is held together by cohesion, but it clings to a glass rod by adhesion.

230. Molecular Theory.—All matter is conceived as made up of separate *molecules* which are the smallest portions of the substances that can exist in a free state, as in gas or vapor. It is be-



lieved that the molecules of any particular substance are all alike, and, in substances not at the absolute zero of temperature, are in more or less active motion or vibration, the energy of vibration depending on the temperature. In solids the vibrating molecules are held by their mutual attractions in such a way that they cannot move far away from their mean relative positions. In liquids the phenomena of diffusion, and the Brownian movement (§273), show that molecules move about in the mass, and are not held in fixed positions relative to each other, though the force of cohesion may be very great. In gases or vapors there is the greatest freedom of motion of the molecules, and their average distance apart is much greater than in liquids or solids, while there is scarcely any cohesion.

It is supposed that any two molecules of matter attract each other, according to the Newtonian law of gravitation, with a force varying inversely as the square of the distance between them for all considerable distances, but when very near each other the force of attraction varies with the distance according to some unknown law, giving rise to the phenomena of cohesion and adhesion, until the molecules come into what is called contact, when a force of repulsion opposes nearer approach.

The experiments of Quincke indicate that molecules must be less than 5×10^{-6} cm. apart in order that the cohesive force may be perceptible.

The idea that matter is molecular in its structure is supported by a great variety of evidence found especially in the phenomena of heat, gases, and radiation, as well as in chemical phenomena.

231. Molecular Equilibrium.—The molecules of a substance may be regarded as in a state of equilibrium under three forces:

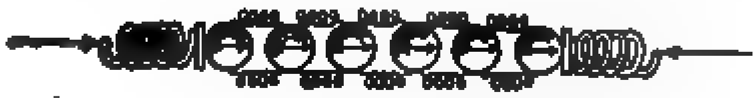


FIG. 129.

external pressure, cohesive force, and an internal pressure due to the rebounding of adjoining molecules against each other as they vibrate to and fro. This latter force may be considered to balance the other two.

We may form a conception of these forces by the following model. Imagine a row of small rubber balls drawn together by springs stretched between them (Fig. 129). Let two outer springs also press them together, and let the balls be thought of as rapidly vibrating to and fro, rebounding against each other, and so keeping a greater distance apart than if they were at rest. The force of the outer springs represents the external pressure and that of the springs joining the balls together the cohesive force,* and these are balanced by the repulsion due to the impacts of the balls against each other.

In solids and liquids the pressure due to cohesion is that which chiefly balances the internal repulsion, the external pressure being usually quite insignificant in comparison.

But in gases the case is different. In consequence of the great average distance between the molecules, the cohesion is so insignificant that the external pressure alone may be said to balance the internal pressure due to the motions of the molecules.

This theory of gaseous pressure is more fully discussed in §270 *et seq.*

232. Structure.—When the properties of any one portion of a mass are exactly like those of any other portion, the mass is said to be homogeneous. Whether a substance is called homogeneous or not depends on the point of view. One part of a brick wall is just like another part, and so it may be said to be homogeneous; but if we compare minute parts we find in some spots brick and others mortar and so there is a limit to its homogeneity. So water is regarded as homogeneous unless we are dealing with portions so small that the molecular structure is significant.

If the various physical properties of a substance are the same in all directions throughout its mass, it is said to be *isotropic*. Water, glass, and mercury are isotropic. Most crystalline substances are not isotropic, and may be called *anisotropic*.

233. Crystals.—In solids which pass slowly into the solid state, either directly from vapor or as the result of the slow cooling of a fused mass or of separation from a solution, there are often formed masses called crystals which have regular and distinctive forms and are bounded by plane faces.

The crystallization begins at certain isolated points and the minute crystals gradually grow in size, until they may meet and form a solid agglomeration.

* The springs representing the cohesive force should be conceived as exerting less force the more they are stretched, for cohesive force diminishes as particles separate.

The study of the fundamental crystal forms has led mineralogists to divide them into six classes or systems.

Some idea of the cause of the formation of crystals may be obtained by considering the forms which may be built up of shot when placed together so that each shall touch as many others as possible. Suppose such a pyramid as that represented in figure 130, where one layer is incomplete; if we think of it as a growing crystal in which the balls represent the molecules and suppose it immersed in a medium in which there are free molecules surrounding it, there will clearly be a tendency for these to fill out the incomplete surface, for a molecule will touch more neighbors when placed along the incomplete edge than anywhere else, and so may be conceived to be more powerfully attracted into that position. In consequence a figure bounded by plane surfaces would result.

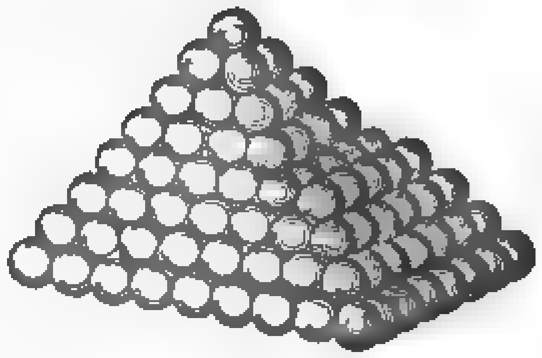


Fig. 130.

The piling of balls would give the crystal forms characteristic of the first or regular system, but to explain the variety of crystal groups it is necessary to suppose that the molecules themselves have properties different in one direction from what they have in another, and that when built up into crystal forms they are all similarly oriented or directed.

Elasticity and Viscosity

234. Stress and Strain.—When a portion of matter is acted on by forces tending to change its size or shape it is said to be under *stress*, and the accompanying distortion or change in volume is called the *strain*.

A stress tending to stretch any portion of matter is called a *tension*, while a stress tending to shorten it is called a *pressure*.

Stress is measured by force per unit surface, as in pounds per square inch, or in grams or dynes per square centimeter.

235. Strain Ellipsoid.—When a body is strained, a small spherical portion of it is in general distorted into an ellipsoid, and the axes of the ellipsoid are the three principal directions of strain at that point.

When the strain is the same everywhere throughout a body, as in case of a stretched wire, it is said to be *homogeneous*. In such a case the strain ellipsoids are all alike and similarly situated, as shown in figure 131.

When a fluid is compressed the strain is homogeneous and the ellipsoids are spheres slightly smaller than in the unstrained state.

The distribution of strain in a bent beam is shown by the ellipsoids in figure 132. The strain in this case is not homogeneous and there is a surface of no strain indicated by the dotted line.

236. Resistance to Strain.—A body is said to be *elastic* if after having been strained it springs back to its original form when the stress is removed. If the stress is the same for a given

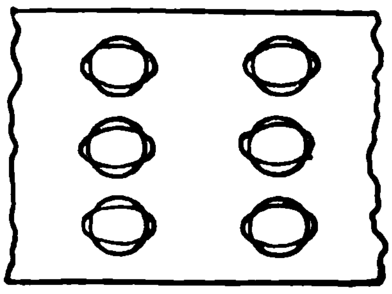


FIG. 131.

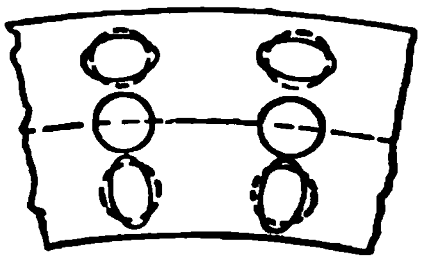


FIG. 132.

amount of strain whether the strain is increasing or diminishing, the body is said to be *perfectly* elastic.

When strained beyond a certain point called *the limit of elasticity*, substances yield permanently and do not return to the original state when the straining forces are removed. In this case there may be a great internal stress while the body is being strained, but on a very slight diminution of strain the stress entirely disappears. Putty, wet clay, and lead all exhibit this permanent distortion under comparatively small forces and even when the strain is small; while india-rubber is remarkable for the great strain which it can experience without passing its elastic limit. It is said to have a *wide* limit of elasticity.

Even within the limits of elasticity most substances show a time lag in returning to their original state after having been strained. Thus when a steel wire is firmly clamped at its upper end, if the lower end is twisted through an arc well within its limit of elasticity, the wire when set free returns at once nearly to its original position, but creeps very slowly back through the re-

maining distance. This lag is found in metals and in glass, but quartz fibers are remarkably free from it.

237. Hooke's Law.—*In small strains of elastic bodies the stress is proportional to the strain.* This is known as Hooke's law, having been enunciated by him in 1676. According to this law, a long spring when stretched 2 cm. will exert twice the force that it would if stretched 1 cm., and the tension required to stretch a spring a small distance is equal to the pressure when the spring is compressed an equal amount.

Careful experiment, however, shows that the law is not exactly true. Most substances offer slightly more resistance to a given small compression than to an equal extension.

An illustration of this law is afforded by the ordinary spring balance in which equal divisions of the scale correspond to equal increments of weight. In this case the elongation or compression of the helical spring may be relatively very great, yet because of its shape the distortion or strain of any little portion is extremely minute and Hooke's law holds very nearly true.

238. Elasticity.—In elastic bodies the elasticity is measured by the ratio of the stress to the corresponding strain.

$$\text{Elasticity} = \frac{\text{stress}}{\text{strain}}.$$

In bodies which are homogeneous and isotropic there are two principal kinds of elasticity, that in virtue of which the body resists change of volume and that resisting change of shape.

The first is called volume elasticity and the second rigidity. Volume elasticity is possessed by all bodies, fluids as well as solids, but rigidity is a characteristic of solids.

In some strains both of these elasticities are involved; for instance, when a wire is stretched there is a sidewise contraction as well as an elongation, so that the resistance to stretching depends on both the rigidity and volume elasticity of the substance. The elasticity of stretching or compression is so important in engineering that it has received a special name and is known as Young's modulus.

239. Volume Elasticity.—When a body is so strained that every little cubical portion is compressed into a smaller cube the corresponding stress must be a pressure equal in all directions,

provided the substance is isotropic or equally compressible in every direction.

This kind of stress is called hydrostatic pressure because it is the only kind of stress that can exist in fluids at rest.

The *volume elasticity* or *bulk modulus* of a substance is the ratio of the increase in pressure to the corresponding compression per unit volume.

Thus this elasticity will be represented by

$$E = \text{Volume elasticity} = \frac{\text{pressure increase}}{\text{change in unit volume}} = \frac{p}{\frac{v}{V}} = V\frac{p}{v},$$

where p is the increase of pressure causing a contraction v in a total volume V.

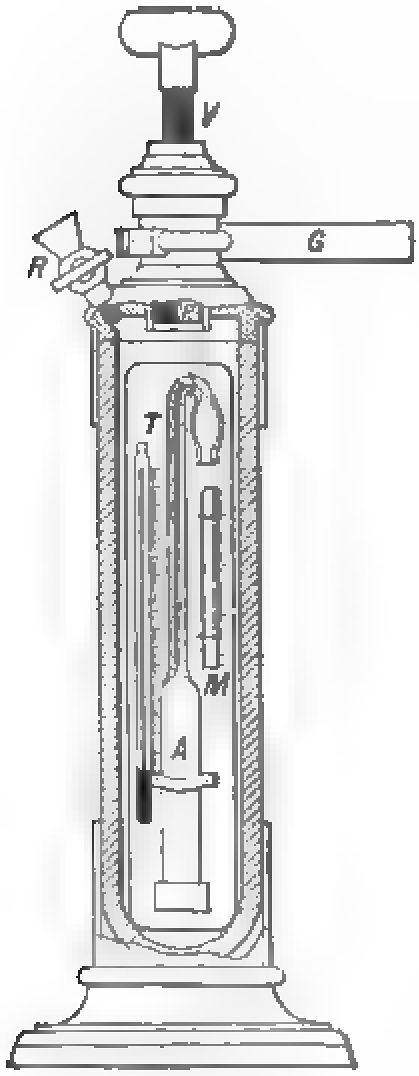


Fig. 133.—Oersted's piezometer.

The volume elasticity a of solid may be found by subjecting a long bar of the substance to hydrostatic pressure in a strong tube having thick glass windows through which its change in length may be observed by fixed microscopes.

240. Compressibility of Liquids.—Liquids are, as a rule, somewhat more compressible than solids, but on the other hand so great is their resistance to compression that for most practical purposes they may be treated as if incompressible.

The compressibility of a liquid may be measured by the apparatus shown in figure 133, known as Oersted's *piezometer*. In this instrument the liquid to be tested is contained in a bulb of glass terminating in a long narrow tube of uniform diameter, open at the end and carefully graduated. This bulb A is surrounded by water in a stout cylindrical vessel of glass and subjected to pressure by means of a piston forced in by a screw. A globule of mercury in the narrow tube separates the liquid in the bulb from the surrounding water. From the number of scale

divisions through which the mercury moves down toward the bulb as pressure is applied, the *apparent* compressibility of the contained liquid is determined, the relation between the volume of the bulb and the volume contained in one division of the capillary tube having been previously ascertained. Although the pressure is the same on the outside of the bulb as on the inside, its volume *diminishes* in consequence of the compression of the glass of which it is made, so that the experiment gives the *difference* between the compressibility of the liquid and that of the glass bulb.

A thermometer gives the temperature of the liquid examined, and the pressure may be determined from the amount of compression observed in a tube M containing air and placed open end downward in the cylinder.

241. Elasticity of Gases.—In case of a gas it is necessary to distinguish between its elasticity when the temperature is kept constant during the compression, and its elasticity when compressed so suddenly that there is no time for the flow of heat to take place. The first is called isothermal elasticity and the second adiabatic elasticity; the latter is always greater, being, in case of air, oxygen, hydrogen, and nitrogen, about 1.40 times as great as the isothermal elasticity.

The isothermal elasticity of a gas may be calculated from Boyle's law. Suppose the pressure is increased from p to p', the decrease in volume will be $v - v'$ and we have

$$E = v\frac{p' - p}{v - v'} \tag{1}$$

But by Boyle's law $pv = p'v'$, therefore

$$\frac{p}{p'} = \frac{v'}{v}$$

and

$$\frac{p' - p}{p'} = \frac{v - v'}{v}$$

substituting in (1) we find

$$E = p'.$$

But the difference between p and p' is supposed extremely small, so that *for gases kept at constant temperature the volume elasticity is equal to the pressure.*

Volume Elasticity and Compressibility

Substance	Temp.	Compressibility in millionths of volume per atmosphere	Volume Elasticity	
			Dynes per sq. cm.	Lbs. per sq. inch
Steel	......	0.55	188.00×10^{10}	27.00×10^{6}
Glass	......	2.44	41.00×10^{10}	6.00×10^{6}
Mercury	0°	3.00	33.00×10^{10}	4.8×10^{6}
Glycerin	20°	25.00	4.0×10^{10}	0.58×10^{6}
Water	20°	46.00	2.2×10^{10}	0.32×10^{6}
Ether	20°	191.00	0.52×10^{10}	0.07×10^{6}
Air {At normal pressure}		1,000,000.00	1.00×10^{6}	14.7

242. Rigidity.—If a cylindrical rod or wire is twisted about its axis without change of length it may be imagined divided into sections of equal thickness, in each of which there has been no change in volume but simply a distortion of the little elements of which it may be conceived as made up.

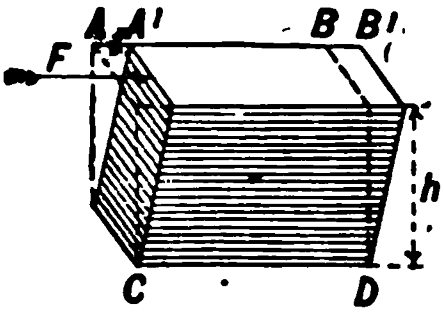


FIG. 134.

Take such a little block as that represented in figure 134. If the base CD is firmly fixed, a force F applied to the upper surface will strain it into the position $A'B'$, just as a thick book lying on a table may be pushed out of shape by force applied to the upper cover. The strain in this case is a pure distortion without any change in volume and is called a *shear*, and the forces bringing it about constitute a *shearing stress*. The strain is measured by the ratio of the displacement AA' or x to the height h, while the stress is the force applied per unit area; or if S is the area of the upper surface of the block the stress is $\frac{F}{S}$. The rigidity n, or elastic resistance to distortion, may therefore be expressed thus:

$$\text{Rigidity} = n = \frac{\frac{F}{S} = \text{stress}}{\frac{x}{h} = \text{strain}} = \frac{Fh}{Sx} \qquad (1)$$

In case of a wire clamped at one end and twisted at the other,

it may be mathematically demonstrated that the moment of the force of torsion T is expressed by the formula

$$T = n\frac{\pi r^4 a}{2l},$$

where n is the coefficient of rigidity of the substance of which the wire is made, r is the radius of its cross section, and l its length, while a is the angle (in radians) through which it is twisted.

By measuring the moment of force required to twist a given wire through a measured angle, the coefficient of rigidity of the substance of which the wire is made may be determined by the use of this formula.

Rigidity

Steel..........	82×10^{10} dynes per sq. cm.,	12.0×10^6 lbs. per sq. inch.
Brass..........	38×10^{10} " " " "	5.5×10^6 " " " "
Glass..........	24×10^{10} " " " "	3.5×10^6 " " " "

243. Young's Modulus.—When a rod or wire is stretched by a weight the elongation is very nearly proportional to the stretching force, and exactly proportional to the length of the wire. In this case the stress is the force per unit cross section and the corresponding strain is the elongation per unit length. The elasticity of stretch for the substance of which the wire is made, or *Young's modulus,* as it is commonly called, may be represented by Y and is determined by the ratio

$$Y = \frac{\frac{F}{S} = \text{stress}}{\frac{e}{l} = \text{strain}} = \frac{Fl}{Se},$$

where F represents the stretching force, e the elongation of the wire, l its length, and S its cross section.

Young's Modulus

Copper........	12×10^{11} dynes per sq. cm.,	17×10^6 lbs. per sq. inch.
Brass..........	11×10^{11} " " " "	16×10^6 " " " "
Iron...........	19×10^{11} " " " "	27×10^6 " " " "
Steel..........	22×10^{11} " " " "	32×10^6 " " " "

244. Beams.—When a floor beam sags under a load the upper part is compressed and the lower part stretched. But the re-

sistance to longitudinal stretching or compression is measured by Young's modulus, so that the stiffness of a beam is proportional to the modulus of elasticity of the material of which it is made.

In case of a beam supported at both ends and loaded at the middle, the sag or deflection y at the middle is expressed by the formula

$$y = \frac{Fl^3}{4h^3bY},$$

where l is the length of the beam, b is its breadth, and h its height, F is the load, and Y is Young's modulus for the material of the beam.

From this it appears that a beam having twice the breadth of another would sag half as much, other things being equal, while if its depth were twice that of the other it would only sag one-eighth as much. For this reason floor beams are placed on edge, the breadth having but little influence on the stiffness compared with the depth.

245. Viscosity.—When a solid is strained beyond its elastic limit the strain may go on increasing indefinitely at a rate which depends on the stress to which it is subjected. Most metals show a viscosity of this kind when the distorting force is great enough, as seen in wire drawing and in the making of lead pipe. A strip of lead when stretched with a moderate weight will continue slowly elongating year after year. A glass fiber fastened at one end and having a small twisting force applied at the other will twist more and more as times goes on.

But when a substance yields continuously in this way to the *very smallest forces*, as in case of tar, pitch, or syrup, it is said to be a *viscous fluid*. In such a fluid one layer slides over another with a velocity which depends on the stress and on the viscosity, the slower the motion for a given stress the more viscous the substance is said to be.

Viscosity may be considered a kind of internal friction between contiguous layers, and the energy spent in overcoming it appears as heat.

All known liquids and even gases are more or less viscous, and in consequence energy is spent in heat and there is loss of pressure (§224) whenever a fluid flows through a long pipe. The outer layers next the wall of the pipe are nearly stationary

in such a case, while the velocity of flow increases toward the center or axis of the stream.

The usefulness of a lubricating oil depends, among other things, upon its viscosity. If not viscous enough it will be squeezed out of the bearing, while if too viscous it will offer needless resistance to the motion.

The viscosity of a fluid may be determined from the time required for a given quantity to escape through a long tube of small diameter, or it may be found by an apparatus called a viscosimeter in which a long inner cylinder is supported by a torsion wire in the axis of an outer cylindrical tube which can be rotated. The space between the two tubes is filled with the oil or other liquid to be tested and the torsion effect on the inner cylinder is measured when the outer one is turning at a constant rate of speed.

The viscosity of a substance depends on its temperature, and it is noteworthy that *heating a liquid makes it less viscous, while the opposite is true of gases.*

246. Energy Absorbed by Viscosity.—The absorption of energy through viscosity is well shown by the following experiment, due to Lord Kelvin (Sir Wm. Thomson).

Take two eggs, one raw and one hard-boiled, and suspend each like a torsion pendulum by means of a fine wire attached to a wire sling enclosing the egg, the long axes of the eggs being vertical; then give each egg a turn or two and let it go. The boiled egg will continue oscillating for a long time, while the raw egg will almost immediately come to rest. The oscillating motion of the shell is so rapid that the inner layers of the raw egg slip on the outer ones by their inertia, and the internal friction or viscosity of the egg causes the energy of vibration to be lost in heat.

This principle has been applied by Lord Kelvin to prevent the violent swinging of a mariner's compass, due to the motion of the ship. The compass box, hung on gimbals so that it can swing freely in any direction, is made with a double bottom, and the space between the two bottoms is partly filled with a viscous liquid, such as glycerin. After any disturbance the glycerin flowing between the two surfaces of the box transforms the energy of motion into heat, and the box is promptly brought to rest.

Diffusion and Solution

247. Diffusion.—If a strong solution of copper sulphate is introduced by a tube into the bottom of a tall vessel containing

pure water, the denser blue solution will at first be sharply separated from the clear water above. By degrees the sulphate will be seen to steal upward into the water until in time it will be uniformly diffused throughout the liquid, just as a gas expands and fills a vessel in which it is set free, though diffusion in liquids is extremely slow.

Stirring a mixture of two liquids increases the surface through which diffusion takes place and so greatly quickens the process of complete mixture. When no diffusion takes place between the liquids they will not mix.

248. Interdiffusion of Gases.—Gases diffuse into each other very freely, as shown by the following experiment. Two globes are connected together, the upper containing hydrogen and the lower carbonic acid gas. In spite of the density of the carbonic acid being 22 times that of hydrogen, it will diffuse upward and the hydrogen downward till finally a uniform mixture will fill both vessels. Each expands and fills the whole space as if it alone were present.

249. Solution of Solids in Liquids.—When a solid is placed in a liquid a certain amount will be dissolved, after which no more will be taken up, and the liquid is said to be saturated. The per cent. that can be dissolved depends not only on the substance, but on the temperature, solubility usually increasing with rise in temperature.

The volume of the solution is usually less than the combined volumes of the two constituents and the process of dissolving is often accompanied by a change in temperature.

Solution of Liquids in Liquids.—Two liquids that diffuse into each other may either mix in any proportion, as in case of water and alcohol, or one may only dissolve a limited amount of the other. Thus if water and ether are stirred together at a temperature of 10°C., the mass will separate into a lower layer of water containing 10 per cent. of ether and an upper layer of ether containing $1\frac{1}{8}$ per cent. of water. At 10°C. ether will dissolve any amount of water less than $1\frac{1}{8}$ per cent. and water will dissolve 10 per cent. or less of ether. As the temperature is raised water will dissolve less ether, while ether dissolves more water.

250. Solution of Gases in Liquids.—Some liquids, such as water, dissolve all gases more or less freely. When there is

simple solution without chemical union the gas is absorbed most freely when the liquid is cold and is driven off when the liquid is heated. Thus when water is heated the absorbed air escapes in bubbles before boiling takes place.

The amount of gas absorbed by a given liquid is proportional to the pressure. Soda water is charged with carbon dioxid gas under pressure, and when the pressure is relieved the gas escapes in bubbles, causing effervescence.

The power of water to absorb various gases is shown in the following table, the figures giving the volume of gas at one atmosphere pressure absorbed by unit volume of water.

Absorption of Gases in Water

Gas	At 0°C.	At 15°C.
Oxygen	0.049	0.030
Hydrogen	0.021	0.019
Nitrogen	0.023	0.015
Carbonic acid gas	1.79	1.00
Ammonia	1140.00	756.00

251. Absorption of Gases in Solids.—Certain porous solids have a great power of absorbing gases. Boxwood charcoal will absorb 90 times its volume of ammonia and 35 volumes of carbonic acid gas. This absorption seems to be due to the condensation of a layer of gas on the surface of the body.

It is by the condensation of a surface film of gas over a body that the so-called Moser's breath figures are explained. If an engraved die lie for some time on a polished plate of metal or glass, on removing the die and breathing on the plate the engraved image is seen.

Platinum in the porous state, known as spongy platinum, absorbs hydrogen gas so powerfully that if placed in an escaping jet of hydrogen the heat developed by the condensation is sufficient to ignite the jet.

There is what seems to be a true solution of gases in some solids discovered by Graham and called by him occlusion. By heating iron wire and then allowing it to cool in an atmosphere of hydrogen, it was found that it occluded 0.44 times its volume of the

... parchment or bl
stances in solution, such as salts, t
other substances such as albumen
could not pass through or only ve
fore, to divide substances into

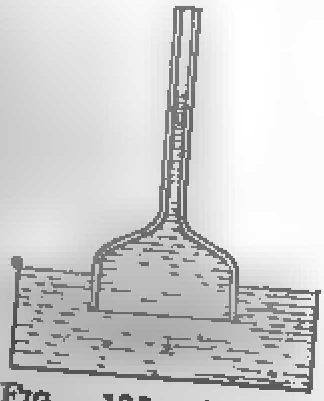


FIG. 135.—Osmotic pressure.

colloids (from t
could be separa

253. Osmotic
tion of sugar or
opening above
bottom with a r
and if the whole i
diffusion of wat
membrane, and t
in the tube until
in the vessel is suf
water from entering. The increase p
equilibrium is reached is known as th
solved substance.

Such a membrane, however, is not suite
pressure, for sugar diffuses through it to som
as water, also it lacks the strength and rigidi
sure developed.

Pfeffer showed how to form by chemical m
porous earthenware cup or cell, a membrane v
to sugar in solution while it freel...

one as in the other, just as equal volumes of different gases at the same temperature and pressure contain equal numbers of molecules (Avogadro's law).

Capillarity and Surface Tension

254. Capillarity.—Under this head are grouped a number of phenomena depending on the force of cohesion at liquid surfaces. Some of these are the upward curvature of the surface of water where it meets the side of a glass vessel, the clinging together of light bodies floating on the surface of water, the forms of drops and bubbles, and the rise of liquids in fine hair-like or capillary tubes. (Latin *Capillus*, a hair.)

To understand these phenomena we must first examine how the conditions at the surface of a liquid differ from those in its interior.

255. Surface Tension.—If a mass of olive oil is floated in a mixture of alcohol and water of the same density as the oil, it will gather itself up into a spherical ball. Draw it out by means of a glass rod into any long or irregular shape, and as soon as it is left to itself it returns to its spherical form, exactly as if covered with an elastic skin. When a camel's-hair brush is immersed in water the bristles stand apart as freely as in air, but when it is withdrawn they cling together by the contraction of the surrounding water surface. Wet threads cling together when drawn out of water. So also a drop of mercury resting on a table is drawn up into a smooth rounded mass by the contraction of the surface, in spite of the weight of the mercury which tends to flatten it out.

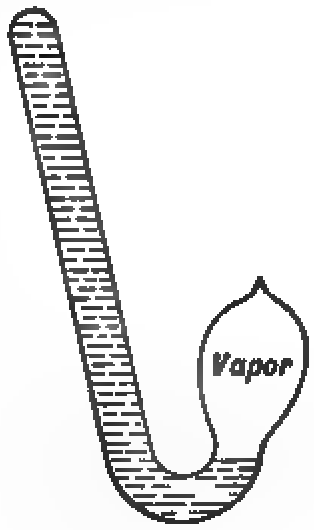


Fig. 136.—Water hammer.

256. Cause of Surface Tension.—That the particles of a liquid attract each other and are held together by a strong force of cohesion may be shown by the *water hammer*, which is a bent tube (Fig. 136) partly filled with water from which the air has all been boiled out, the tube having been sealed up while boiling so that it contains simply water and its vapor with scarcely any air. The absence of air causes the water to strike the end of the tube with a sharp metallic click when it is shaken, hence its name. If the water is all run into one arm of the tube, completely filling it, and is jarred into good contact with the sides of the tube, it may

then be held in the position shown in the figure and the water will remain in the full arm in spite of the fact that the pressure of the vapor in the other branch is quite insufficient to sustain the column of water; for if a slight jar is given to the tube, the liquid sinks to the same level in each branch. In this case the column of water is sustained by clinging to the walls of the tube and by the cohesion of one particle to another, and it is under *negative pressure* or *tension* as much as is a rope supporting a weight.

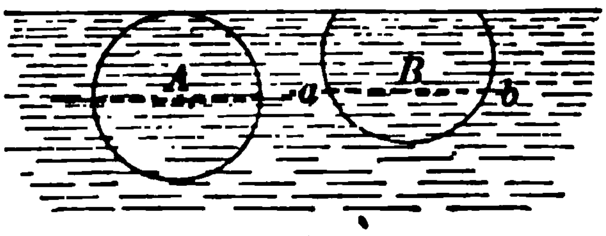


FIG. 137.

But this force of cohesion can be detected only between particles that are exceedingly close together.

The small distance within which all those particles lie that have any sensible attraction for a given particle may be called the radius of the sphere of molecular attraction. If the spheres of action are represented by the circles about A and B, it is clear that the particle at A is in equilibrium, so far as the cohesive forces are concerned, being equally drawn in all directions, while B, which is nearer to the surface than the radius of the sphere of action, has the downward attraction of liquid below the line ab balanced only by the upward attraction of the medium above the liquid surface. If the latter is a free surface with air or vapor above, the downward attraction will be in excess and the tendency is to drag particles away from the surface and into the interior of the liquid. In consequence of this the surface tends to contract. If the upper surface of the liquid were in contact with a substance whose attraction for the particle B was greater than that of the liquid below ab, the upward attraction would be in excess and the surface would tend to enlarge. Thus a drop of oil on a clean glass plate will spread out over the whole surface of the glass.

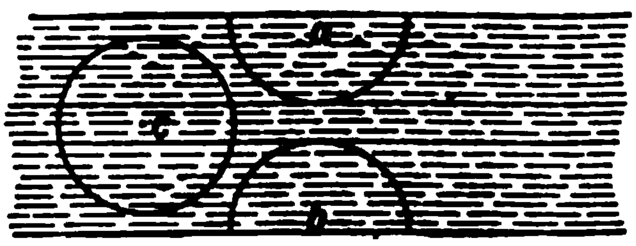


FIG. 138.

Some idea of the size of the spheres of attraction may be formed from the study of soap films.

Let figure 138 represent a section of a soap film in which the circles indicate the spheres of attraction. Particles in the middle of the film, between the two dotted lines, are farther from the surface than the radius of the sphere of action, and no change in

the contractile force of the film is to be expected so long as the two surfaces are thus comparatively independent of each other, but if the film is made thinner than the diameter of the sphere of action a change in the tension of the film is to be expected. Such a change—a sudden decrease in tension—is observed when the thickness of a soap film is about 100 millionths of a millimeter, indicating that the radius of the sphere of action in the soap solution is about 50 millionths of a millimeter.

257. Measure of Surface Tension.—The surface tension of a liquid is the force with which the surface on one side of a line one centimeter long pulls against that on the other side of the line. Thus it is the contractile force which a square centimeter of surface (Fig. 139) exerts on each of its bounding sides. In the diagram the arrows marked T indicate the outward forces due to the tension of the surrounding surface required to balance the contractile force of the surface inside the square.

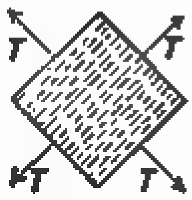


Fig. 139.

258. Surface Energy.—If a strip of surface one centimeter wide is made one centimeter longer than at first, the work done is measured by the contractile force or surface tension T multiplied by the distance that it is drawn out. Thus if it is drawn out one centimeter, increasing the area of the surface by one square centimeter, the work expended is T ergs. This work is stored up as energy of the surface and is expended when the surface contracts. **Particles of liquid near the surface have thus more energy than particles in the interior, and the increase in energy in ergs per square centimeter of surface is numerically the same as the surface tension in dynes per linear centimeter.**

The following table gives some values of surface tensions at 20°C.

Surface Tensions in Dynes per Centimeter

Substance	Air	Water	Mercury
Water..........	73.5	....	412
Mercury.......	539.0	412.0	....
Olive oil.......	[redacted]	20.6	335
Alcohol........	24.5	....	....
Ether..........	17.6	....	....

The tensions just given are for the interface between the substance at the top of the column and the one at the side.

259. Variations of Surface Tension.—The surface tension of a liquid depends on temperature, in general being less with higher temperatures; it is also in case of water greatly affected by impurities. Pour a little water into a flat-bottomed porcelain tray, but not quite enough to cover the bottom. If a few drops of alcohol are added at any point, the water will rush away from that spot in every direction leaving the porcelain surface bare. If a drop of ether or alcohol is held near the surface of clean water on which lies some lycopodium powder, this will be dragged away from near the drop. The surface tension of the liquid is weakened by alcohol and even by the vapor of alcohol or ether and the surrounding uncontaminated liquid with its greater surface tension contracts and draws the other after it. In the same way are to be explained the lively movements that are noticed when minute fragments of camphor are dropped on clean water. The surface tension is weakened by the impurity at the first point of contact and as the liquid is drawn away from that point the camphor also moves, leaving a trail of contaminated surface behind it.

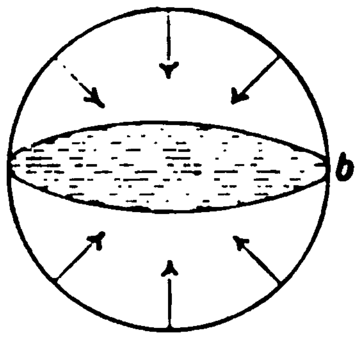


Fig. 140.

260. Pressure Due to Surface Tension.—The contractile force of a curved surface produces a pressure inward on the concave side. Let the figure represent a drop of water which we may imagine free from gravity. It will at once assume a truly spherical form and the liquid will be under pressure in consequence of the tension of the surface.

To determine the amount of this pressure consider the drop as in two halves separated by the plane ab. All around the circumference of this section the surface tension is pulling the two halves together. The amount of this force will be $2\pi rT$ if r is the radius of the drop and T the tension.

But this force is balanced by the pressure of one-half of the drop against the other. Calling p the pressure per square centimeter, the total pressure is $\pi r^2 p$. But since these two forces balance we have

$$2\pi rT = \pi r^2 p;$$

and therefore

$$p = \frac{2T}{r}.$$

Notice that this pressure increases as the size of the drop diminishes, and in a water drop one-hundredth of a millimeter in diameter it amounts to 150 grams per square centimeter.

This same expression gives the pressure, due to surface tension, of the air or vapor inside of a bubble in a mass of liquid.

In a soap bubble there is a thin film of the soap solution having two surfaces to be considered. Each of these has contractile force and consequently the total pressure inside of the bubble is

$$p = \frac{2T}{r_1} + \frac{2T}{r_2},$$

where r_1 and r_2 are the radii of the outer and inner surfaces of the bubble, respectively. Since these radii are practically equal, we have

$$p = \frac{4T}{r}.$$

261. Contact Angle.—When a clean plate of glass is dipped into water the liquid rises in a curve against the glass. The free surface of the water is here enlarged in spite of its contractile force by the *expanding force* of the surface of contact between the water and glass.

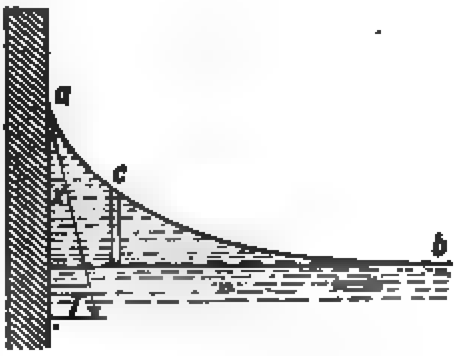


Fig. 141.—Contact angle.

This expanding force or *negative* tension is due to the great attraction between the water and glass, as explained in paragraph 256. Let E represent the amount of this expansive force or negative tension of the surface between water and glass, and let T represent the tension of the water surface. Then it is clear that the liquid will rise until the upward and downward forces are in equilibrium; that is, until $E = T \cos x$.

The particular angle x at which equilibrium takes place is known as the contact angle for the two substances involved. In case of kerosene oil and glass E seems to be greater than T, and hence there can be no equilibrium, the angle x becomes 0°, and still the edge of the oil creeps up. It is in this way that a film of oil spreads over the whole surface of a glass lamp.

The case of mercury and glass is different, the mercury curves *down* instead of *up* at the line of contact and the angle x is about 140°, indicating that the surface between mercury and glass has a positive tension, or contractile force.

262. Rise in Capillary Tubes.—If a large glass tube, say, 2 in. in diameter, is introduced into water, the water rises around the edge on the inside until the weight of the water raised above the original level is just equal to the total upward pull of the contracting surface. This upward pull is equal to the tension per unit length multiplied by the inner circumference of the tube or $2\pi rT$, where r is the radius of the tube and T the surface tension.

If small tubes are taken the weight of liquid required to balance this force cannot be secured without raising even the middle of the liquid above the original level, and the smaller the tube the greater the height to which the liquid will rise. Let h represent the average height of the column, then $r^2\pi h$ = the volume of liquid raised; and if d is its density, its weight in grams will be $r^2\pi hd$ and its weight in dynes $r^2\pi hdg$, and this is equal to the sustaining force $2\pi rT$.

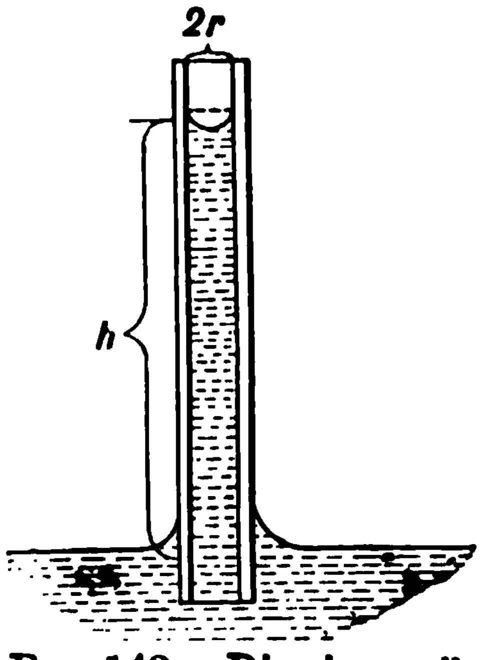


Fig. 142.—Rise in capillary tube.

$$2\pi rT = r^2\pi hdg,$$

$$h = \frac{2T}{rdg}.$$

Hence the height to which liquid rises in a capillary tube varies inversely as the radius of the tube, a relation known as Jurin's law.

Of course if the contact angle is not zero, we must write $T \cos x$ in the above formula instead of T.

The pressure within the liquid in a capillary tube is less than the atmospheric pressure at all points above the level surface of the liquid in the open vessel, decreasing according to the hydrostatic law, toward the top. The curved surface at the top exerts a back pressure against the atmosphere equal to the pressure of a column of liquid of the height h.

The height at which a liquid will stand in a capillary tube is independent of its shape, depending only on the size of the tube at the point where the curved surface or meniscus stands.

A liquid cannot rise and overflow the top of a capillary tube, however short it may be, for as it reaches the top the curvature of the surface changes, becoming less until its upward pressure is just balanced by the column of liquid below.

The rise of oil in lamp wicks and of sap in vegetable fibers are familiar instances of capillary action.

263. Depression of Liquids in Tubes.—In cases where the contact angle is greater than 90°, as between mercury and glass, the surface of the liquid is rounded upward in a small tube and the level of the liquid is depressed. In this case the surface being concave downward produces a downward pressure. Thus in a barometer the mercury column stands lower than it would normally do unless the tube is so large that the center of the column is sensibly flat. (See §192.)

264. Effect of Curvature of Surface.—In a conical tube a drop of water will be concave outward at both ends, but since the

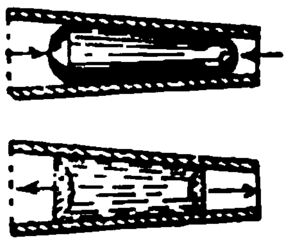


FIG. 143.

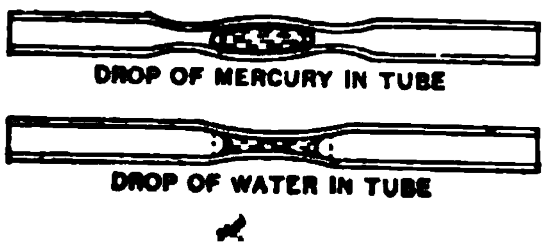


FIG. 144.

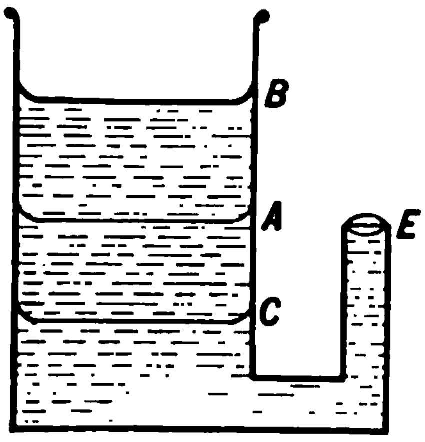


FIG. 145.

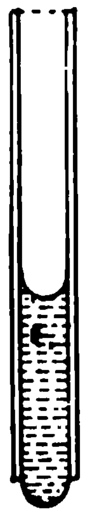


FIG. 146.

smaller surface has the smaller radius of curvature, it will exert the greater pressure against the air and the drop will move toward the small end of the tube, while a drop of mercury which rounds outward at both ends will be driven toward the larger end (Fig. 143). Consequently a drop of mercury will be in stable equilibrium at a widening in a narrow tube, while a drop of water will seek the narrowest point (Fig. 144).

If a capillary tube is connected at the bottom with a larger vessel of water (Fig. 145), when the water in the large vessel is at A, level with the top of the capillary tube, the surface of the water at E is flat. If the level is raised to B in the large vessel, the surface at the end of the capillary tube will round up, taking exactly the curvature necessary to balance the hydrostatic pres-

sure due to the height of B above E. On the other hand, if the level is lowered to C the surface at E will become concave with a curvature that will give an upward pressure equal to that of a column of water from A to C.

When a narrow glass tube is dipped in water and withdrawn a short column of water will be held in the lower end, the lower convex surface acting together with the upper concave one to support the liquid.

265. Small Floating Bodies.—When a floating body is wet by a liquid, the liquid rises around it and drags it down by the weight of this raised mass. So a hydrometer with a glass stem around which the liquid rises will sink lower than it otherwise would.

Around bodies which are not wetted the liquid curves down, and consequently the body is buoyed up by the weight of the liquid displaced in consequence of the curvature. For instance, if a clean needle is laid carefully on water it will float. The liquid is bent down where it meets the needle and therefore the volume of water displaced is much greater than the volume of the needle itself and so the weight of the displaced water may be equal to the weight of the needle.

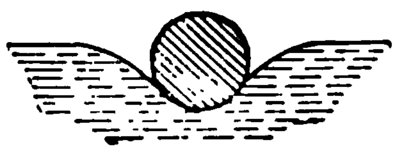


FIG. 147.

266. Attraction and Repulsion.—When the liquid rises around floating bodies they are drawn together as soon as they are near enough for the curvature of the surface due to one to affect the other. Notice how small floating pieces of wood or cork cling together as soon as they are wet, so also bubbles in a cup of cocoa cling together, and are also drawn to the sides of the cup where the surface curves up.

If the cup is filled to the brim and enough added to make the surface curve down slightly at the edges, the bubbles will at once rush away from the edge.

Bodies which are not wetted and around which the surface curves down are also drawn together, but are driven away from bodies around which the surface curves up.

267. Explanation of Attraction of Floating Bodies.—When two small floating objects are both wetted the liquid rises higher between them than it does on the outside, as shown in the upper part of figure 148, and since the pressure at any point in the liquid

higher than the level surface is less than the atmospheric pressure, the pressure on the outside is greater and they are forced together.

In the second case the liquid stands higher around the floating bodies on the outside than it does between them, and since the pressure in the liquid at points below the level surface is greater than the atmospheric pressure, they are pushed toward each other in this case also.

But when the liquid wets one and not the other, the surface is lowered on the inside of the wetted one and raised on the inside of the other so that in each case the pressure on the inside is greater than on the outside and the two are urged apart.

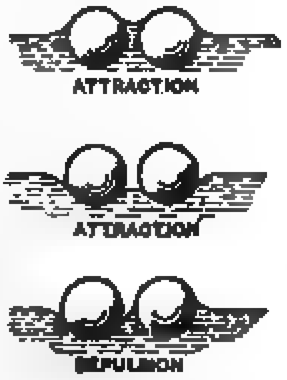


FIG. 148.

268. Soap Films.—Some most interesting illustrations of surface tension are found in the phenomena of soap films. When a loop of thread is laid on a soap film formed in a wire ring and the film is broken inside the loop, the latter will be drawn into an exact circle, for it is pulled equally in every direction by the contracting film. And this circular loop may be moved from one part of the film to another without changing shape, showing that the tension does not depend on the width of the film.

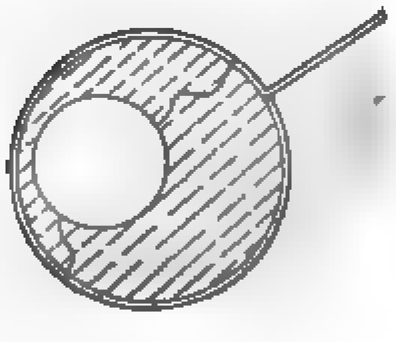


FIG. 149.—Loop in soap film.

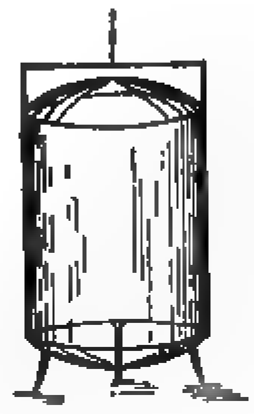


FIG. 150.—Cylindrical bubble.

If wire frames forming the outlines of cube, tetrahedron, or cylinder are dipped into a soap solution and then carefully withdrawn, symmetrical figures of great beauty are formed by the films.

By blowing a bubble between two rings and then drawing the rings apart until it becomes cylindrical, the ends will be seen to bulge out, showing that the air within is under pressure, and

the radius of curvature of the spherical ends will be found to be twice that of the cylindrical surface. Why is this?

269. Equilibrium of Cylindrical Film and Formation of Drops.— If such a cylindrical film is short relative to its length, it is in stable equilibrium; but if it is longer than its circumference, it is unstable and will collapse at one end and bulge out at the other because by so doing the surface will become smaller. This will result in its breaking into two bubbles, a large and a small one (Fig. 151), with a very small one between them; a result which is of interest, because it illustrates why a thin jet of water breaks up into drops. Imagine a thin cylindrical jet escaping from a vessel of water. It is under pressure sidewise from the contrac-

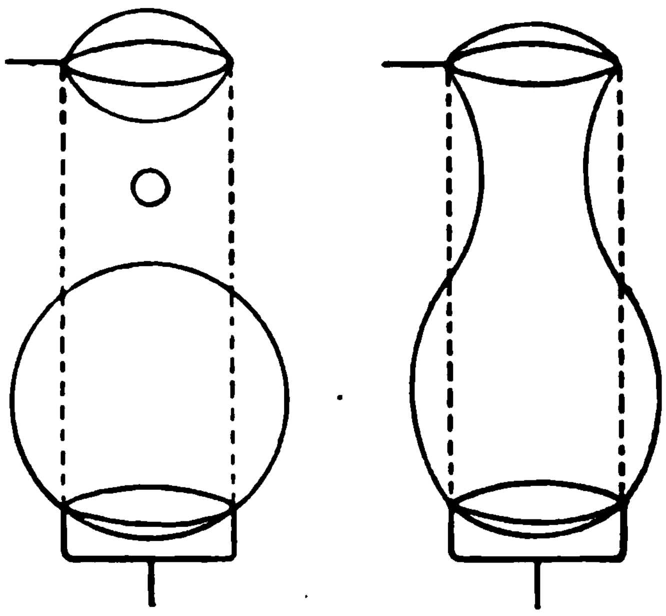


Fig. 151.—Breaking of unstable cylindrical bubble.

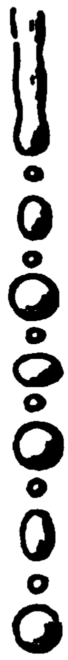


Fig. 152.—Jet breaking into drops.

tile force of the surface, but since it is long enough to be unstable it yields, becoming first undulatory, as shown in the upper part of the figure, then finally breaking up into alternate big and little drops which are elongated when first separated and vibrate from this to the flattened form, finally settling down to spherical shape. These details were first made out by Savart; they may best be studied from photographs made when the stream is instantaneously illuminated by an electric spark.

Problems

1. Find the diameter of a drop of water in which the pressure is twice the atmospheric pressure on its surface, taking surface tension of water as 74 dynes per cm.

2. Two flat glass plates, 10 × 10 cm., placed face to face in a vertical position and separated only by bits of tinfoil, have the lower edges immersed in water which rises and fills the space between the plates. Find how much less the average pressure is between the plates than on the outside and thence find the force with which they are pressed together.
3. In case of a soap bubble 5 cm. in diameter, how much greater is the pressure within the bubble than without? Take surface tension 70 dynes per cm. Give answer in dynes and also in grams per sq. cm.
4. A glass hydrometer having a stem 8 mm. in diameter floats in water. With what force due to surface tension of water wetting its stem is it pulled downward?
5. How much deeper will the hydrometer in the last problem sink than if it had floated in a liquid of the same density that did not rise on its stem?
Ans. 3.8 mm.

Kinetic Theory

270. Kinetic Theory of Gases.—It was shown by Daniel Bernoulli (1700–1782) that the pressure of a gas could be best explained as due to the impacts of its molecules against each other and the walls of the vessel. In recent years Clausius and Maxwell especially have developed this theory, showing that the characteristic properties of gases are in harmony with it, and it is now generally accepted as giving a true conception of their structure.

In this theory it is assumed that the molecules of gas are constantly striking against each other or the walls of the vessel and rebounding. When two molecules approach each other, at a certain distance they experience a repulsive force which increases as they come nearer until further approach is stopped by the force and they are repelled apart or rebound. The distance between their centers when they are nearest together and about to rebound is called the diameter of the molecule. The molecule on rebounding soon gets out of the influence of the other and then flies in a straight line until it meets another from which it rebounds, either directly or glancing off sidewise, changing both its own motion and that of the molecule against which it strikes, and so it continues its path zig-zagging about. The average distance that a molecule travels between two successive impacts is called its *mean free path*. The velocity of a particular molecule is doubtless changed at every impact not only in direction, but in amount, sometimes increased and sometimes diminished, but

there is no loss of energy on the whole; whatever one molecule loses the one impacting against it gains. The average kinetic energy of the molecule, and consequently its average velocity, remains unchanged unless energy is in some way communicated to the gas from outside.

271. Pressure of a Gas.—An expression for the pressure of a gas may be deduced in an elementary way by neglecting the *size* of the molecules and their impacts against each other and considering each molecule as rebounding only from the walls of the vessel. Imagine a cubical vessel one centimeter each way, and for simplicity conceive the whole number of molecules N contained in it to be divided into three equal groups, one group rebounding between the sides AD and BC and producing pressure against them, the other groups being directed against the other pairs of sides. If V is the velocity of a molecule, it will strike against the side BC once every time that it travels across the vessel and back again, a distance of 2 cms. The number of impacts per second of one molecule against the side BC will therefore be $\frac{V}{2}$. The momentum of the molecule before impact is mV toward the side, after impact it is mV in the opposite direction; the total change in momentum of the molecule in one impact is $2mV$ where m is the mass of the molecule. But there are $\frac{V}{2}$ impacts per second, so that each molecule in rebounding from one side experiences a change of momentum per second $2mV \times \frac{V}{2} = mV^2$ and since the whole number of molecules impacting against the side BC is $\frac{N}{3}$ the total *change in momentum produced by that side in one second is* $\frac{1}{3}NmV^2$, and this is, therefore, the *force* against the side. But since the side BC has unit area, the force against it equals the pressure, hence

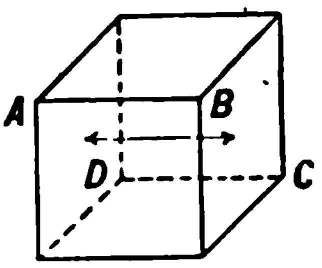


FIG. 153.

$$p = \tfrac{1}{3}NmV^2 \qquad (1)$$

where p represents the pressure of the gas.

Now, the product Nm is the total mass of gas in one cubic centimeter, or its density d, and hence:

$$\frac{p}{d} = \tfrac{1}{3}V^2 \qquad (2)$$

Maxwell's Law.—It has been shown on mechanical grounds, by Maxwell and others, that **when two masses of gas are at the same temperature, the average kinetic energy of a molecule of the one is equal to the average kinetic energy of a molecule of the other.**

That is

$$\tfrac{1}{2}m_1V_1^2 = \tfrac{1}{2}m_2V_2^2 \tag{3}$$

where m_1 and m_2 are the masses of the molecules of the two gases and V_1 and V_2 are their velocities.

Boyle's Law.—According to the law just stated the kinetic energy of the molecules in a mass of gas is determined by its temperature, and hence V changes only when the temperature of the gas changes. **Formula (2) above, then, is in agreement with Boyle's law and expresses the fact that the pressure of a gas is proportional to its density when the temperature is constant.**

This formula may be used to calculate the average molecular velocities, giving as follows:

Mean Velocity of Molecules in Gases at 0°C.

Hydrogen	1843	meters per sec.
Nitrogen	492	" " "
Oxygen	461	" " "
Carbon dioxide	392	" " "

Avogadro's Law.—When two different gases have *the same pressure* we have by equation (1)

$$\tfrac{1}{3}N_1m_1V_1^2 = \tfrac{1}{3}N_2m_2V_2^2 \tag{4}$$

If the two masses of gas are also at *the same temperature*, we have by (3)

$$\tfrac{1}{2}m_1V_1^2 = \tfrac{1}{2}m_2V_2^2 \tag{5}$$

and combining the two equations we find

$$N_1 = N_2;$$

that is, **the number of molecules per cubic centimeter is the same in all gases at the same temperature and pressure.** This is known as Avogadro's law, and was reached by him from purely chemical considerations.

272. Molecular Magnitudes.—The number of molecules in a gas at one atmosphere pressure and 0°C. is found to be 26.5×10^{18}

per cubic centimeter, or 434 million million million per cubic inch. Several different methods lead to approximately this result, but the most accurate determination is by an electrical method to be explained later (§614).

The *mean free path* or average distance that a molecule travels before striking against another may be deduced by the kinetic theory when the viscosity of the gas is known.

Also by several different lines of reasoning that cannot here be discussed the effective diameters of gaseous molecules have been approximately determined.

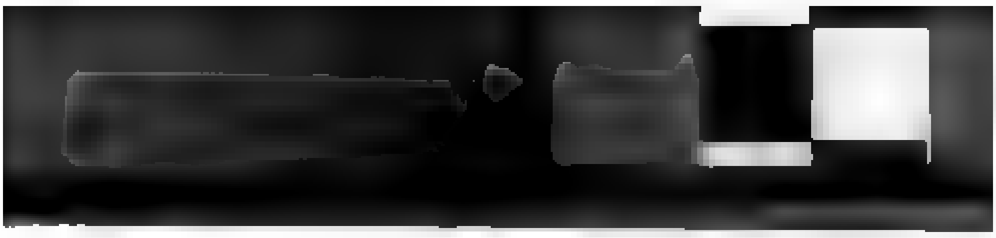
Molecular Magnitudes in Gases at Atmospheric Pressure and 0°C.

Gas	Mean free path	Diameter of molecule in millionths of a millimeter	Number per c.c.
Nitrogen........	.0000098 mm.	0.28	$26.5 \times 10^6 \times 10^6 \times 10^6$
Hydrogen.......	.0000185 mm.	0.21	" " " "
Carbon dioxide..	.0000068 mm.	0.37	" " " "

The numbers representing the diameters must be regarded as only approximations to the truth, but which doubtless express the true *order* of magnitude, being probably neither 10 times too large nor too small.

In case of nitrogen at atmospheric pressure the mean free path is about 13 times the diameter of the molecule. More than one million such molecules in a row would be required to make a length of 1 mm. Lord Kelvin has estimated that if a drop of water were magnified to the size of the earth, "the structure of the mass would then be coarser than that of a heap of fine shot, but probably not so coarse as that of a heap of cricket balls."

273. Brownian Movement.—The English botanist Brown in 1827 on observing with the microscope very fine particles held in suspense in a mass of water, discovered they were in constant irregular motion, and the smaller the particle the more lively was the motion observed. It is a spontaneous motion that never ceases, and is believed to be caused by the incessant motion of the molecules of the liquid, which bombard the particle on all sides driving it hither and thither.



The French physicist Perrin has made a careful study of this phenomenon using an emulsion in water of exceedingly fine grains of mastic. He finds by exact measurement of the distribution of the grains and the amount of their motions that they distribute themselves just as should be expected from the kinetic theory, and even deduces by inference from his measurements the number of molecules in a cubic centimeter of gas under standard conditions, finding 30.5×10^{18}, in good agreement with determinations by other methods.

References

Soap Bubbles, C. V. Boys.
Brownian Movement and Molecular Reality, J. Perrin, translated by F. Soddy.

WAVE MOTION AND SOUND

Surface Waves

274. Wave Motion.—The phenomena of wave motion are perhaps most easily grasped from the study of water waves. Looking upon a series of waves coming across a smooth lake from a passing vessel, we notice the steady advance of a definite form of motion, having a velocity which is quite independent of that of the vessel, and carrying energy, for the water in a wave is in motion and has kinetic energy.

The water over which the waves have passed is left calm, and if a floating cork is observed it will be seen to rise and move forward with the crest of the wave, then sink and move backward in the trough, repeating the motion with the next wave, and coming to rest in its original position when the disturbance has passed. The motion of the cork, which is that of the water in which it lies, shows that the wave does not carry along with it a mass of water, but that the motion and energy are passed along from one mass of liquid to the next.

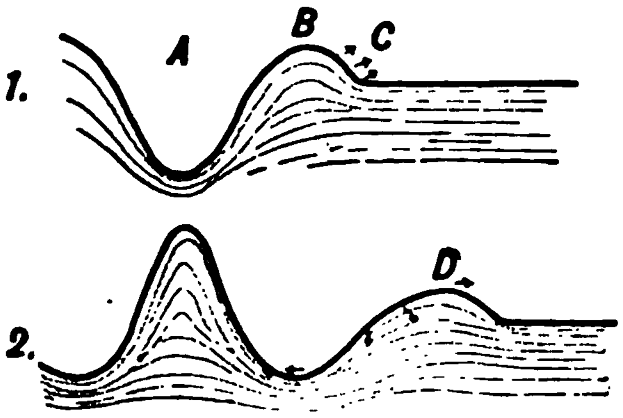


Fig. 154.—Origin of a water wave.

A wave may be defined as a form or configuration of motion advancing with a finite velocity through a medium.

By means of waves energy is transmitted, being passed along from one part of a medium to the next by the interaction of adjoining parts.

275. Origin of a Series of Water Waves.—When a stone is dropped into a smooth pond, water is carried down by the motion of the stone, as shown at A, figure 154, and also thrust out in a ridge at B; beyond C the surface is undisturbed. Then it begins to rush back toward A from the surrounding parts and B sinks, at the same time the forward part of the wave between B and C is urged forward, in part by its forward momentum

and in part by the pressure. The rush toward A does not stop when A has risen to the level, but continues until the kinetic energy of the flow toward A is spent in heaping up water at A at the expense of the hollow at B, as shown in 2, figure 154. Thus there are set up oscillations at A, the energy of which is gradually spent in sending out waves. *Each wave takes with it a certain definite quantity of energy which remains with it as it advances.*

276. Motion in a Water Wave.—In a series of water waves the motion of the particles is as shown in figure 155, each particle moving around in a closed curve, which in the simplest form of wave is a circle. In the diagram are shown the paths of motion of nine water particles which were originally equidistant and one-eighth of the whole wave length apart. Each moves clockwise in a circle and all with the same uniform velocity; but while particle a is at the top of its path, b is back of the top by one-

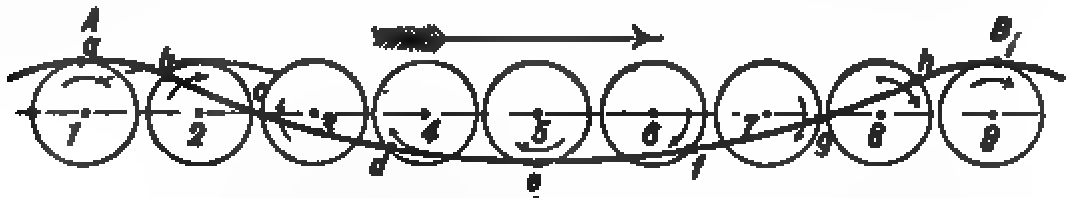


FIG. 155.—Motion in a simple oscillatory water wave.

eighth of a circumference. The position of each in its path is called the *phase* of its motion; a and i are said to be in the *same phase*, while the phase of e is opposite to that of a and i; also c and g are opposite in phase because they are a half-circumference apart in their motion.

Each particle in the diagram differs in phase from the next one by one-eighth of a complete revolution. Of course all the particles which were between a and b in the undisturbed condition of the surface will still be between a and b and will have intermediate phases, thus forming the surface of the wave between those points.

It will be seen that the wave is advancing in the direction of the long arrow at the top, for an eighth of a period later b will be at the top and a will have passed beyond, and the position of the crest of the wave will be as shown by the dotted line. In the time of a complete revolution or *period* of the particles the wave will advance from a to i and a new crest will have come to a.

The **wave length** is the distance between particles in the same phase of motion, in this case from a to i.

The **amplitude** of the wave is the radius of the circles, which is the distance that a particle is displaced from its equilibrium position; it is one-half the vertical height of the wave from trough to crest.

The **velocity** of a wave is the velocity with which a particular phase of motion moves along; for example, it is the velocity with which the crest of the wave moves along. Since a wave travels the whole wave length λ, in the period of revolution of a particle P, we have

$$V = \frac{\lambda}{P}$$

where V represents the velocity of the wave.

The **frequency** of a series of waves is the number passing a particular point per second. If n waves pass per second there must be n complete waves in the distance V traversed by the waves in one second, or

$$V = n\lambda.$$

The velocity of the particle in its circular orbit must not be confused with the velocity of the wave. It is always less than the wave velocity and depends on the amplitude of the wave. A mechanical wave model devised by Lyman exhibits the motions in figure 155 and admirably illustrates the motion in water waves.

277. Decrease of Amplitude with Depth.—In consequence of their difference of phase, the particles a and b, near the crest of

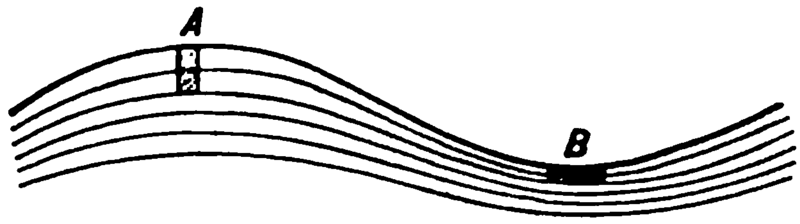


FIG. 156.—Amplitude of wave decreases with increased depth.

the wave, are nearer together than their positions of equilibrium, while near the trough of the wave e and f are farther apart. But since water is practically incompressible its volume does not change and a little mass of water which was originally of cubical form must become elongated vertically near the crest and horizontally near the trough as shown in figure 156. The water particles on the lower surface of the cube must, therefore, have less

amplitude of motion than those at the surface, as is evident from the figure. At a depth equal to the wave length the amplitude is only $\frac{1}{535}$ of that at the surface.

278. Velocity of Oscillatory Waves.—The type of water wave described is known as a simple oscillatory wave and the varieties of form observed in ocean waves are due to a number of such waves superposed. In a smooth pond it may easily be observed that two independent series of waves may cross each other, producing a complicated resultant motion; but after they have passed, each is found to have kept its original motion undisturbed.

It is shown in treatises on hydrodynamics that the velocity of a simple oscillatory surface wave is expressed by the formula

$$V = \sqrt{\frac{g\lambda}{2\pi}}.$$

From which the following velocities are found:

Wave length	*Velocity*
75 ft.	13.3 miles per hour
300 ft.	26.6 miles per hour
1200 ft.	53.2 miles per hour

From this it is clear that a group of waves, as we see them in the ocean, resulting from the superposition of a variety of waves of different lengths, must continually change in form, as the component simple waves travel with different velocities.

279. Ripples.—The formula just given for the velocity of a wave assumes that forces due to the weight of the liquid are the only ones involved. But the surface tension of the liquid also plays a part, though it is entirely insignificant except in very short waves or *ripples*. The complete formula for the velocity of a wave is

$$V = \sqrt{\frac{g\lambda}{2\pi} + \frac{2\pi T}{\lambda d}},$$

where T represents the surface tension of the liquid and d its density. The effect of surface tension is to increase the velocity of shorter waves. When water waves are about 17 mm. long their velocity is a minimum, being 23.3 cms. per second. Longer waves travel faster because gravitation force predominates, while in shorter waves surface tension has the principal effect.

Compressional Waves

280. Compressional Waves.—Water waves of the type just considered are *surface* waves, and can only exist at the surface of a medium. But the kind of wave now to be studied can travel in every direction through an elastic medium.

Consider the model shown in figure 157, which represents a series of equal masses resting in a frictionless groove and connected by springs. If the first mass is moved toward the second, the latter will move *because* the spring between the two is compressed. *But it will not begin to move until after the first mass has approached it*; for *if* the two moved exactly together there would be no compression of the spring between them, and consequently no force exerted on the second mass to move it. As the second mass moves forward there is compression of the second spring, followed by motion of the third mass, and so on, the masses being set in motion one after the other as the wave of compression reaches spring after spring.

Fig. 157.—Illustrating elasticity and inertia of medium.

So also if the first mass is drawn away from the others, the first spring is stretched, causing motion of the second mass which stretches the second spring. The motion is therefore communicated through the whole series as a wave accompanied by stretching or expansion of the springs.

Such a wave of expansion or compression is set up whenever a material object is set in motion or brought to rest; for all bodies may be considered as made up of massive particles in elastic equilibrium with each other, like the balls and springs in the diagram. Thus, when a chair is lifted, a wave of expansion runs down through it, and when it is set on the floor a wave of compression runs up.

281. Newton's Formula for Velocity of a Compressional Wave.—The velocity of such a wave depends on the elasticity and density of the medium. Recurring to the illustration, it is evident that making the springs between the balls stiffer will increase the speed with which the motion will be communicated from one ball to the next, while if the masses of the balls are made greater the effect will be to make the speed of the wave less.

It was proved by Newton from the principles of mechanics that

the velocity of a wave of compression or expansion in a medium of which the volume elasticity is E and the density d is expressed by the formula,

$$V = \sqrt{\frac{E}{d}}.$$

A simple proof of this formula will be found on page 683.

282. Motion in a Series of Compressional Waves.—If the first of the series of balls represented in figure 157 is made to oscillate regularly backward and forward, now moving toward the second ball and now away from it, a *series* of waves will be sent along the row of balls, alternately waves of compression and expansion; and each ball will oscillate just as the first one does, though the second will always be in a phase of motion a little behind the first, the third will lag behind the second and so on.

This is precisely the kind of motion which is set up in air by a tuning-fork or other rapidly vibrating body, and which excites

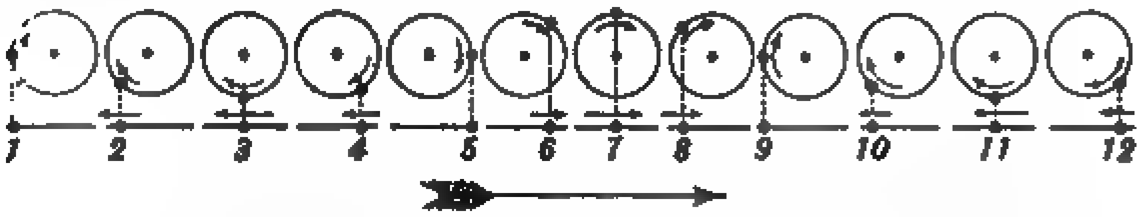


FIG. 158.—Motion of particles in a sound wave.

in our ears the sensation of sound. Such waves in air are accordingly known as *sound waves.*

The details of the motion in a sound wave may be understood from figure 158.

The diagram illustrates the relative phases of motion of a series of particles in the wave one-eighth of a wave length apart. Each particle is shown as a black dot on a short straight line which represents the path in which it oscillates, the center of the line being its equilibrium position. Suppose the first particle is made to oscillate in simple harmonic motion, then that will be the mode of vibration of all the particles, and each will move back and forward keeping vertically under an imaginary companion particle that is supposed to move with uniform velocity around in the corresponding circle shown in the diagram.

It will be seen from the positions of the companion particles in the circles that, taken in the order in which they are numbered,

each is one-eighth of a complete vibration behind the phase of the preceding particle; indeed, the associated circles are only used to show the relative phases of the numbered particles below, which represent actual particles in the medium.

Particle 1 is at the end of its path, while the second particle is moving toward the end and will be there an eighth of a period later, when 3 will be in the phase now shown by 2, and so on; therefore the wave will have moved forward in the direction of the long arrow underneath. It will be seen that particles 1 and 9 are in the same phase, and accordingly the distance between them is the *wave length.* The particle at 7 is in the center of a condensed region, where the particles are closer together than normal, while those at 3 and 11 are the centers of rarefied or expanded regions. *In the condensed region the particles are moving forward in the direction in which the wave is advancing; in the rarefied region they are moving opposite to the wave.* There are intermediate points where

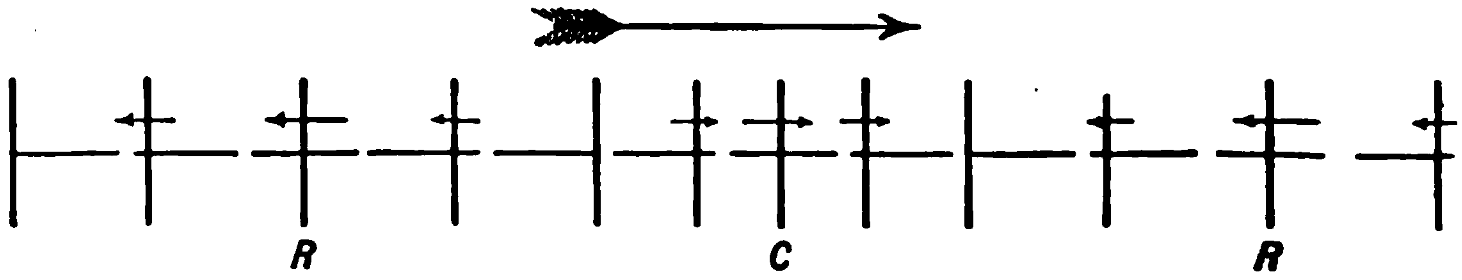
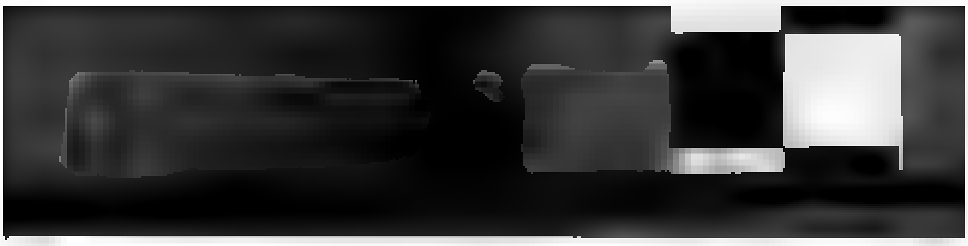


FIG. 159.—Motion of air layers in sound wave.

the medium is neither condensed nor rarefied where the particles are for the instant at rest at the end of their paths of vibration, as at 1, 5, and 9.

It must not be forgotten that each of the particles considered is only one of a *layer* all vibrating in the same way. This is represented in figure 159, where the dots of the previous diagram are replaced by heavy lines which represent successive layers of particles, differing in phase by one-eighth of a period. The small arrows indicate the velocities of the layers at the given instant, and the instantaneous position of the regions of condensation and rarefaction are marked by the letters *C* and *R*, respectively.

To sum up, in a compressional wave the particles vibrate longitudinally, or back and forth in the direction of advance of the wave, and there is a progressive change in phase, in consequence of which alternate regions of compression and rarefaction are produced.



The *amplitude* in such a wave is the distance that a particle oscillates on each side of its equilibrium position, or half the whole distance through which it vibrates.

283. Illustration.—The propagation of a wave of compression or rarefaction may be very well shown in a regular spring a meter and a half long which is supported by threads in a horizontal position, as shown in figure 160. The turns of the spring should be rather large and it should be of such a stiffness that a wave will take a second or two to travel its length.

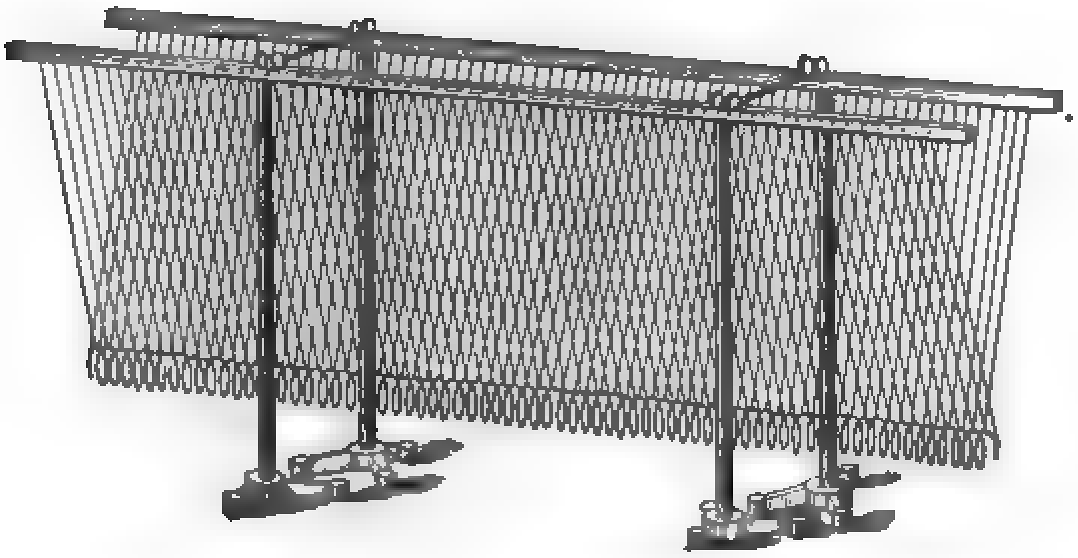


FIG. 160.—Spring wave model.

SOUND

284. Sound Communicated by Waves.—There are three principal evidences that sound is communicated by compressional waves through material bodies. First, sound is not communicated through a vacuum; second, the motions of sounding bodies are such as might be expected to set up compressional waves; and, third, the observed velocity of sound is the same as that of compressional waves, both in air and in other media.

285. Sound Requires a Material Medium.—Place an alarm bell rung by clockwork under the receiver of an air pump, as in figure 161, so that it may rest on a mass of soft cotton, or is otherwise supported so that no vibrations can be transmitted through its supports to the plate of the air pump. When the air is exhausted from the receiver the bell is no longer heard, however vigorously it may be ringing. *Sound waves*, therefore, *do not pass, through a vacuum, they require a material medium.*

286. Sound Originates in Vibrating Bodies.—All sources of sound are vibrating bodies capable of setting up air vibrations. A brass plate supported at the center and covered with sand if set in vibration by a bow may be made to give out a variety of different sounds, but in each case there is a characteristic arrangement of the sand showing that a particular mode of vibration of the plate corresponds to each sound. (See Fig. 199.) Tuning-forks are set in vibration by being struck, strings by being bowed, the vibrations of the string being evident to the eye or causing a buzzing sound when the string is touched with a piece of paper. In reed instruments the metal tongue of the reed vibrates strongly when sounding, and even in flutes and organ pipes it may easily be shown that the air is set in strong vibration.

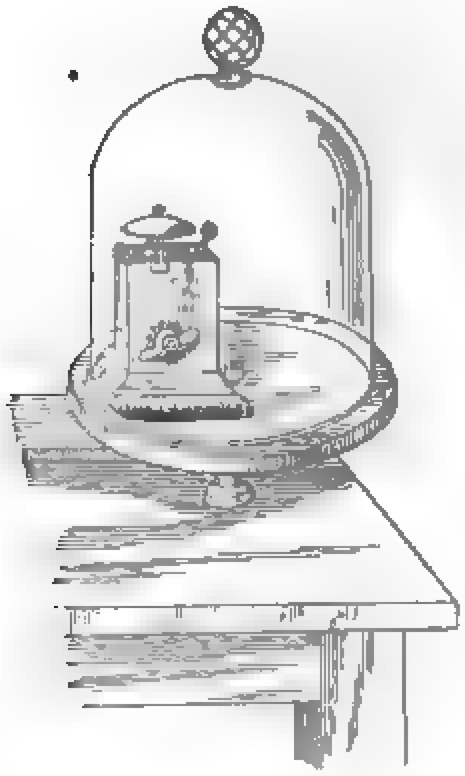


Fig 161.—Bell in vacuo.

287. Velocity of Sound.—The velocity of sound in air was determined by two Dutch observers, Moll and Van Beck, in 1823, by timing the interval between seeing the flash of the discharge of a distant cannon and hearing its report.

Cannons were set up on two hills nearly 11 miles apart, and by observing alternately first from one hill and then from the other the observers sought to eliminate the influence of any air currents which might exist. At the same time the temperature of the air was observed at a number of points between the two stations. The velocity was thus found to be 1093 ft. or 333 meters per second at 0°C.

Regnault (1810–1878) conducted an extensive series of investigations on the velocity of sound in the Paris water mains, which afforded large tubes free from wind disturbance and at a uniform temperature. He made use of an automatic apparatus by which the instant of discharge of a pistol was recorded electrically on the rotating drum of a chronograph, while the arrival of the sound at the distant station, where it caused a thin stretched membrane to vibrate, was electrically recorded on the same drum. By these

experiments he found that the velocity was influenced by the size of the pipes to a small extent, and also that very intense sounds traveled slightly faster than feebler ones. He also conducted experiments in the open air, using the same recording apparatus, and found the velocity of sound in dry air at 0°C. to be 330.6 meters per second.

Bosscha determined the velocity of sound by causing two little hammers to give simultaneous taps at regular intervals, the frequency of the taps being determined by a pendulum which made electrical connections at every swing. If one of the sounding instruments is placed beside the observer while the other is moved away, the taps are no longer heard simultaneously, but those from the more distant one come later; if moved far enough apart so that the sound from the tap of the distant hammer reaches the ear at the same instant as the next succeeding tap of the nearer hammer, the two are again heard simultaneouly, and the distance between the two sounders divided by the time interval between the taps gives the velocity of sound.

288. Velocity of Sound in Water and in Solids and Gases.—Colladon and Sturm measured the velocity of sound in the water of Lake Geneva by causing a bell to sound under water and using as a receiving instrument a sort of ear trumpet with the outer end closed by a rubber diaphragm and placed beneath the surface of the lake. The velocity was found to be 1435 meters per second. In solids the velocity of sound is usually measured by the longitudinal vibrations of rods or wires as explained later, §342.

The velocity of sound in various gases and vapors has been determined by comparison with that in air by the method of Kundt, §335.

The velocities of sound in some common media are given in the following table. The velocity of sound in wood and steel is so great that a person standing near one end of a long beam or rail that is struck at the farther end hears two sounds in quick succession, first that transmitted by the solid and then that through air.

Velocities of Sound

Medium		Meters per sec.	Ft. per sec.
Air at 0°C. and 76 cms. pressure	Regnault....	330.6	
	Bosscha.....	331.6	
	mean	331.1	1,086.7
Hydrogen at 0°C. and 76 cms. pressure........		1,286.0	4,219.0
Carbon dioxide at 0°C. and 76 cms. pressure...		261.0	856.0
Water at 13°C............................		1,437.0	4,715.0
Brass rod................................		3,600.0	11,800.0
Iron rod.................................		4,950.0	16,240.0
Steel rod................................		5,000.0	16,410.0
Pine-wood rod (along the grain)..........		3,300.0	10,830.0

289. Velocity of Compressional Waves.—The velocity of a compressional wave in air may be readily calculated by Newton's formula

$$V = \sqrt{\frac{E}{d}}.$$

It was shown by Newton that the elasticity of a gas at constant temperature is equal to its pressure (see §241). But on substituting pressure for elasticity in the above formula the calculated velocity was found to be too small.

Laplace pointed out that though the *average* temperature of air is not changed by the passage of sound waves, yet in the compressed part of a wave the air is heated for the instant, and where it is rarefied there is cooling, and that these changes take place so rapidly that there is no time for heat to flow from one part to another, so that the air is practically in an *adiabatic* condition (§241). The effect of heating during compression is to resist the compression, and cooling during expansion acts to oppose the expansion, the effective elasticity in this case is therefore increased and in case of air has been found to be 1.40 times as great as if the temperature had remained constant. The formula thus becomes for a gas like air,

$$V = \sqrt{\frac{p}{d}1.4.}$$

Substituting the values for air at normal temperature and pressure, and expressing both pressure and density in C. G. S. units, we have

$$V = \sqrt{\frac{76 \times 13.6 \times 980 \times 1.40}{0.001293}} = 33{,}120 \text{ cms. per sec.,}$$

which is in good agreement with the velocity of sound as found by experiment.

In case also of solids and liquids the results obtained by the formula agree with velocities obtained by direct experiment. The elasticities of these substances are so much greater than that of air that the velocities of sound in them are large in spite of their great densities.

Thus in water the elasticity or ratio of pressure increase to corresponding decrease in volume is, in C. G. S. units,

$$\frac{76 \times 13.6 \times 980}{0.000047} = 2.16 \times 10^{10} \text{ dynes per sq. cm.}$$

or 15,230 times that of air, while it has only 773 times the density of air.

290. Influence of Temperature and Pressure on Sound Velocity in Air.—From the formula in the preceding paragraph it is clear **that the velocity of sound in air is independent of the pressure,** for when the pressure is increased the density increases in the same proportion, by Boyle's law, and the ratio $\frac{p}{d}$ remains constant, and consequently the velocity is constant so long as the temperature is not changed.

But if the temperature is raised, pressure being constant, the density diminishes and the ratio $\frac{p}{d}$ increases. **Hence the velocity of sound in air is increased $\frac{1}{546}$, or about 2 ft. or 0.60 meters per second per degree Centigrade rise in temperature.**

291. Influence of Pitch on Velocity of Sound.—It may be easily noticed that the notes of music coming from a distant band are heard in the same relation to each other as if the band were near. There is no confusion of the melody such as would result if high-pitched sounds traveled faster or slower than low ones. Regnault made careful observations on this point and concluded that *the velocity of sound is the same whatever the pitch may be.*

It will be shown later that the pitch of a sound depends upon wave length, hence we conclude that *the velocity of sound is the same for all wave lengths.*

Reflection and Refraction of Waves

292. Reflection of Water Waves.—When a water wave meets an immovable obstacle it is turned back or reflected. Since the obstacle does not move, it cannot receive energy from the incident wave, and therefore the reflected wave carries the energy away. Each point of the obstacle reacts against the waves which meet it and so produces a periodic disturbance and may be regarded as a center from which waves are sent out. The reflected wave as a whole is the resultant of these little waves coming from each point of the obstacle.

Suppose a wave from a center *O*, figure 162, meets a straight wall *BC*. When in the position *AED* the disturbance has just reached the wall at *E* and is about starting back. By the time the wave at *A* has advanced to *B* and at *D* has reached *C*, the part of the wave reflected at *E* will have returned an equal distance to *F*.

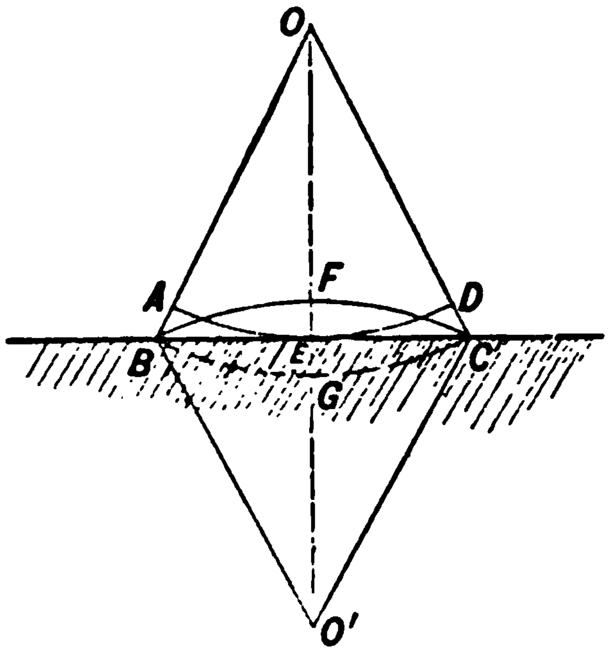


Fig. 162.—Reflection of waves.

If the wall had not been there the wave would have advanced to the position of the dotted line *BGC*, but since after reflection it has the same velocity as before, the reflected wave will at each point have gone back from the wall as far as it would have passed the line *BC* if the wall had not been there. The front of the returning wave *BFC* has therefore the same curvature as *BGC* if the wall is *flat*. The returning wave is therefore circular, having its center at a point *O'* which is as far back of the wall as the center *O* is in front of it; and the line *OO'* is at right angles to the wall.

Another method of looking at this subject is interesting. The effect of the vertical wall is to oppose any forward or backward motion of the water particles next to it without interfering with vertical motions. Let us now imagine the wall removed and that whenever a wave starts from *O* an

exactly equal wave sets out from O'. The waves will meet along the line BC and the forward or backward movement due to the one will be exactly balanced by that of the other, while their vertical movements will be added. There results, therefore, an up-and-down oscillation along the line BC exactly as if the wall were there.

On each side of the line BC there will be waves coming toward the line and others going back from it exactly as if reflected from it. And indeed they may be properly regarded as reflected, for there is no transfer of energy across the line BC because there is no forward or backward motion across that line, and if a thin wall were slipped in along BC separating the two systems of waves the motion would not be changed on either side.

293. Angle of Reflection.—When a wave front meets a reflecting surface obliquely, the direction of the wave front and its direction of propagation are changed as shown in figure 163. At W_1 is shown a portion of a wave front approaching P where it is reflected, afterward advancing as shown at W_2 as if it came from O'. The angle i between the direction of advance of the incident wave and the normal to the surface is called the angle of incidence, while the angle r between the direction in which the reflected wave moves and the normal N is called the angle of reflection. **The angle of reflection is equal to the angle of incidence.** For the angles a and b are [illegible]rly equal and the angle i is equal to a, and r is equal to b, [illegible]e the lines OO' and NP are parallel.

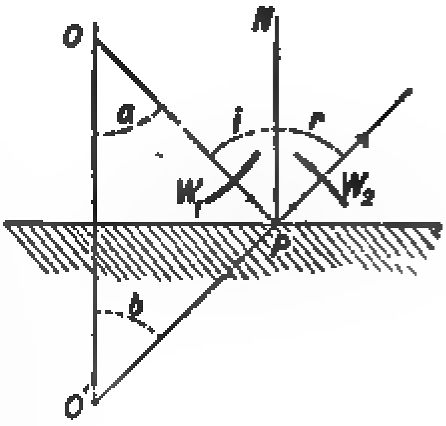


Fig. 163.

[illegible] is interesting to see how the reflected wave may be regarded as the resultant of little waves coming back from each point of the reflecting surface. Let AB (Fig. 164) be a wave front meeting the reflecting surface AC at A. The disturbance at A causes a little circular wave to go back which will have a radius AD equal to BC by the time that the wave front at B has reached C, since the velocity of the returning wave is the same as that of the advancing one. So also the circular wavelet starting backward from F when the advancing wave has reached the position FG, will have a radius FK equal to GC when the wave front at B reaches C.

From each point in succession of the reflecting surface between A and C these elementary waves spread out just as from A and

F, in arcs of circles whose centers lie on the line AC. The *envelope* of these circles is the line DC which is tangent to all of them. All these elementary waves therefore act together and combine to produce a new wave front along the line DC. Since AD is equal to BC and is at right angles to the tangent DC, just as BC is at right angles to AB, the two triangles ADC and ABC are equal and the angle of incidence BAC is equal to the angle of reflection DCA.

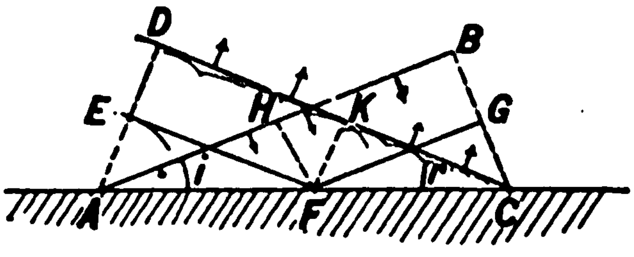


Fig. 164.

While the elementary waves are conceived as spreading out in circles in every direction, they produce an effect only along the wave front DC where all act together, for it may be shown that they interfere (§319) with each other in other directions, such as off to one side, *unless the reflecting surface is very small.* If the distance AC is not more than the length of a wave, regular reflection will not take place, but the reflected wave will spread out in all directions as if the reflecting surface were the center of disturbance.

294. Refraction of Waves.—When waves pass from one medium into another there is generally a change in velocity, which causes the direction of the wave to change when it meets the surface of separation obliquely.

Let AB represent the advancing wave front, meeting at A the second medium where the velocity is less. While the wave front advances from B to D in the first medium, the wave from A will have gone a *less* distance AC in the second medium in consequence of the smaller velocity. The new wave front will be CD, tangent to the elementary waves from A and from all points between A and D. The direction of advance of the wave is changed *toward* the perpendicular, from BD to DE. An intermediate position of the wave, where part is in one medium and part in the other shows a sharp bend at G.

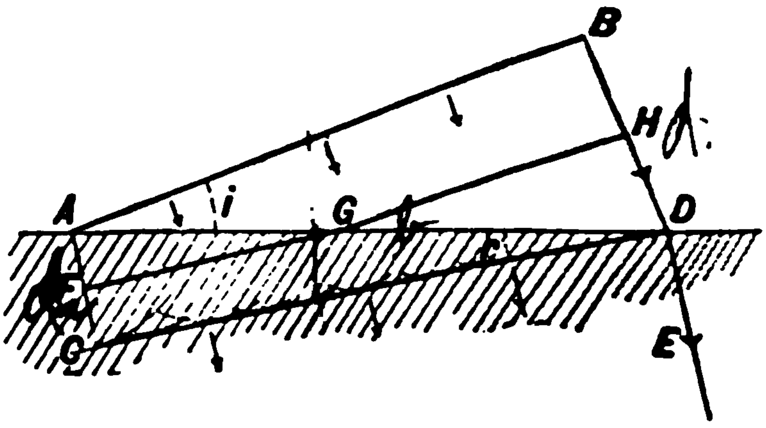


Fig. 165.

If the velocity in the second medium were *greater* than that in the first the waves would become by refraction more oblique

to the surface of separation instead of less oblique as in the case illustrated.

The law of refraction will be more fully discussed in connection with the study of light.

The refraction most commonly noted in water waves is when they run obliquely into shoal water near shore, where their velocity is retarded. The effect is to swing the wave front around more nearly parallel with the shore.

295. Reflection of Sound.—In the reflection of sound the same principles apply as in case of water waves. Sound waves reflected from a large flat surface appear to come from a point as far behind the surface as the sounding body is in front of it. Echoes from buildings, cliffs, and even from a wooded hillside are familiar examples of the reflection of sound. If there are a series of cliffs or shoulders of rock at different distances multiple echoes are heard. A pistol shot from a boat on a smooth lake comes as a single sharp sound followed by faint echoes from the distant shores, but if the water is rough the shot is followed by a reverberating roar as the sound comes back reflected from wave after wave.

By means of a large parabolic mirror the tick of a watch placed at its focus is reflected so that it may be heard 50 ft. away by an observer having his ear at the point on which the reflected waves are converged. The proper position to hold the ear may be found by observing where the image is formed of a light placed at the focus of the mirror, showing that the law of reflection is the same for sound as for light.

A watch is used in this experiment because the waves of sound which it gives out are so short, even relative to the size of the mirror, that the law of regular reflection holds.

In rooms with arched ceilings focal points may sometimes be found such that sounds going out from one point are converged toward the other. A person holding his ear at one point can hear the slightest whisper coming from the other.

There are other kinds of whispering galleries in which the effect depends not on regular reflection, but on the gradual deflection of a wave as it runs along a smooth surface.

296. Reflection and Refraction of Sound Waves.—Whenever sound waves meet the surface between two media usually both

reflection and refraction take place. If there is a very great change of density and elasticity most of the energy goes into the reflected wave, and the refraction will be slight. On the other hand, if the two media, like two strata of air at different temperatures, differ only slightly in their properties, most of the energy will be transmitted in the refracted waves into the second medium and but little will be reflected back from the surface.

If a lenticular bag of thin rubber is filled with carbonic acid gas (CO_2) in which sound travels more slowly than in air, the sound from the ticks of a watch will be concentrated at a focus conjugate to the position of the watch, just as light is converged by a lens of glass which retards its waves. For in passing through such a lens the middle part of the wave AB is more retarded than its edges, so that it is transformed into the form CD which is concave toward the ear at E.

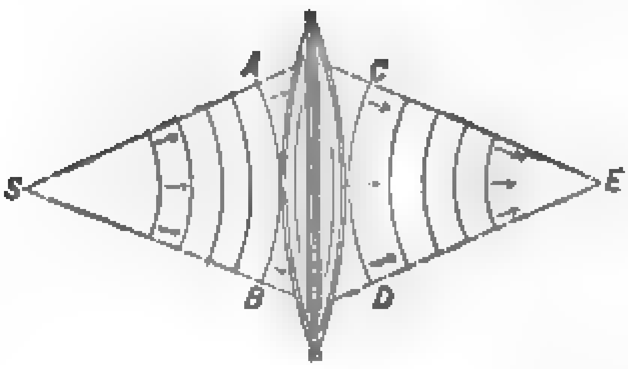


Fig. 166.

297. Effect on Fog Signals.—On account of reflection and refraction from strata of air of different temperatures, or from foggy layers, the sound of a fog horn may be entirely unheard by a vessel near the shore and in danger. If the lower portion of a horizontally moving sound wave is in warmer air than the upper part it will travel faster and cause the wave front to change its direction and may even cause it to curve upward.

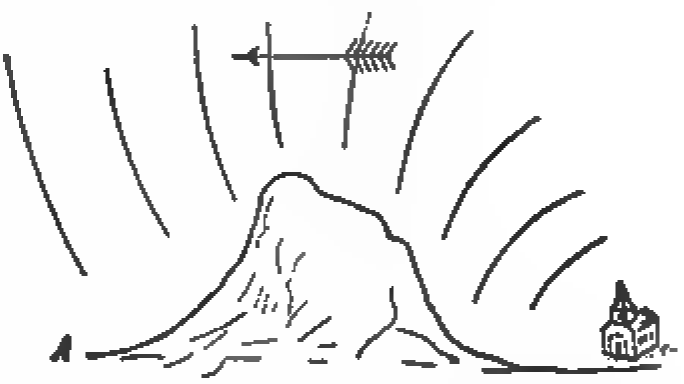


Fig. 167.—Sound waves changed by wind.

So also currents of air, causing one part of a wave front to move faster than another will change its form and consequently the direction in which it advances. Thus the observer at A (Fig. 167) might not hear the church bell when the air was still because of the screening effect of the ridge, but if a breeze were blowing which being stronger above would carry the upper

›f the waves along faster than their lower part, the wave would be tipped over so that they might come down to A g the bell to be heard.

Sound Characteristics

. **Sound and Noise.**—Sounds that have a sustained and character and do not seem to be a mixture of various nt sounds may be called *tones* or *musical sounds*. Abrupt idden sounds that do not last long enough to convey any f musical pitch, or mixtures of discordant sounds are *noises*.

Tone Characteristics.—A musical sound or tone has *'ty*, *pitch*, and *quality* or *timbre*, and each of these depends a physical property of the sound wave. The **intensity** ound depends upon the amplitude or the energy of the ion, the **pitch** depends on the frequency of the waves, and **ality** depends on the particular manner of vibration of the les.

Intensity.—Intensity of a sound, as the term is ordi- used, refers to the strength of the sensation excited by the wave. It depends upon the amplitude of vibration in ıve, for increasing the amplitude of vibration of the sound- ›dy increases the loudness of the sound. But one is not *proportional* to the other, and if two sounds of different s are equally intense, it by no means follows that the ampli- of vibration are the same. Usually the higher pitched vill be more intense for a given amplitude than one of lower

term intensity as applied in physics to sound waves to the energy of the motion and is measured by the energy ıitted per second through 1 square centimeter of surface. how the *intensity of the sensation* is related to the *energy sound vibration* is a question for the psychologist.

energy per cubic centimeter in a sound wave depends density of the medium d, the square of the frequency of ion n^2, and the square of the amplitude a^2, and is ex- d by the formula

$$E = 2\pi^2 d n^2 a^2.$$

as been found by Lord Rayleigh that when the amplitude ation of the air particles is as small as one-millionth of a

millimeter the sound is barely audible, while an amplitude as great as a millimeter would occur only in the very loudest sounds.

301. Decrease in Intensity with Distance.—When sound waves can spread out in every direction from a sounding body forming a series of spherical waves the intensity varies inversely as the square of the distance from the source. For the same amount of energy is transmitted across every spherical surface having its center at the source of sound, and the larger the surface of such a sphere the smaller will be the energy transmitted per unit surface. The spherical wave front at C, for example, will have four times the area of a sphere at one-half its distance from the source as at A, consequently the energy transmitted per square centimeter of surface will be only one-fourth as great at C as at A. The energy or intensity therefore varies inversely as the square of the distance from the source.

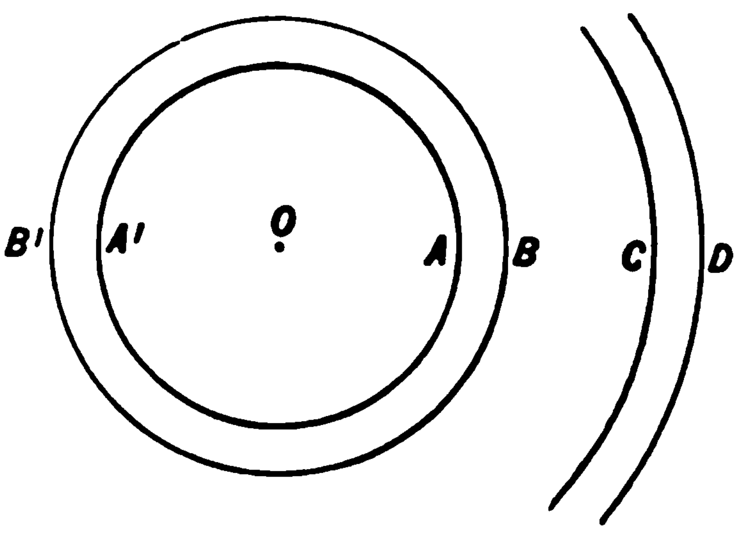


FIG. 168.

Of course if the wave is prevented from spreading out, as in a speaking tube or when the source of sound is near the surface of a smooth lake, this law does not hold.

302. Speaking Tubes and Ear Trumpets.—The ordinary *speaking tubes* connecting distant rooms in buildings depend not on regular reflection, but on the fact that the air particles next the inner surface of the tube vibrate most easily parallel with the surface, this causes the direction of vibration to be deflected by gradual bends in the tube, and consequently the wave runs along the tube without reflection. Sharp bends should be avoided in such tubes as they give rise to reflected waves which run back. In such a tube since the wave is prevented from spreading out, the vibrations do not become less energetic as the wave advances, except from back reflections and from friction against the walls of the tube, and therefore the sound is heard with only slightly diminished intensity at the distant end.

When one end of a rod or wire of metal or of a long uniform

beam of wood is struck the sound is carried along the rod or beam just as in a speaking tube with very little loss through waves sent out sidewise, and is therefore very distinctly heard at the farther end.

In ear trumpets, by the constraint of the smooth walls of the tube, the wave entering the wide end is gradually diminished in area till it emerges at the small end conveying all the energy that entered at the large end. Thus if the large end is 100 times that of the small end *the energy per cubic centimeter* in the emergent wave is 100 times as great as in the wave which entered the trumpet, neglecting the loss by friction, etc.

303. Megaphone and Speaking Trumpet.—In the megaphone sound waves coming from the speaker instead of spreading out in all directions from the mouth are limited by the walls of the instrument, so that the wave emerging at the wide end has the whole energy of the voice. It will be shown in connection with the diffraction of light that when light waves pass through an opening which is not more than a wave length in diameter, the waves spread out in every direction from the opening, while if the opening is much larger, the waves, on account of interference, will not spread out so much but will travel straight forward, illuminating a spot directly opposite the opening. For precisely the same reason sound waves coming directly from the mouth spread out in every direction while waves from the larger opening do not spread out so much, and produce a more intense effect directly ahead.

Precisely how this effect is caused by the interference of waves will be better understood after studying the diffraction of light.

304. Pitch.—The pitch of a sound depends on the frequency of the vibrations.—This is well shown by Savart's wheel. If a card is held so that it is struck by the teeth of a rotating cogged wheel a sound is given out which rises steadily in pitch as the speed of the wheel increases. If a device indicating the number of revolutions is attached to the wheel the number of taps per second producing a sound of a given pitch is readily determined.

Another instrument by which the number of vibrations may be determined is the *siren* devised by Cagniard de la Tour, shown in figure 170. A disc having a circular row of equidistant holes is mounted on an axis so that it can rotate almost in contact with

the upper surface of a flat circular box in which holes are made exactly corresponding to those in the disc, so that as the disc

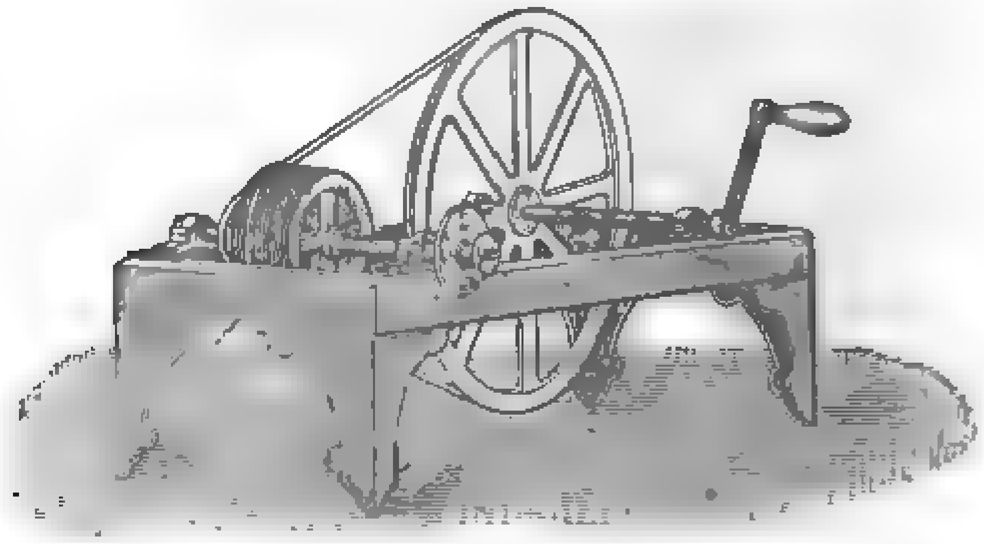


FIG. 169.—Savart's wheel.

rotates the holes are alternately opened and closed as many times in each revolution as there are holes in the series. The box is connected by a tube with a bellows and the puffs of air that come through the holes of the disc as it rotates give rise to a tone which is higher in pitch the faster the disc rotates. A revolution counter is attached to the axle so that the speed may easily be determined.

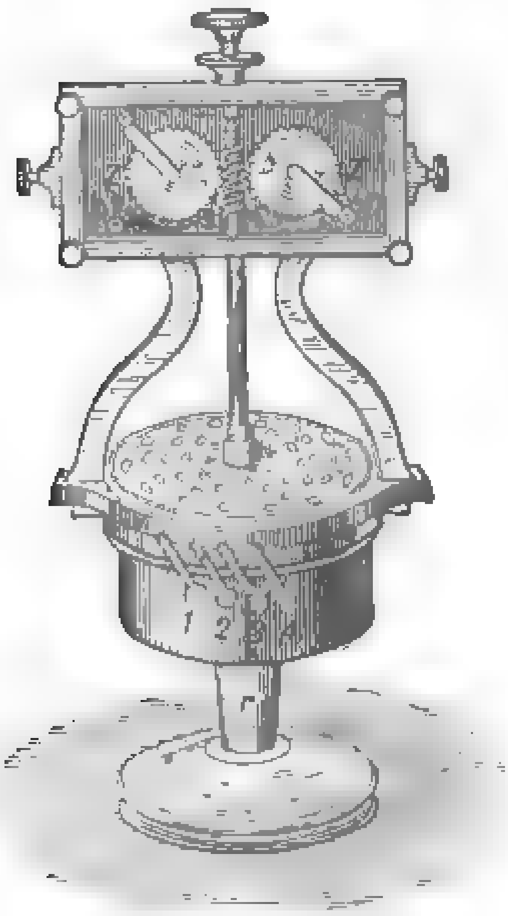


FIG 170.—Siren.

The holes of the disc are inclined one way and those in the upper plate of the box are oppositely inclined, so that the blast of air through the holes causes the rotation to take place automatically, the speed being controlled by the strength of the blast and a brake if necessary.

For finding the number of vibrations of a tuning-fork *the graphic method* may be used, illustrated in figure 171. A point or stylus is fixed to one prong of a tuning-fork which is mounted so that the stylus just touches a sheet of smoked paper stretched over

a cylindrical drum. The axle of the drum is a coarse screw by which the drum is moved slowly lengthwise as it rotates. If the fork is set vibrating, on rotating the drum a wavy curve will be drawn in helical form around the drum, each wave corresponding to a vibration of the fork. To find the number per second a second curve may be simultaneously drawn alongside of the first by a tuning-fork whose frequency of vibration is known; or a small electric marker connected with a clock may be mounted

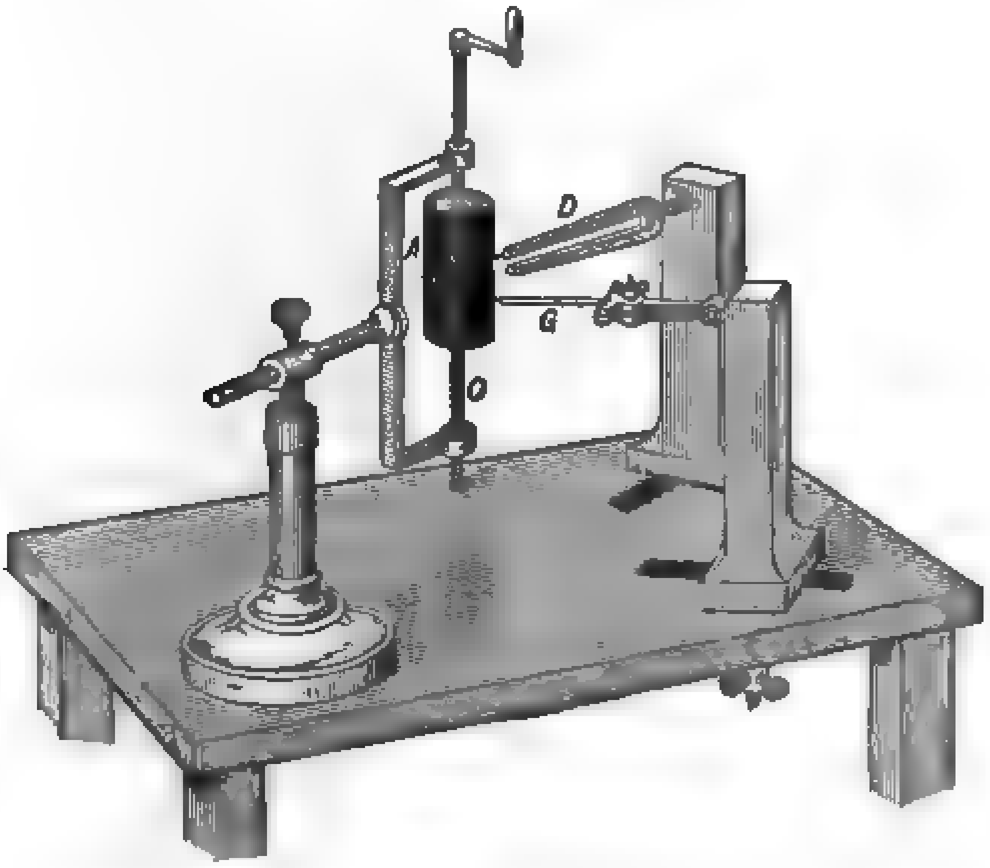


Fig. 171.—Vibrations recorded on blackened drum.

with its point touching the drum close beside the stylus of the fork, so that its marks made every second lie close to the curve drawn by the fork. The number of vibrations of the fork per second are found by counting the undulations in the curve between two consecutive time marks.

These various methods of experiment show that the pitch of a sound *depends only on the frequency of vibration*, and that it makes no difference whether the sound comes from a tuning-fork or from the puffs of a siren or the taps of Savart's wheel, all will have the same pitch if the frequency is the same.

305. Doppler's Principle.—The pitch of a sound as heard *depends on the number of waves that reach the ear per second.* Consequently if the ear is moving toward the sounding body the apparent pitch will be raised, since more waves per second will meet the ear; and conversely if the ear is moving away from the sounding body it will receive fewer waves per second than if it were at rest and the pitch will appear lower.

Let E (Fig. 172) represent the position of the ear and S that of a sounding body making n vibrations per second, and let the

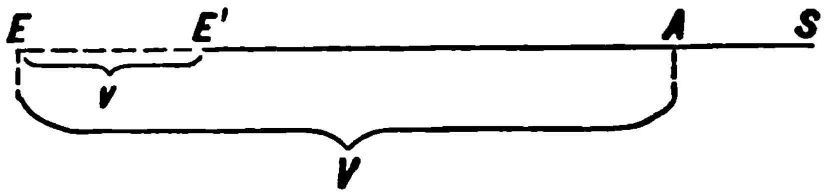


FIG. 172.—Case of ear moving toward a sounding body.

distance from A to E be V, the distance that sound travels per second. Then between A and E there are n sound waves which will reach the ear in one second if it remains at E, but if in one second the ear advances a distance v to E' it will meet in addition the waves between E and E'. Let x be the number of waves between E and E', and n' the number reaching the ear per second, then

$$n' = n + x$$

and

$$x : n = v : V \quad \therefore \quad x = \frac{nv}{V}$$

and

$$n' = n \pm \frac{nv}{V} = n\left(1 \pm \frac{v}{V}\right)$$

the signs being plus or minus according as the ear has a velocity toward, or away from, the sounding body.

A similar change in pitch is observed when the sounding body is moving toward or away from the observer. But in this case the formula is somewhat different as the wave length of the sound is changed in consequence of the motion.

Let S, the source of sound, have a velocity v toward the observer at E. In one second as it advances from S to S' it gives out n waves. The first of these waves leaving it at S has reached A, having advanced a distance V equal to the velocity of sound

in the medium, by the time that the sounding body giving out the nth wave has reached S'. All n waves, therefore, lie between, A and S', and the wave length, λ', is $\frac{V-v}{n}$.

But the number of waves that will reach E per second will be the number of wave lengths that are contained in the distance that the waves travel per second, or $n' = \frac{V}{\lambda'}$;

hence

$$n' = \frac{nV}{V \mp v}$$

where the minus sign is to be taken when the velocity of the sounding body is toward the hearer.

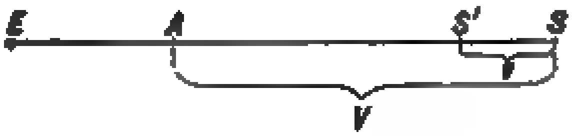


FIG. 173.—Moving source of waves.

Doppler's principle explains the sudden lowering in pitch observed in a locomotive whistle as it passes, and why a bicycle bell on an approaching wheel is heard of a higher pitch than when it is receding. It has also a most interesting application to light waves (§909).

Resonators and Analysis of Sound

306. Quality of Sound.—The ear readily observes the difference in *quality* or *timbre* between the sound of a violin and that of a flute or between notes of an organ and those of a piano though of the same pitch.

This individual character of tones depends in part on certain superficial characteristics. The note of the piano comes impulsively, suddenly strong, and then rapidly dying out, while the tones of an organ do not come instantly to full strength, but are then sustained and steady. But tones equally sustained and steady may yet differ greatly in *quality*, as for example, the tones of tuning-fork, organ-pipe, and violin. To investigate the cause of this difference we shall need *resonators* which vibrate in sympathy with the tones studied.

307. Sympathetic Vibration.—A bell ringer by timing his pulls on the rope to correspond to the swing of the bell is able to set a heavy bell strongly swinging, while mere random pulls would accomplish very little; so it is that sound waves or other comparatively slight impulses may set up strong vibrations in a body if they are exactly timed to correspond to its natural period of vibration. This fact of *sympathetic vibration* may be illustrated by tuning two strings on a sonometer to the same pitch, and then sounding one strongly; the other will be set in vibration by the impulses communicated to it through the supporting bridges.

Again, if the dampers are raised from the strings of a piano and a clear strong note is sung near the instrument, the corresponding string will be heard sounding after the singer's voice is silent.

A very interesting case of sympathetic resonance is that in which a tuning-fork is set in vibration by the sound waves from a similar-fork placed 20 or 30 ft. away. The two forks are mounted on suitable resonance boxes and must be of exactly the same pitch; if they are thrown out of unison even very slightly, as may be done by affixing a bit of beeswax to a prong of one of them, they will no longer respond to each other.

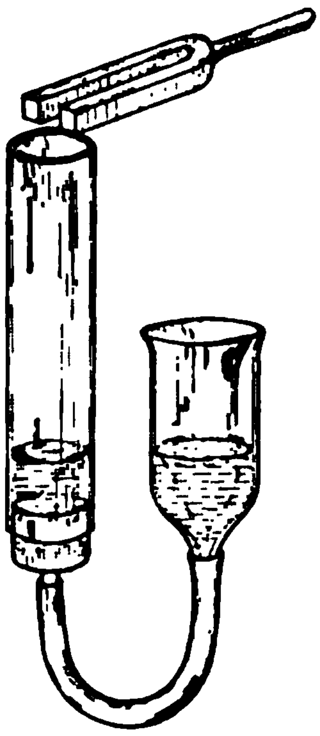


Fig. 174.—Resonance.

308. Resonators.—When water is poured into a tall cylindrical jar the noise produced has a noticeable *pitch* which grows higher as the water level rises in the jar. This pitch is due to the air column in the jar, which has a natural time of vibration of its own and responds to any component vibration of the same pitch which may exist in a noise produced in its vicinity. On blowing sharply across the mouth of the jar the same pitch is noticed, the confused rustling noise having some component to which the jar can respond. The roaring heard in sea shells is explained in the same way.

If an ordinary tuning-fork, not mounted on a resonance box, is held by its stem and struck, it will scarcely be heard a few feet away, but if it is held, as shown in the figure, over the mouth

of a jar tuned to respond, a strong tone will be given out. The arrangement shown in the figure permits the tuning to be easily effected by raising or lowering the connected water reservoir, thus changing the level of the water in the resonance tube until the response is most powerful.

The pitch of the air column may be lowered also by partially closing the opening of the jar.

Such an air column being easily set in vibration by the proper tone is known as a *resonator* and may be useful in detecting in a mass of tones the presence of the particular one to which it is tuned.

309. Helmholtz Resonators.—Helmholtz, in the analysis of composite tones, made use of spherical resonators, each having a large opening and also a small one adapted to the ear.

A resonator of this form is particularly useful because it **responds easily to vibrations of one pitch only** and so is well suited to the analysis of sounds.

310. Complex and Simple Tones.—The following experiment will now give us a clue to the cause of the difference in quality of tones. Take a series of tuning-forks mounted on resonating boxes, the frequencies of the forks being in the order of the series of whole numbers, 1, 2, 3, 4, 5, etc., which is known as the *harmonic* series. If the deepest toned fork in the series makes 250 vibrations per second, the next will make 500, and the next 750, etc. Provide also a set of Helmholtz resonators, one adapted to each fork.

Now, on sounding the lowest pitched fork alone, a deep tone is obtained to which *the corresponding resonator alone will respond.* If the next lowest is now sounded at the same time with the other, the tones blend and come to the ear as a single tone of the same pitch as before but of a different quality. And so by sounding along with the deepest or fundamental fork any or all of the others, making some of the component tones strong and some weak, great variety of tones may be obtained differing in quality but all of the same pitch.

But if any of these tones is tested by the resonators it is found that all those resonators respond which correspond to the forks used in producing the tone. Such a tone is called *complex*, while a tone to which only one resonator will respond is called a *simple tone.*

311. Analysis of Sounds.—In the case just considered it is evident from the way in which the sounds of various qualities were produced that they were complex and consisted of sounds of different pitches blended together. But if we now sound an open organ pipe of the same pitch as the deepest toned resonator we find that not only does that resonator respond, but so also to a greater or less degree do the whole series of resonators, showing that though the sound comes from a single pipe it is just as truly complex as though originating in a series of tuning-forks.

The component simple tones which unite to form a complex tone are known as its partial tones, the lowest of these in pitch is the fundamental, and the others are the upper partial tones or upper harmonics. The latter term is especially applicable when the upper partial tones are members of the harmonic series which starts with the fundamental.

From the laws of dynamics as well as from experiment there is reason to believe that **a simple tone, to which a resonator of only one certain pitch will respond is one in which the vibrations of the air are simple harmonic** (§121).

The ear seems to hear the simple harmonic components of a complex tone as separate simple tones, for persons with ears trained to the analysis of sound can often detect the different harmonics in a tone without the aid of resonators.

312. Synthesis of Sounds.—Helmholtz devised an interesting apparatus by which complex sounds might be built up from their simple components. This consisted of a set of ten tuning-forks, corresponding to the first ten terms of a harmonic series, which were kept continuously vibrating by means of electro-magnets, each fork being mounted in front of an appropriate resonator as shown in figure 175.

The resonators were cylindrical brass boxes, each mounted with its opening close to the prongs of the corresponding fork, the openings being closed by covers which could be drawn back by pressing the keys of the key-board. When the resonators were closed scarcely any sound came from the forks, but drawing back the cover from any resonator by depressing its key brought out the corresponding tone with an intensity which depended upon the amount that the key was depressed. By means of

such an apparatus the sound of an open or closed organ pipe, a violin, or reed instrument can be closely imitated.

An interesting modern instance of the synthesis of sounds is found in the ingenious "telharmonium" of Mr. Cahill in which the separate harmonics are transmitted by means of alternating currents of electricity of different frequencies which combine to

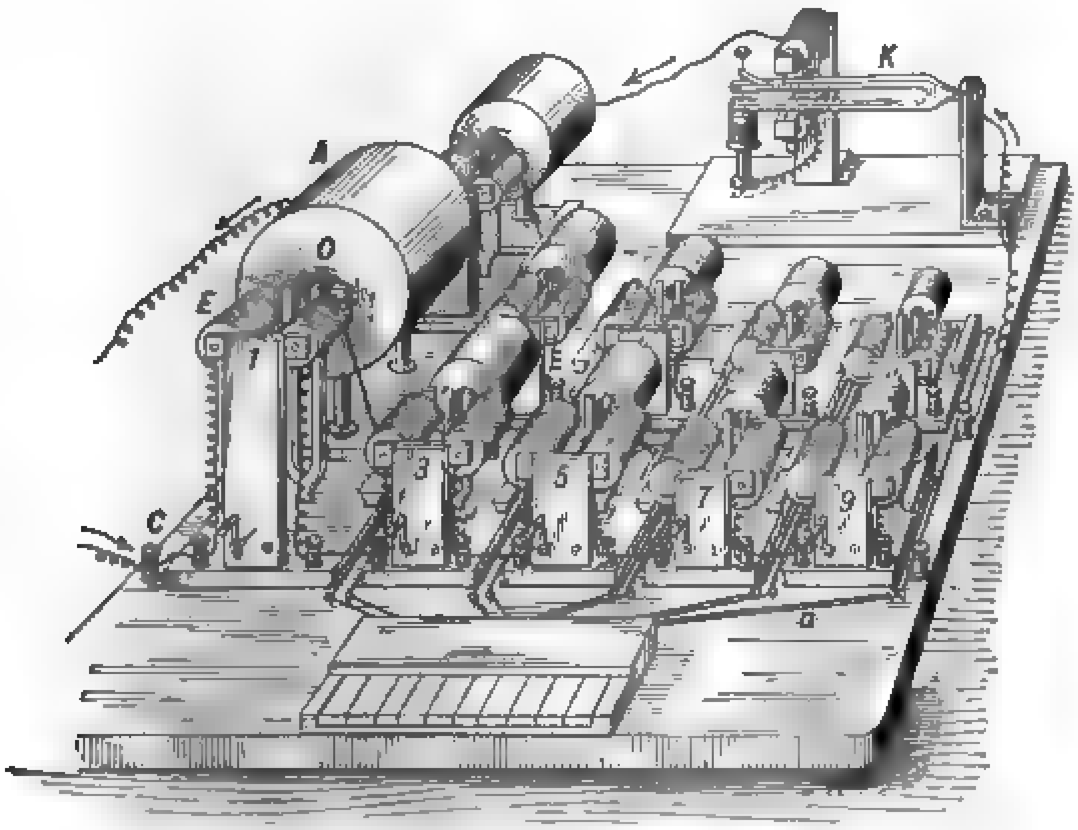


FIG. 175.—Helmholtz apparatus for the synthesis of sound.

form a single resultant current which acts on the telephone receiver at the end of the line. By combining the proper simple harmonic components all the instruments of an orchestra are imitated.

313. Quality of a Musical Tone.—The quality of a musical tone may then be said to be determined by the pitch and intensity of the different simple tones or harmonics into which it may be resolved.

314. Fourier's Analysis.—It was shown by the distinguished French mathematician Fourier that any regular periodic vibration, such as can take place in a sound wave, may be resolved into a sum of simple harmonic components all of which belong to a harmonic series, in which the fundamental has the same

period as the vibration analyzed. Thus, according to this theorem of Fourier, it is possible to analyze any sound wave into its simple harmonic components, and it is these simple harmonic components which are the simple partial tones detected by resonators.

For example, the upper curve in figure 176 represents a simple sine wave. The three lower curves represent waves having the

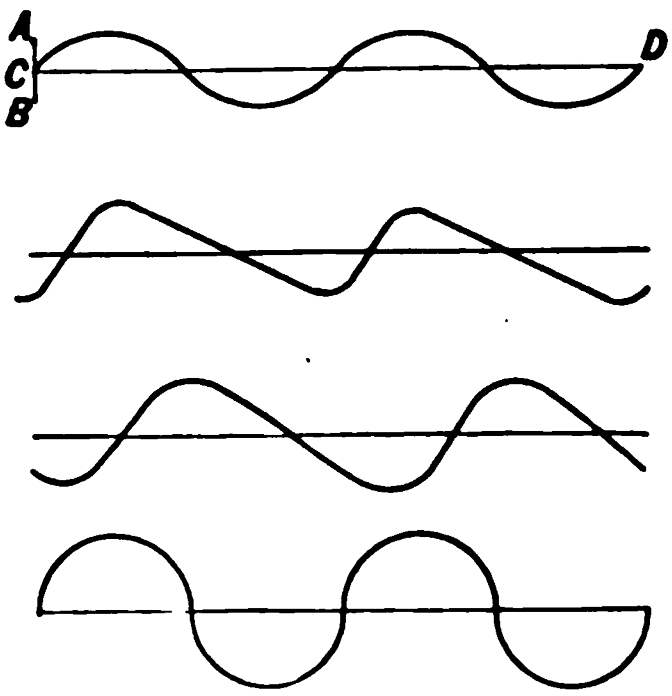


FIG. 176.—Vibration curves.

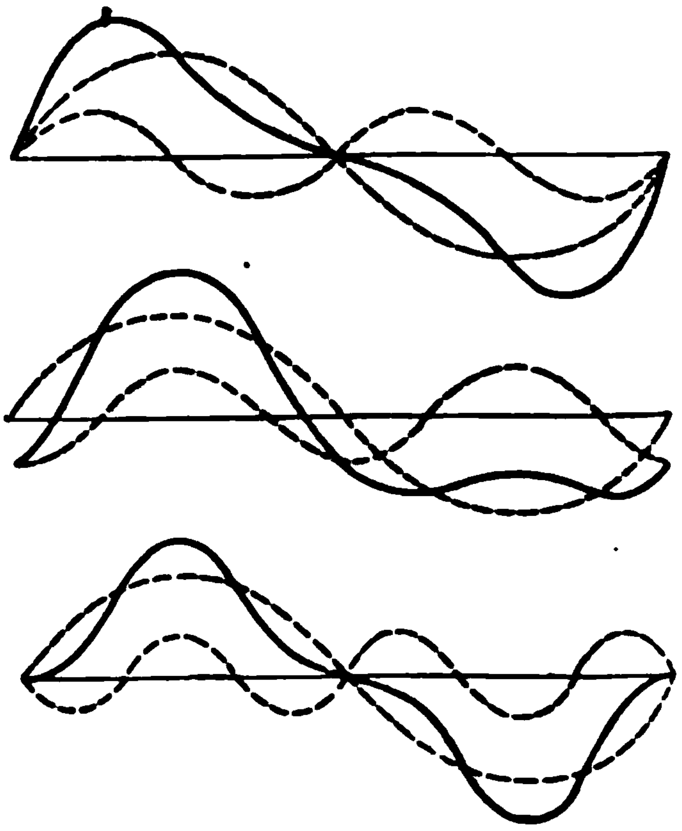


FIG. 177.—Combination of vibrations.

same wave length and therefore the same periodicity as the upper curve, but they represent entirely different modes of vibration.

Now, according to Fourier's theorem, each of these curves can be resolved into simple harmonic components. In figure 177, for example, the wave forms expressed by the heavy lines are the resultants of the simple harmonic waves represented by the dotted sine curves. The resultant curve in each case is obtained by adding the corresponding ordinates of the two component curves. In the first two cases one component has the same wave length as the resultant, while the other has half that wave length. These two components have the same amplitude in the first case as in the second, but their relative phases are different in the two cases and hence the resultant curves are different. In the lower curve one component has one-third the wave length of the resultant, while the component having half the wave length is absent or has zero amplitude.

315. Musical Tones.—In tones suitable for music the upper *partial tones* almost exactly fall into the harmonic series, start-

ing with the fundamental; that is, their frequencies are very nearly exact multiples of the frequency of the fundamental.

But the partial tones given out by bells and plates when struck do not correspond, even approximately, to the lower terms of the harmonic series, and are quite unsuited for music.

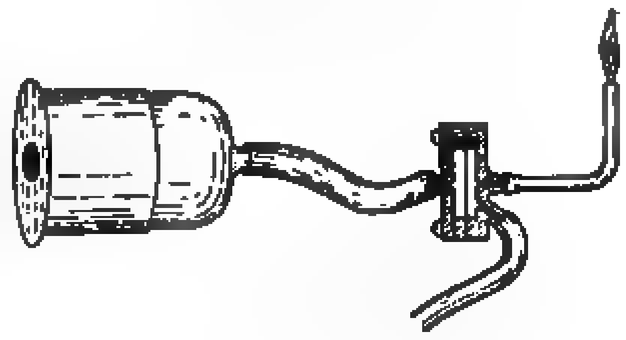


Fig. 178.—Resonator and manometric flame.

316. Koenig Resonators and Manometric Flames.—The French acoustician Koenig made use of resonators in which the front part was cylindrical and could be pushed in or drawn out so that each could easily be adjusted in pitch.

To observe the vibrations of the resonators he employed *manometric flames.* A small, flat, disc-shaped box or capsule of wood, was divided into two chambers by a thin membrane, such as gold beater's skin. The cavity on one side of the diaphragm was connected by a short tube with a resonator, while the cavity on the other side had two openings, through one of which illuminating gas was admitted, while the other was connected with a fine jet where the gas burned in a small flame. The vibrations of the air in the resonator were transmitted through the diaphragm in the manometric capsule to the illuminating gas, causing the flame to dance.

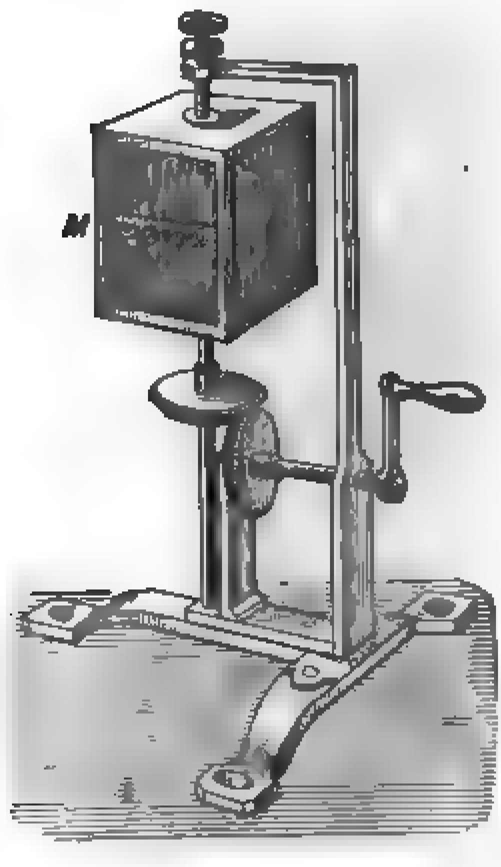


Fig. 179.—Rotating mirror.

The image of such a flame viewed in a rotating mirror, is drawn out in a band of light which when the flame is in oscillation shows serrations like saw teeth, as shown in figure 179; the particular form of the serrations revealing the mode of vibration of the flame.

317. Sensitive Flames.—Under some conditions a gas flame may be very sensitive to sound. A small cylindrical jet is required having an aperture about 0.5 mm. in diameter, and the pressure must be such as to produce a long flame *just on the brink of roaring.* A pressure of about 9 in. of water is commonly required. Such a flame is sensitive to the vibrations of exceedingly short waves of sound, and breaks into a shorter flaring or roaring flame when a bunch of keys is jingled in its vicinity or a sharp hiss given or a very high-pitched whistle sounded.

By means of sensitive flames sound waves so short as to be quite inaudible may be detected, and their interference and reflection studied.

Interference and Beats

318. Superposition of Waves.—When the same portion of fluid is traversed by two waves, the motion of the particle will be the resultant of the two and may thus become very complicated.

In case of surface waves in a liquid the eye can readily observe a series of short waves running over longer ones and preserving their motion as though over an undisturbed medium.

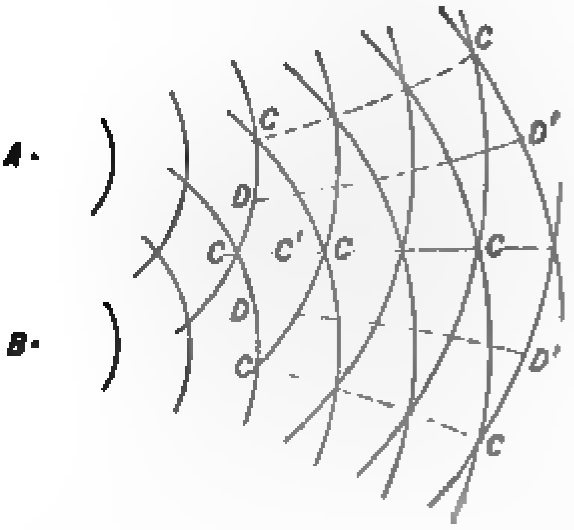


Fig. 180.—Interference of waves.

319. Interference of Waves.—Suppose A and B are the centers of two series of waves of the same wave length and amplitude. Then there will be certain points, as at C, where waves from the two sources act together so as to produce great disturbance. Suppose the lines in the diagram represent the crests of waves, then at the points C the crest of one wave is superposed on another, while at C' the troughs of two waves come together; a half period later the crests will be at C' and the troughs at C. There will result, therefore, along the line CC' a series of waves of double the amplitude of the original waves. At the points D, however, the crest of a wave from one center coincides with the trough of a wave from the other, there-

fore there will be at least a partial neutralization at points along the line DD'. If the waves coming together at D have equal amplitudes and equal wave lengths and are also simple harmonic waves they would separately produce at D equal and opposite displacements at every instant and will therefore completely neutralize each other.

This interaction of two sets of waves by which at certain points one is more or less completely neutralized by the other is known as *interference.*

320. Energy is Not Lost in Interference.—When there is complete interference at any point there is no motion of the medium and no energy at that point, but the energy of the two interfering waves is not lost or destroyed but appears at neighboring points (such as C, Fig. 180) where the amplitude of the component waves are added. For at these points the energy of the resultant vibration is four times what it would be if one of the trains of waves were suppressed. There results from the interaction of the two wave systems *a different distribution* of energy, but the total energy remains unchanged.

321. Interference of Sound Waves.—The interference of sound waves is well shown in the following experiment. The sound waves from a tuning-fork (Fig. 181) enter a suitable receiver which is connected to an earpiece by means of two tubes, one of which has a sliding portion by which its length can be varied. When the tubes are adjusted to be of equal length the sound of the fork is distinctly heard by the observer at E. As the sliding tube is drawn out, making one tube longer than the other, the sound grows fainter and reaches a minimum when one tube is longer than the other by a half wave length of the sound waves sent out by the fork, for in this case the waves reach the ear through the two tubes in opposite phases and interfere. If the slider is drawn out still farther the sound *increases in strength* reaching a maximum when one *tube is just a whole* wave length longer than the other.

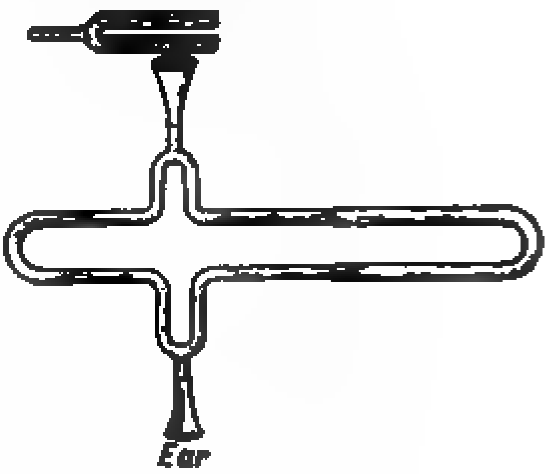


Fig. 181.

If two similar organ pipes of the same pitch are mounted on a rather small air chest, as shown in figure 182, and sounded simultaneously they will usually sound in opposite phases, owing to an oscillation of the air in the air chest itself. The sound waves coming from one pipe will thus interfere with those from the other and the *fundamental* tones will be almost completely neutralized, the higher harmonics will, however, still be heard.

322. Beats.—If two organ pipes sounding together are not exactly of the same pitch the sound comes in pulses or throbs called *beats.* For in this case one pipe is giving out more vibrations per second than the other and, consequently, the relative phases of the two are constantly changing, as shown in the following figure where the dotted curves represent the waves from the two pipes, one of which is supposed to give out eleven vibrations for every ten of the other. The full line represents the resultant motion. It is clear that while one pipe is gaining one complete vibration on the other, there will be an instant when the waves are in opposite phase and interfere and another instant when they will be in the same phase and stengthen each other. There will, therefore, be one hundred beats in the time in which one pipe has made one hundred more vibrations than the other. Or if one pipe makes m vibrations per second and the other n, the number of beats per second is $m-n$.

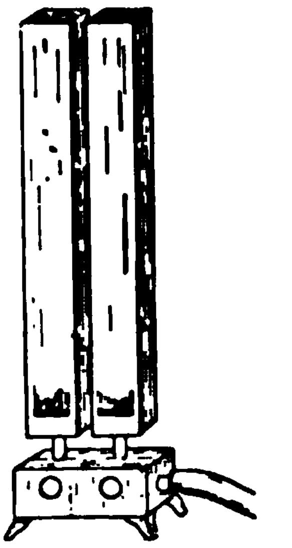


FIG. 182.—Interference between organ pipes.

Beats are easily heard when two adjoining notes on the piano or organ are simultaneously struck, and the lower the notes are on the scale the slower will be the beats. Beats do not occur however, between notes that are very different in pitch. For example, no beats would be heard in case of two simple tones, one making 200 vibrations per second and the other 300 vibrations, though if one made 2000 vibrations and the other 2100 there would be 100 beats per second, heard as a distinctly jarring roughness. The explanation of this was given by Helmholtz (§358).

In tuning two strings or two forks to unison they are adjusted until no beats are heard. If it is required that two tuning-forks shall be very accurately of the same pitch they may be tuned to

make the same number of beats per second with a third fork, as it is easier to count accurately three or four beats per second than to distinguish between no beats at all and very slow beating.

FIG. 183.—Formation of beats.

STANDING WAVES AND VIBRATING BODIES

323. Transverse Vibration of a Cord.—Take a long flexible rubber tube or other elastic cord fixed at one end, as at *P* (Fig. 184), and, holding it slightly taut, let the hand give a sudden movement to one side and back again.

A wave is set up as at *A* which runs the length of the cord, is reflected at *P*, and returns on the opposite side of the cord as shown at *B*. On reaching the hand it is reflected to *A* and the

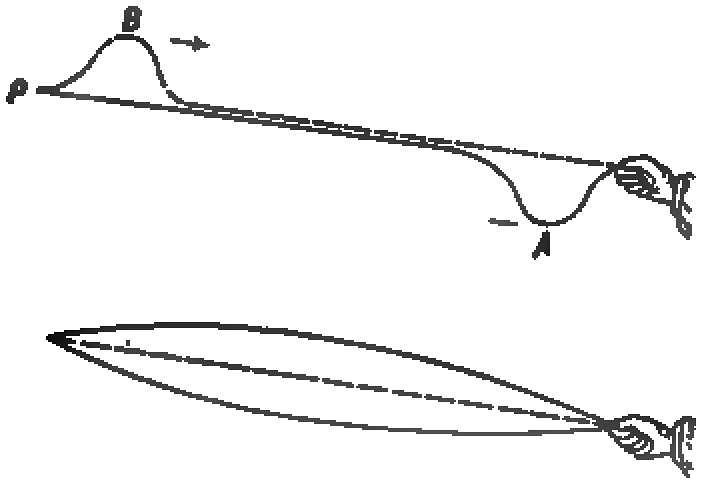


FIG. 184.—Waves in cord.

motion is repeated. **Thus the cord makes a complete vibration and returns to its original form in the time in which a wave runs the length of the cord and returns.**

If the wave is not sent out by so sharp a movement it may take the form shown in the lower part of the figure where the cord simply swings to and fro or vibrates sidewise.

But this vibration is just as truly due to a wave running along the cord and back again as in the first case and *the period of one complete vibration is the time required for a transverse wave to run the length of the cord and return.*

Now, suppose waves are sent out by the hand with *twice* the former frequency, a wave will start from the hand at the same instant that the preceding wave starts back after reflection. The two will meet at the middle and, as they are on opposite sides of the cord in consequence of reflection, they will exactly neutralize each other at the middle point, which will, therefore, remain at rest and the cord will vibrate in two segments. The vibrating segments are called *loops* and the points of rest *nodes*. The period of a complete vibration is in this case only half that of the fundamental period when there is only one loop.

If waves are sent out with three times the frequency of the first case, the cord will break up into three vibrating segments with intermediate nodes, and with greater frequency of vibration a still greater number of segments may be produced.

324. Standing Waves.—These vibrating segments are a particular case of what are called **standing waves, which are set up in water or air or in other elastic bodies by the interaction of similar trains of waves running in opposite directions,** and are usually due to reflected waves meeting those which are advancing.

Standing waves are easily observed on the surface of water in a circular vessel in the center of which a periodic disturbance is produced. If the period of the disturbance is so adjusted that the wave length produced has the proper relation to the size of the vessel, a steady state is produced in which waves going outward meet the reflected waves, causing nodal rings where the water is at rest. Between these rings the surface oscillates up and down.

Standing waves are also produced in organ pipes by the reflection of the air waves from the ends of the pipes.

325. Formation of Standing Waves in Cord.—The diagram (Fig. 185) illustrates the mode of formation of nodes and loops in a cord.

The dotted line represents a wave traveling from right to left along the cord, while the broken line represents an equal train of waves moving from left to right as indicated by the arrows. The resultant wave is shown by the continuous curved line, and its ordinate at any point is the sum of the ordinates of the two component curves at that point. It will be observed

that the crests of the two component waves are approaching each other at the points marked L, and a moment later will coincide. The resultant wave will then be at a maximum. A quarter period later the two component waves will exactly neutralize each other, the crest of one coming exactly over the trough of the other, and the cord which takes the resultant form will at that instant be straight. As the waves move still further the crests A and C will come together in the middle of the diagram and the

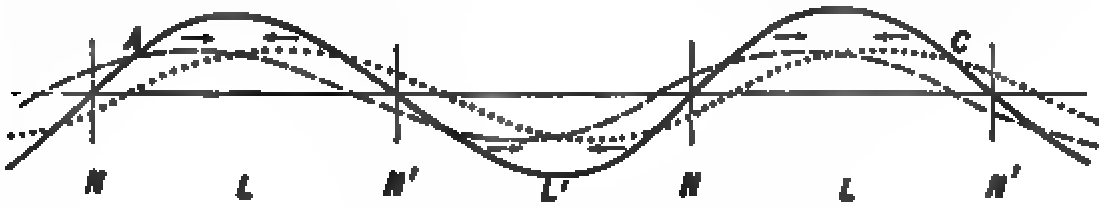


FIG. 185.—Formation of nodes and loops.

resultant wave will then show a form just opposite to that in the figure, being bent up in the middle and down on each side. A little consideration will show that there will never be any displacement at the points N, N'; for in any position of the two component waves one is always as much above such a point as the other is below it; these points are threfore nodes, and **the distance between consecutive nodes is one-half the complete wave length.**

326. Velocity of Wave in a Cord.—The velocity of a transverse wave along a stretched cord may be deduced as follows. Suppose that an infinitely long cord having tension T and a mass per unit length m is drawn rapidly through a bent glass tube as shown in the figure. If the cord were

FIG. 186.—Solitary wave in cord.

at rest it would produce a pressure against the tube in consequence of its tension. At a point where the radius of curvature of the tube is r the pressure against the tube, or force per unit length, is $\frac{T}{r}$, being greater the more sharply curved the tube is at that point. But if the cord is drawn through the tube with velocity V, its centrifugal force per unit length as it runs over the curved part of the tube is $\frac{mV^2}{r}$, and this acts to diminish the pressure.

If the speed is just right, one will exactly balance the other and we shall have

$$\frac{mV^2}{r} = \frac{T}{r} \text{ or } V^2 = \frac{T}{m}.$$

Since the radius of curvature cancels out of the result, the speed at which there will be no pressure against the tube will be the same whatever its radius of curvature may be, and consequently whatever its shape. Suppose the cord is now drawn along at this critical speed, the tube may be made to vanish and the bend in the cord will remain unchanged. If, now, the observer moves along at the same rate as the cord from left to right, the cord will appear to him to be at rest while the bend will be seen to travel along the cord as a wave from right to left with a velocity V, where

$$V = \sqrt{\frac{T}{m}}.$$

The velocity with which a transverse wave runs along a perfectly flexible cord is thus determined from the relation

$$\text{Velocity of the wave in cm./sec.} = \sqrt{\frac{\text{Tension in dynes}}{\text{Mass per cm. in grams}}}.$$

327. Transverse Vibration of Cord.—It has already been shown that the time of vibration of a stretched cord when it vibrates as a whole is the time required for the wave to run from one end of the cord to the other and back again. Or if it is vibrating in segments the period of vibration in one of the segments is the time required for the wave to run twice the length of the segment. If l be the length of the cord or the distance between consecutive nodes, the period of vibration P is

$$P = \frac{2l}{V}$$

and if n is the frequency of the cord, or number of vibrations per second, we have

$$n = \frac{1}{P} = \frac{V}{2l}$$

where V is the velocity of a transverse wave along the cord. Substituting the value of V from the preceding paragraph we have

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}},$$

a formula which expresses in compact form the following three laws:

1. The number of vibrations per second made by a string

under a given tension is inversely proportional to the length of the vibrating segment.

2. In case of two strings of the same length and mass per unit length, the frequencies are proportional to the square roots of the tensions; thus if one has four times the tension of the other it will make twice as many vibrations per second.

3. If two strings have the same length and are under equal tensions, their frequencies will be inversely proportional to the square roots of their masses per unit length. Thus if one is four times as heavy as the other, it will have only half the frequency.

328. Upper Harmonics of Cord.—It has already been seen that a stretched cord may vibrate not only as a whole, but it may also vibrate in two segments or in three, and so on. These modes of vibration may be easily established by touching the cord lightly at a point where a node is desired and at the same time bowing it at a loop. For instance, if a cord is touched at one-fourth of its length from one end and then bowed half-way between the end and the point where it is touched it will vibrate in four segments and give a tone which has a frequency four times that of its fundamental mode of vibration. That the cord actually vibrates in this way was prettily shown by Tyndall as follows: Little riders of bent paper were hung on the cord at the points where nodes were to be established and others midway between them on the loops. On sounding the cord as above described all the little riders on the loops were "unhorsed," while those at the nodes remained undisturbed.

These partial modes of vibration of a cord are known as its *harmonics*, because if the fundamental mode of vibration of the cord has a frequency of n vibrations per second the partial modes of vibration will have frequencies $2n$, $3n$, $4n$, $5n$, etc., according as the cord vibrates in 2, 3, 4, or 5 segments, respectively; thus the frequencies of the partial modes of vibration are related to the fundamental as the terms in a harmonic series.

If cords were *perfectly flexible* this would be *exactly* the case, but as actual cords always have a certain amount of stiffness, which affects the higher harmonics where the vibrating segments are short more than the lower harmonics where the vibrating segments are longer, the above is simply a close approximation to the truth, and the cord when vibrating in four segments, for

example, will have slightly *more* than four times the frequency of the fundamental.

329. Superposition of Vibrations.—When a string is struck or bowed, a number of these different possible modes of vibration are in general set up *simultaneously*. The form of vibration assumed by a cord in which two such modes of vibration coexist is shown in figure 187. The dotted lines show three positions of

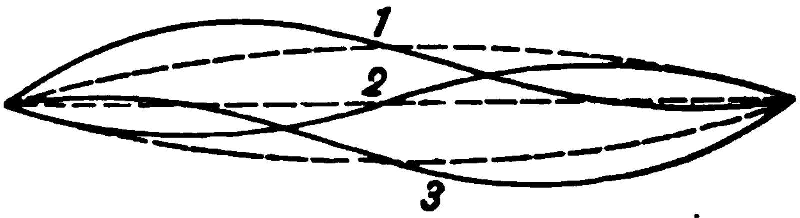


Fig. 187.—Two simple vibrations combined.

the cord simply vibrating as a *whole* in its fundamental mode. The full lines, however, show its form when it is at the same time vibrating in two segments, the two halves vibrating with reference to the dotted lines just as they would have done about the middle straight line if the fundamental mode of vibration had been absent.

330. Experimental Demonstration.—That such a coexistence of different modes of vibration commonly exists in a string may be shown by the following experiment. Pluck the string of a sonometer strongly at, say, one-fourth of its length from one end; then touching the string lightly at the middle, its fundamental mode of vibration will be damped, but it will still be free to vibrate in two segments and the tone characteristic of that mode of vibration, an octave higher than the fundamental, will be heard still sounding. Or if struck one-sixth of its length from one end and then touched lightly at one-third of its length, the fundamental mode will be damped while it will still be heard sounding a tone having three times the frequency of the fundamental.

331. Young's Law.—When a string is struck at any point only those modes of vibration are set up which do not have nodes at that point.

When a string is touched at any point all vibrations are damped that do not have nodes at that point.

It is clear from these laws that in order to stop all the vibrations of string it should be damped at the same point where it is struck. This is done in the piano.

332. Quality of Tone.—Strings are suited for musical instruments because the partial tones that they give out have frequencies which form a harmonic series with the fundamental, and all the lower tones of a harmonic series as far as the seventh harmonic are pleasing when sounded together. The seventh and ninth harmonics are, however, decidedly inharmonious with the others, and therefore it is desirable that in musical instruments the strings should be struck or bowed in such a way that these harmonics may not be developed. This is accomplished in the piano by striking the strings about one-seventh or one-eighth of their length from one end, so that all the partial modes of vibration having nodes near that point are weak.

The hardness of the hammer in a piano also has a decided influence on the tone. The harder the hammer the more sharply the string is bent when struck and the more prominent are the higher harmonics. If the hammer is too soft the tone is soft and lacking in the richness that comes from the proper strength of the harmonics.

Organ Pipes and Wind Instruments

333. Organ Pipes.—An organ pipe may be considered as made up of two parts—a vibrator and a resonator. There are two types in use, *flute* pipes, in which the vibrations are caused by a stream of air rushing against an edge, and *reed* pipes, in which the vibrator is a thin strip of metal.

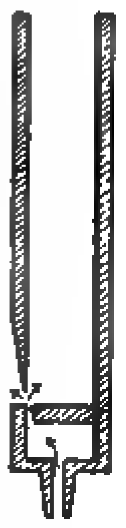


Fig. 188.—Section of organ pipe.

The construction of a *flute* pipe is shown in figure 188. At the lower end of the pipe is the embouchure or mouth which is like that of an ordinary whistle. Air, forced into the air chamber at the bottom, escapes through a narrow slit against an edge just opposite. The upper part of the pipe is a tube which may be either open or stopped at the upper end, and constitutes a resonator which reinforces the vibrations set up at the embouchure.

If a blast of air is sent through a skeleton pipe, which has a mouth-piece but no resonating chamber, a soft whistling noise is heard which rises in pitch with the force of the blast. The pitch of this tone also depends on the bluntness of the edge against

which the blast strikes and its distance from the opening. If the pipe is now provided with a resonating chamber of a proper shape and size to reinforce the vibration, a strong, clear tone will be given out.

The vibrations appear to be caused by the friction of the stream of air against the edge, together with its inertia, just as little waves are formed on the surface of a stream of water in front of a wire or rod which cuts the surface.

The vibrations caused in this way are taken up by the air column in the pipe, which as it vibrates reacts on the stream of air at the mouth-piece, causing it to deflect alternately inward and outward in rhythm with the vibration of the air column; in this way regular impulses are received by the air column, and a strong vibration is maintained.

In case, then, of *flute* pipes it appears that **the size and shape of the resonating cavity is what chiefly determines pitch,** though a certain adaptation in the form of mouth-piece and strength of blast is required in order to evoke a good tone.

334. Nodes and Loops in Organ Pipes.—The vibration of the air column in an organ pipe is a case of standing waves and is due to the

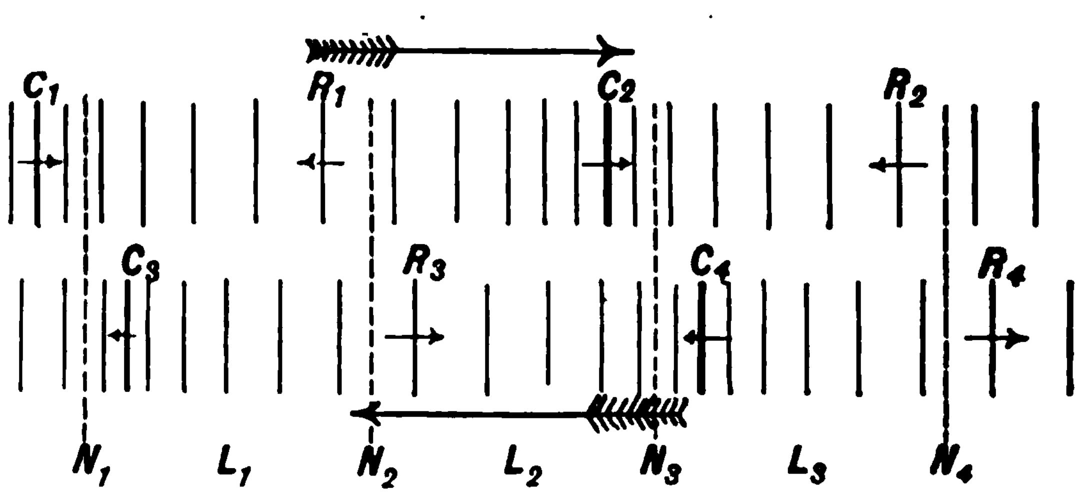


FIG. 189.—Formation of nodes and loops in organ pipe.

interaction between waves running up the pipe and reflected waves moving in the opposite direction, forming nodes and loops just as in case of the vibrations of a cord.

The upper part of figure 189 represents sound waves advancing from left to right in the direction of the upper large arrow. In condensed portions of the waves at C_1 and C_2 the particles are moving forward in the direction of advancement of the wave. In the rarefied portions at R_1 and R_2 the particles have a backward velocity. Immediately under this is represented the state of things in waves returning from right to left. The particles in the condensed portions of these waves have a velocity from right to left as shown by the small arrows, and from left to right in the rarefied regions. If, now, these two sets of waves pass simultaneously through the same mass of gas the par-

ticles take the resultant motion and nodes and loops are formed in the positions indicated in the lower part of the diagram. For it is clear that the rarefactions R_1 and R_2 will reach N_2 simultaneously, tending to produce opposite displacements, and a half period later the condensations C_4 and C_1 will come together at the same point, tending also to produce opposite displacements, and a little consideration will show that at every instant the two sets of waves will balance each other at N_2 so that the particles there

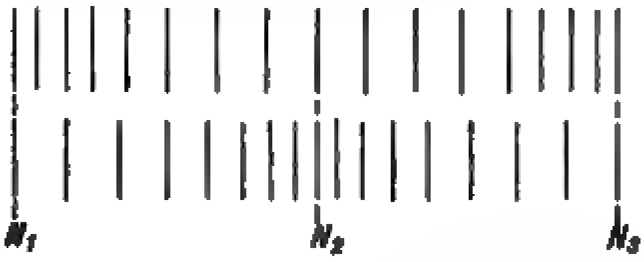


FIG. 190.—Opposite phases at nodes.

will remain at rest. So also at N_1, N_3, and N_4, which are thus *nodes*. But at L_2 the rarefaction R_1 of the advancing wave will arrive at the same instant as the condensation C_4 of the returning wave, and since in both of these the velocity of the particles is from right to left the resultant velocity at L_2 will be from right to left at that instant. A half period later when C_1 and R_4 come together at L_2 the particles there will have a maximum velocity from left to right.

Thus the air between nodes surges back and forth, in one-half vibration swinging toward N_2 on both sides and producing a compression there as shown in the lower part of figure 190. While in the next half vibration the air layers swing away from N_2 and toward N_1 and N_3, producing rarefaction at N_2 and condensations at N_1 and N_3, as in the upper part of figure 190.

At nodes, therefore, the greatest changes in pressure take place, although the nodal layer itself remains at rest, while the motion of the particles is greatest in the loops midway between nodes.

Successive nodes are a half wave length apart and are in opposite phases, one being a point of rarefaction at the instant when the other is a point of condensation. So also *the phases of motion in successive loops are opposite.*

335. Kundt's Experiment.—The nodes and loops in a vibrating column of gas are beautifully shown in the following experiment due to Kundt.

FIG. 191.—Kundt's tube experiment.

A glass tube about 5 cm. in diameter and a meter long is tightly stopped at one end, while in the other is fitted a light piston of cardboard attached to the end of a glass rod which is clamped firmly at the middle. The rod is set in longitudinal vibration by

drawing along it a wet cloth held firmly clasped around the rod. By adjusting the position of the piston *A* in the large tube a point is found where the air column between *A* and *B* is in resonance with the vibrations of the rod. The air is then set in such powerful vibration that any light dust in the tube, such as lycopodium powder, is driven out of the loops and gathers in little heaps in the nodes. This will occur when the sound waves run the length of the air column and back in a certain whole number of vibrations of the rod. In the diagram there are six loops indicating that the rod makes six vibrations while the wave runs the length of the tube and returns.

The stopped end is exactly a node, while the piston end, where the motion is communicated, is *very nearly* a node.

The distance between nodes is a half wave length and may easily be measured with considerable accuracy. The tube may now be filled with some other gas and the distance *AB* again adjusted and the distance between nodes found for this gas also, and since the frequency of vibration of the glass rod is the same in both cases the velocity of sound in the gas is to that in air in the same ratio as the distances between nodes in the two cases.

In this way the velocity of sound has been measured in a large number of gases and vapors.

336. Reflection in Organ Pipes.—Reflection of waves takes place in general when a wave meets a boundary where there is a change of medium.

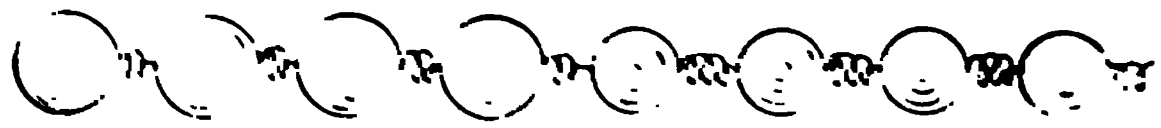


FIG. 192.—Reflection model.

In a stopped organ pipe, a wave running up the pipe meets the unyielding end, and must therefore be reflected in such a way that the reflecting surface is a node, or point of no motion. In an open pipe it is quite the reverse, the wave advancing in the pipe on coming to the open end finds a freer medium, unconstrained by the walls of the pipe, and therefore reflection takes place, but in such a way that the end is a loop or point of great motion.

The mechanical model represented in figure 192 will serve to make clear the nature of these two kinds of reflection. The figure represents a series of balls resting in a frictionless groove.

Half are larger and of greater mass than the other half, and all are connected together by springs of equal stiffness. Suppose the left-hand ball is given an impulse forward, the spring between it and the next will be compressed and the motion transmitted as a wave of compression from one to the other and so on along the line, each coming to rest after giving up its motion to those ahead. But the last of the row of large balls will move forward more freely than it would have done if there had been no change in the size of the balls, and, therefore, it stretches the spring behind it, which gives a forward pull to the ball next behind it and so sends back a wave of rarefaction. This is the kind of reflection which takes place at the *open* end of an organ pipe.

When, on the other hand, a compressional wave is sent from right to left along the row of small balls, the last one does not move forward as far as it would have done if there had been no change of medium, and the spring behind it is, therefore, more compressed and gives a backward impulse to the preceding ball, sending a compressional wave back through the series. This is the kind of reflection which takes place at the *stopped* end of an organ pipe.

337. Open Pipes.—In open organ pipes there must, therefore, be a loop at the top and also a loop at the mouth for there is great motion of the air at these points. Between them near the middle of the pipe is a node.

The position of the node may be demonstrated by lowering into the pipe a horizontal tray of thin membrane covered with sand. If the tray is exactly in the node the tone is not affected, but if it is either raised or lowered a loud buzzing is heard in consequence of the vibration of the membrane.

An open pipe sounding in this way is giving out its deepest or fundamental tone. Since consecutive loops are a half wave length apart, it follows that **the length of an open pipe is half the wave length of its fundamental tone.**

Thus an open pipe 1 meter long gives out waves 2 meters long and accordingly makes $\frac{331}{2} = 165.5$ vibrations per second, since $n = \frac{V}{\lambda}$.

But an open pipe may vibrate in other ways consistent with the condition that the two ends must be loops. Thus it may vibrate having loops and nodes, as shown in figure 193, so that the

distance from loop to loop or node to node may be only one-half the length of the pipe, or it may be one-third the length, etc. The frequency of vibration in the first of these partial modes is, therefore, twice that of the fundamental, that of the next three times, the next four times, etc.

Thus, **any of the harmonics of the fundamental may be produced by an open organ pipe.**

The partial modes of vibration generally coexist to a greater or less degree with the fundamental, giving character and richness to the tone. The relative strength of the harmonics depends on the shape of the mouth-piece and the force with which it is blown and the shape of the pipe itself. The higher harmonics are more emphasized when the pipe is strongly blown.

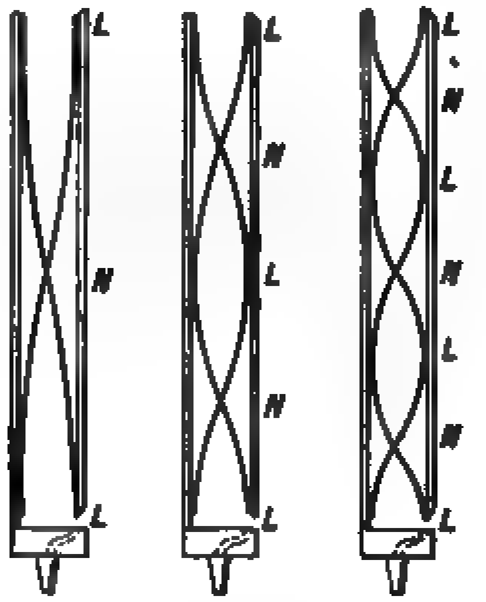


Fig. 193.—Nodes and loops in open pipes.

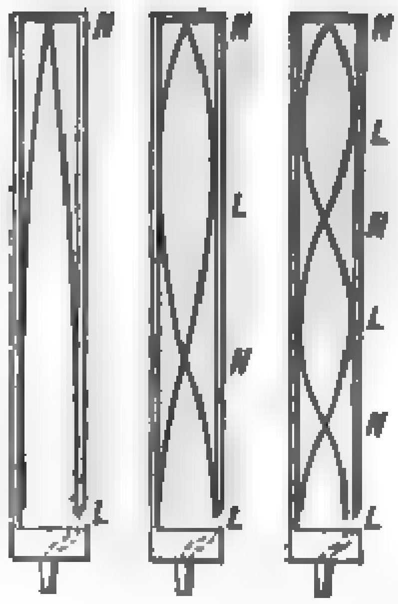


Fig. 194.—Nodes and loops in stopped organ pipes.

The position of the nodes and loops just discussed is only approximately correct, as the open end is not exactly a loop, still less is the mouth exactly at a loop and the node is nearer the mouth of the pipe than it is to the top, the variation being most marked in wide pipes.

338. Stopped Pipes.—In stopped pipes the stopped end is exactly a node, while the mouth is nearly a loop. **Thus the length of the pipe is one-fourth the wave length of the fundamental tone.**

Suppose a compressional wave starts at the mouth of the pipe, it runs up to the top, is there reflected back as a compressional wave, but on reaching the mouth again is reflected in the opposite

way as a rarefaction, then traveling up and being reflected as a rarefaction it returns to the mouth where it is again reflected as a condensation; the wave has thus traversed the length of the pipe four times before returning to its original phase. A stopped pipe one-half meter long would therefore have a wave length two meters long and would vibrate with the same frequency as an open pipe one meter long.

Stopped pipes also may vibrate in other modes than the fundamental, but there must in every case be a node at the top of the pipe and a loop at the mouth. In figure 194 is shown the distribution of nodes and loops in the fundamental and next two upper partial modes of vibration. It will be observed that the distance from node to loop in the first partial mode is one-third that in the fundamental and consequently the frequency of this mode of vibration is three times the fundamental. The next higher mode of vibration has five times the frequency of the fundamental, etc. **Hence a stopped organ pipe may sound its fundamental and odd harmonics,** the frequencies of its proper tones being related to each other as the series 1, 3, 5, 7, etc. The absence of the even harmonics causes the tones of stopped pipes to differ decidedly in quality from those of open ones.

FIG. 195.—Free reed.

339. Reed Pipes.—In reed pipes the vibrations are caused by a metal tongue or reed. Two forms are used, the *free* reed and the *striking* reed. A **free reed,** represented in figure 195, consists of a tongue of thin metal riveted firmly at one end to a plate in which there is an aperture just under the tongue large enough to admit of its vibrating freely through it without touching.

The tongue when at rest is slightly above the aperture and when the reed is blown the stream of air catches it and carries it down, nearly closing the opening, this stops the rush of air and the tongue springs back, when the action is repeated. In this way a vibration is maintained.

Reeds of this type are used in cabinet organs, harmonicas, and accordions.

In **striking reeds** the tongue is a little larger than the opening

and when at rest stands slightly above it. When blown it is carried down and clapping over the opening stops the rush of air, then rebounding is again carried down, thus being maintained in vibration. The tones from striking reeds are stronger and more penetrating than from free reeds. They are used in ordinary tin horns and in the clarinet and in some stops of pipe organs.

When reeds are used in pipe-organs they are provided with resonators which strengthen and improve the tones.

The tones of reed pipes are rich in the higher harmonics, and the shape of the resonator used greatly influences the relative strength of these harmonics and hence determines the quality of the tone produced.

340. Effect of Changes of Temperature.—When the temperature rises the velocity of sound in air increases and consequently the pitch of the flute pipes in an organ is raised. On the other hand, the effect of higher temperature is to diminish the elasticity of the metal tongues of reeds so that they vibrate more slowly, lowering the pitch of the reed pipes. An organ is thus thrown out of tune by great change of temperature.

341. Other Musical Instruments.—In the flute and piccolo the vibrations are produced by blowing across an opening or embouchure near one end, the pitch produced being determined by the strength of blast and by the effective length of the resonating cavity which is regulated by opening or closing holes in its side. The deepest tone of a flute, as of an open organ pipe, is one whose wave length is double the length of the instrument. When blown hard the higher harmonics are sounded.

The mouth-piece of a fife is like an ordinary whistle or flute organ pipe, while the clarinet and oboe have mouth-pieces in which a thin slip of wood mounted over an opening forms a striking reed.

In the bugle the vibrations are due to the air being blown between the tightly drawn lips of the player as they are placed upon a suitable cup-shaped mouth-piece, the pitch being determined by the tension of the lips and by the resonance of the tube. The long coiled tube in such instruments has a very deep fundamental tone the numerous upper harmonics of which can be easily evoked. The cornet is also provided with little valves by

which the effective length of the tube is varied and an additional number of tones made possible.

Vibration of Rods and Plates

342. Longitudinal Vibration of Rods.—If a rod of steel, say a meter long and a centimeter in diameter, is held firmly at the middle point and if a cloth dusted with powdered rosin and folded over the rod is grasped firmly with the hand and drawn off the end with a quick strong pull, a clear, high-pitched sound may be produced due to the *longitudinal* vibrations of the rod. That the vibrations are of this nature may be demonstrated by means of a small ivory ball hung by a cord and resting against the end of the rod. The ball will be violently driven off, swinging out as shown in the figure.

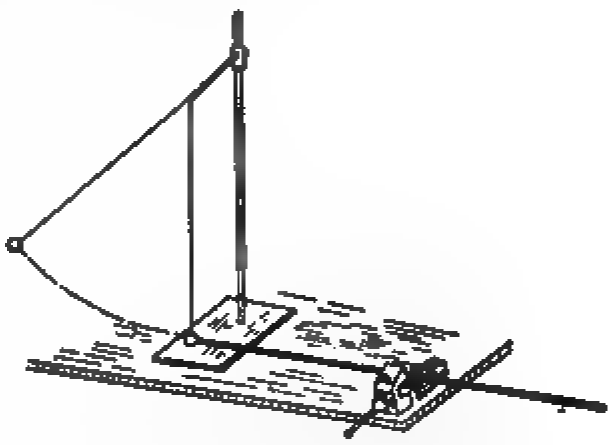


Fig. 196.—Ball driven from end of rod.

Glass tubes held at the middle may be similarly set in vibration, using a wet cloth instead of one dusted with rosin. Tyndall was able to set a large glass tube so powerfully in vibration by this means that the tube was shivered to pieces.

The middle point where the rod is held or clamped is a node and the ends vibrate lengthwise to and fro simultaneously toward the middle or away from it so that the bar is alternately lengthened and shortened. The vibrations are thus precisely like those in an open organ pipe where there is a node in the middle and loop at both ends, and, as in the organ pipe, the period of a complete vibration is the time required for a compressional wave to travel the length of the bar and back again. Thus if the velocity of sound or of compressional waves in steel is 5000 meters per second a bar 1 meter long will make 2500 vibrations per second, since the wave length is 2 meters.

The area of cross section of the bar does not affect the result, the same pitch is obtained from bars of different diameters and shapes of cross section if they are of the same material and length.

By a little dexterity such a rod may be made to give a higher harmonic, vibrating with a node in the middle, and two others, each one-sixth of the length of the bar from the end. The wave length in this case is evidently one-third of that in the former, and the frequency of vibration three times as great.

Rods of other metals or of wood or glass may be caused to vibrate in this way and the velocities of sound in them may be compared by their frequencies of vibration as shown by the tones which they give out.

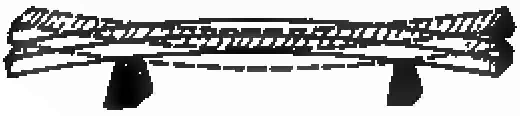


FIG. 197.—Tranverse vibration of rod.

343. Longitudinal Vibration of Wires.—Longitudinal vibrations may also be set up in wires firmly clamped at both ends by rubbing them lengthwise with a bit of rosined cloth.

The clamped ends of the wires are nodes in this case and the middle is a loop. The pitch depends only on the velocity of sound along the wire and on its length and is quite independent of its tension except in so far as the tension affects the elasticity of the wire.

344. Transverse Vibrations of Bars.—The *transverse* vibrations of bars are determined by their mass and stiffness, and hence depend on Young's modulus of elasticity, since it is this coefficient which determines a bar's resistance to bending. If a uniform free bar is struck at the middle point it tends to vibrate as shown in the figure, with a node near each end, and it may be supported at the nodes on wooden bridges without materially affecting its vibration.

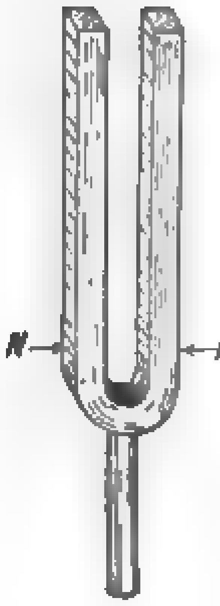


FIG. 198.

In the xylophone or kaleidophone the bars of wood or metal vibrate transversely and are supported at their nodes.

345. Tuning-forks.—A tuning-fork may be considered a bent bar vibrating in the mode shown in figure 197. For there are two nodal points, one on each leg of the fork near the bottom. The prongs swing alternately toward and away from each other, while the stem of the fork, being attached to the vibrating segment between the nodes, vibrates up and down. This is made apparent by the loud tone given out when the stem of a vibrating fork is touched to a wooden table top or sounding board.

Tuning-forks are often mounted on wooden resonators, boxes enclosing an air chamber capable of responding to the vibrations of the fork.

346. Law of Similar Systems.—When two vibrating systems are made of the same material and are exactly similar in dimensions, though not of the same size, their periods of vibration are proportional to their linear dimensions. This law is shown by mathematical reasoning to be a consequence of mechanical principles, and is illustrated in many familiar instances.

For example, if two stopped organ pipes are constructed with cubical resonating chambers, but one having half the dimensions of the other, the smaller will vibrate with twice the frequency of the larger. Two tuning-forks of equally stiff steel and exactly similar in shape will be an octave apart if one is twice as large as the other. And so, also, if we take two straight steel bars, one of which has half the dimensions of the other in each direction, the smaller will make twice as many vibrations per second as the larger when vibrating in the same manner.

347. Vibration of Plates.—The vibrations of flat plates of various shapes were studied by Chladni who scattered sand on

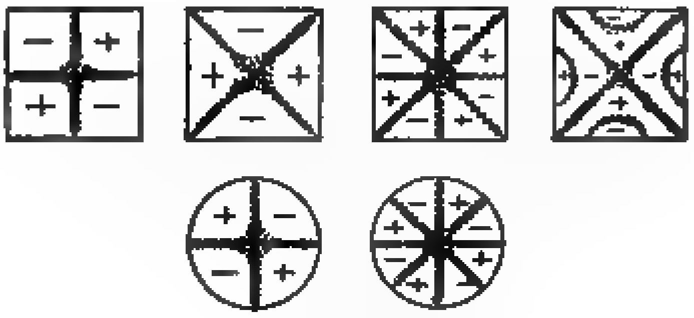


FIG. 199.—Chladni's figures.

the plates and observed the figures formed by the nodal lines in which the sand gathered when the plates were bowed. Some of these forms, known as Chladni's figures, are shown in figure 199. The upper row shows different modes of vibration that may be set up in a square plate supported at its center and bowed at some point on the edge. The slowest mode of vibration is the first, in which the vibrating segments are the four corners. **Segments separated by a nodal line must always be opposite in phase,** one vibrating up, while the other swings down. This opposition of phase is indicated by marking them alternately plus and minus.

If a resonator or wide-mouthed bottle which can respond to the vibrations of the plate is held with its mouth over any vibrating segment it will respond strongly, but if moved over a nodal line so that it is simultaneously

acted on by two adjoining segments it is silent because the segments are in opposite phases.

So also when the plate is vibrating as shown in the first or second diagram in figure 199, if the hands are held just above two similarly vibrating segments so as to quench the sound waves coming off from them, the sound from the plate will be heard louder than before.

348. Bells.—The blow of its tongue on a bell causes the circular rim to spring out into slightly elliptical shape, from which it springs back passing through the circular form into an ellipse with its greater axis at right angles to the first; thus it oscillates in four segments with four intermediate nodes as shown in the figure. Making the rim of the bell thicker causes it to oscillate more quickly by reason of its increased stiffness and thus raises its pitch.

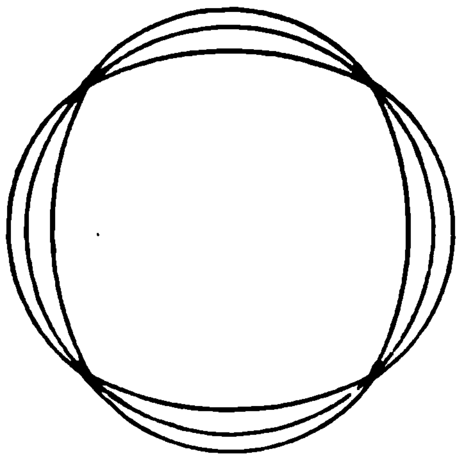


FIG. 200.—Vibration of bell.

The above is its fundamental or slowest mode of vibration, but simultaneously with this the blow of the hammer sets up higher modes of vibration in which the rim may vibrate in 6, 8, or 10 segments with intermediate nodes. These higher tones are not in the harmonic series of the fundamental and hence the tones of bells are unsuitable for music. When the bell is first struck the higher tones are more prominent than the fundamental, but as the sound dies away the fundamental tone persists the longest.

The beating or throbbing heard as the tone of a bell dies away is due to want of uniformity in the rim, in consequence of which there are *two* fundamental tones of slightly different pitch. One or the other of these is excited according to the point struck by the hammer, though in general both are simultaneously set up.

MUSICAL RELATIONS OF PITCH

349. Musical Intervals Depend on Ratios.—The musical effect of two tones when sounded tgether depends upon the *ratio of their frequencies.* This is well shown by means of the siren (§304). If four rows of holes in the siren are simultaneously used, in which the numbers of holes are proportional to 4, 5, 6, and 8, respectively, a combination of tones will be produced which will be recognized as the *major chord—do mi sol do.* And this musical

relationship holds whatever may be the speed of the siren, showing that whatever the pitch may be it is the *ratio* of the frequencies of two tones which determines their musical relationship.

350. Harmonious Ratios.—Tones are harmonious whose frequencies are proportional to any two of the simple numbers 1, 2, 3, 4, 5, 6. The most important harmonious ratios and their musical names are here given:

1 : 1 unison
1 : 2 octave
1 : 3 twelfth
2 : 3 fifth
3 : 4 fourth
4 : 5 major third
5 : 6 minor third

The names are derived from the ordinary musical scale; thus the *octave* is the relation of the first and eighth tones of the scale; the *fifth*, that of the first and fifth; the *fourth*, that of the first and fourth, etc.

351. Major Scale.—Three tones whose frequencies are in the ratio 4 : 5 : 6 form what is known as a *major triad*.

The major scale is a sequence of tones so related that the first, third, and fifth tones form a major triad; also the fourth, sixth, and eighth, and the fifth, seventh, and ninth. The first note of the sequence is called the key-note and the triad starting with the key-note is the triad of the *tonic*. The fifth tone is known as the *dominant* and the fourth as the *subdominant*, and their triads are, respectively, known as the triads of the dominant and of the subdominant.

If the tones of the scale are represented by letters as in ordinary musical notation, their ratios for the key of *C* will be as follows:

Designation	*C*	*D*	*E*	*F*	*G*	*A*	*B*	*c*	*d*
Triad of tonic	4	...	5	...	6			8	....
Triad of subdominant	...	.	....	4	...	5	...	6	..
Triad of dominant	....	3		..	4	.	5	.	6
Ratio to tonic	1	9/8	5/4	4/3	3/2	5/3	15/8	2	9/4

352. Tones and Half Tones.—If the ratio of the vibration frequency of each tone to that of the one immediately preceding it is taken, we find

C	*D*	*E*	*F*	*G*	*A*	*B*	*c*	*d*

$9/8$	$10/9$	$16/15$	$9/8$	$10/9$	$9/8$	$16/15$	$9/8$	etc.
Tone	Tone	Half-tone	Tone	Tone	Tone	Half-tone	Tone	

These ratios determine the musical character of the intervals. When the ratio of the frequencies of two tones is $9/8$ or $10/9$, they are said to differ a *whole tone*, while those whose ratio is $16/15$ are said to be a *half tone* apart.

353. Minor Triad.—In the major triad, three tones whose ratios are 4 : 5 : 6, the interval between the first and second tone is a major third, while that between the second and third is a minor third. If we had three tones in the ratio 10 : 12 : 15, the interval between the first and second would be a minor third (5 : 6) while the interval between the second and third would be a major third (4 : 5). Such a combination of tones is known as a *minor triad.*

354. Minor Scale.—A scale based on minor triads in the same way that the major scale is based on major triads is known as the minor scale. In the key of *C* the tones *C*, *D*, *F*, *G* are the same on both scales, while *E*, *A*, *B* each differs from the corresponding note of the major scale by the interval $25/24$, the minor tone being lower in each case. These tones of the minor scale may be designated *E* flat, *A* flat, *B* flat.

355. Temperament.—Since there are two kinds of whole-tone intervals ($9/8$ and $10/9$) and also two kinds of half-tone intervals ($16/15$ and $25/24$), and since a note a half tone higher than *D*, for example, which is called *D* sharp, would not be the same as *E* flat, it is clear that many notes are required to admit of playing music accurately even in a single key; and this number must be greatly increased if we are also to be able to play correctly in other keys.

In such an instrument as the violin the artist may indeed use true intervals, but in keyed instruments, like the piano or organ, the number of keys that would be required in such a case would make the key-board unmanageably complicated. Hence what is known as the *equally tempered* scale is used. In this scale the

whole tones are all equal and each equal to two half-tones. And as there are five whole tones and two half-tones in an octave, the octave must be equivalent to six whole-tone intervals or twelve half-tone intervals; hence since two notes an octave apart are in the ratio 1 : 2, notes a whole tone apart must be in the ratio $1 : \sqrt[6]{2}$, and those a half tone apart in ratio $1 : \sqrt[12]{2}$.

The following table gives in the upper row the vibration frequencies of notes in the true or diatonic scale, beginning with middle *C* of the piano, while in the lower row are shown the corresponding frequencies in the equally tempered scale.

C	*D*	*E*	*F*	*G*	*A*	*B*	*c*	
261	293.6	326.2	348.0	391.5	435.0	489.4	522	Diatonic scale.
261	292.9	328.8	[illegible]	391.0	[illegible]	492.6	522	Equally tempered.

The Ear and Hearing

356. The Ear.—To make clear the physical basis of hearing a short account of the structure of the ear will be required.

Referring to figure 201, three principal parts will be noticed: the *external ear channel* closed at the end by the tympanum or drum skin, the *middle* ear in which is the chain of little bones or ossicles which connect the tympanum with the inner ear, and the *inner* ear itself in which the auditory nerve terminates and which is contained in a cavity in the massive part of the temporal bone. The small bones of the middle ear are situated in the upper part of a tube containing air, which is known as the *Eustachian tube* and which opens into the back of the mouth through a small valve which opens in the act of swallowing. The air pressure in the Eustachian tube is thus kept the same as that on the outside of the drumskin. Persons going down in diving bells often experience a pain in the ears owing to the difference of pressure, which is relieved at once by swallowing.

The inner ear consists of a long chamber coiled up like a snail shell, and hence known as the *cochlea*, the *three semicircular canals*, and the *vestibule*. There are two openings from the Eustachian tube into the inner ear, one of which is closed by a membrane and the other by the stirrup bone or *stapes*, one of the

four ossicles of the middle ear. The interior of the inner ear is filled with liquid, the *endolymph.*

The cochlea is probably that part of the ear by which musical sounds are distinguished in pitch and quality. It is divided into two parts by the *basilar membrane* which runs lengthwise through all its convolutions dividing it into two chambers. This membrane is strongly fibrous in structure, the fibers running across from one side of the tube of the cochlea to the other. Along its inner edge, where it is attached to the walls of the cochlea, the fibers of the auditory nerve terminate, so that a disturbance of

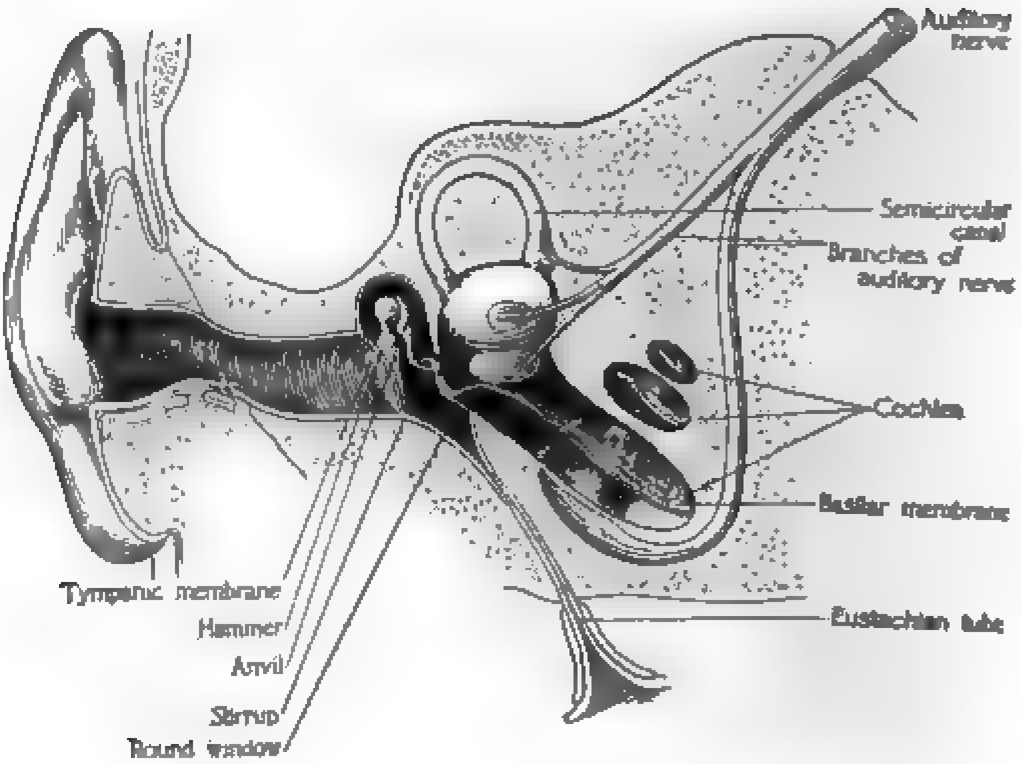


FIG. 201.—Diagram showing tympanum, ossicles, and internal ear.

any part of the basilar membrane causes a stimulus to the corresponding filament of the auditory nerve.

As this membrane also gradually varies in width from one end to the other, its fibers vary in length like the strings of a piano and have their own periods of vibration, which are slower for the long fibers and quicker for the shorter ones.

When sound waves falling on the tympanum cause it to vibrate, these vibrations are transmitted through the ossicles to the liquid on one side of the basilar membrane, then through the membrane itself to the liquid on the other side of it which is in contact with the flexible membrane closing the second opening

into the Eustachian tube. The vibrations are transmitted most easily by that part of the basilar membrane which can vibrate in sympathy with the impressed vibration, and therefore the corresponding nerve filaments are stimulated. The arrangement is such that sounds of different pitch, awaking sympathetic vibrations in different portions of the basilar membrane, stimulate different nerve filaments and so give rise to different sensations.

It is now easy to understand why the ear should analyze complex sounds, hearing each simple harmonic component as a separate simple tone. For it is in accordance with the mechanical laws of sympathetic resonance that when in a complex vibration there is a simple harmonic component which has the same period as the resonator, then the latter will respond. If there are, therefore, three different harmonic components in the vibrations communicated to the basilar membrane, the three corresponding portions of the membrane will be set in vibration, and consequently three different nerve filaments will be stimulated, exciting three distinct sensations of pitch.

357. Influence of Phase.—From the above theory of audition developed by Helmholtz, it is to be expected that the relative phases of the components in a complex tone will have no influence on the resulting sensation, for the same parts of the basilar membrane are set in vibration whatever the phases of the component tones.

358. Beats.—There is one important exception to this. In case of beats the pulsations of tone are certainly due to the changing relative phases of the two components which alternately act together and against each other. But this case is exceptional because the two tones are so near in pitch that they affect closely adjacent portions of the basilar membrane. It is not to be supposed that only a single fiber of the basilar membrane vibrates in response to a particular tone, but the adjoining portions are also set in vibration to some extent.

Suppose that two tones very near together in pitch are sounded and one excites the strongest response in the basilar membrane at *a*, figure 202, and the other at *b*.

The membrane on each side of *a* will also respond to the first tone and on each side of *b* to the second. If *a* and *b* are sufficiently near together, the fibers midway between them will vibrate

simultaneously in sympathy with both tones; they will therefore take the resultant motion and will vibrate alternately strongly and feebly according as the two component vibrations are in the same or opposite phases. Hence the nerve filaments connected with these fibers receive an intermittent stimulus which produces the disagreeable jarring sensation of beats, just as the intermittent stimulus of a flickering light is painful to the eye (Helmholtz).

It is quite in accordance with this theory of audition that rapid beats are heard as a distinct roughness and do not merge into a tone. Thus if two Koenig forks, one making 2816 vibrations per second and the other 2560, are strongly sounded the beats are heard as an extremely disagreeable buzzing, though the number is 256 per second, while a *tone* of that frequency is wholly agreeable when sounded with either of the forks. The beating is due to the disturbance of the basilar membrane between the points where it responds to 2560 and 2816 vibrations per second while to excite the sensation of a tone having a frequency of 256 an entirely different portion of the membrane must be set in vibration, viz., that part which has a natural frequency of 256 per second.

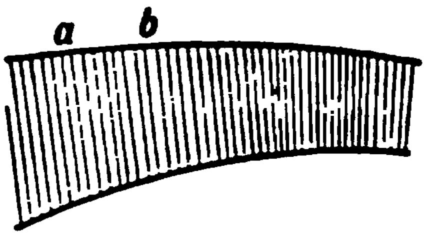


Fig. 202.

359. Combinational Tones.—Under some circumstances when two tones are strongly sounded, a tone is also heard whose frequency is equal to the difference between the frequencies of the two generating tones; its frequency is thus the same as that of the beats between the tones, though it is an entirely separate phenomenon. These *differential tones*, as they are called, may be very distinctly heard when two high-pitched forks are strongly sounded together, such as the two Koenig forks referred to in the last paragraph.

Helmholtz showed on mechanical principles that when two simple harmonic vibrations act on a membrane to set it in vibration, if the displacement of the membrane is so great that it is not simply proportional to the displacing force, then the resulting motion of the membrane will not be simply the sum of the two impressed harmonic vibrations, but will also include other components, one of which has a frequency equal to the *difference* of the frequencies of the two original vibrations, and the other a *frequency* equal to the *sum* of their frequencies.

Now, the tympanum of the ear is attached at the center to a small bone which is drawn inward by a muscle, thus keeping the drum skin tense, hence it is stretched in slightly conical form, and is therefore unsymmetrical and resists inward displacement more than it does outward. Hence, according to Helmholtz, when two strong vibrations are simultaneously impressed upon the tympanum, the motion which it communicates to the inner ear consists not simply of these, but includes also a vibration whose frequency is their difference and another whose frequency is their sum. These are called the *differential* and *summational* tones. The latter were first observed by Helmholtz after he had shown theoretical reasons why they should exist. They are not so easily observed as the differential tones, and some observers have disputed their existence.

360. Helmholtz Theory of Dissonance and Consonance.—Helmholtz showed that **dissonance was explained by beats taking place between either the tones themselves or their upper harmonics or the differential tones that they gave rise to, and that a clearly marked consonance occurs when the ratio of two tones is such that there are no beats, but when a slight change in the ratio gives rise to disagreeable beating.**

For example, the most perfect consonance is *unison*, for then the fundamental tones and upper harmonics all agree and there is no beating, but a slight mistuning causes beating not only between the fundamental tones, but between each pair of harmonics. Suppose, for example, one tone and its harmonics have the vibration frequencies shown in the series

$$100 \qquad 200 \qquad 300 \qquad 400 \qquad 500$$

if the other tone, instead of making exactly 100 vibrations, makes 106, then it with its harmonics will form the series

$$106 \qquad 212 \qquad 318 \qquad 424 \qquad 530$$

and there will be 6 beats per second between the fundamental tones, 12 between the first harmonics, 18 between the second harmonics, etc., The ear, therefore, selects a unison as a well-marked consonance. So also with the *octave*: suppose two tones which with *their harmonics* are given by the two series

Octave	100	200	300	400	500	600
		200		400		600

here the fundamental of the tone making 200 vibrations per second is of exactly the same pitch as the first harmonic of the other tone, and there are no beats between any of the harmonics. But suppose the octave is mistuned, as, for example, below

Mistuned octave	100	200	300	400	500	600
		210		420		630

here there are 10 beats per second between the fundamental of one and the first harmonic of the other, 20 per second between the harmonics 400 and 420, etc., and the result is great dissonance.

It is clear from the above that the richer tones are in harmonics, the more dissonant they will be when mistuned. It is thus much easier to judge whether an octave is tuned correctly in case of two reed pipes than with two wide stopped pipes almost free from harmonics.

Problems

1. How long must a water wave be to travel with a velocity of 20 miles per hour?
2. What relation is there between the lengths of two water waves one of which has twice the velocity of the other?
3. Find the velocity of sound in dry air at 20°C. and pressure 73 cm. of mercury, when its velocity at 0°C. and 76 cm. pressure is 332 meters per sec.
4. What must be the amplitude of motion of the particles in a water wave, if the velocity of the particles at the wave crest is equal to the velocity of the wave?
5. If the height of a water wave from crest to trough is 3 ft. and its length is 50 ft., find its velocity, its frequency or the number of waves that pass per sec., and the direction and amount of the velocity of the water particles on the crest of the wave.
6. How many vibrations per second will be received from a bicycle whistle giving out 500 vibrations per sec. and approaching at the rate of 10 miles per hour?
7. If an observer were to move with the velocity of sound toward a sounding body at rest, what pitch would be heard? What if the observer were at rest while the sounding body approached him with the velocity of sound?
8. A tuning-fork having a frequency of vibration of 1000 per sec. is moved away from an observer and toward a flat wall with a velocity of 5 meters per sec. Find how many beats per second will be heard by the observer.
9. A cord 30 ft. long is stretched between two fixed supports with a force of

40 pounds' weight. How many transverse vibrations per sec. will the cord make if it weighs ½ lb.?

A very long cord weighing 5 gms. per meter and stretched with a weight of 5 kgs. has one end made to oscillate sidewise 4 times per sec. Find the length of the waves set up in the cord.

A brass wire and a steel wire of the same diameter are stretched by equal weights and their lengths adjusted to give the same pitch when vibrating transversely. When the steel wire is 1 meter long between supports, how long will the brass wire be?

How many vibrations per sec. will be given out by an open organ pipe 76 cm. long. Give also the frequencies of its first three upper harmonics. Take temperature of air as 20°C.

How long must a stopped organ pipe be in order to have the same frequency of vibration as the open pipe in problem 12; also what are the frequencies of its first three upper harmonics?

What rise in temperature would raise the pitch of a flute pipe in an organ one semitone? Take original temperature as 0°C. and semitone ratio as $^{16}/_{15}$.

In a Kundt's tube filled with air the distance between the dust heaps is 17 cm., but when the tube is filled wih carbon dioxid gas, the distance between nodes is 13.4 cm. Find the velocity of sound in the carbon dioxid if that in air is 340 meters per sec., both gases being at 15°C., and vibrations being produced by the same rod in both cases.

References

NDALL: *Lectures on Sound* (Appleton).
A delightfully written exposition of the subject by a brilliant experimenter and lecturer.

LMHOLTZ: *Sensations of Tone*, translated by Ellis (Longmans).
A thorough treatise on the scientific basis of music.

HEAT

Thermometry

361. Temperature Sense.—The idea of temperature is obtained directly from our sense of touch. We speak of bodies as hot or cold according to the way in which they affect our temperature sense; and though temperatures cannot be accurately compared in this way, we may yet roughly estimate whether one body is hotter than another or whether a body is growing warmer or colder.

362. Transfer of Heat.—When a hot body is brought into contact with a cold body the former is cooled while the latter is warmed. When a layer of copper is interposed between the hot and cold bodies the change goes on rapidly, but when a layer of felt is interposed the change is much slower. Hot water in a thermos bottle changes its temperature very slowly indeed, so that it is easy to imagine an ideal receptacle in which no change whatever in temperature could occur.

These facts indicate that the temperature of a body changes only when something passes into it from without or escapes from it to other bodies. This something is called heat.

Heat is said to pass from the hot to the colder body rather than that cold passes from the cold to the hot body, because experiment shows that when a body cools it loses something, namely, energy or power to do work, and hence heat rather than cold is considered the entity.

363. Other Effects of Heat.—As bodies change in temperature other accompanying changes take place. As they grow hotter they increase in size, an enclosed mass of gas or vapor exerts a greater pressure, if heated enough a solid melts to a liquid, or a liquid is changed to vapor; also the elastic, electric, and magnetic properties of substances are seriously modified.

364. Equal Temperatures.—When two bodies are placed in contact and no change takes place in either one such as would indicate a transfer of heat, they are said to be at the same tem-

perature. When one grows hotter and the other colder, the latter is said to be at a higher temperature than the other.

Temperature may be defined as that property of a body which determines the flow of heat. If there is no transfer of heat between two bodies when placed together, they are at the same temperature.

Thus temperature plays the same part in the flow of heat that pressure does in the flow of fluids.

365. Thermometers.—To accurately compare temperatures instruments are employed called *thermometers*. Thermometers may be based on the expansive effect of heat, on the changes in pressure in a gas or vapor that are produced by change of temperature, on changes in the electrical properties of bodies, or, in short, on any easily measurable property of a substance, which depends on temperature.

Ordinary thermometers depend on the expansion of a liquid, such as mercury, alcohol, or ether, contained in a bulb of glass having a long tube or stem in which the liquid rises or sinks as it expands or contracts.

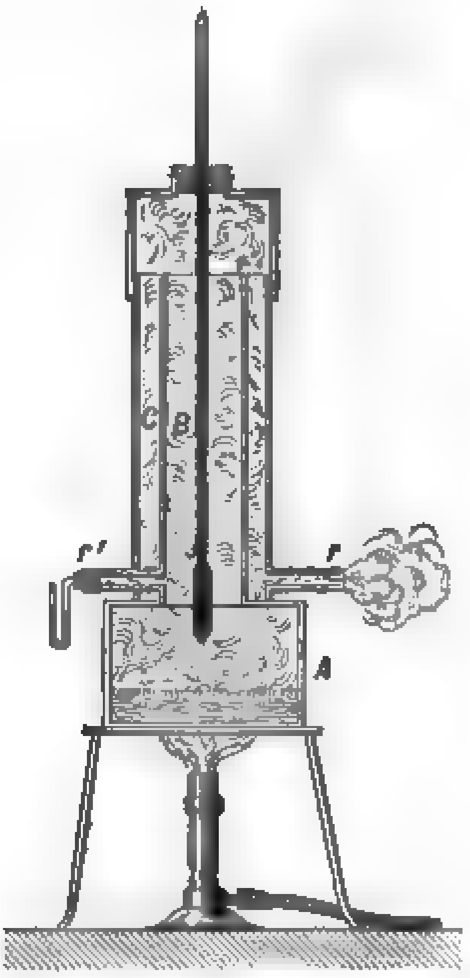


FIG. 203.—Boiling-point apparatus.

366. Fixed Points.—In order that temperature observations by different observers may be comparable, **all thermometric scales are based on two fixed temperatures. These are the temperatures at which ice melts and that at which water boils under standard atmospheric pressure.**

1. The chief precaution to be taken in determining the freezing point is to see that the ice is free from salt. Ice from ponds frequently has traces of salts from the soil. Changes in barometric pressure do not affect the freezing point of water by as much as 0.001°C., and may, therefore, be disregarded.

2. The boiling point is determined by the use of some apparatus, such as that shown in the figure, so that the whole ther-

mometer up to the point at which the mercury stands in the stem is bathed in steam as it escapes from the boiling water. The escaping steam is made to pass down around the outside of the vessel so as to prevent the steam in contact with the thermometer from being cooled.

Impurities in the water may cause it to boil at a temperature slightly above the point at which pure water boils, but the escaping steam will have the same temperature as that from pure water if the pressure is the same. It is for this reason that the thermometer bulb is kept in the steam and is not allowed to dip into the water itself.

The boiling point is decidedly influenced by changes in atmospheric pressure. **An increase of 27.0 mm. in the barometric height raises the boiling point by one whole degree Centigrade.**

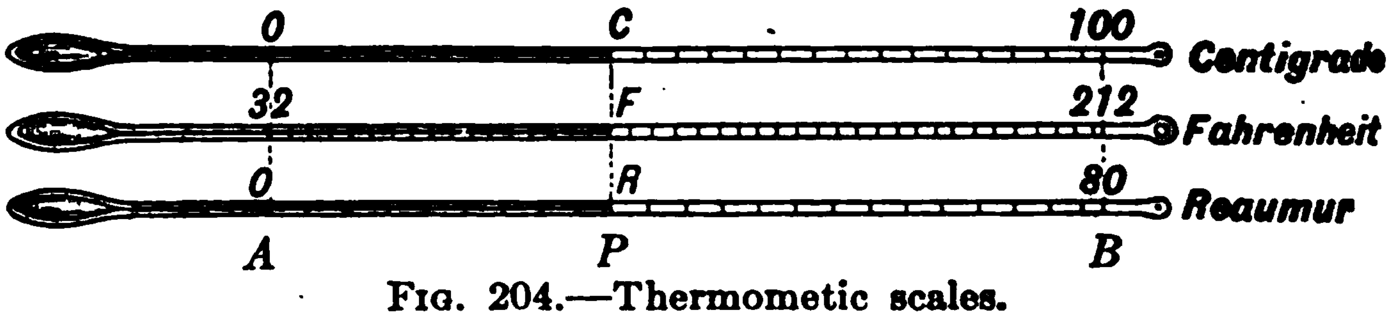


Fig. 204.—Thermometic scales.

367. Scales of Temperature.—In order that a thermometer may be useful in determining intermediate temperatures it must be graduated or divided into intervals or degrees.

Three scales are in general use: the Centigrade or Celsius scale, used extensively on the continent and in most scientific investigations; the Fahrenheit scale, used chiefly in English-speaking countries; and that of Reaumur, used to some extent on the continent of Europe.

In the Centigrade scale the freezing point is marked zero and the boiling point 100, the interval being divided into 100 degrees.

In Fahrenheit's scale the freezing point is 32° and the boiling point is 212°, so that there are 180 degrees between the two.

In Reaumur's scale the freezing point is 0° and the boiling point 80°. The relation of the three scales is shown in figure 204.

Since 180 Fahrenheit degrees correspond to 100 Centigrade degrees, a Fahrenheit degree is $\frac{5}{9}$ of a Centigrade degree. To change Fahrenheit temperatures to Centigrade we, therefore, subtract 32° and take $\frac{5}{9}$ of the remainder. On the other hand, to change Centigrade temperatures to the Fahrenheit scale add 32° to $\frac{9}{5}$ of the Centigrade temperature.

Or since the ratio of the number of degrees between *A* and *P* (Fig. 204) to

the whole number of degrees betwen A and B is the same in each case, we have the equations

$$\frac{C}{100} = \frac{F - 32}{180} = \frac{R}{80}$$

by which relations temperatures on any one scale may be changed to either of the others.

368. Graduation of the Scale.—A thermometer is ordinarily graduated so that each degree corresponds to an equal apparent increase in the *volume* of the mercury. Thus if the thermometer tube is perfectly cylindrical the degree marks are equidistant, but if the tube is not uniform in diameter the degree marks should be so spaced that the *volume* between consecutive marks is the same everywhere throughout its length. The graduations will, therefore, be closer together where the tube is wider and farther apart in the narrower portions of the tube. The 50° mark should be so placed that it divides in half the volume between the 0° and 100° points.

369. Arbitrary Feature of Thermometric Scales.—But it is evident that even such a scale depends upon the properties of the expanding substance and if we were to take two thermometers, one containing alcohol and the other mercury, and were to graduate them in this manner, while they would agree at the fixed points they might not agree anywhere else.

This arbitrary element enters into every scale of temperature. Suppose, for example, we were to define 50° as that temperature which results from mixing equal weights of water at 0° and 100°, respectively. It would be found that if mercury had been taken instead of water the resulting temperature would have been different. And even if we were to define 1° as the rise in temperature of a given mass of water due to the addition of $\frac{1}{100}$ of the whole amount of heat required to raise it from 0° to 100°, the scale of temperature obtained would be different from that obtained by using in a similar way some other substance than water.

370. Peculiarities and Defects of Thermometers.—The rise of the mercury in a thermometer when it is heated is due to the *difference* between the expansion of the mercury and that of the glass bulb; for if the mercury and glass expanded equally the mercury would not rise at all in the tube. Therefore, two mer-

curial thermometers may not agree except at the fixed points, unless they are made of the same kind of glass.

The most serious defect in mercurial thermometers is the change in the zero point. When the bulb is cooled after having been heated it takes a long time to return to its original dimensions. Thus if a thermometer is heated to 100° and then quickly cooled, the zero point will be found lower than before it was heated; this is known as the *depression* of the zero point. The bulb continues slowly to contract, but it may be weeks before the original zero point is reached.

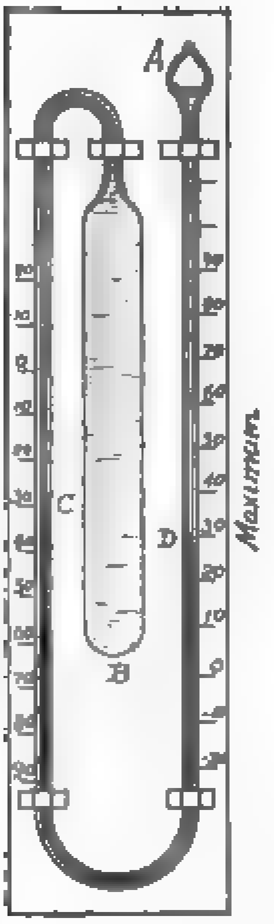


Fig. 205.—Six's maximum and minimum thermometer.

This depression of the zero point may amount to two- or three-tenths of a degree and is a source of error that affects more or less all temperature observations with such instruments, for the reading at a particular temperature depends on whether or not the thermometer has recently been heated to a higher temperature.

Researches carried on at Jena have resulted in the production of a special glass for thermometers, known as the Jena normal glass, which is almost free from this defect and is, therefore, employed in making thermometers for exact work.

371. Spirit Thermometers.—Alcohol and ether thermometers can be used at temperatures so low that mercury would freeze, their expansions also are so much greater than mercury that so fine a tube is not required. But these liquids wet the tube and if the upper part of the stem is cooler than the surface of the liquid column the liquid will distil and condense in the upper end of the tube.

Alcohol expands 6 times as much as mercury and ether 8½ times as much, they are therefore suitable for sensitive thermometers, though on account of the pressure of the vapors of these liquids an alcohol thermometer should not be used above 100°C. and an ether thermometer not above 60°C.

372. Maximum and Minimum Thermometer.—For registering maximum and minimum temperatures the form of instrument devised by Six (Fig. 205) is found convenient. In this instrument the bulb B and the tube as far as the mercury column at C is filled with phenol or some liquid having a large expansion coefficient. The mercury column fills the lower part of the tube between C and D while the tube above D is also filled with phenol reaching up to the bulb A which is only partly filled. When the tem-

re rises the expansion of the liquid in the bulb *B* causes the mercury
ı to sink at *C* and rise at *D*, pushing upward a little index of iron in the
bove *D* which in consequence of friction remains where it is pushed and
the maximum temperature. On cooling the contraction of the liquid
auses the mercury to rise at *C* pushing upward a little index at that
which marks the minimum temperature. To set the instrument the
are drawn down against the mercury column by means of a small
t.

. **The clinical thermometer** used by physicians is a maximum
ometer having a short scale ranging from about 95° to 108°F.; the
made very flat and narrow just above the bulb. The mercury will
pass through the constriction in rising, but as it contracts capillary
auses the column to separate at that point, leaving the upper part of
ercury column to mark the maximum point. To set the instrument
ercury is brought back to the bulb by a vigorous

FIG. 206. Galileo's air thermometer.

. **Air Thermometer.**—Since mercury-in- thermometers can be used only between C. and 450°C., and their readings are so influenced by the peculiarities of the kinds s of which they are made, they are not suited used as independent standards. For stan- purposes air or hydrogen or nitrogen may ed as the thermometric substance, because porcelain bulb such a thermometer can be rom −200°C. up to 1500°C., and the ex- on of these gases is so great (more than y times that of mercury) that the expan- f the glass or porcelain bulb containing the quite insignificant in comparison, and can be allowed for ut sensible error.

rude form of air thermometer which is interesting because used in 1597 by Galileo, the inventor of the thermometer, wn in figure 206. The bulb containing air terminates in a dipping into a vessel of colored liquid which rises or sinks tube, according as the enclosed air contracts or expands. nost evident defect of this arrangement is that the liquid n will rise and fall as the atmospheric pressure changes, though the temperature of the gas may remain constant.

. **Standard Air Thermometer of Constant Volume.**—For xact *measurement of* temperature by the air thermometer

it is found most convenient to keep the *volume* of the air constant and use its *pressure* to measure temperature. A form of instrument devised by Jolly is much used. The bulb A contains the gas to be used, which may be hydrogen or nitrogen or air that has been dried and freed from carbon dioxide. The bulb is connected by a capillary tube with the wider tube at B.

The vertical tubes B and DE are connected by a flexible rubber tube, which is full of mercury, the mercury column extending up into the glass tubes at B and E. The tube DE is attached to a slide and can be raised or lowered along a fixed scale and clamped at any point.

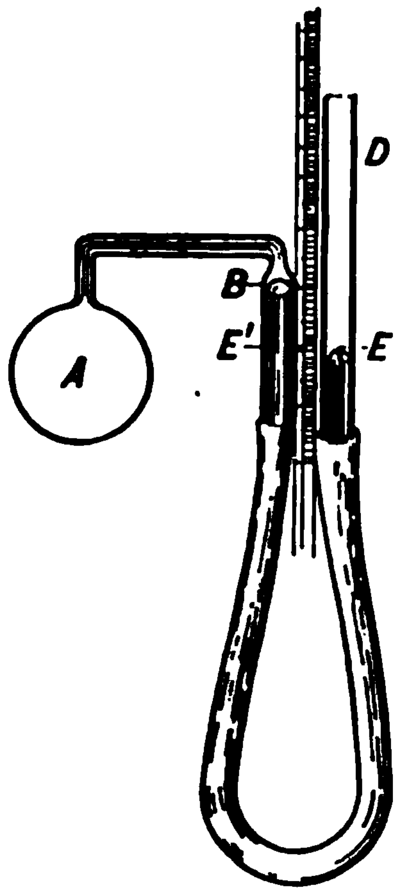


Fig. 207.—Jolly's air thermometer.

In using the instrument the air in the bulb is first cooled to 0° in melting ice and the tube DE adjusted in height until the mercury at B comes exactly to a fixed mark at the end of the capillary tube. The pressure of the enclosed air is then obtained by subtracting the height of the mercury column BE' from the barometric height which gives the pressure of the external air on E. In a similar way the pressure of the enclosed air may be measured when the bulb is heated to 100° in steam. In this case also the mercury level at B must be adjusted to the same point as before, keeping the volume of the air constant except for small changes in the size of the bulb itself. The pressures of the enclosed gas in these two cases may be represented by p_0 and p_1, respectively.

If it is now desired to determine the temperature of a bath in which the bulb is immersed it is only necessary to measure the pressure p exerted by the gas just as in the other cases. If this pressure is found to be half-way between p_0 and p_1 the temperature of the bath is 50°. Or, in general, if t is the temperature to be determined corresponding to the pressure p

$$t : 100 :: p - p_0 : p_1 - p_0$$

In the most refined work we must make corrections for the expansion of the glass bulb itself due both to changes in temperature and pressure, and also take account of the fact that the gas just above B is not at the same temperature as the bulb A.

Such a process would evidently be too cumbrous to employ except for the purpose of standardizing some more convenient working form of instrument, such as the mercurial thermometer or the electrical resistance thermometer.

376. Electrical Methods.—Some very important methods of measuring temperatures are based on electrical phenomena and will be more particularly described in that connection.

The thermo-electric method determines temperature by measuring the electromotive force set up when the junction of two wires made of different metals is heated. For low temperatures a copper-iron junction may be used, while for high temperatures the junction of a pure platinum wire with one of platinum-rhodium alloy is used.

The resistance method depends on the increase in the electrical resistance of a coil of pure platinum wire with rise in temperature (§651).

A resistance thermometer consists of a coil of platinum wire mounted in a glass or porcelain tube to protect it from injury and contamination, and provided with connections by which its electrical resistance may be tested. By means of suitable accessory apparatus the temperature of the coil may be read directly without calculations. On account of the range of temperatures that can be measured in this way (from −270° to 1500°C.), and the accuracy and ease with which the determinations may be made, this is one of the most valuable of all methods of temperature measurement.

377. High Temperatures.—For measuring high temperatures the gas thermometer, or electrical methods, or radiation pyrometers may be used.

The hydrogen gas thermometer having a porcelain bulb may be used up to 1500°C. (§375).

The platinum-rhodium thermo-couple and the electrical resistance thermometer may be used up to 1500°C. if protected by porcelain tubes.

For the highest temperatures *radiation pyrometers* are used. These are of two types. One depends on the heating power of the radiation from a mass of molten metal or from the interior of a furnace, and is so devised that it may be used at quite a distance from the hot body if the radiating surface is large.

The other type depends on a measurement of the intensity of the light from the glowing hot body or interior of a furnace.

Problems

1. Find the Fahrenheit temperatures corresponding to 80°, 20°, −10°, and −50°C.
2. Find the Centigrade temperatures corresponding to 1000°, 98.6°, 0°, and −50°F.
3. What temperature reads the same on both Fahrenheit and Centigrade scales, and at what temperature is the Fahrenheit scale-reading twice that on the Centigrade scale?
4. A temperature interval of 35° on the Centigrade scale is an interval of how *many degrees Fahrenheit?*

5. The absolute zero of temperature is $-273°$ on the Centigrade scale; what is it on the Fahrenheit scale?
6. Calculate the Fahrenheit temperatures of the melting points of iron, copper, lead, and mercury. (See p. 288.)

Expansion of Solids

378. Expansion of Solids.—Almost all solids expand when heated. Isotropic bodies, such as glass and all liquids, expand equally in every direction. Crystals in general expand differently in different directions, and may even contract along one direction and expand in another, but in most cases the expansion more than makes up for the contraction so that there is on the whole an increase in volume with rising temperature.

379. Coefficient of Linear Expansion.—**The fractional part of its length that a rod elongates when raised one degree in temperature is called its coefficient of linear expansion.** Let the length of a bar at 0° be l_0, and let a be its coefficient of linear expansion, then its increase in length for a rise in temperature of 1° will be l_0a, and for t degrees its increase in length is l_0at, so that its total length l at the higher temperature is:

$$l = l_0 + l_0at \quad \text{or} \quad l = l_0(1 + at).$$

In this formula l may be taken as the length of the bar at a temperature t degrees higher than that at which its length is l_0, even though the latter may not be its length at 0°C.

It must not be supposed that the coefficient of expansion of a substance is the same at all temperatures, for in general it increases as the temperature rises. In the above formula a represents the average value of the coefficient throughout the rise in temperature represented by t.

380. Coefficient of Volume Expansion.—If a cube of substance is taken measuring 1 cm. each way at 0°, and having a coefficient of linear expansion a, then its linear dimensions at $t°$ will be $1 + at$ and its volume will be

$$(1 + at)^3 = 1 + 3at + 3a^2t^2 + a^3t^3$$

but the coefficient a is so small that the terms involving a^2 and a^3 may be neglected and the volume may be expressed as

$$1 + 3at.$$

$3a$, therefore, respresents the increase in volume of a unit cube for one degree rise in temperature and may be called the coefficient of cubical or volume expansion; hence **the coefficient of cubical expansion is three times the coefficient of linear expansion in an isotropic body.**

381. Measurement of Coefficients of Expansion.—When the substance whose coefficient of expansion is to be obtained has the form of a long rod, its expansion may be measured by a comparator such as that shown in the figure.

Two microscopes are set on two marks on the bar, one near each end. The microscopes are firmly clamped to a solid base which is kept free from temperature change. The bar to be examined is enclosed in a box provided with glass windows

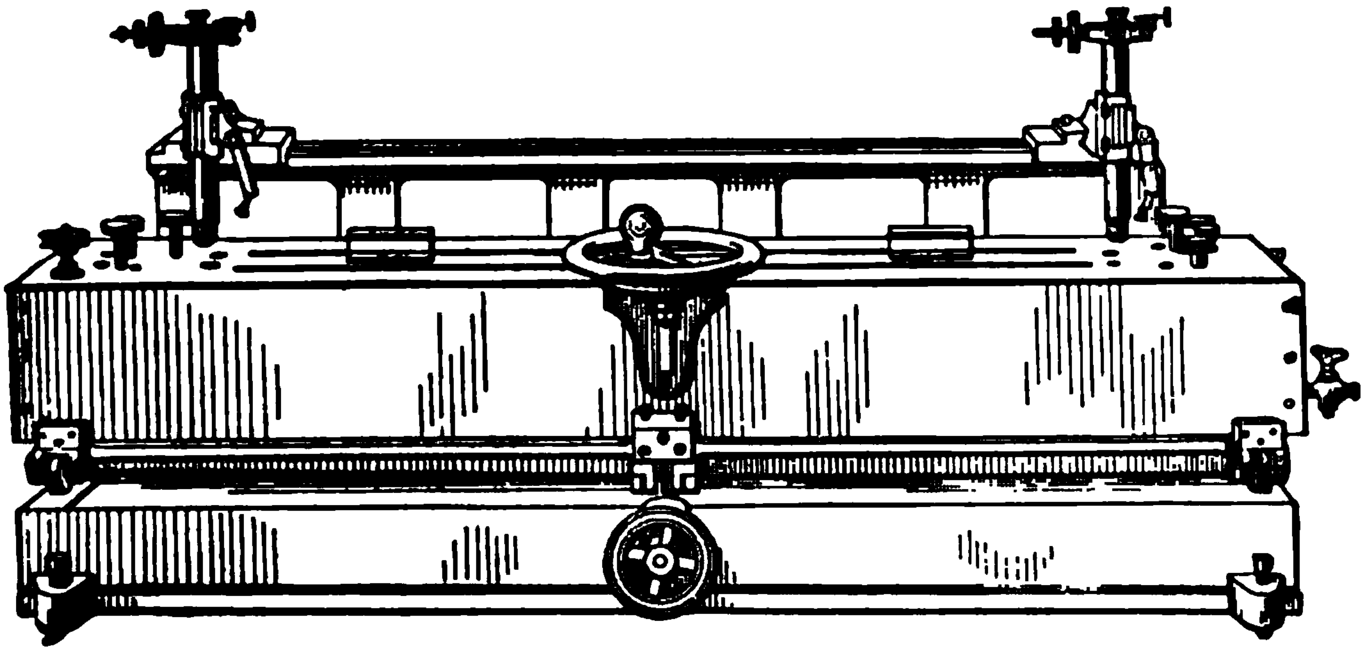


FIG. 208.—Comparator.

through which the microscopes are set on the marks. The bar is first packed in melting ice and the micrometers attached to the microscopes are set on the two marks. Then water at a higher temperature is caused to circulate through the box, maintaining a constant higher temperature, and the micrometers are again set on the two marks. The difference between the micrometer readings gives the elongation of the bar and accurate thermometers give the change in temperature. The whole length of the bar between the marks is then carefully determined.

If this length is l and the elongation is e when the temperature is raised from t to t', the coefficient of expansion a is found from the relation $e = la(t'-t)$, or

$$a = \frac{e}{l(t'-t)}.$$

This is the average value of the coefficient between the temperatures t and t'.

382. Expansion of Crystals.—Crystals that do not belong to the *regular system* expand differently in different directions. A sphere cut out of such a crystal will become an ellipsoid when its temperature is raised. In some cases two of the axes of the ellipsoid would be found of the same length and in some cases all three would be different. The directions in the crystal corresponding to the axes of the ellipsoid are called the axes of thermal expansion. In quartz the expansion at right angles to the axis of the crystal is nearly twice the expansion in the direction of the axis.

Table of Coefficients of Linear Expansion, per Degree Centigrade

Invar	.00000096	Brass	.0000189
Glass	089	Silver	194
Platinum	089	Aluminum	222
Steel	110	Lead	280
Iron	117	Zinc	298
Copper	167	Ebonite	770

These values are approximate. The exact value for any substance depends on the state of hardness, purity, and temperature of the specimen.

383. Some Illustrations.—An iron tire when heated expands so that it can easily be slipped over the wooden rim of the wheel, which it binds firmly on cooling. So the breeches of cannon are strengthened by having a series of tubes shrunk over the inner core, in this way producing an outside compression of the core which enables it to withstand the enormous pressure of the powder gas.

Allowance has to be made for expansion in case of bridges. In a steel bridge 1000 ft. long the change in length between extremes of summer and winter may amount to 8 in.

The aggregate length of the rails in a mile of track may be 4 ft. longer when hottest than when coldest, so that an allowance of about 0.3 of an inch is needed for each 30-ft. rail. The grate bars of furnaces rest loosely in their supports in order to allow expansion, and long steam pipes are provided with sliding or "expansion" joints unless the bends in the pipe are such as to yield elastically to elongation and contraction.

Quartz crystals have very large expansion, and when unequally heated fly to pieces because of the great strains which result in that case. When quartz is fused, however, into a glass, its coefficient of expansion is extremely small, and vessels made of fused quartz may, when red hot, be suddenly quenched in water without breaking.

A specially prepared nickel-steel, having 36.1 per cent. of nickel and known as *invar*, has a temperature coefficient of only 0.0000009 or $\frac{1}{10}$ as large as platinum. It is of great value for measuring bars and tapes, and for pendulums.

Since the expansions of glass and platinum are nearly equal, platinum wires are used wherever wires are to be hermetically sealed into glass, as in case of the connections of an incandescent-lamp filament. Wires of another metal having a greater coefficient of expansion would shrink away from the glass on cooling, leaving a crack through which air could pass.

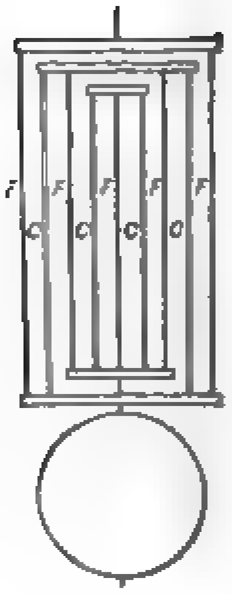


Fig. 209.

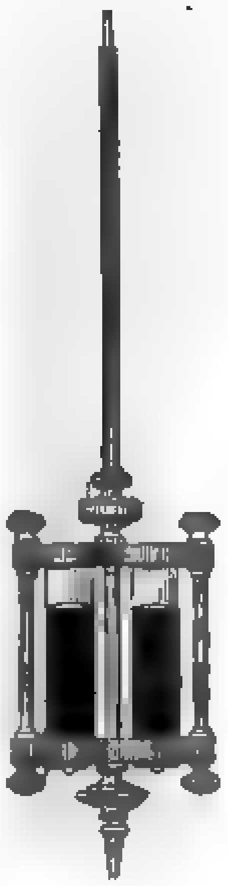


Fig. 210.

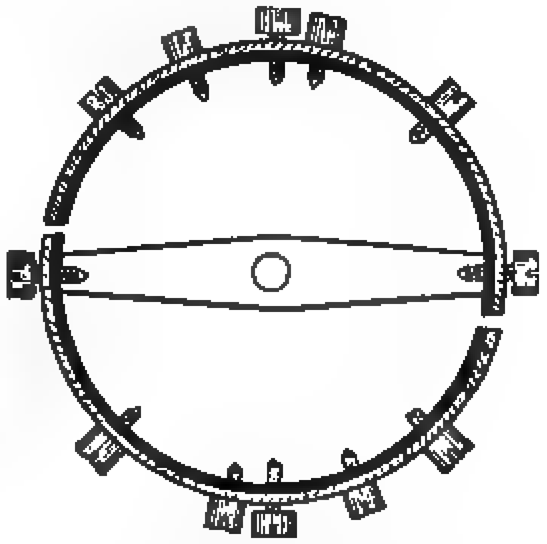


Fig. 211.

384. Compensated Clock Pendulums.—The elongation of a clock pendulum with rising temperature causes it to swing more slowly and the clock loses time. Dry wood pendulum rods have very small expansion and so are sometimes used, but they are affected by moisture. For the most accurate clocks compensated pendulums are used. One of the best forms is *Graham's mercurial pendulum* (Fig. 210), where a reservoir of glass or steel containing mercury is hung by a steel rod. If properly designed, the raising of the center of oscillation (§142) due to expansion of the mercury is balanced by the lowering due to the elongation of the steel suspending rod, so that the effective length remains constant.

In Harrison's *gridiron pendulum* (Fig. 209) the expansion of the steel bars *FF* will lower the bob, while the expansion of the brass rods *CC* will tend to raise it. If the upward elongations of *C* and *C* for a given change in temperature are together equal to the combined downward elongations of *FFF* the bob will neither be raised nor lowered.

385. Watch Compensation.—The balance-wheel of a watch if uncompensated will run slower as the temperature rises, because the elasticity of the hair spring is less at higher temperatures, and also the expansion of the wheel makes its moment of inertia greater.

Compensation is secured by making the balance-wheel as shown in figure 211. The rim is made of brass on the outside and steel on the inside, and instead of being continuous it is cut in two segments which are connected rigidly by a cross-bar. When the temperature rises the brass outer side of the rim expands more than the steel inner side so that the free ends of the segments bend inward, thus carrying part of the mass in toward the axis and so tending to compensate the outward expansion of the cross-bar, and the diminished elasticity of the hair-spring. The adjustment is completed by means of little screws set in the rim of the wheel. Those near the free points tend to increase the compensation, while those near the fixed ends of the segments have the opposite effect.

386. Force of Contraction.—The force produced by the shrinking of a bar on cooling is the same as would be required to stretch it by the same amount at the same temperature.

Problems

1. What is the change in length of the steel cables of a suspension bridge 2000 ft. long between the extremes −20°F. and 97°F.?
2. A brass meter bar is correct at 15°C.; what will be its length at 20°C.?
3. What is the coefficient of expansion of a 30-ft. steel rail on the Centigrade scale and also on the Fahrenheit scale if it changes in length 0.234 in. when the temperature ranges from −17°F. to 100°F.?
4. At 20°C. a brass plug 5 cm. in diameter is $\frac{1}{100}$ of a millimeter too large to fit a hole in a steel plate. At what temperature will it just fit?
5. A glass specific-gravity bottle has a capacity of exactly 300 c.c. at 15°C.; what will be its capacity at 0°C.?
6. A cylindrical zinc pendulum bob has a hole running lengthwise through it in the direction of its axis through which the steel pendulum rod passes, and rests on a cross-piece at the lower end of the rod. How long must the rod and the bob be that the center of gravity of the bob may remain constant at 95 cm. below the point of support while the temperature changes? Take expansion coefficient of steel as 0.000010 and for zinc 0.000029.

Expansion of Liquids

387. Expansion of Liquids.—When a liquid contained in a bulb provided with a long neck is heated, it rises in the stem by

an amount which depends on the difference between the expansion of the liquid and that of the bulb. The rise indicates what is known as the **apparent expansion.** If a bulb containing liquid is suddenly plunged into a vessel of hot water the liquid in the stem may be observed to sink at first because the bulb expands before the liquid within is fully heated.

To determine the expansion of a liquid take a bulb with a graduated stem like the tube of a thermometer and calibrate the stem, or determine the relation between the volume of the whole bulb and the volume of the divisions of the stem. This may be done by filling the bulb with mercury and weighing it, and then separately weighing the amount of mercury required to fill a certain number of divisions of the stem; the relative weights give the relation between the volumes. The bulb is now filled with some liquid up to a certain mark on the stem and then packed in ice or cooled to some steady low temperature and the point to which the liquid contracts is observed. It is then warmed to some higher temperature and the point at which the liquid stands is again observed. From the divisions of the stem between these two points the apparent increase in volume is determined, and if this is divided by the original volume and then by the rise in temperature, the *apparent coefficient of expansion* is obtained.

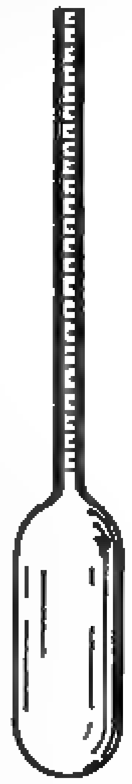


Fig. 212.—Bulb with graduated stem.

The expansion of a glass bulb of volume V, is Vat where a is the coefficient of volume expansion of glass and t is its rise in temperature, while the expansion of the contained liquid is Vbt where b is its coefficient of expansion.

Since the rise of the liquid in the stem is due to the excess of its expansion over that of the bulb the apparent expansion is

$$Vbt - Vat = Vt(b - a).$$

The apparent coefficient of expansion is therefore $b - a$, or the difference between the coefficients of expansion of the liquid and the bulb.

Hence the coefficient of expansion of the bulb must be determined *before that of the contained* liquid becomes known. This

may be accomplished either by studying the expansion of a bar made of identically the same glass or by observing the apparent expansion in the bulb of some liquid whose coefficient of expansion is already known.

388. Weight Thermometer Method.—In case of a liquid such as mercury which has great density and does not wet the glass the apparent coefficient of expansion in a bulb may be determined as follows. The bulb and stem are both completely filled to the very top at the lower temperature, and when the temperature is raised the expanding liquid escapes in drops at the end of the stem, where it is caught and weighed and the amount of the expansion thus determined.

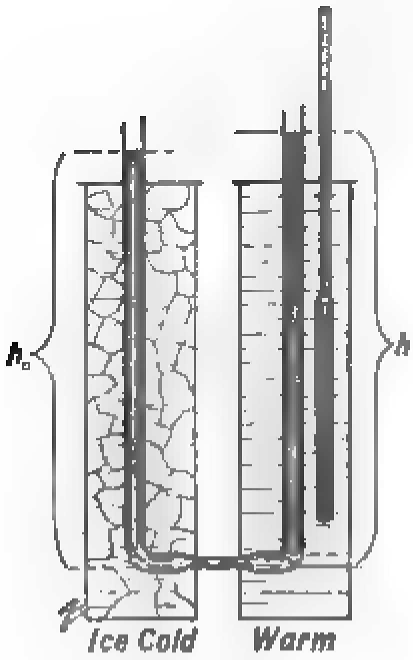


Fig. 213.—Expansion of mercury.

389. Absolute Expansion of Mercury.—The expansion of mercury has been studied with great care because it is the liquid best adapted for use by the weight thermometer method (§388) in determining the coefficients of expansion of bulbs to be used in the study of other liquids. Its coefficient of expansion was determined by Dulong and Petit by the following method which is *independent of the expansion of the tube containing the mercury.*

Two vertical tubes (Fig. 213) connected at the bottom by a very thin horizontal cross tube, contain mercury. One is packed in ice and the other is heated to some known temperature t. Then by the laws of hydrostatics the less dense liquid will stand higher, and the height of the cold column of mercury multiplied by its density is equal to the product of the height of the hot column by its density, or

$$hd = h_0 d_0 \tag{1}$$

But as a given mass of mercury expands in volume it diminishes in density, so that

$$V : V_0 = d_0 : d \tag{2}$$

and since

$$V : V_0 = 1 + at : 1$$
$$d_0 : d = 1 + at : 1$$

or

$$d_0 = d(1 + at)$$

and by equation (1)

$$\frac{h}{h_0} = 1 + at \tag{3}$$

So that by measuring the heights h and h_0 and determining the temperature t of the hot column the coefficient of volume expansion a of the mercury becomes known.

390. Expansion of Water.—The expansion of water has been determined with great accuracy at the German National Laboratory or *Reichsanstalt* by the method just described.

The curve of expansion (Fig. 214) shows that when water is heated from 0°, it first contracts and then expands, reaching its maximum density at almost exactly 4°C.

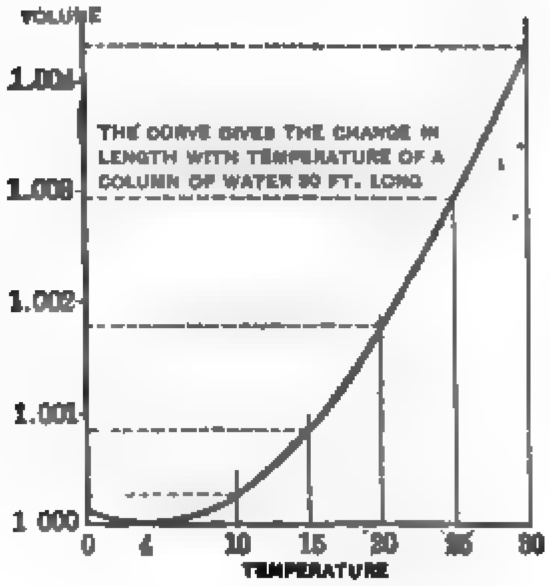


FIG. 214.—Expansion of water from 0°–30°.

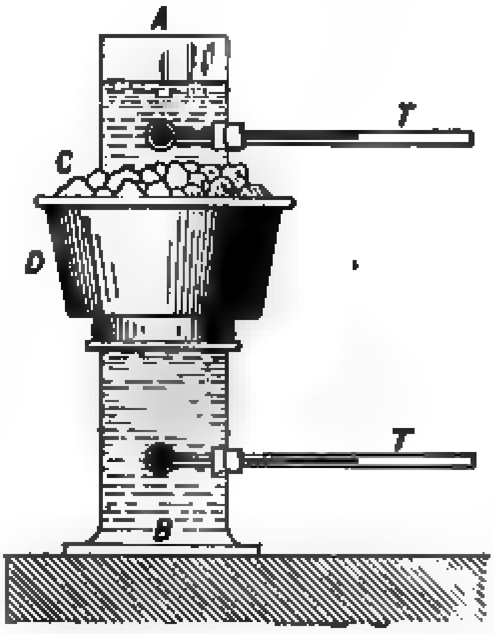


FIG. 215.—Hope's apparatus for determining the temperature of maximum density of water.

This fact is of great importance in nature, for the cooling of a lake goes on rapidly at first, the cooled surface water settling to the bottom, thus aiding the cooling of the whole by convection currents. But when the water has reached 4°C. any farther cooling must be accomplished by the slow process of conduction, for the colder water being less dense will remain at the top. So ice forms at the top and only gradually thickens downward, and if the lake or pond is not too shallow the bottom does not fall below 4°C. for there is a small supply of heat flowing out from

the earth which makes up for that lost by conduction toward the surface.

Hope made use of the apparatus shown in figure 215 to determine the temperature of maximum density of water. A vessel of water provided with thermometers at the top and bottom is cooled above by being surrounded by ice. The lower part of the vessel is carefully jacketed with cotton or felt to prevent the inflow of heat through the sides. The upper thermometer will at first stand higher than the other, but finally the lower will stand steadily at 4°C. while the upper will cool below that point.

The temperature of maximum density of water is lowered when salt is dissolved in it. Sea-water attains its maximum density only at −3.67° which is below its normal freezing point.

Density and Volume per Gram of Water

Temperature	Density	Volume per gram	
0°C.	0.999867	1.000132 c.c.	As found at the Reichsanstalt by the method of balancing columns.
3.98	1.000000	1.000000	
10.00	0.999727	1.000272	
15.00	0.999126	1.000874	
20.00	0.998229	1.001773	
25.00	0.997071	1.002937	
30.00	0.995673	1.004345	
35.00	0.994057	1.005977	
40.00	0.992241	1.007819	
60.00	0.9834	1.0169	Approximate values.
80.00	0.9719	1.0289	
100.00	0.9586	1.0431	

Coefficients of Expansion of Some Liquids

	at 0°	at 20°	at 40°	Average between 0–40
Water	−0.000067	0.000206	0.000388	0.000192
Mercury	0.000179	0.000180	0.000181	0.000180
Alcohol	....	....	....	0.00112
Ether	0.00151	0.00165	0.00189	0.00167

It will be observed that these coefficients are larger than those of solids, *and that in* general they increase with the temperature.

Expansion of Gases

391. Expansion of Gases.—The expansion of gases with heat is much greater than that of solids or liquids and is remarkable for being *nearly the same for all gases.* On account of the great compressibility of gases there are two distinct conditions under which their expansion by heat may be determined. First, the pressure may be kept constant and the volume expansion of the gas measured as the temperature rises, or, second, the volume of the gas may be kept constant and the increase in pressure with rising temperature may be measured.

If the gas perfectly obeyed *Boyle's law* its coefficient of expansion at constant pressure would be equal to that with constant volume.

392. Expansion at Constant Pressure.—Gay-Lussac was the first to carefully study the expansion of gases at constant pressure, but Regnault by the apparatus indicated in the diagram obtained far more accurate results.

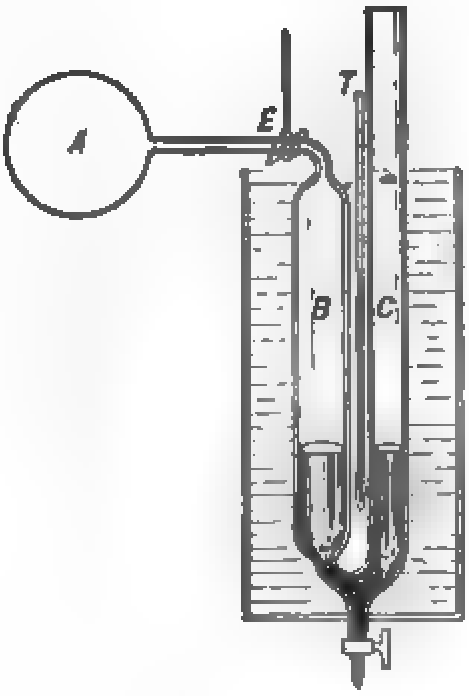


Fig. 216.

The bulb A is filled with the gas to be studied and cooled to zero by means of melting ice. By the stopcock E it is then shut off from the gas supply and connected with B which is completely filled with mercury up to the opening of the small tube at the top, and if the gas in the bulb is at the same pressure as the outer air the mercury will stand in the open tube C at the same level as in B. The bulb A is then heated to any desired temperature, say to 100°, and as the gas expands mercury is allowed to flow out of the stopcock at the bottom so that it is kept at the same level in B and C, thus maintaining the pressure constant. Part of the expanded air is in A at 100° and part in B at the temperature of the water bath which surrounds the tubes. The tube B is graduated, so that the exact volume of the expanded gas may be determined.

393. Increase in Pressure at Constant Volume.—Regnault also was the first to make accurate measurements of the increase in pressure of a gas when the volume is kept constant. The apparatus used is the same as that described above (Fig. 216) but when the bulb A is heated and the expanding gas begins to force down the mercury in B, more mercury is poured into C until the additonal pressure again causes the mercury to exactly fill B. In this way the heated gas is kept confined in the bulb A, and its pressure is measured by the height of the mercury in C above that in B together with the height of the barometer. In this experiment the bulb is expanded slightly both by the rise in temperature and by the increased pressure in the interior, and on account of this change in volume a small correction must be applied.

Coefficients of Expansion of Gases

Gas	Increase in volume at constant pressure per degree C.	Increase in pressure at constant volume per degree C.
Air	0.003671	0.003668
Oxygen	....	3674
Nitrogen	3671	3668
Hydrogen	3661	3660
Carbon monoxide	3669	3667
Carbon dioxide	3710	3687
Sulphurous acid	3903	3845

One cubic foot of air at 0° would expand to 1.367 cu. ft. at 100°, an increase of more than $\frac{1}{3}$ of its volume at 0°C.

From the above table it is clear that **different gases have nearly equal coefficients of expansion. This is known as the law of Charles or Gay-Lussac.**

The increase in volume of a gas per degree rise in temperature is about $\frac{1}{273}$ of its volume at 0°C.

394. Absolute Scale of the Air Thermometer.—According to Charles' law, gases, at constant pressure, expand nearly 0.00366, or $\frac{1}{273}$ of their volume at zero for a rise in temperature of one degree Centigrade. Consider a cylinder filled with air or hydrogen and closed by a piston which always exerts the same *pressure on the* enclosed gas. When the gas is at 0° suppose

iston stands at A, then when the gas is warmed to 100° it ıds and the piston rises to B. If we divide the space from B into 100 equal parts and continue the graduation down· r A, marking off equal spaces for every degree, we shall hat there will be 273 degrees below the zero. If we now he bottom of the cylinder the zero point iall have a scale of temperature in which will be the freezing point of water and 373° e the boiling point. This scale is called bsolute scale of the air thermometer, and ro is called the absolute zero. It is only sary to add 273° to any Centigrade temure to obtain the corresponding temperaon the absolute scale. It will be seen the way in which the scale is obtained ɛ, that the volume of the gas in the cylins proportional to its temperature on the ute scale, and since all gases have nearly ame coefficient of expansion, it may be l as true in general, for all gases that are oo near their points of condensation, that **olume of a gas is very nearly proportional absolute temperature when the pressure t constant. So also when the volume is kept ant the pressure of a gas is nearly proportional to the absoemperature.** At the absolute zero the pressure would be

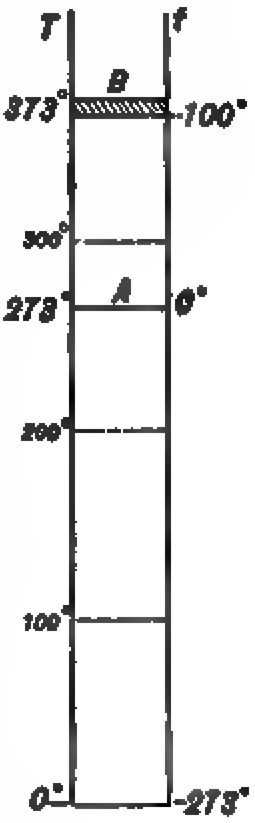


Fig. 217.

There are good reasons for believing that the pressure of is proportional to the energy of vibration of the molecules herefore that at the absolute zero the molecules of gas have ıergy of motion. Consequently this is the lowest possible erature, for if a substance has no energy of motion to give , cannot give out any heat and be cooled farther.

ourse no gas would actually be reduced to zero volume, however much ht be cooled, though its pressure might be reduced to zero. It would ıse into a liquid and cease to behave as a gas before reaching zero e.

entirely independent and more conclusive line of reasoning ɔd to the establishment of *the absolute thermodynamic scale nperature.* (*See* Appendix 1.) This is independent of the

properties of any particular substance, and its zero is the lowest possible temperature. Experiment shows that the absolute scale based on the expansion of gases agrees almost exactly with the thermodynamic scale except at the very lowest temperatures. The zero of the gas scale is therefore properly called the *absolute* zero.

By means of liquid air, temperatures as low as $-200°$C. may be obtained, and by the evaporation of liquid hydrogen $-258°$C. has been reached, only 15° above the absolute zero, while the boiling point of liquid helium is found by Onnes to be $-268.5°$C., only 4.5° above the absolute zero. At these low temperatures rubber and steel become as brittle as glass, lead becomes stiff and elastic, while the electrical resistances of metals are greatly reduced.

395. General Gas Formula.—As was shown in the last paragraph, the volume of a given mass of gas kept at constant pressure is proportional to its temperature on the absolute scale; that is

$$\frac{v}{T}=\frac{v_0}{T_0}$$

where $T=273+t$ and $T_0=273+t_0$, T and T_0 being the absolute temperatures corresponding to t and t_0 of the ordinary Centigrade scale.

By taking account of Boyle's law also, it may be shown that *in general* **whatever changes may take place in the pressure volume and temperature of a given mass of gas, the initial pressure volume and temperature are connected with their final values by the relation**

$$\frac{p_0v_0}{T_0}=\frac{pv}{T}.$$

For, suppose a given mass of gas in a cylinder is in the state A having volume v_0 pressure p_0 and temperature T_0, and is to be brought into a state C in which the volume pressure and temperature are all changed. Let the gas first have its temperature raised to T, *keeping the pressure constant* at p_0, it will come to a volume v' such that

$$\frac{v'}{T}=\frac{v_0}{T_0}$$

:harles' law. Now *keep the temperature constant* at T and ge the pressure to p, the volume will change from v' to v, and ;oyle's law we have $pv = p_0v'$ or

$$v' = \frac{pv}{p_0}.$$

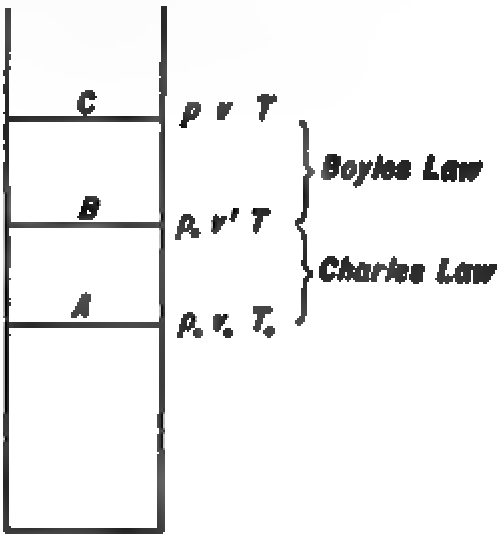


FIG. 218.

tituting this value in the pre- ; equation, we obtain

$$\frac{pv}{T} = \frac{p_0v_0}{T_0}.$$

value of this quantity $\frac{pv}{T}$ is ›ntly proportional to the mass as in the cylinder, for it is that the volume would be ; as great if the mass of gas were to be doubled, keeping temperature and pressure constant. Therefore we have

$$\frac{pv}{T} = mR$$

e m is the mass of the gas and R is a constant which depends on the *kind* of gas. The most convenient form of this ula for use is

$$\frac{pv}{mT} = R \quad \text{or} \quad \frac{pv}{mT} = \frac{p'v'}{m'T'}$$

ie product of the pressure by the volume, divided by the ; and by the absolute temperature is constant for a given of gas.

iis formula is exact only to the degree that the gas obeys e's law. It is, however, a very close approximation to the ı for the more perfect gases when far from their points of ensation.

›roblem will illustrate the use of this formula. Suppose it is required to he pressure that will be produced by 13 gms. of air in a vessel whose ity is 1000 c.c., at 12°C. when it is known that 1 c.c. of air and 76 cms. pressure, weighs, 0.001293 gms. Substituting in the › formula we have,

$$\frac{p \times 1000}{13 \times (273 + 12)} = \frac{76 \times 1}{0.001293 \times (273 + 0)}$$

:e p = *797 cms. of mercury.*

Problems

1. If a column of mercury at 100°C. and 90 cm. high balances the pressure of a column of mercury at 0°C. and 88.4 cm. high, find the density of mercury at 100°, that at 0° being 13.6.
2. Find the average coefficient of expansion of mercury between 0° and 100° from the data given in problem 1.
3. A barometer at 20° has a height of exactly 76 cm.; at what height would it stand if the mercury were at 0°C.?
4. A glass bulb has a capacity of 200 c.c. when placed in melting ice. How many grams of mercury will it contain at that temperature? What does the volume of the bulb become if placed in steam at 100°? Calculate the density of mercury at 100° and find how many grams will now be contained in the bulb. What weight of mercury flows out in the temperature change?
5. A glass bulb at 0° contains 544 grm. of mercury; what weight of mercury will flow out when it is heated to 90°C.?
6. How does the temperature of water affect the depth to which a hydrometer sinks in it?
7. A weighted glass bulb having a volume of 700 c.c. at 20°C. weighs 1.01 gms. when completely immersed in water at that temperature. What will it weigh in water at 4°C., taking the density of water as given in the table on page 254 and taking 0.000025 as the volume coefficient of expansion of glass? Ans. 0.05 grm.
8. If 13 cu. ft. of air at pressure 76 and temperature 20°C. weighs 1 lb., find weight of 900 cu. ft. at 15°C. and pressure 55.
9. A mass of air at 100°C. and pressure 76 has a volume of 5 cubic meters; what will be its volume at 15°C. and pressure 90 cm.?
10. If a liter of air at 0°C. and pressure 76 weighs 1.293 gms., find how much a cubic meter will weigh at 25°C. and pressure 72.

Calorimetry

396. Basis of Heat Measurement.—The quantitative measurement of heat rests on two assumptions. The first is that when bodies at different temperatures are put in contact the cooling of one and heating of the other are due to a transference of something which we call heat from one to the other, and that what one receives is precisely equivalent to what the other loses.

A second assumption is that heat is distributed uniformly throughout the mass of any homogeneous body all at the same temperature. That is, in a mass of water at one temperature every cubic centimeter contains as much heat as any other. These assumptions are justified by experiment, for the results of measurements based on them are so strikingly consistent that

it seems almost as though we were dealing with a subtile substance. Indeed the chemists of 100 years ago were accustomed to think of heat as a substance and called it **caloric.** On account of this the process of measuring heat is called **calorimetry.**

Unit of Heat.—Various units of heat are in use, but the one generally used in physical measurements is the heat required to raise the temperature of a gram of water one degree Centigrade. This unit is known as the *calorie,* but to be precise the exact temperatures must be specified. No general agreement has been reached on this point, but there are advantages in adopting as the unit **the heat required to raise the temperature of a gram of water from 15° to 16°C.,** because 15°C. is somewhat near the average temperature at which experimental work is carried on, and this unit of heat is about equal to the one-hundredth part of the heat required to raise a gram of water from 0°C. to 100°C.

The following table shows the relative values of the calorie at different temperatures, taking that at 15° as the unit.

Temperature	*Value of Calorie*
5°	1.0049
10°	1.0021
15°	1.0000
20°	0.9982
25°	0.9973
30°	0.9971

In engineering practice the kilogram calorie or large calorie is used as a unit of heat on the continent of Europe, while in English-speaking countries engineers usually employ the British thermal unit (written B. T. U.) which is the heat required to raise the temperature of a pound of water one degree Fahrenheit.

397. Specific Heat.—It was discovered by the Scotch chemist Black (1728–1799) that the heat given out by a gram of lead in cooling one degree was by no means equal to that given out by a gram of iron when cooled the same amount, and that in general substances differed from each other in this respect.

The ratio of the heat given out by a mass of any substance in cooling one degree to the heat given out by an equal mass of water in cooling through the same range of temperature is known as the *specific heat* of the substance.

It may also be defined thus. **The specific heat of a substance is the number of calories required to raise the temperature of a gram of the substance one degree Centigrade.**

The following experiment illustrates how substances differ in their specific heats.

A number of balls of different metals, iron, zinc, copper, lead, and tin, of the same mass, are heated in a bath to a temperature of about 150°C. and then placed on a thin cake of paraffin supported above the table. The iron ball having the largest specific heat gives out the largest amount of heat in cooling and so melts the most paraffin. It therefore sinks deepest into the plate and perhaps drops clear through. The zinc and copper balls come next, while the lead having the smallest specific heat sinks in less than any of the others.

The specific heats of some substances are given in the table below.

Table of Specific Heats

Water at 4°	1.0049	Copper	0.0931
Water at 15°	1.0000	Zinc	0.0935
Water at 30°	0.9971	Iron	0.114
Ice at 0°	0.502	Sulphur	0.176
Steam at 100°	0.421	Aluminum	0.217
Lead	0.0310	Lithium	0.941
Mercury	0.0331	Crown glass	0.16
Tin	0.0562	Flint glass	0.117
Silver	0.0570	Normal ther. glass	0.199

It is remarkable that of all ordinary substances except hydrogen water has the greatest specific heat. The specific heat of a substance is in general greater at higher temperatures.

398. Calorimetry.—The measurement of quantities of heat is called calorimetry. If the specific heat or gram calories of heat required to raise one gram of a substance one degree is represented by s, then the heat required to raise m grams of the substance one degree will be ms. And if the temperature is raised from t to t' the rise in temperature is $(t' - t)$ degrees, and the heat taken in by the substance is $ms(t' - t)$. This expression gives also the gram calories of heat given out when the substance cools through the same range of temperature.

Since the specific heat of a substance changes slightly with the temperature, s represents the *average* value of the specific heat between the temperatures t and t'.

The product ms is known as the **heat capacity** of the given mass of substance, **it is the number of calories of heat required to raise the whole mass one degree in temperature.**

Some methods of measuring quantities of heat are discussed in the following paragraphs, while certain other methods based on the melting of ice or the condensation of steam will be discussed later §§425, 426, 441.

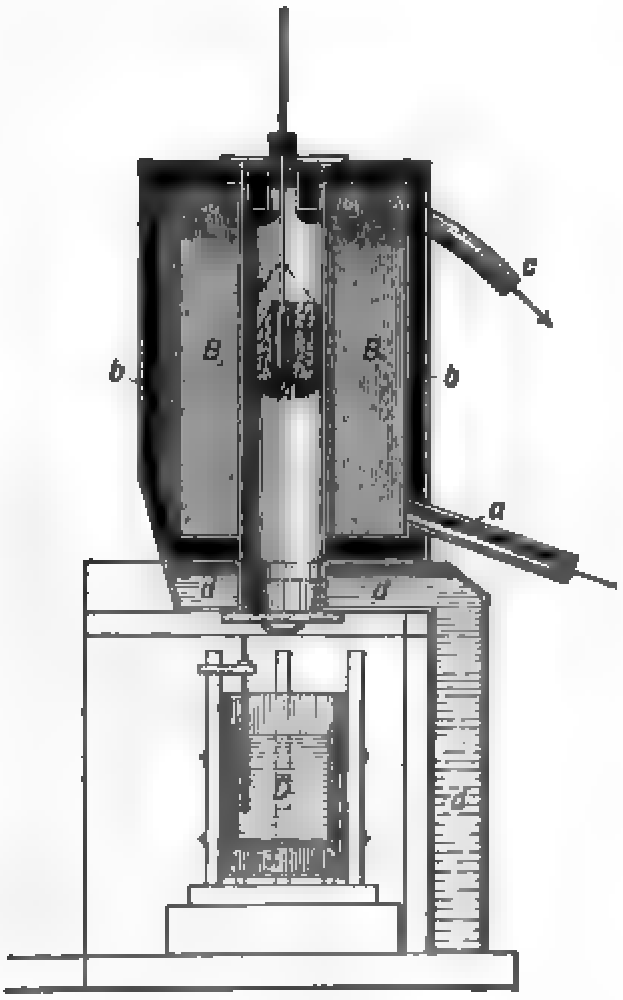


FIG. 219.

399. Method of Mixtures.—Suppose the specific heat of a mass of lead is to be measured by the method of mixtures. A weighed quantity of water is placed in a thin metallic cup or *calorimeter D* (Fig. 219) which is supported on little legs of cork or wood or some poor conductor of heat and is surrounded by an outside vessel to protect it from air currents and radiation from external objects. The mass of lead A having been heated to, say, 100° is suddenly plunged into the water and the water stirred till its temperature has risen as high as it will.

The heat given out by the lead in cooling from its initial temperature t' to the final temperature of the water t'', is $ms(t' - t'')$ where m is the mass of the lead and s is its specific heat. So also the heat taken in by the water as it warms from its original temperature t to t'' is expressed by $WS'(t'' - t)$, where W is the weight of water and S' is its specific heat, which in ordinary work *is taken as 1.*

But the heat given out by the lead must be equal to that received by the water, therefore

$$ms(t' - t'') = W(t'' - t)$$

In the above discussion the heat that went into the cup containing the water has been neglected. But clearly the cup must have experienced the same change in temperature as the water that it contains, and so must have received an amount of heat equal to $m's'(t'' - t)$, m' and s' representing its mass and specific heat. This heat also came from the lead and so must be added to the right-hand side of the above equation; the result is then

Heat given out by the lead in cooling		Heat received by water		Heat received by calorimeter vessel
$ms(t' - t'')$	$=$	$W(t'' - t)$	$+$	$m's'(t'' - t)$

therefore

$$s = \frac{(W + m's')\,(t'' - t)}{m(t' - t'')}$$

The quantity $m's'$ represents the quantity of heat measured in gram calories required to raise the temperature of the calorimeter cup one degree. It is called the *water equivalent* of the calorimeter because it represents the number of grams of water that would require as much heat to raise its temperature one degree as the calorimeter cup requires. It will be noticed that the water equivalent of the calorimeter is added directly to the mass of water which it contains. The water equivalent of the stirring rod and thermometer should be included also, as they, too, are raised in temperature by heat coming from the lead.

The form of heater shown in figure 219 was devised by Regnault. The substance to be heated is suspended in the central tube surrounded by the steam jacket. A thermometer with its bulb in a cavity in the middle of the substance serves to show the steady temperature to which it finally comes. The calorimeter is slipped under the heater and the substance lowered into the calorimeter cup without exposure to cold air currents.

If the substance to be tested is in small fragments they may be held in a basket of light wire gauze whose heat capacity has been previously determined. If the solid is soluble in water some other liquid in which it does not dissolve must be used in *the calorimeter*. The specific heat of liquids as well as solids

may be found in this way, provided there is no chemical action between the liquid and the water in the calorimeter cup which would cause either a development or an absorption of heat.

400. Compensation Calorimeters.—In certain cases, especially where the calorimeter is of necessity large, it is important that the temperature of the instrument may be kept constant so that its heat capacity or water equivalent need not be known.

In the Junkers calorimeter, for instance, used to measure the heat developed in the combustion of gas or oil, a stream of cold water flows through a long copper pipe which is coiled around the combustion chamber. When all comes to a steady state the heat removed by the stream of water per second must be just equal to the heat arising from the combustion in the same time. The temperature of the water is measured at the inlet and also at the outlet by delicate thermometers, and the gain in temperature multiplied by the number of grams of water flowing through per minute gives the heat carried away by the stream of water in that time.

401. Electrical Calorimeters.—The specific heats of liquids may be compared by heating first one and then the other in a calorimeter vessel by means of a current of electricity passing through a coil of wire immersed in the liquid. If the heat developed per second by the electric current is just the same in one case as in the other, and if the masses of liquid used in the two cases are such that the temperature rises at exactly the same rate in both cases, then the heat capacities of the two liquid masses must be the same; that is

$$m_1 s_1 = m_2 s_2$$

where m_1 and m_2 are the masses of the two liquids and s_1 and s_2 are their respective specific heats.

402. Two Specific Heats of Gases.—The specific heat of a gas may be measured while its pressure is kept constant, or it may be measured when the gas is enclosed in a bulb and kept at constant volume. **Experiment shows that the specific heat at constant pressure is greater than the specific heat at constant volume**, and when we come to discuss the relation of heat to work we shall find why this is so. (§412.)

Regnault measured the specific heats of various gases at con-

stant pressure by causing a stream of gas to flow first through a long copper tube coiled in a vessel of hot oil, and then through a copper tube coiled in the calorimeter vessel and surrounded by water. The gas was heated by the oil bath and gave up its heat on passing through the calorimeter so that when the mass of gas which passed through the calorimeter was known its specific heat could be determined.

Great difficulty was found in measuring the specific heat of a gas at constant volume, because its heat capacity is very much less than that of the vessel in which it is enclosed; but the determination was successfully made by Joly using the steam calorimeter (§441).

Specific Heats of Gases

Gas	Specific heats		Molecular weight	Molecular heat	Heat per 1000 c.c.
	Constant pressure	Constant volume			
Air	0.237	0.169	....	....	0.222
Oxygen	0.217	0.155	32	4.96	0.221
Hydrogen	3.409	2.421	2	4.84	0.217
Nitrogen	0.244	0.173	28	4.85	0.222
Carbon dioxide	0.217	0.158	44	6.96	0.312
Chlorine	0.121	0.096	71	6.81	0.427

The relative molecular weights of the gases are given in the fourth column. Evidently 32 grams of oxygen will contain the same number of molecules as 2 grams of hydrogen or 28 grams of nitrogen. The heat capacities of these weights of the various gases is shown in the next column, which indicates that the more perfect gases require nearly the same amount of heat per molecule to raise their temperatures 1°.

In the last column is shown the heat required to raise *equal volumes* of the different gases one degree in temperature, a volume of 1000 c.c. at 0°C. and at atmospheric pressure being taken in each case. Here again is to be noted the equality of the values for the more perfect gases, as would be expected from Avogadro's law that *equal volumes of gases at the same temperature and pressure contain equal numbers of molecules.*

403. Change of Specific Heats with Temperature.—The specific heats of the more perfect gases are nearly constant. The specific heats of *solids and liquids* are in general greater at high temperatures than at low.

e of most metals the change is small, but carbon, boron, and silicon narked increase. These substances have been studied by H. F. Weber, nds for the diamond at $-50°$ specific heat 0.0635, while at 985° it is , the value changing very rapidly at low temperatures and becom- nost constant about 800°. Also the various forms of carbon, graph- d diamond differ greatly in their specific heats at low temperatures, me to nearly the same value as the temperature is raised.

nst finds that the specific heats of substances at very low temperatures proximately proportional to T^3, where T is the absolute temperature, the temperature rises they approach a maximum limit which nearly with Dulong and Petit's law.

. Dulong and Petit's Law.—Dulong and Petit in 1819 ed that **the product of the specific heats of elements in the state by their atomic weights was approximately constant.** constant is proportional to the heat required to raise one one degree and is therefore known as the *atomic heat.* Certain ed exceptions are boron, carbon, and silicon, but all of these ances have specific heats which vary greatly with the tem- ure, and are marked by particularly high melting points.

e following table shows the atomic heats in case of some ances:

Substance	Atomic weight	Specific heat	Atomic heat
minum	27.4	0.2170	5.94
nuth	210.0	0.0308	6.47
alt	58.8	0.1067	6.27
per	63.4	0.0931	5.90
ı	56.0	0.1138	6.37
ne	127.0	0.0541	6.87
ium	7.0	0.9408	6.59
ıganese	55.0	0.1217	6.69
d	207.0	0.0310	6.42
inum	197.4	0.0325	6.42
er	108.0	0.0570	6.16
hur	32.0	0.1776	5.68
	118.0	0.0548	6.46
	65.2	[illegible]	6.10
on (amorphous)	10.9	0.254	2.77
bon (graphite)	12.0	0.174	2.09
bon (diamond)	12.0	0.147	1.76
on (crystalline)	28.0	0.165	4.62

Problems

1. When 10 lbs. of water at 12°C. is mixed with 17 lbs. of water at 20°C. find the temperature of the mixture.
2. If 10 lbs. of water at 12°C. is mixed with 18 lbs. of mercury at 20°C. what will be the temperature of the mixture?
3. If 3 kgms. of copper at 100°C. placed in 3 kgms. of water at 10°C. raise the temperature of the water to 17.7°C., find the specific heat of the copper.
4. A mass of 300 gms. of platinum heated to the temperature of a furnace is dropped into 1000 gms. of water and raises its temperature from 15°C. to 25°C. Find the temperature of the furnace, taking the average specific heat of the platinum as 0.033.
5. A mass of 150 gms. of copper heated to 100° is dropped into 350 gms. of water at 12° contained in a thin copper vessel weighing 30 gms. Find the resulting temperature, taking the specific heat of copper as 0.094.
6. How much heat is required to warm the air in a room 3 × 6 × 5 meters in size, from 0°C. to 20°C., the pressure being constant?
7. A mass of 750 gms. of iron at 100°C. is dropped into a copper calorimeter containing 557.8 gms. of water at 15°C. and warms it up to 25°C. Find the specific heat of the iron if the copper vessel weighs 50 gms.
8. When 150 gms. of copper at 80°C. and 200 gms. of iron at 100°C. are dropped into 400 gms. of water at 12°C. contained in a copper calorimeter weighing 50 gms. find the resulting temperature.
9. How many calories in one British thermal unit (B.T.U.).

Sources and Mechanical Equivalent of Heat

405. Sources of Heat.—There are three principal sources of heat—chemical action, electric currents, and mechanical work. When two substances combine chemically the process is usually accompanied by a giving out of heat. When the action goes on slowly, as when iron oxidizes or rusts, there is but slight rise in temperature, though the actual heat developed is the same as when the same amount of iron is burned in oxygen.

In ordinary combustion there is a rapid combination of the burning substance with the oxygen of the air. Heat must be supplied to start the process, but once started the heat of combination is sufficient to maintain it. Combustion may take place without the presence of air or oxygen. Copper and other metals will burn in chlorine gas. Gunpowder and other explosives contain within themselves all the elements which are to form the new combinations, so that when the spark or jar comes which precipitates the change it goes on with a rapidity which is explosive.

The development of heat by electric currents has become familiar to everyone in arc and incandescent lights and in the various industrial processes which use this source of heat. The laws which govern the heating effect of currents will come into our later study. But it should be noticed here that electric currents are always produced either by chemical action, by mechanical work, as in case of a dynamo driven by an engine, or by the direct action of some other source of heat, as in case of thermo-electric currents. So that in heating by electricity the ultimate source of the heat is the chemical action in the battery cells or the work done by the engine or water power driving the dynamo.

406. Heat of Combustion.—The following table shows the heat developed in the combustion of a gram of various fuels.

Heats of Combustion in Calories per Gram

Hydrogen gas	34,500	Wood	4,000
Anthracite	7,800	Charcoal	8,000
Alcohol (absolute)	7,180	Gasoline	12,000

407. Heat Produced by Work.—Heat may also be developed in a variety of ways from mechanical work, either against friction or in distorting viscous or plastic bodies or in compressing gases.

The savage obtains a fire by twirling, by means of a bow, a pointed stick pressed into a socket where it is surrounded by inflammable material. Everyone is now familiar with the great heat developed by the brakes on car wheels or in imperfectly lubricated bearings, a "hot box" on a railway car often causing a blaze.

Those who held that heat was a *substance*—caloric—in order to explain the production of heat by friction, held that the frictional rubbing of substances caused some "latent heat" to become "sensible."

But Sir Humphrey Davy in 1799 caused two pieces of ice to be rubbed together by clockwork in a vacuum thereby melting some of the ice, which showed that the current explanation was untenable since it was known that ice in melting *takes in* heat instead of giving it out.

In 1798 Count Rumford, who was in charge of the Bavarian cannon shops, being struck by the great development of heat in

turning and boring cannon, caused a blunt boring tool to be turned when pressing into a socket in a metal block immersed in water. In this way water was made to boil and it was shown that heat was developed so long as work was expended in driving the tool. This experiment showed that heat could not be a substance forced out of the metal by the action of the boring tool, and Count Rumford remarks, "it appears to me extremely difficult, if not quite impossible, to form any distinct idea of anything capable of being excited and communicated in the manner the heat was excited and communicated in these experiments, except it be motion."

408. Mechanical Equivalent of Heat.—The honor of establishing the equivalence of heat and work on a solid basis of experiment must be given to the English physicist James Prescott Joule, who, in a series of careful experiments conducted between 1843 and 1850, measured by a variety of methods the amount of work required to heat a pound of water one degree Fahrenheit. In some of these experiments heat was developed by churning water, in others by churning mercury or by rubbing two plates of iron together, or by compressing air, or in rotating a bar of iron between the poles of a powerful magnet, in which case the iron is heated by electric currents developed within it.

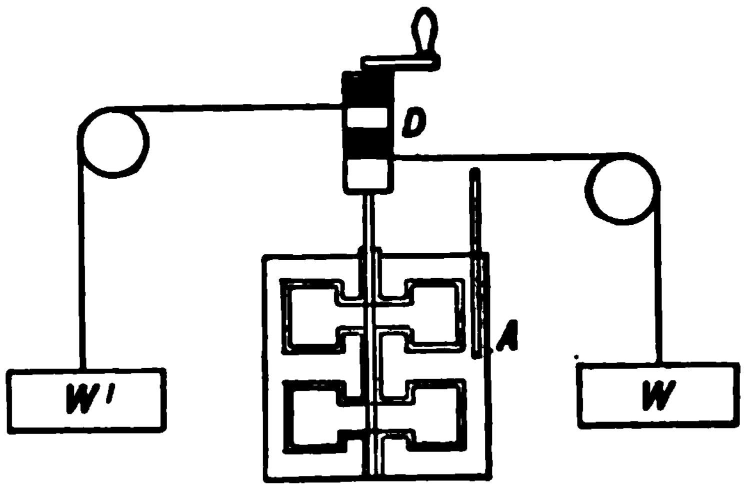


Fig. 220.—Joule's mechanical equivalent apparatus.

And all these diverse methods led to the same result, namely, that the energy required to heat one kilogram of water one degree Centigrade is equal to the work done in raising a weight of one kilogram to a height of 427 meters, or, in other units, 778 foot-pounds of work are required to raise the temperature of one-pound of water one degree Fahrenheit.

For the most exact determination of the relation between heat and work Joule adopted the following method.

A closed calorimeter *A*, filled with water, was provided with a set of paddles attached to a central axle which could be rotated by means of the weights *WW'* which were suspended from cords

wound around the axle D. Fixed vanes projected inward from the sides of the calorimeter vessel so that between the fixed and rotating paddles the water was violently stirred.

The temperature of the water having been taken, the weights were wound up to their full height and then allowed to drop to the floor, turning the paddles as they descended. This was repeated twenty times and the temperature of the calorimeter again read.

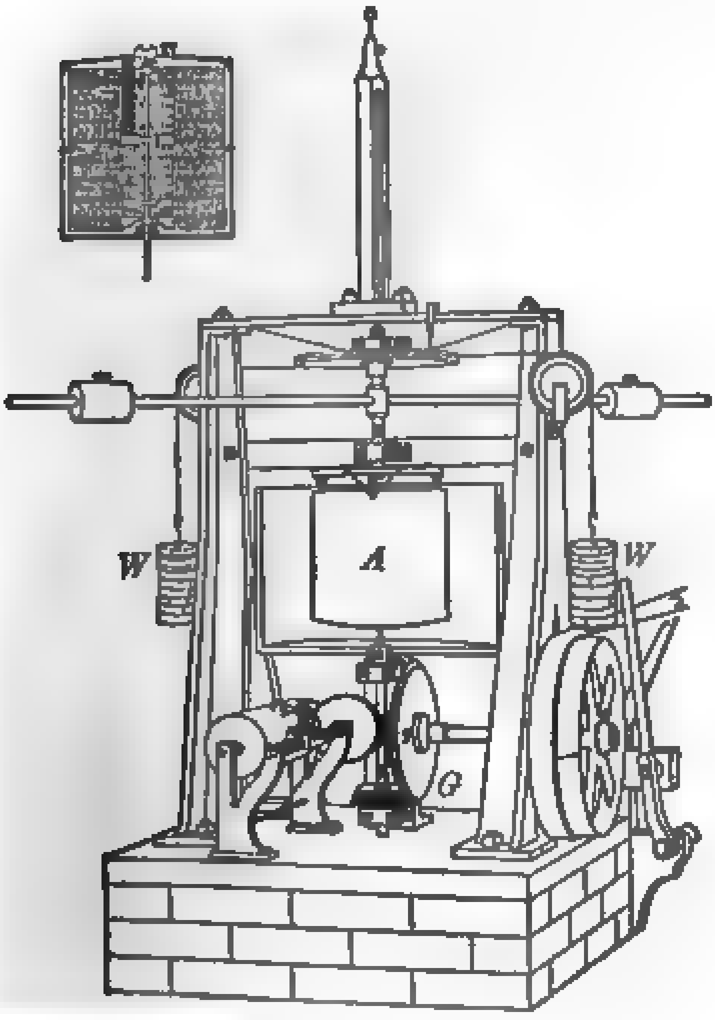


FIG. 221.—Rowland's apparatus for measuring mechanical equivalent of heat.

The total work done was found by multiplying the amount of the weights by the distance through which they fell; but since the weights have some energy of motion when they reach the bottom this as well as the energy required to overcome the friction of the pulleys must be subtracted from the total work in order to obtain that spent in heat in the calorimeter.

A modification of this method used by Rowland made it possible to stir the water continuously, and at the same time measure the work. In this apparatus which is shown in figure 221, the

calorimeter is suspended by a steel wire, while the shaft driving the paddles enters it from below. In making an experiment the paddles are driven at a uniform rate by an engine, and the tendency of the calorimeter to turn is exactly balanced by the weights WW, which are hung from cords attached to the rim of a wheel which is fastened to the calorimeter.

The work done is $2\pi nL$, where n is the number of revolutions of the paddles and L is the moment of the force exerted by the weights WW in balancing the calorimeter. The calorimeter was enclosed in an outer vessel to protect it from air currents and to enable the loss due to radiation to be accurately determined.

The mechanical equivalent of heat as determined by such experiments is found to be as follows:

1 gram calorie = 41,870,000 ergs or 4.187×10^7 ergs.
1 kilogram calorie = 427.3 kilogram-meters of work, taking $g = 980$.
1 British thermal unit = 778 foot-pounds of work.

409. Transformation of Heat into Work.—The experiments of Joule showed that whenever mechanical work is apparently lost through friction, there is always a precisely equivalent amount of heat developed. It remained to show that whenever work is obtained from heat, as in any form of heat engine, *a quantity of heat disappears* which is equivalent to the work done by the engine. This was experimentally proved by the French physicist and engineer Hirn, who in an elaborate series of experiments showed that when an engine was doing work the total heat given out by it in the escaping steam, together with that lost by radiation and conduction, was less than that which it received from the boiler; and that **for every 427 kilogram-meters of work done by the engine, enough heat disappeared to raise the temperature of a kilogram of water one degree Centigrade.**

We therefore conclude that **heat is a form of energy** and that when work is done against friction there is a transference of energy, but no loss of it, as truly as in all other cases of work.

410. Heating of Gases by Compression.—When a mass of gas contained in a cylinder is expanded by drawing out the piston the gas exerts a pressure against the piston as it moves outward and consequently does *work*. But in doing work it expends energy

and consequently *is cooled*. On the other hand, if a mass of gas is compressed the work of compression is done *upon* the gas and its energy is correspondingly increased, and it is *heated*.

The heating effect of compression may be readily shown by the experiment of the *fire syringe*. This instrument consists of a strong glass tube or syringe, closed at one end and provided with a close-fitting piston. If the tube is full of air and the piston is suddenly thrust home, the heat developed will be sufficient to ignite a bit of tinder. Instead of tinder a pellet of cotton soaked in ether may be used, in which case the flash is readily seen as the piston is forced down.

FIG. 222.—Fire syringe.

This experiment shows that the dynamical heating of a gas when compressed is very considerable. When air at 0°C. is compressed in a non-conducting cylinder, its rise in temperature is 90° when compressed to half its original volume, 429° when compressed to one-tenth, and 1084° when compressed to one-fiftieth of its volume.

In air compressors the heat developed in this way has to be removed by a stream of cold water.

411. Cooling Due to Work of Expansion.—A gas when it expands is cooled because it does work; but *is all of the cooling due to the external work done?*

That is, **if it were possible to pull out the piston of a cylinder containing gas so suddenly that the gas could not follow it and exert pressure against it as it moved back, would the gas be cooled or not?**

This question was asked by Joule, and answered by an ingenious experiment in which he connected a copper receiver containing air at a pressure of 22 atmospheres, with another from which the air had been exhausted. On opening the stop-cock between the two vessels the air expanded and filled both, but of course *it did no external work in expansion* since there was no piston to push back.

It was found after the expansion that the mass of gas as a whole had not changed in temperature, the gas rushing into the vacuum being heated just as much as the expanding gas in the *other vessel was cooled.*

More exact determinations show, however, that most gases when expanded are *very slightly* cooled even when no external work is done and it is this cooling of which advantage is taken in the process of making liquid air (§451).

412. Specific Heats of a Gas.—We can now see why the specific heat of a gas at constant pressure must be greater than that at constant volume. For when a mass of gas is warmed while the pressure is kept constant, it *expands*, doing external work. *The heat supplied must, therefore, furnish the energy for this work* as well as that which simply increases the energy of motion of the gas molecules. But when a gas is kept at constant volume there is no external work done and the heat supplied all goes to increase the molecular energy of the gas. According to Joule's experiment (§411), the increase in molecular energy is just the same in one case as in the other, so that **the difference between the heats required in the two cases is entirely due to the external work done, and is mechanically equivalent to that work.**

413. Convective Temperature of the Atmosphere.—The change of temperature caused by the compression or expansion of air plays a most important part in the atmosphere. Masses of air moving upward expand and cool, while descending air masses are heated by compression. This in part serves to determine the distribution of temperature in the atmosphere, the temperature at any height tending to be equal to that which a mass of air rising to that point from the surface of the earth acquires in consequence of its expansion.

The presence of water vapor modifies what may be called the convective temperature at a given height, for the latent heat given out as the moisture in a rising mass of air condenses retards the cooling. A mass of dry air at 20°C. at the earth's surface, will be cooled to -53°C. in rising $3\frac{1}{2}$ miles.

When there is a downward current of air, as in case of the wind blowing over mountains and sweeping down into the valleys beyond, the compression of the air as it descends raises its temperature so that it becomes a warm wind, as in the so-called "foehn" wind of the Alps or the "dry chinook" of Montana.

414. The Nature of Heat Energy.—When a body is heated it radiates heat to surrounding bodies. The rate at which a given body gives off radiation depends on its temperature. As

it grows hotter the radiation may become so intense that the body glows or is incandescent. Later it will be shown that this radiation is made up of waves in which the vibrations are almost inconceivably rapid. These waves originate in the hot body and take energy from it so that it cools as it radiates. It is believed that these waves are a consequence of rapid vibratory motions in the molecules of the body, and that the heat energy of a body exists, in part at least, in the form of energy of motion of the molecules.

Radiation comes from all bodies even those that we ordinarily speak of as cold. The molecules of all bodies are therefore considered to be in rapid vibration, though we are ignorant of the exact nature of this vibration.

But heat energy exists in bodies in another form than energy of vibratory motion, for bodies usually expand when heated, and consequently the particles or molecules are slightly moved apart. And since in case of solids and liquids there is a strong cohesive force between the particles, work must be done in separating them, and the energy which does this work comes from the vibratory energy of the molecules, which is thus transformed and stored up in the body as potential energy.

Another instance of such a transformation is in case of *change of state*, as when ice is melted. Here also particles which are held firmly in a comparatively fixed position in the solid state are dragged away from each other and set free to slip past each other in the liquid state, and to effect this change work must be done, and consequently in this case also a certain amount of vibratory heat energy must be changed into energy of separation or potential energy.

Experiment is in complete agreement with this conclusion and shows that a considerable amount of heat energy must be given to a body to change it from the solid to the liquid form.

Therefore, heat energy is thought of as existing in the body both in the form of kinetic energy or energy of motion of molecules, and as potential energy due to the separation of molecules in opposition to their mutual attractions.

415. Temperature Depends on the Kinetic Energy of the Molecules.—When two bodies at different temperatures are put in contact *there is a transfer* of molecular energy from one to the

other until equilibrium is established. When there is no longer any change taking place, it is said that both are at the same temperature. This transfer of energy is doubtless due chiefly to the energy of motion of the molecules, as it is difficult to see how the potential energy of the molecules of one body could affect appreciably the condition of a neighboring body. When ice and water are mixed together until both come to the same temperature, all flow of heat from one to the other entirely ceases and yet a gram of ice has very much less potential energy than a gram of water at the same temperature. It appears, therefore, that *temperature* is chiefly, if not entirely, determined by the energy of motion, rather than the potential energy, of the molecules.

Problems

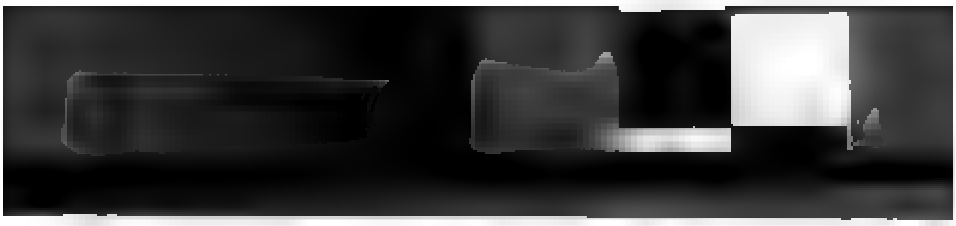
1. The cylinder of an air compressor is cooled by a stream of water in which the flow is 1 gallon per minute. If 10 H. P. is expended in compression, find how many degrees the water is raised in temperature. 1 gallon = 3785 c.c. 1 H.P. = 746×10^7 ergs per sec.
2. How much is the water of Niagara raised in temperature by the fall of 160 ft.?
3. What would have to be the velocity of a lead bullet that it may be melted on striking the target, supposing all its energy to be transformed into heat within the bullet? It takes 5.86 calories to melt 1 gm. of lead.
4. What is the heat of combustion of anthracite coal in British thermal units per pound?
5. How much more heat is required to raise the temperature of a kilogram of air from 0°C. to 30°C. constant pressure, than if the volume were kept constant, and why is more required?
6. How many British thermal units of heat are developed by the brakes when a 100-ton train having a velocity of 20 miles per hour is brought to rest?
7. One liter of air at 0°C. is warmed to 10°C. at constant pressure. Compute the amount of heat required, and the external work done by its expansion, taking the pressure as 76 cm.

   Also compute the heat that would have been required if its volume had been kept constant.

   From these two results deduce the mechanical equivalent of heat (see (§412).

Transmission of Heat

416. Different Modes.—Three modes of transferring heat energy from one place to another are recognized, ***conduction, convection,* and *radiation*.**



When heat energy gradually diffuses through a mass of matter, passing from particle to particle from the warmer toward the colder parts of a body, the process is called conduction. In this case energy of motion is conceived as communicated from molecule to molecule progressively throughout the mass.

When heat is carried along by the motion of a stream of gas or liquid, the process is called convection.

In the above two cases the transference takes place in and through *matter*, but a hot body surrounded by a perfect vacuum may give out energy and warm neighboring objects. **In this case the energy is transmitted by waves in the ether and the process is called radiation.** The term radiation is also applied to the ether waves themselves coming from the hot body.

When one end of a bar of iron is heated the other end becomes hot by conduction; the circulation in a vessel of water which is being heated carries heat from one part to another by convection; while the warmth received from a hot stove comes to us largely as radiation.

Conduction and convection are relatively slow processes while radiation is transmitted with the speed of light.

In transparent bodies, such as glass or water, heat is communicated from one part of the substance to another by conduction and radiation combined, for energy is radiated through the body directly from one part to another at the same time that it is being communicated from molecule to molecule by conduction.

While radiation originates in hot bodies and heats any body which absorbs it, **radiation itself cannot be regarded as heat; for unless it is absorbed it does not affect the temperature of the bodies through which it passes.** We shall study the nature of radiation in connection with *light;* while radiation considered as an *effect* of heat will be discussed in §§461–472.

417. Conduction.—In general solids conduct heat better than liquids, and liquids than gases. Silver and copper are the best conductors of heat, having about 7 times the conducting power of iron, while iron conducts 100 times as well as water, and water has 25 times the conductivity of air.

Among solids the metals are the best conductors, and it is remarkable that, ***generally speaking*, the best conductors of heat *are also the best conductors* of electricity.**

In crystals heat may be conducted more rapidly in one direction than another. If a thin plate of quartz is coated with wax or paraffin and if a wire kept hot by an electric current is passed through a hole in the center of the plate, the wax will melt outward in elliptical form if the plate is cut parallel to the axis of the crystal, showing that heat is conducted more rapidly in the direction of the axis, than at right angles to it.

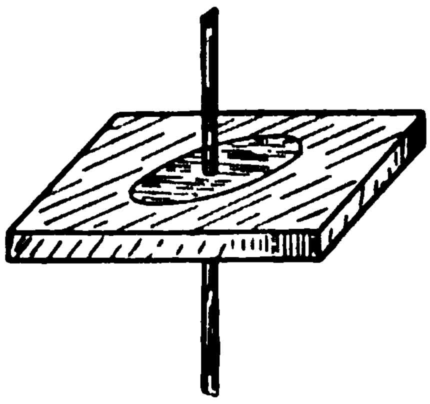


FIG 223.

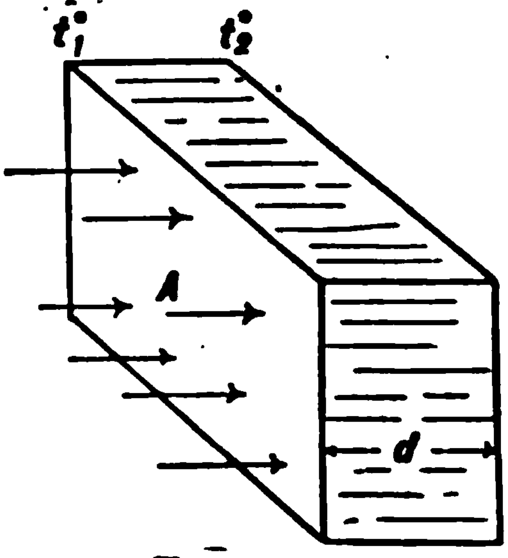


FIG. 224.

418. Conductivity.—If a slab of substance of uniform thickness d and faces of area A has one surface at temperature t_1, and the other at t_2, the heat H which is transmitted per second will be proportional to the area of the faces A and to the difference in temperature of the surfaces $t_1 - t_2$, while it will be inversely proportional to d the thickness of the plate. Thus,

$$H = \frac{kA(t_1 - t_2)}{d}$$

where k is a constant which depends on the substance of which the slab is made, and is known as its coefficient of conductivity or simply its *conductivity*. When A is one square centimeter and d is one centimeter, and $t_1 - t_2$ is one degree, then $H = k$; that is, **the conductivity or conducting power of a substance is measured by the number of gram calories of heat which are transmitted in 1 second through a plate one centimeter thick and having surfaces one square centimeter in area when the opposite faces differ in temperature by one degree Centigrade.**

The drop in temperature per centimeter between one side of the plate and the other is called the *temperature gradient* and is expressed in the above formula by

$$\frac{t_1 - t_2}{d}.$$

419. Measurement of Conductivity.—The conductivity of a metal which is a good conductor may be measured by the following method due to Searle. A short bar, say 1 in. in diameter and 6 in. long, is heated at one end by steam while the other end is kept cool by a stream of water which flows through a pipe closely wound around the bar in a helix as shown in the figure.

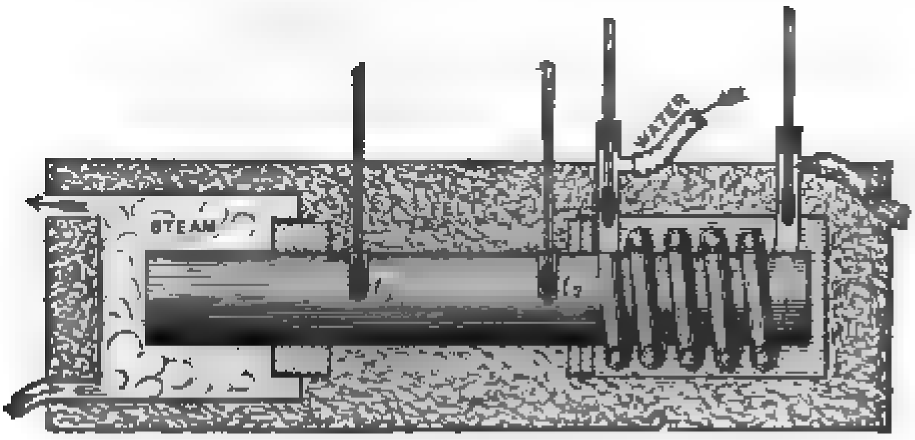


FIG. 225.

The whole is thoroughly packed in hair felt, which is a very poor conductor, so that heat cannot escape from the sides of the bar but must flow from the hot toward the cold end, where it is removed by the stream of water. When a steady condition of flow is reached, the heat passing along the bar in any time will be equal to that removed in the same time by the stream of water. Thermometers are mounted at the ends of the helical tube, by which the temperature of the water may be observed as it flows in and flows out, and the gain in temperature of the water as it flows through the helix multiplied by the total weight of water that passes through in, say, 10 minutes, gives the gram calories of heat transmitted along the bar in that time. The temperatures at two points on the bar, such as t_1 and t_2 are measured by thermal junctions or thermometers, the distance d between those points is measured and also the area of cross section A of the bar. All the elements are then known for computing the conductivity from the formula of the preceding article.

In case of poor conductors the flow of heat through a thin slab may be measured by such a method as that of Lees. A very broad and thin disc-shaped box of copper, heated by means of a current of electricity which passes through a resistance coil contained in the box, is placed between two thin plates of the sub-

stance of which the conductivity is to be determined, say these are plates of glass. Outside of the glass plates are placed solid copper discs, so that each plate of glass comes between the central heating box and an outer disc, as shown in section in figure 226. The temperatures of the outer discs and also of the inner box are determined by thermometers or thermal junctions (§670), and since copper is so good a conductor compared with glass the surfaces of the glass plates may be supposed to be at the same temperatures as the copper plates with which they are in contact. A band of thick felt encircles the edge of the discs to prevent loss in that way and the heating is maintained constant until a steady state is reached where there is no longer any change in the temperatures and the heat flows out through the glass plates as fast as it is generated.

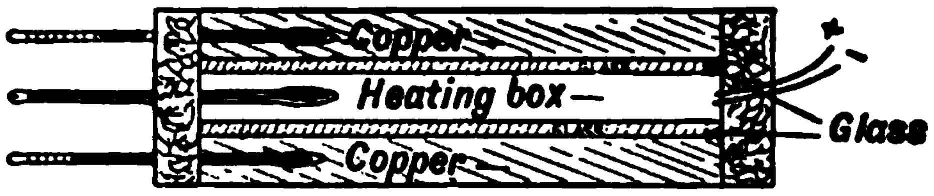


Fig. 226.

Then the heat developed per second is easily determined by electrical measurements and knowing the temperatures of the surfaces of the glass plates and their areas and thickness the conductivity of glass may be calculated.

420. Conductivity of Liquids.—The conductivity of liquids is small, that of mercury being only one-tenth that of iron, and the determination of their conductivities is complicated by convection currents. They may be determined by the method described in the last article. A thin layer of liquid rests on a copper disc and has the copper heating box in contact with its upper surface. By this arrangement convection currents are not established, and if suitable precautions are taken to prevent loss of heat upward from the heating box, the conductivity of the liquid may be measured.

421. Conductivity of Gases.—The conductivity of a gas is measured by the rate of cooling of a heated bulb enclosed in a spherical vessel of the gas to be studied, the outer surface of the large vessel being kept cool in a water-bath. The determination is complicated both by convection and radiation.

But the kinetic theory shows that the conductivity of a gas should be the same at low pressures as at high, provided the rarefaction is not so great as to make the mean free path of the molecules appreciably large compared with the size of the vessel.

By diminishing the pressure of the enclosed gas the effect of convection is diminished, while the true conductivity is not affected. In this way the two may be separated. To determine the effect of radiation the gas is exhausted as far as possible, so as to do away with both convection and conduction.

Table of Conductivities

Heat Conductivities in C. G. S. Centigrade Units

Silver	1.096	Paraffin	0.0002	Air	0.000056
Copper	1.000	Hair felt	0.00009	Hydrogen	0 000327
Iron	0.167	Cork	0.0007	Carbon dioxide	0 000030
Zinc	0.265	Water	0.0014		
Glass	0.0020	Mercury	0.0152		

Problems

1. A glass window pane 170 cm. long and 90 cm. wide is 3 mm. thick. How much coal must be burned per hour to compensate for the loss of heat by conduction when the outer surface is at −5°C. and the inner surface at 20°C.?
2. In the preceding problem if the temperatures given are those of the outer air and within the room, respectively, will the flow of heat be as great as found above? Explain answer.
3. A partition of iron 2 cm. thick and 10 cm. high and 15 cm. wide divides a vessel into two compartments, one of which contains ice, while steam at 100° is passed into the other. Find how much ice is melted in 5 minutes, when 80 calories are required to melt 1 gram of ice.
4. An iron boiler has 1 square meter of heating surface. How much water will be evaporated in 1 hour when the outer surface is kept at 150°C. while water is boiling at 100°C. if the iron is 0.7 cm. in thickness, and if 536 calories are required to evaporate 1 gram of water?

Change of State

Fusion

422. Changes of State.—Among the most interesting and important effects of heat are the changes of state which it produces in matter. Solids if sufficiently heated are changed to

[redacted]

liquids, or may pass directly into the gaseous state, and liquids are transformed into vapors or gases. Even with the temperatures that can be artificially produced almost all known substances can be made to assume any one of these three conditions. Thus nitrogen may be liquefied and solidified, while, on the other hand, platinum may be liquefied and volatilized.

There are three principal changes to be considered: that from the solid to the liquid state, that from liquid to gas or vapor, and that directly from solid to vapor or gas.

Melting.—When ice below zero is slowly heated it first warms to 0° and then melts, the temperature at the surface of the ice remaining constant at 0° until it is all melted. If heat is now slowly taken from the mass, solidification will take place at the same temperature and it will remain at 0° till all is frozen. That the temperature should remain constant while the substance is melting, although it is steadily receiving heat, is characteristic of all cases where there is a well-marked melting point. Some substances, such as selenium, pass from the solid to the liquid state through a soft pasty condition and without there being any point at which the temperature is stationary. Iron passes through such a pasty stage, on which account it is easily welded. So also glass, beeswax and paraffin become very soft as the melting point is approached. There is, however, usually some temperature at which the pasty mass becomes fluid and a considerable absorption of heat takes place, and this is the melting point.

Melting Points

Platinum	1710°C.	Aluminum	657°C.	Sulphur	114°C.
Iron	1503°	Zinc	419°	Ice	0°
Copper	1084°	Lead	327°	Mercury	−39°
Silver	955°	Tin	232°		

Change of Volume.—Most substances occupy a larger volume in the liquid state than in the solid, and therefore contract when they solidify. Some, however, among which are water, bismuth, and cast iron, expand when they solidify. This property is of importance in making castings, as all parts of the mould are filled and its details are sharply reproduced in the casting. The volume of 1 gram of ice at 0° is 1.09082 c.c., while that of the

same amount of water at 0° is 1.00012 c.c., so that the increase in volume of a cubic centimeter of water on freezing is 0.0907 c.c.

In consequence of the expansion of water in freezing, ice floats in fresh water with about one-twelfth of its volume exposed. The practical importance of this property of water can hardly be overestimated. If it contracted in freezing, ice would sink to the bottom of a lake, and ultimately the whole mass of water would be frozen solid, to the destruction of all forms of life that it contained.

423. Effect of Pressure on Melting.—If a substance contracts when it melts, increase in pressure will aid melting; that is, the melting will take place at a slightly lower temperature under increased pressure. If, on the other hand, a substance expands when it melts, the effect of increased pressure will be to raise the melting point, as the pressure in a sense resists the melting.

In case of water an increase of pressure amount to 1 atmosphere will lower the freezing point 0.0075°C. hence an increase of pressure of 133 atmospheres, or about 1900 lb. to the square inch, will be required to lower the freezing point one degree.

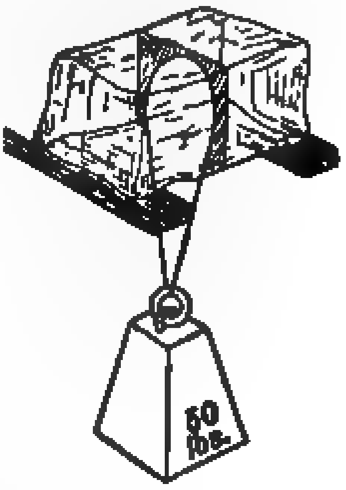


FIG. 227.—Wire cutting through a block of ice.

In some experiments on the expansive force of water in freezing made by Major Williams at Quebec, iron bomb shells were filled with water and exposed to cold. In one case the plug was driven violently out and a short column of ice protruded from the opening. In another experiment the shell was burst and a sheet of ice was forced through the crack. In these cases probably the water was still unfrozen under the enormous pressure just before the shell burst, and froze instantly as the pressure was relieved by the bursting.

If a weight of 40 or 50 lbs. is suspended by a wire loop hung over a block of ice at 0°C., the wire will cut slowly through the ice, the pressure causing ice to melt under the wire; but the water flowing around the wire freezes again above it, leaving the block as solid as before. As this action goes on heat is taken up by the water in melting at the lower side of the wire and given out again

in freezing on the upper side, so that there is a steady flow of heat across the wire as it cuts its way through the ice. And as the action cannot take place without this transfer of heat, a copper wire, being a good conductor of heat, will move faster than an iron wire, other things being the same.

Regelation, or the clinging together of pieces of ice when pressed together, is doubtless due to melting at the points of contact where there is pressure, followed by instant freezing when the pressure is relieved. In this way may be explained the packing of a snow ball, so also Tyndall explains the motions of glaciers; for where the ice is under special pressure, as where it meets a projecting point of rock, it melts and the water flowing around the obstacle freezes again.

424. Latent Heat of Fusion.—If a vessel containing ice and water at 0°C. is kept in a region where everything is at 0°, there will be no change, the ice will not melt nor will the water freeze. But if the temperature of the surrounding bodies is below 0° there will be a flow of heat out of the water accompanied by freezing; if, on the other hand, heat is given to the mass ice will melt, but the temperature *at the surface of contact of ice and water* will remain steady at 0° until all is either melted or frozen. It appears, then, that **a substance may be at the melting point, but it will not melt unless a definite amount of heat is received for each gram of substance melted.**

The latent heat of fusion is the quantity of heat required to change one gram of a substance from the solid to the liquid state without change of temperature.

If equal weights of water at 0° and at 100° are mixed together the temperature of the mixture will be 50°. But if equal weights of ice at 0° and water at 100° are mixed the temperature of the resulting mass of water is 10°.

Every gram of water cooling from 100° to 10° gives out 90 calories of heat, but for every such gram cooled from 100° to 10° there is a gram of ice melted into water at 0° and then raised from 0° to 10°. For the latter change just 10 calories is required, therefore 80 calories must have been used in transforming 1 gram of ice at 0° into water at 0°. The latent heat of fusion of ice is therefore 80.

Suppose m grams of ice at 0°C. are put into a calorimeter containing W grams of water at a temperature t and the temperature of the mixture after the ice is melted is t'; then, calling the latent heat of fusion of the ice L, the heat required to melt it to water at 0° will be Lm, and mt' calories more will be required to raise it to temperature t'. The water in the calorimeter will

give out in cooling $W(t - t')$ calories, where W includes the water equivalent of the calorimeter. Then

$$Lm + mt' = W(t - t')$$

Of course if Wt is not as great as Lm, only a portion of the ice will be melted and the final temperature will be 0°.

The latent heat of fusion doubtless represents the energy required to separate the molecules from the close association which prevails in the solid state; it probably exists in the form of potential energy. So when a body is heated and *expands* it is no doubt true that only a part of the heat given to it contributes to its rise in temperature, the rest of the heat energy doing the work of expansion in opposition to the internal forces of cohesion and also to the external pressure. This portion of what is called the specific heat exists as potential energy and might appropriately be called the *latent heat of expansion.*

Latent Heats of Fusion

Ice	80.00	Lead	5.86
Sulphur	9.37	Zinc	28.13
Tin	14.25	Silver	21.07
Bismuth	12.64	Mercury	2.82

425. Ice Calorimeter.—Some important calorimetric processes make use of the latent heat of fusion.

The ice calorimeter of Lavoisier and Laplace is shown in the figure. An inner vessel to receive the heated substance is surrounded by ice in a vessel which is again surrounded on all sides by ice contained in an outer vessel. Heat from outside will melt ice in the outer vessel, but the ice in the inner vessel will melt only very slowly and its rate may be determined by the drip from the lower stopcock. When a mass of heated metal is introduced into the inner vessel it cools down to 0° and gives out heat which causes an increased flow from the lower stopcock. The water escaping

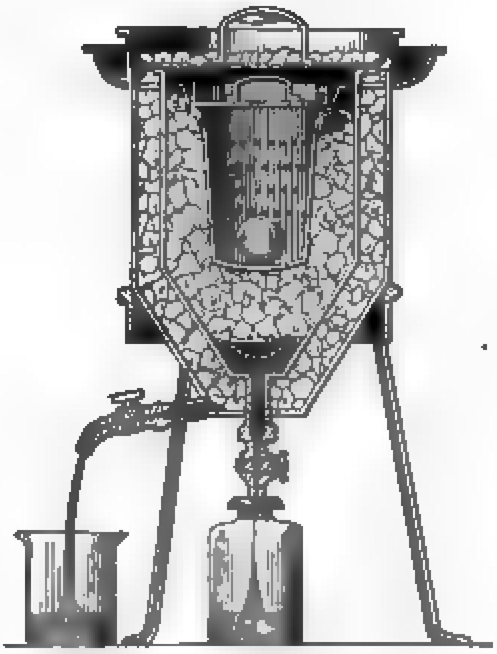


FIG. 228.—Ice calorimeter.

[redacted]

from this, in excess of the steady drip observed at first gives the weight of ice melted by the heat from the substance.

Let m be the mass of the substance which was heated to a temperature t, and introduced into the calorimeter; and let L be the latent heat of fusion of ice, and W the weight of ice melted; then if s is the specific heat of the substance

$$ms(t - 0°) = LW.$$

This method is only adapted to determine the specific heat of rather large masses, as the water from the melting ice clings to the fragments of ice and escapes only gradually.

426. Bunsen Ice Calorimeter.—A form of ice calorimeter which can be used to measure small quantities of heat with much precision was devised by Bunsen and is represented in figure 229. It depends on the change in *volume* which takes place when ice melts. The vessel PWQ is made of one piece of glass in the form of a bulb W enclosing a test-tube P, and provided with a curved neck Q. The bulb W is filled nearly to the bottom with distilled water, the remainder of the bulb and the neck Q being filled with mercury. The whole instrument is now carefully packed in snow and cooled to 0°C.; after which a freezing mixture is introduced into the test-tube P and a sheath of clear ice frozen around it. When the ice is sufficiently thick the freezing mixture is removed and the test-tube filled with water at 0°C.

Attached to Q is a long capillary tube of uniform cross section extending horizontally. As the water in freezing expands it forces the mercury out to the end of this tube. Now let a fragment of substance heated to 100° be dropped into the test-tube: it gives up its heat to the water, and then to the ice surrounding the tube and causes some ice to melt. But in melting a contraction in volume takes place of 0.0907 c.c. per gram of ice melted; and the contraction may be determined by observing how far the mercury column has moved back along the tube D. From this contraction the weight of ice melted may be determined and so the heat given out by the substance in cooling from 100° to 0°C. becomes known. The volume of the contraction when the mercury moves back a certain distance along the tube D may be found by weighing the mercury required to fill a measured length of the tube.

[redacted]

427. Retardation of Freezing Point.—Substances may often be cooled below the temperature at which they normally solidify, and still remain in the liquid form. As soon as solidification begins, however, the temperature of the mass rises owing to the latent heat given out by the part that is becoming solid, and when the temperature has been raised in this way to the melting point no further solidification takes place.

428. Supersaturated Solutions.—The formation of crystals from a concentrated solution is somewhat analogous to solidification. In many cases when a crystal is dissolved heat becomes latent, and when it forms from a solution latent heat is given out. Sodium sulphate crystals may be melted at 48°C. in their own water of crystallization. If the solution is now allowed to cool slowly in a clean flask closed by a cork, it may be brought down to 15° or 20° without crystallizing. Dropping in a minute crystal of the salt will at once precipitate the crystallization, which will go on so rapidly that the temperature may rise 10° or 15°, but not higher than 48°C., the rise in temperature being produced by the giving out of what may be called the latent heat of crystallization.

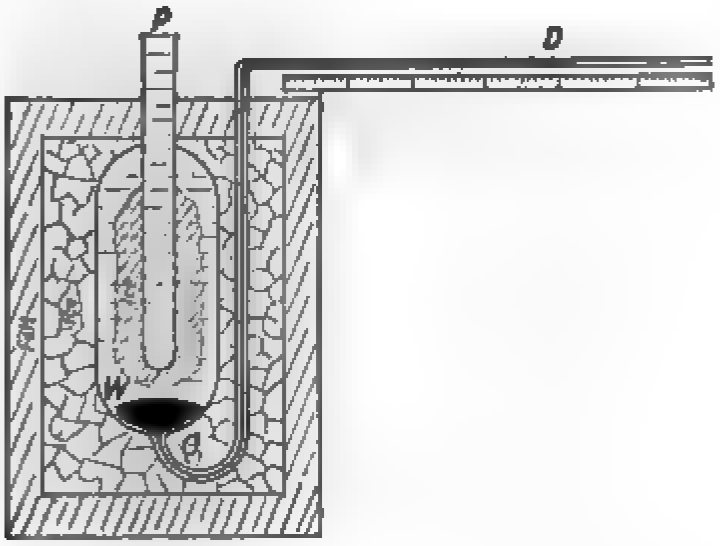


Fig. 229.—Bunsen ice calorimeter.

429. Freezing Mixtures.—When a very dilute solution of common salt in water is cooled below 0°C., ice crystals are formed, leaving the remaining solution stronger; while if a saturated solution of salt is cooled in the same way, the salt crystallizes out, leaving the solution weaker. This goes on progressively as the temperature is still further lowered, one solution becoming stronger and the other weaker, until at −22°C. the two solutions reach the same strength, and when cooled further each solidifies into a mass which the microscope shows to be an agglomeration of minute crystals both of ice and of salt.

The final solution, which is of such strength that on cooling neither component crystallises out without the other, is said to be *eutectic.*

Similarly, there may be a eutectic alloy of two metals, the melting point of which is a minimum for the given metals.

Now, *suppose that a freezing* mixture of common salt and ice at 0°C. is

enclosed in a non-conducting vessel. The affinity of the two causes both salt to dissolve and ice to melt, and in each of these changes heat becomes *latent;* and this heat energy must come out of the mixture, which accordingly is cooled. But it appears from the first part of this article, that *ice and salt cannot both be in equilibrium with the same brine solution, unless it is a saturated solution at −22°C.* Consequently ice and salt continue to dissolve until this final state is reached. No lower temperature than this can be produced by a mixture of these substances. By using calcium chloride and ice, a temperature of −54°C. may be attained.

Problems

1. How much ice is melted when a mass of 500 gms. of copper at 100°C. is dropped into a hole in a block of ice at 0°C.?
2. How much more energy has 1 kgm. of water at 70°C. than the same mass of ice at −10°C?.

   Note.—The specific heat of ice is not the same as that of water.
3. A mass of ice weighing 30 gms. is in a tube with water enough to make the whole volume 50 c.c. at 0°C. What change in volume takes place when 100 grm. calories of heat are given to the mixture?
4. When 400 gms. of ice at 0°C. are put into 500 gms. of water at 60°C., what is the final temperature of the mixture?
5. If 5 lbs. of snow are mixed with 2 lbs. of water at 60°C., how much snow will be melted?
6. If 3 gms. of iron at 100° are dropped into a Bunsen ice calorimeter, find the resulting change in volume.
7. When 2 lbs. of snow at 0° are mixed with 3 lbs. of water at 80°, find the resulting temperature.
8. A mass of 100 gms. of ice at −16°C. is put into water at 0°C. and 10 gms. of the water are frozen, all coming to 0°C. Find the specific heat of ice.
9. If 100 gms. of lead cools from 340° to 327° in 2 minutes and then the temperature remains steady for 25.8 minutes while the mass is solidifying, find the latent heat of fusion of lead, assuming that heat is lost at a uniform rate and that the specific heat of lead at 330° is 0.032.

Vaporization

430. Evaporation.—The change from the liquid to the gaseous state is known as *evaporation,* or if accompanied by the formation of bubbles of vapor throughout the mass it is called *boiling* or ebullition.

Ordinary open-air evaporation is complicated by the presence of the gaseous atmosphere above the surface of the liquid. The simple case where a vessel contains nothing but a liquid and its own vapor will first be considered.

1. **Saturated Vapor.**—Take a barometer tube about a r long, fill it carefully with mercury so as to exclude all air invert it in a deep cistern of mercury (Fig. 230). The mer- in the tube will stand at the barometric height if the tube is d high enough. Now introduce into the tube a few drops ther. On reaching the vacuum the r will at once evaporate and the mer- column will be forced down perhaps 40 by the pressure of the vapor. If there ough ether all will not evaporate, but a will remain as a liquid on top of the ury column; **in this case the vapor is to be saturated, and its pressure is the test possible for ether vapor at the given erature.** For if the volume occupied he vapor is diminished by pushing the downward some of the ether vapor will lense, but the height of the mercury mn will remain unchanged, showing that has been no change in pressure. So if the tube is raised, thus increasing volume of the vapor, ether evaporates, the pressure does not change until all the r is evaporated.

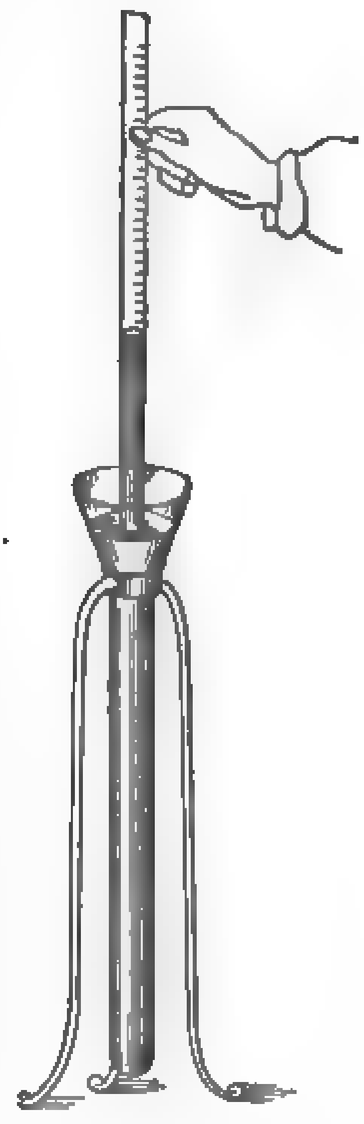


FIG. 230.—Pressure of ether vapor.

saturated vapor is one which is so dense it cannot be farther compressed with- condensation. *If the volume of a satu- l vapor is diminished, an exactly corre- ding amount of vapor condenses, so that pressure and density of the remaining r continue unchanged so long as the temperature is constant.* **saturated vapor is in equilibrium with its liquid, the tendency e liquid to evaporate being exactly balanced by the tendency ie vapor to condense.** Indeed it is probable that molecules onstantly escaping from the liquid and passing into the vapor, e other molecules of vapor striking down into the liquid are ht and held by its attraction, and *when the vapor is saturated two processes exactly balance.* (See §435.)

2. **Non-*saturated Vapor.***—When a vessel containing liquid

and vapor is enlarged so much that all the liquid is evaporated, a further enlargement of the vessel causes the pressure of the vapor to diminish very nearly according to Boyle's law for gases, and the more it is expanded and so removed from its point of condensation, the more exactly does it conform to Boyle's law.

Fig. 231. Measuring vapor pressure.

433. Influence of Temperature on the Pressure of Saturated Vapors.—The series of changes considered in paragraph 431 is supposed to have taken place at a constant temperature. **The pressure of a saturated vapor increases as the temperature rises.** The effect of temperature on vapor pressure may be determined by the apparatus of figure 231. Two barometer tubes are surrounded by a waterbath whose temperature may be varied. A few drops of the liquid to be studied are introduced into one of the tubes and float on the mercury column, filling the upper part with vapor, the pressure of which causes the mercury to stand lower than in the other barometer. The difference in height of the two mercury columns thus measures the pressure of the vapor. Evidently this method can be applied only when the pressure of the vapor is less than one atmosphere.

The following tables give the vapor pressure of water and of a few other liquids.

Vapor Pressure of Water

Temperature	Press. mm. of mercury	Temperature	Press. mm. of mercury
−10°C.	2.16	99.9°C.	757.30
0	4.58	100.0	760.00
+10	9 18	100.1	762.71
20	17 41		
30	31.56	Temperature	Pressure in lb. per sq. inch
40	55.0		
50	92.2	100°C.	14.7
60	149.2	110	20.8
70	233.8	150	69.1
80	355.5	200	225
90	526.0	250	576
100	760.0		Subtract 14.7 from above to get steam-gauge pressure.

Vapor Pressures of Some Other Liquids

Temperature	Alcohol	Ether	Mercury
−20°C.	0.334 cm.	6.92 cm.	
0	1.27	18.23	0.00002 cm.
20	4.40	43.48	0.0001
50	22.03	126.8	0.0015
100	168.5	492.0	0.027
Boiling points....	78°	34.6°	357°

Notice how small the pressure of mercury vapor in a barometer must be.

434. Density of Saturated Vapor.—The higher the temperature of a given mass of vapor, the smaller the volume into which it must be compressed before condensation begins.

The density of a saturated vapor therefore increases with rise in temperature.

435. Evaporation in Air.—If water is introduced into a large vessel full of air, very nearly the same amount will evaporate as if the air had not been there. The water vapor exerts its own pressure independent of that of the air, making the total pressure the sum of the two.

This is a particular case of the general law stated by Dalton as follows: **when a liquid is contained in a vessel with air or any other gas or vapor that has no chemical action upon it, the amount that will evaporate and the pressure of its saturated vapor will be the same as though the other gas were not there, and the total pressure will be the sum of the pressures of the gas and of the saturated vapor.**

The law thus stated is a close approximation to the truth, but it does not hold *exactly*, especially when the gas or vapor is very dense.

The presence of air or gas has a very marked effect, however, in *retarding* evaporation. The layer of air next the liquid becomes filled with vapor which is gradually carried away by currents and by diffusion and as it is removed further evaporation takes place till saturated vapor fills the vessel. The fact that the *pressure of another gas or vapor does not stop the evaporation of a liquid*, but that *the process ceases as soon as the vapor reaches*

a certain definite density points to the conclusion that evaporation stops in the presence of a saturated vapor not because of its *pressure* but because *molecules passing from it into the liquid compensate for those escaping into the vapor.*

436. Boiling.—When a liquid is exposed in an open vessel it evaporates more or less at all temperatures, since, as we have just seen, the pressure of air on its surface, even though it may be greater than the vapor pressure of the liquid, cannot prevent evaporation, though it retards it. This evaporation takes place more rapidly the greater the vapor pressure of the liquid, hence the greater volatility of ether than of water; so also water at high temperatures evaporates more rapidly than at low. If, however, the liquid be heated sufficiently bubbles of vapor will form in its interior and rise to the surface and escape. The liquid is now said to be *boiling* or in a state of *ebullition*, and its temperature remains constant so long as it keeps boiling at a given pressure.

Boiling can take place only when the pressure of the vapor is equal to that of the atmosphere on the liquid surface, otherwise bubbles could not be formed.

The tabulated *boiling point* of a substance is that temperature at which its vapor pressure is equal to the standard atmospheric pressure, viz., 76 cms. of mercury.

If boiling takes place in a *closed* boiler the temperature does not remain constant but rises as the pressure of the contained air and vapor increases.

Boiling points at one Atmosphere Pressure

Zinc	958.0°C.	Ammonia	−33.6°C.
Sulphur	444.5°	Carbon dioxide (sublimes)	−78.0°
Mercury	357.0°	Oxygen	−182.5°
Water	100.0°	Nitrogen	−195.0°
Alcohol	78.0°	Hydrogen	−252.0°
Ether	34.6°		

437. Effect of Pressure on the Boiling Point of Water.—From the previous paragraph it appears that a table of vapor pressures shows the boiling points corresponding to different pressures. Referring to the table on page 290, it will be seen that near 100° C. the boiling point of water changes by one-tenth of a degree for 2.7 mm. change in pressure.

A vessel of water under the bell jar of an air pump may be made to boil by exhausting the air until the pressure is slightly less than the vapor pressure of the water as given in the table.

On high mountains the temperature of boiling is so low that eggs cannot be cooked, and in some high altitudes closed vessels provided with safety valves are used in cooking in order that it may be possible to heat the water to a sufficiently high temperature. On Mt. Blanc water boils at 84°C. By observing the boiling point the barometric pressure, and hence the height of a mountain, may be estimated. This process is known as hypsometry.

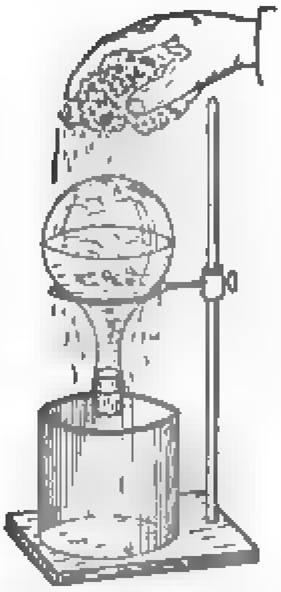


Fig. 232.—Boiling at reduced pressure.

If a flask half-full of water vigorously boiling is taken from the heating flame, and instantly corked air-tight with a rubber cork and inverted as shown in the figure, it may be made to continue boiling by cooling the upper part of the flask, for in this way vapor is condensed and the pressure on the interior is so much reduced that the liquid boils even though its temperature is decidedly below 100°C.

438. Geysers.—The explanation of the action of geysers given by Bunsen is based on the dependence of the boiling point on pressure. Suppose a deep fissure or well into which water flows at the bottom and where it is gradually heated from below. The boiling point at the surface is 100°. At 36 cms., or a little more than 1 ft. below the surface the added pressure of the water column will make the boiling temperature 101°. Suppose at this point the actual temperature is 100°. Again at 10 ft. below the surface the boiling point is about 107° and the actual temperature may be a little less; and at 50 ft. below the surface the boiling point will be 127° and the actual temperature may here also be supposed a little below this. Thus at each point the temperature of the water

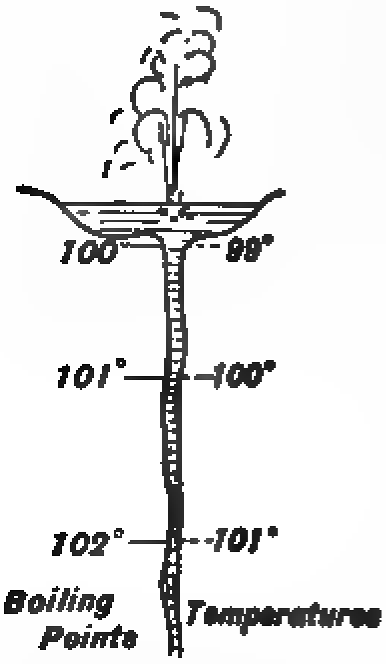


Fig. 233.—Geyser.

column filling the well will be just below that required to produce boiling. Now suppose that at some point down in the well the water becomes heated to its boiling point. The bubbles of steam in forming lift the whole upper column of water quickly so that each point in the column finds itself under a pressure less than that at which it boils and instantly steam bursts out at every point driving the mass of water violently out of the well.

439. Effect of Dissolved Salts on Vapor Pressure.—When a liquid contains salts in solution its vapor pressure at a given temperature will be less than in case of the pure solvent, the amount of the change depending on the nature and concentration of the solution. A saturated solution of common salt in water boils at 108.4°C., while one of calcium chloride boils at 179.5° and contains 76.4 per cent of the salt. The vapor escaping in these cases is pure water vapor at a pressure of 76 cms., and though the temperature of the bubbles of steam as they escape from the liquid may be higher than 100°, it was discovered by Rudberg that *a thermometer in the steam a little above the liquid will record exactly as it would in steam from pure water.* The vapor as it escapes has a pressure of 76 cms., if that is the external pressure, and a temperature *above* 100°C.; it is, therefore, *non-saturated* and at once begins to cool, but when it reaches 100° it is saturated and any further loss of heat causes condensation on the upper part of the vessel through which the steam is escaping, so that the vessel is soon heated to 100° and the escaping steam is kept at that temperature.

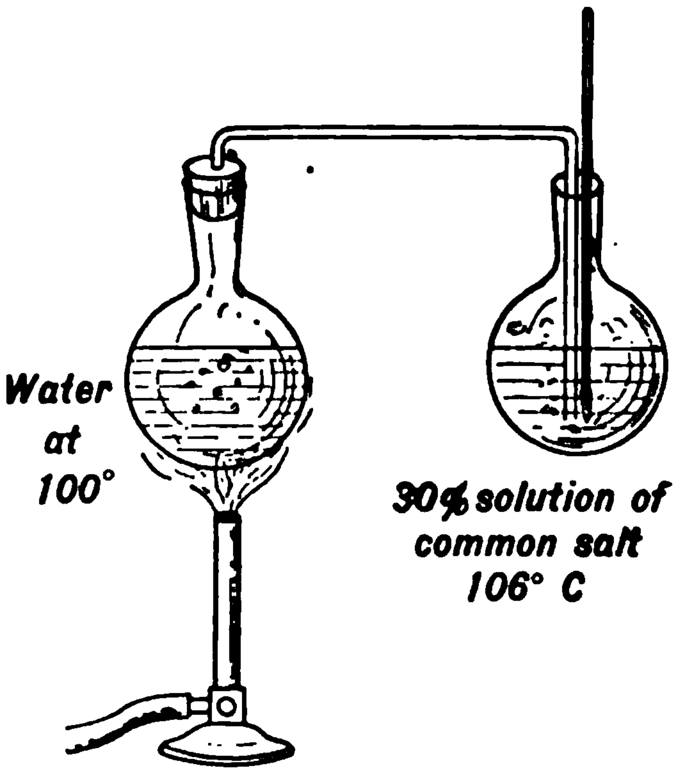


FIG. 234.—Heating by condensed steam.

On the other hand if steam at 100° is continuously passed into a solution of salt, as in figure 234, the salt solution will be heated up to its own boiling point though that may be several degrees above 100°C.

440. Latent Heat of Vaporization.—Heat is required for evaporation, just as for melting. The molecules of the liquid are torn away from each other in opposition to their cohesion and this requires work. Hence vapor has more energy than an equal mass of the liquid at the same temperature. **The heat required to change one gram. of liquid into vapor at the same temperature is known as its latent heat of vaporization.** If dry steam is

ed into a condensing vessel made of thin metal and sur-
ıded by water in a calorimeter, as shown in figure 235, the
m will condense in the worm tube and collect in the bottom
ıe condenser. In condensing, its latent heat
aporization is given up, and the condensed
ɜr is cooled from 100° to the final temperature
he calorimeter. If from the whole heat re-
ed by the calorimeter we subtract the heat
n out by the water in cooling *after* it is con-
ɪed, the remainder is the latent heat obtained
ı the condensation of the steam. The amount
ɔndensed water is obtained by weighing the
r apparatus before and after the experiment,
so the heat per gram of condensed steam may
ɔund.

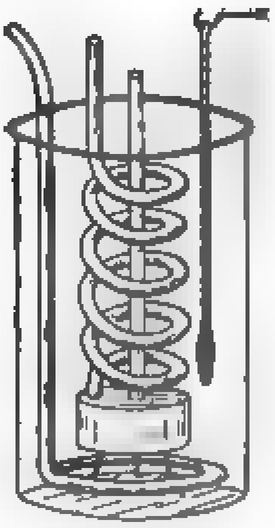


Fig. 235.—Calorimeter for vapor.

e latent heat of vaporization of water is less at high
eratures than at low as might be expected; thus to evaporate 1 gram.
ıter at 100° takes 536.6 gram-calories of heat, while 596.7 calories are
red if the evaporation takes place at 0°C.

e latent heat L required to vaporize a gram of water at any temperature
ʳ be determined from the following formula which expresses the results
iffith's experiments,

$$L = 596.73 - 0.601t$$

ɜ t is the temperature of evaporation on the Centigrade scale.

Heats of Evaporation (at normal boiling points)

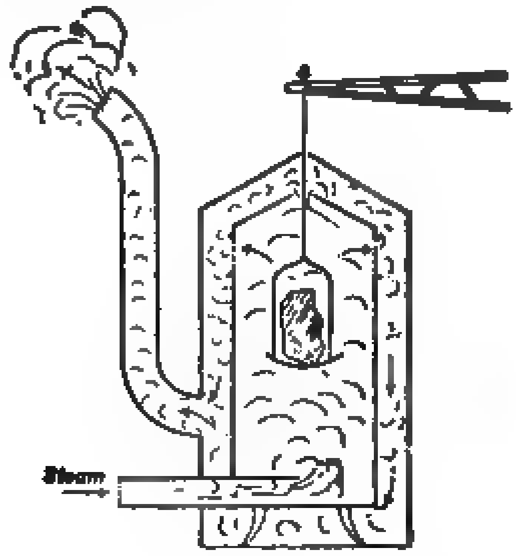
r	537	Ether	91
l alcohol	264	Ammonia	326
ıol	208	Liquid air	51

1. Joly's Steam Calorimeter.—Dr. Joly has devised a very
ıl method of measuring specific heats which is based on the
that if a body is immersed in steam at 100° condensation
ɜake place on the body until its temperature is raised to 100°
ı no further condensation will take place. The heat received
he body is the latent heat of vaporization given out by the
n in condensing. The apparatus is shown in figure 236.
body the specific heat of which is to be determined is hung
the pan of a delicate balance, and the amount of water
ensing on it is found by its gain in weight when steam is
ɜd through the inner vessel.

Let w be the mass of condensed water, t the original temperature of the suspended body, m its mass, s its specific heat, and L the latent heat of vaporization of steam; then

$$ms(100^\circ - t) = Lw.$$

442. Cooling by Evaporation.—If a vessel of water is placed under the bell jar of an air pump and the air exhausted, the water will boil as the pressure is reduced and at the same time its temperature will rapidly fall owing to its giving up heat energy to supply the latent heat of vaporization of the vapor coming off. The process may even be carried so far as to freeze the water. This is illustrated by a device due to Wollaston and known as a *cryophorous* (cold transferrer) which is shown in



236.—Steam calorimeter.

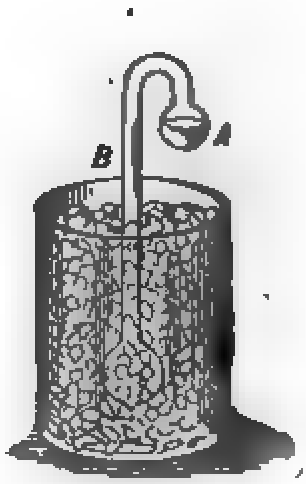


Fig. 237.

figure 237. It consists of a tube having a bulb at each end and containing only water and its vapor, the air having been driven out by boiling the water before sealing the tube. The water is all run into the upper bulb while the lower one is surrounded by a freezing mixture. Vapor arising from the water in the upper bulb takes away heat in evaporation and is condensed in the lower bulb. As this process continues the water in the upper bulb is soon frozen if protected from outside sources of heat by being surrounded with cotton.

In tropical countries water is kept in porous earthen jars put in a breezy place protected from the sun. The rapid evaporation from the moist surface of the jars keeps the water cold.

If a few drops of ether or alcohol are poured on the hand the chilling as it evaporates is very noticeable. If a test tube partly filled with ether is placed in a glass of ice-cold water and the ether rapidly evaporated by causing a stream of air to bubble through it by a foot bellows, a thick shell of ice will soon be frozen around the test tube.

The ordinary wet bulb thermometer, §445, affords another instance of cooling by evaporation.

443. Refrigerating Machines.—The refrigerating machines used for ice making on a large scale and for cooling rooms for cold storage depend on the condensation and evaporation of ammonia, the arrangement employed being as follows. Ammonia gas is compressed by a pump into a condenser *B* where the heat developed by the compression and condensation is removed by a stream of water. From the condenser it is conducted in liquid form through a pipe leading to the region to be cooled. There it escapes through a valve *C* into a long pipe *D* which winds about the room to be cooled, and affords large cooling surface. In this pipe the ammonia evaporates and expands and so takes up heat from the surrounding region which is accordingly chilled. The expanded gas is conducted back to the pump where it is again compressed into the condenser.

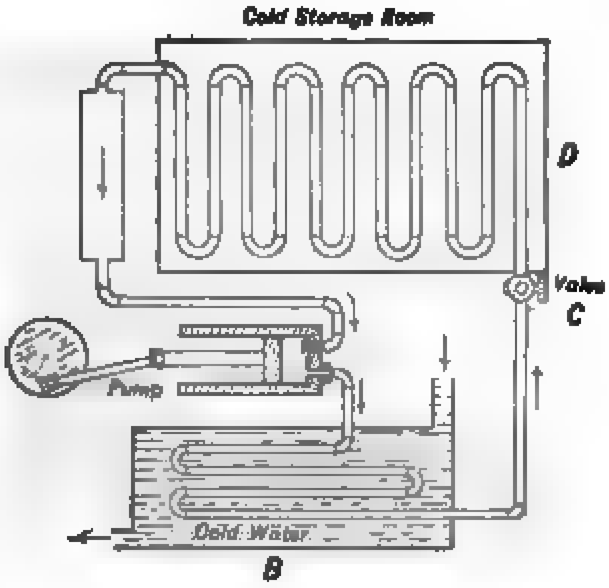


Fig. 238.—Refrigerating by ammonia.

The valve *C* has only a small opening and chokes the flow so that while the pressure in the pipe leading from the condenser to *C* is sufficient to keep the ammonia in the liquid form, beyond *C* the pressure is very small, permitting rapid evaporation and expansion.

444. Sublimation.—A solid may evaporate directly without passing through the liquid state. This is known as *subliming.* Gum camphor sublimes very freely, also the naphthaline balls so often used for protection from moths. Ice slowly evaporates when below the *freezing point,* and carbon dioxide not only

passes directly into vapor from the solid form but *cannot exist in the liquid state* at atmospheric pressure.

Heat is absorbed or becomes latent in sublimation just as in other changes of state, the latent heat of sublimation being the heat required to cause one gram of the substance to sublime at a given temperature.

445. Atmospheric Moisture.—The determination of the moisture in the atmosphere is known as *hygrometry*, and is of much importance in meteorology.

When a mass of dry air in the free atmosphere receives water vapor the added pressure of the vapor causes the whole to expand, since its pressure cannot be greater than that of the surrounding atmosphere. The expanded air is less dense than dry air at the same pressure and temperature, for the water vapor which it contains is only $\frac{5}{8}$ as dense as the dry air which it displaces.

The following are some methods employed in determining the moisture in the atmosphere:

1. A measured volume of air is drawn through a tube containing some drying substance, such as calcium chloride or phosphoric anhydride, and the gain in weight of the drying substance gives the amount of moisture which the air contained.

2. A bright polished metal vessel is cooled till moisture from the air just begins to condense on its surface. The temperature at which this occurs is known as the *dew point*. If the dew point is found to be 10° then the pressure of water vapor in the air is such that it is saturated at 10°. Hence the vapor pressure is 9.1 mm. as shown in the table on p. 290 which gives the pressure of saturated water vapor at different temperatures.

3. The temperature of the air as read by a wet bulb thermometer, one having the bulb covered with a thin piece of cloth kept wet by a wick dipping into a vessel of water, may be compared with the atmospheric temperature as given by a thermometer with a dry bulb.

The wet bulb thermometer will read lower than the one with dry bulb in consequence of evaporation, and the more rapid the evaporation the greater the difference in temperature will be.

When the two temperatures are known the hygrometric state of the atmosphere may be determined by reference to *psychrometric tables.*

The method is exceedingly convenient but is not reliable unless the air is drawn by the wet bulb thermometer at a regular rate, or unless the thermometer is whirled through the air with sufficient velocity to secure the maximum evaporation.

4. A human hair when treated with ether to remove oily substances is very sensitive to moisture, contracting when moist and elongating as it dries.

When one end of such a hair is wrapped around a slender axle to which a pointer is attached, the varying moisture condition of the air may be read by the motion of the pointer; and the instrument is known as a hair hygrometer.

A piece of catgut when stretched by a light weight twists and untwists as the moisture in the air varies.

446. Humidity.—The sensation of dampness is due to the degree of saturation of the air. When the air is cold a comparatively small amount of moisture will make it feel damp, because at a low temperature but little moisture is required to make a saturated vapor. Only 4.7 grams per cubic meter are required at the freezing point, while at 77°F. or 25°C. 22.75 grams may be contained in a cubic meter before saturation.

On this account the *relative humidity* is usually sought in meteorological observations.

The hygrometric state, or Relative humidity of the air is the ratio of the water vapor actually contained in a volume of air to the amount that it would contain at the observed temperature if the vapor were saturated.

Problems

1. If a Bunsen burner can heat 2 kgms. of water from 10° to 80° in 10 minutes, how much water can it boil away per hour?
2. How many grams of water are required to fill a room 3 × 5 × 4 meters in size, with saturated water vapor at 20°C.? Water vapor has 5/8 the density of dry air at the same temperature and pressure.
3. The barometric height on Mt. Washington is about 60.5 cm.; at what temperature will water boil there?
4. A flask half-full of water boiling vigorously is corked tight and immediately immersed in a bath having the temperature 50°C. What will the pressure in the flask become, and when will it stop boiling?
5. When water boils at a pressure of 35.55 cm. of mercury, find the temperature and also the heat that must be supplied to evaporate 100 gms.
6. What is the weight of a cubic meter of saturated steam at 100°C. if water vapor has 5/8 the density of dry air at the same temperature and pressure?
7. Find the temperature of the water at the bottom of a pail of water 30 cm. deep which is boiling while the barometer stands at 76.
8. Find the relative humidity when the air is at 20°C., the dew point having been found to be 10°C.
9. How much coal is needed to evaporate a cubic foot of water (28.3 kgm.) in a boiler at atmospheric pressure, supposing the efficiency of the boiler to be 50 per cent. and the heat of combustion of coal to be 8000 gram-calories per gm. of coal?

10. Find the total amount of heat required to change 100 gms. of ice at $-20°C.$ into steam at 100°.
11. A mass of 100 gms. of copper at 20° is suddenly enveloped in steam at 100°. Find the amount of steam that will condense on the copper.
12. A preserve jar containing only water and its vapor is sealed up and put into a kettle of water which is kept boiling. Will the jar burst, and what will the pressure within it become?
13. A preserve jar half-full of water and half-full of dry air at pressure 76 is sealed up at temperature 20°C. and put into a kettle of water which is kept boiling. Find what the pressure in the jar will become, neglecting the expansion of water and glass.
14. In the cryophorus how much water must distil over from the upper to the lower bulb in order that 50 gms. of ice may be frozen? And what value of the latent heat of vaporization should be used?
15. How much ammonia must be evaporated per hour in the refrigerating coils in a cold-storage room if the temperature is to be maintained at 0°C. when 500 gram-calories of heat are flowing into the room per second?
16. How much heat in British thermal units is required to evaporate a pound of water at 100°C.?
17. How much coal is required to evaporate 100 kgms. of water in a boiler in which the gauge pressure is maintained at 54.4 lbs., supposing no heat wasted?

CONDENSATION OF GASES

447. Triple Point.—The particular temperature and pressure at which a substance may exist as solid, liquid or vapor, is called the *triple point*. If a vessel containing only water and its vapor is cooled until the water begins to freeze it will then be at its triple point, for all three states or phases (solid, liquid, and vapor) are in equilibrium with each other.

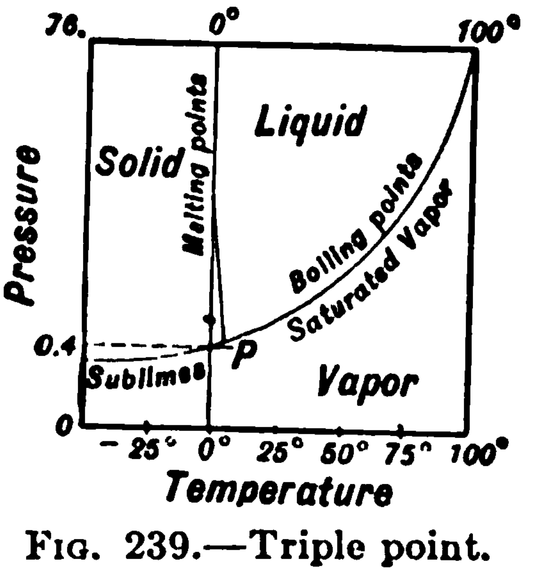


Fig. 239.—Triple point.

In the pressure-temperature diagram for water (Fig. 239) the curve of *boiling points* between the liquid and vapor regions shows the condition of temperature and pressure at which the vapor is *saturated* and in equilibrium with its liquid.

The curve between the solid and liquid regions shows melting points at different pressures, while that between the solid and vapor regions shows the pressure of the vapor in equilibrium with ice at different temperatures.

: triple point is where these lines meet. In case of water .elting point rises very slightly as pressure is diminished, ıt at the triple point the temperature is about 0.0075°C. zero, and the pressure is 4.6 mm. of mercury.

case of carbon dioxide the pressure at the triple point is :r than one atmosphere, hence that substance cannot exist liquid form at atmospheric pressure.

. Condensation of Carbon Dioxide and Critical Point.—transition from the us to the liquid state rst thoroughly studied fferent temperatures pressures by Andrews), a diagram of whose s is given in the figure. point on the diagram ponds to a certain of the substance. ıbscissa or distance of oint from the side line the volume in units of cale at the bottom, the distance of the from a horizontal line measured by the at the side gives the ure in atmospheres. base line correspond- o *zero* pressure is far ' the diagram, which de only large enough

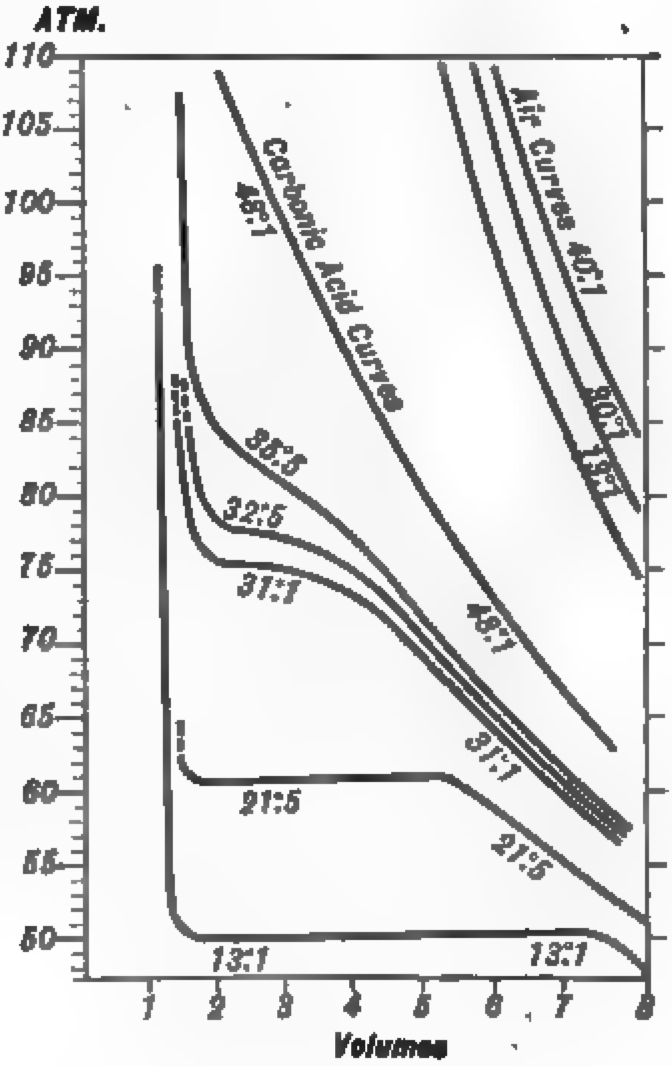


FIG. 240.—Isotherms of CO_2.

:lude the actual observations. A line on the diagram rep- ting the series of states through which the substance may t at a given temperature is called an isothermal line. Thus ne marked 13.1° indicates that if a gram of CO_2 be taken .1° and at a pressure less than 50 atmospheres the volume e greater than 7.1 c.c. As the volume is diminished the ure increases until when the volume is 7.1 c.c. the pressure nes 50 atmospheres, but at this point condensation begins

and the pressure remains constant until all is condensed. This part of the isothermal line, where the substance is part liquid and part saturated vapor, is horizontal, since the pressure is constant.

After the vapor is entirely condensed any further decrease in volume is accompanied by rapid rise in pressure as indicated by the nearly vertical branch of the isothermal line. Where the isothermal line is horizontal the substance is part liquid and part saturated vapor, becoming wholly saturated vapor at the right end where the line begins to drop from the horizontal; beyond this point the vapor is non-saturated, departing considerably from Boyle's law at first, but conforming to it more closely as its volume increases and pressure diminishes.

At 21.5° it is noticeable that the volume of the saturated vapor is less, about 5.2 c.c., while the volume of the liquid is greater than at the lower temperature, and condensation does not occur till the pressure has reached 60 atmospheres, this being its vapor pressure at 21.5°C.

Thus as the temperature is raised the density of the saturated vapor increases while that of the liquid decreases until at 31° they seem to come together, the saturated vapor having the same volume as the liquid. In this case there is no visible condensation with separation of liquid and vapor, and the isothermal line shows no straight horizontal part, but simply a point of inflection where the tangent is horizontal.

The next isothermal, that for 35.5°, shows a point of inflection, a point where the compressibility is a maximum, as indicated by the large decrease in volume for small rises in pressure, but there is no condensation. And at 48° there is scarcely any evidence even of a point of special compressibility, the pressure rising steadily and rapidly as the volume is diminished.

The point at which the density of the liquid becomes equal to that of its saturated vapor is called the critical point. The temperature and pressure of the substance at that point are known as its *critical temperature and pressure,* and the volume of one gram as its *critical volume.*

The critical temperature may also be defined as that temperature above which the substance cannot exist as a liquid having a free surface.

449. Gas and Vapor.—It thus appears that there is no sharp distinction between gases and vapors. What are ordinarily known as gases are substances whose critical temperatures are so low or critical pressures so great that under ordinary conditions they cannot exist as liquids with a free surface, while vapors arise from substances whose critical temperatures are so high that they ordinarily exist even at atmospheric pressure in the liquid state.

Critical Temperatures, Pressures, and Volumes

Substance	Temperature in degrees C.	Pressure in atmospheres	Volume of 1 grm. in c.c.
Water	365.0°	195.0	2.3
Ether	194.4	35.6	3.8
Sulphur dioxide	155.4	78.9	1.9
Ammonia	130.0	115 0	....
Carbon dioxide	30.92	77.0	3.4
Oxygen	−118.0	50.0	1.5
Nitrogen	−146.0	35.0	2.7
Hydrogen	−242.0	20.0	....
Helium	−266.0	2.3	

450. Condensation of Gases.—Faraday, about 1823, began a series of experiments in which he liquefied nearly all the known gases except oxygen, nitrogen, hydrogen, and carbon monoxide. The form of apparatus used by him for chlorine and other gases is shown in figure 241. It consists of a strong bent glass tube hermetically sealed, in one end of which is placed the substance or mixture from which the gas is to be evolved, while the other end is placed in a freezing mixture to induce condensation. When heat is applied the gas is given off on one side and the pressure due to its own evolution causes it to condense on the other. In cases where heat was not required the two ingredients were placed separately in the two branches of the tube which was then sealed. The tube was then tipped up so that the substances were mixed in

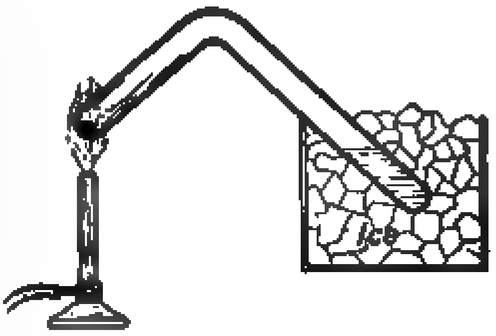


Fig. 241.

one branch of the tube while the other was introduced into the freezing mixture as before.

Solid Carbon Dioxide.—When carbon dioxide, after being liquefied by pressure, is cooled, and then allowed to escape from a small opening, the evaporation and expansion produce such a degree of cold that a considerable part of the escaping substance is frozen into snow. If the jet is enclosed in a woolen bag this snow may be collected. It slowly sublimes, passing directly from the condition of solid to vapor only so fast as the necessary heat of vaporization is obtained from surrounding bodies.

The temperature of solid CO_2 at atmospheric pressure is $-78°$C.; if a little is placed on the hand it is kept from close contact at first by the gas given off due to the heat of the hand. If pressed into contact it burns like a hot iron. If mixed with ether a freezing mixture is obtained giving a temperature of $-78°$C. at atmospheric pressure, and if the pressure is reduced by an air pump $-116°$C. may be reached. The mixture even at atmospheric pressure readily freezes mercury.

451. Liquefaction of Air.—Many attempts were made to liquefy the permanent gases, as they were called, by Faraday, Natterer, and others, but without success until, in 1878, Cailletet and Pictet, working independently, one at Paris and the other at Geneva, almost simultaneously achieved the desired result. Both subjected the gases to great pressure and then cooled them to the lowest point attainable by evaporating liquid sulphur dioxide or carbon dioxide under diminished pressure. But even at the low temperatures thus secured no condensation was observed until a stopcock was opened and the compressed gas suddenly permitted to expand. The cooling due to this sudden expansion caused a cloud of particles of condensed gas to appear. In this way oxygen, nitrogen, and carbon monoxide were shown to be liquefied.

Wroblewski and Olszewski obtained a still lower temperature by cooling liquid ethylene first with ice and salt, then with carbon-dioxide snow mixed with ether, and finally the cooled ethylene contained in a triple-walled glass vessel was made to boil at diminished pressure, the gas as it evaporated being pumped out by an exhaust pump. (Fig. 242.) In this way a temperature of $-136°$C. was reached, and when oxygen

was compressed into a tube dipping below the surface of the boiling ethylene it was condensed at a pressure of about 20 atmospheres. In this manner considerable quantities of liquid oxygen and nitrogen were first obtained.

Linde's Apparatus.—The present methods of obtaining liquid air on a large scale are based on the progressive cooling of a stream of escaping gas by its own expansion, and the first apparatus of this kind was devised by Dr. Linde in 1895. Compressed air at a pressure of about 200 atmospheres, and dried and purified from carbon dioxide, passes into the inner tube of the *interchanger* which contains long coils of tubing one within the other and

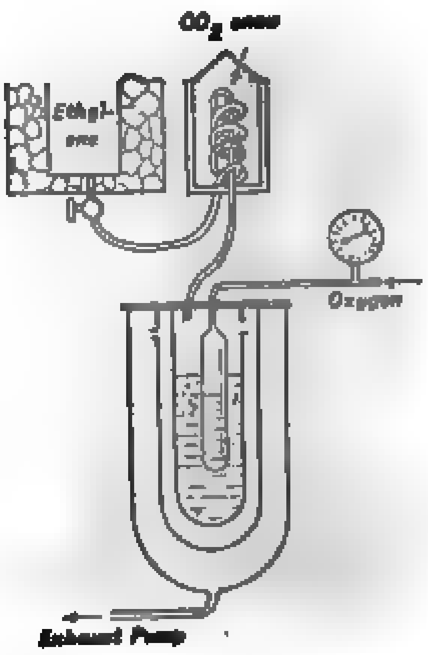


Fig. 242.—Wroblewski's apparatus.

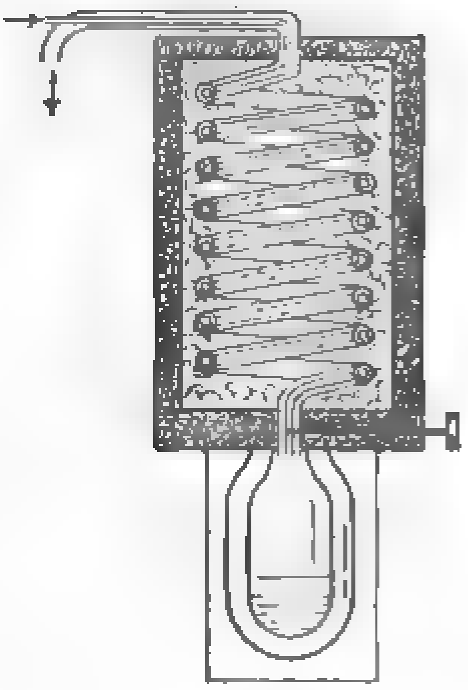


Fig. 243.—Liquid air interchanger.

packed in felt to prevent the inflow of heat from outside. At the lower end of the interchanger is a needle valve through which the compressed gas is allowed to escape in a steady stream. The escaping gas cooled by expansion passes out through the outer tube of the interchanger, thus cooling the inflowing stream of compressed gas in the inner tube. But this on expansion is still further cooled and so the cooling goes on progressively until the temperature becomes so low that a part of the air is liquefied as it escapes at the valve and falls into the receiver below, where it is collected. (See note §453.)

This receiver is a *double*-walled glass vessel (such as are used

in thermos bottles) known as a Dewar flask. The space between the walls of the flask is thoroughly exhausted of air to prevent conduction of heat to the inner vessel. If a drop of mercury is contained in the vacuum space its vapor will condense over the surface of the inner vessel forming a bright metallic mirror that reflects radiation and thus still further aids in preventing the liquid air from receiving heat from outside.

Liquid air when first produced contains both oxygen and nitrogen, but as the boiling point of nitrogen (−195.5°C.) is lower than that of oxygen (−183°C.) the former soon boils off leaving nearly pure oxygen.

452. Liquefaction of Hydrogen and Helium.—The liquefaction of hydrogen has been accomplished by Dewar using an improved form of Linde's apparatus in which two separate interchangers were used, one entirely surrounded by the other. The outer one was first used for the production of liquid air and in this way the whole apparatus was cooled to −180°C. Hydrogen was then passed through the inner apparatus and still further cooled by its own expansion until it finally collected as a clear liquid boiling under atmospheric pressure at −252°C. or 21° above the absolute zero.

On reducing the pressure the temperature of the boiling hydrogen was lowered until it froze into a solid at −258°C.

A cubic centimeter of liquid hydrogen weighs 0.086 grm.; it is therefore the lightest liquid known.

If a bulb containing air has a long neck which is sealed up and surrounded by liquid hydrogen the air will condense and freeze in the neck leaving the bulb highly exhausted.

If fragments of box charcoal are contained in the cooled neck their absorption is so powerful that the bulb becomes almost a perfect vacuum.

Helium, the last gas to yield to condensation, was finally liquefied in 1908 by the Dutch physicist Onnes. Liquid helium, according to Onnes, boils at −268.5°C., and has a density 0.15.

453. Note on Cooling by Expansion in Linde's Apparatus.—The cooling of a steady stream of gas escaping under pressure through a small opening, as in Linde's apparatus for the liquefaction of air, is by no means as great as when a mass of gas is expanded in a non-conducting cylinder as explained in §411. For while the expansion of the gas tends to cool it,

the kinetic energy of the gas rushing out of the opening tends to heat the expanded gas and one effect nearly balances the other so that the cooling is but slight in case of air at room temperature. As air is cooled, however, the effect increases and when a sufficiently low temperature is reached to liquefy the air, the latent heat of vaporization is involved just as in an ammonia refrigerating machine.

When a stream of *hydrogen* gas at room temperature is forced in this way through a small opening the gas is slightly *heated* instead of being cooled and it is only after being cooled below −80°C. to begin with, that the escaping jet is cooled at all by its own expansion.

Melting and Boiling Points of Condensed Gases

Substance	Melting point	Boiling point
Helium	Unknown	−268.5°C.
Hydrogen	−258°C.	−252.0°
Nitrogen	−210	−195.5
Oxygen	−227	−183.0
Air	....	−191.0

Heat Engines

454. Heat Engines.—The coversion of heat into mechanical energy is of the greatest importance to man, since vast stores of fuel existing in the earth as coal, petroleum and gas are thus made available for useful work.

It is interesting to consider that these deposits are really storehouses of the energy of sunlight which fell on the earth in ages long gone by and effected the separation of carbon from oxygen in plants, thus storing up potential energy which is ready to be given back to us as energy of heat under the magic touch of flame.

The principal kinds of heat engines are the steam engine, the hot-air engine, and engines that burn gas or vapors explosively.

455. The Steam Engine.—A simple double-acting steam engine is shown in the diagram (Fig. 244). Steam from the boiler is admitted to the steam chest S, passes through one steam port into the cylinder A and forces the piston toward the left. Whatever steam or air is in the other end of the cylinder B escapes through the other port, passes under the cup-shaped slide valve and out at the *exhaust* E. But the slide valve is so connected

to the main shaft through the eccentric that as the piston moves toward the left the slide valve moves toward the right, closing the first port and opening that at the other end of the cylinder. Steam is thus admitted first into one end of the cylinder and then into the other, forcing the piston back and forth. The fly-wheel F which has a large moment of inertia steadies the motion and carries the crank C past the "dead centers."

456. High-pressure and Condensing Engines.—When the exhaust E opens directly into the atmosphere the engine is called *high pressure*, because the power depends on the excess of the stream pressure in the boiler above the atmospheric pressure outside. Ordinary locomotives and most small engines are of this type.

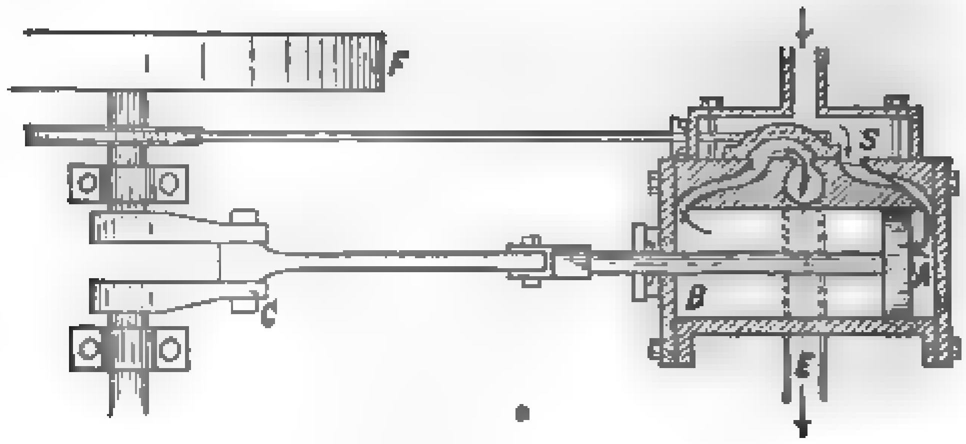


Fig. 244.—Steam engine.

But greater economy is obtained by connecting the exhaust to a vacuum chamber in which the steam as it comes from the engine is condensed by a jet of cold water or in tubes surrounded by cold water, a small pump being provided to pump out from the vacuum chamber the condensed steam as well as any air that may have leaked in. Such engines are known as *condensing* engines, and since the pressure in the vacuum chamber may be less than 1 lb. to the square inch the back pressure against the piston is less by 14 lbs. to the square inch than if the exhaust had opened into the atmosphere, and the effective pressure is consequently just so much greater.

457. Compound Engines.—To get as much work as possible out of steam it should be used *expansively* in the engine, and should not pass into the exhaust until its pressure in consequence of expansion has diminished almost to that in the exhaust, otherwise it escapes with explosive puffs which represent lost energy.

If the expansion takes place in one cylinder the steam which is
lmitted at high pressure and temperature does not escape until
s pressure and temperature are both greatly reduced by ex-
ansion. **To avoid this great change in pressure and tempera-
ure in a single cylinder, compound engines are used in which the
eam passes successively through several cylinders, a part of
e expansion taking place in each one.** Each cylinder must be
rger than the preceding one to allow for the expansion of the
eam, and as the steam pressure in one cylinder is less than in the
receding one the area of the piston head is made correspondingly
rger in the second, so that the total force exerted by each
iston may be about the same.

458. Steam Turbine.—In the DeLaval steam turbine one or
ore jets of escaping steam are directed against a series of blades
t in the rim of a wheel, driving it with great velocity.

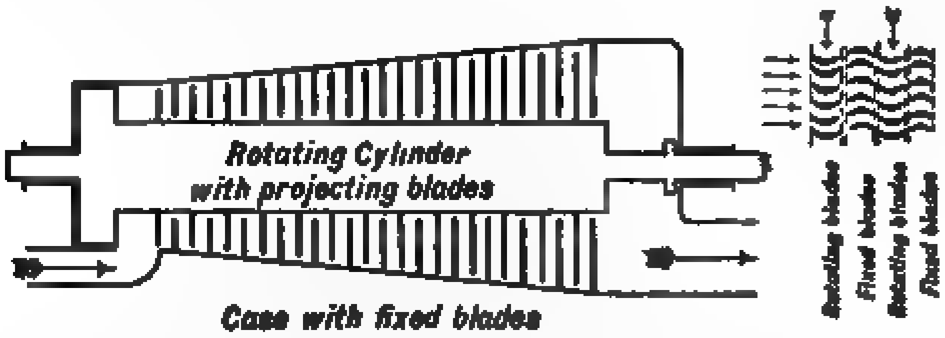


Fig. 245.—Steam turbine.

In the Parsons turbine (Fig. 245) the blades are set in rows or
ands around the circumference of a long cylindrical drum, which
otates inside of an outer case. Steam is admitted around one
nd of the cylinder and impinges obliquely on the first row of
lades. These blades are curved so that they deflect the stream
f escaping steam as it passes between them and direct it against
second row of blades fixed to the outer case. These in turn
eflect the stream so that it strikes obliquely against the second
ow of blades on the rotating cylinder, thus the escaping steam acts
n row after row of blades successively from one end of the cylin-
er to the other where it escapes into the exhaust. The space
etween the rotating cylinder and the outer case widens from one
nd toward the other to allow for the expansion of the steam as it
asses through, and consequently the blades are short at the end
here the steam *enters* but are longer as the other end is ap-

proached. The turbine is thus in some respects like a compound engine, the successive rows of blades in the former corresponding to the successive cylinders in the latter.

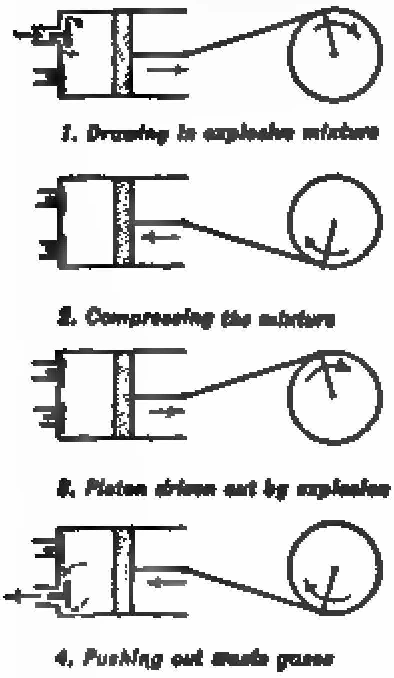


FIG. 246.—Four-cycle gas engine.

459. Gas and Gasolene Engines.—In these engines an explosive mixture of gas or gasolene vapor and air is drawn into the cylinder and there ignited, power being obtained from the expansive force of the hot gases which result from the explosion. Figure 246 shows the series of operations in the "four cycle" or Otto type of engine. In the first outward stroke the mixture of air and gas in proper proportion is drawn in; the valve then closes and the mixture is compressed on the return stroke. When the crank is on the "dead center" and the compression is maximum the mixture is ignited by flame or electric spark and power is obtained from the thrust of the expanding gas on the outward stroke. The exhaust valve then opens and the waste gases are driven out as the piston moves back. The engine is made *single acting* to avoid undue heating and the cylinder is also kept cooled by a circulation of water around it. It will be observed that power is obtained only in the third operation, or on every alternate outward stroke. A heavy flywheel is, therefore, used, or in automobiles four such engines may act on one shaft, the explosions taking place succesively in the several cylinders, one to every half revolution of the shaft.

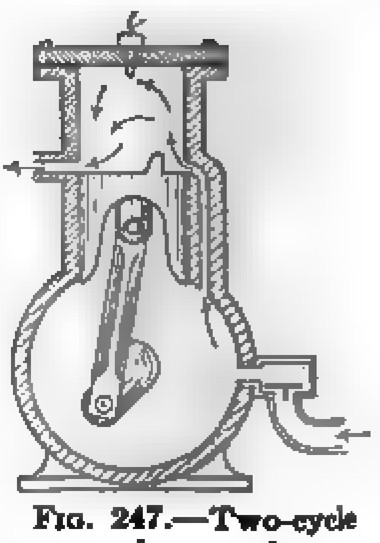


FIG. 247.—Two-cycle gasolene engine.

In so-called "two-cycle" engines the explosive mixture which has been compressed in the crank case by the outward movement of the piston is admitted to the cylinder near the end of the stroke while the exhaust valve is open, and by its inrush helps to displace and drive out the spent gases. The mixture is compressed as the piston moves back and then exploded, giving power on the

ard stroke. Of course in this case there is some mixture of pent gases with the fresh charge and some of the latter is also through the exhaust. The two-cycle engine is growing in · for motor vehicles and boats because of its great simplicity, because the thrust comes at every stroke.

0. Efficiency of Heat Engines.—The efficiency of an engine ratio of the work done in a given time to the mechanical valent of the heat which is supplied to it during that same It is shown by thermodynamic reasoning that no enigne have a greater efficiency than

$$\frac{T - T_0}{T}$$

e T is the highest temperature of the working substance as it s through the engine and T_0 is its lowest temperature, both ured on the *absolute scale.*

us if an engine takes in steam at 163 lbs. pressure, the temure of which is 185°C. and if the temperature of the exhaust 0°C., then $T = 458$ and $T_0 = 373$ and its efficiency cannot eater than $^{85}\!/_{458} = 18\% +$.

king account also of the loss of heat in the furnace and boiler, ound that a good engine (multiple expansion and condensing) give about 1 horse-power-hour per pound of coal, but ory non-condensing engines require 2 or 3 lbs. of coal per -power-hour.

r a further discussion of the relations of heat to work, taking e Second Law of Thermodynamics, Carnot's cycle, and the in absolute or thermodynamic scale of temperature, see *ndix I.*

Reference

account of development of the steam engine in *Heat as a Form of y*, by R. H. Thurston.

Radiation and Absorption

1. **Radiation.**—A person standing near an open fire is cons not only of the light coming from it, but also of a sensation rmth which is felt in the skin wherever it is directly exposed e glow. This sensation is lost when an opaque screen is

interposed, and returns as instantaneously as the light whei screen is withdrawn. This is shown by the fact that af solar eclipse the warming effect of the radiation from th reappears as soon as the light itself.

The radiation may be felt even through a sheet of thin ic by means of a lens of ice it may be converged into a focus ciently intense to ignite gun cotton.

But since both sides of the ice are at the same temper (the temperature of melting ice) no heat can be transmitt ordinary conduction. Radiant energy is, therefore, transm by a very different process. This is also shown by the fact t passes with the greatest facility through a vacuum.

The process of emitting energy in this way is called *radi* and the total stream of energy coming from the body in this is called its radiation.

We shall discuss here some circumstances which influen giving out and absorbing of this radiant energy, but radi itself, its nature and varied phenomena, will be taken up in later study of light, for light is but that part of the total strea radiation to which the human eye responds.

462. Instruments for Detecting Radiation.—The hea effect of radiation is detected usually either by the thermo radio-micrometer, bolometer, or radiometer. The therm and radio-micrometer are explained (§663, 670) in the sectio thermo-electricity.

In the bolometer, devised by Langley, a thin strip of plati perhaps 0.01 mm. thick and 0.5 mm. wide and having a blacke surface, is mounted in connection with a Wheatstone's bridge galvanometer so that its resistance may be balanced. W radiation falls on the strip it is heated, and in consequence electrical resistance changes slightly, which disturbs the bala of the bridge and causes a current to flow through the gal nometer. The mass of the platinum strip is so small that change in temperature takes place almost instantaneously w radiation falls upon it. Langley was able to make the arra ment so sensitive that a change in temperature of the strip small as one-millionth of a degree could be detected.

It was found by E. F. Nichols that the principle of the ra ometer (§465) might be used in the construction of an instrum

was exceedingly sensitive for the detection and measure- f radiation.

ais instrument a light cross arm of wire carrying on each small disc of mica blackened on one side, and having a airror hung from it, is suspended by a fine quartz fiber in a from which the air can be completely exhausted. The iscs are vertical with their edges toward the axis of sus- ı, and the blackened sides of both face toward the same When a high exhaustion is reached the slightest radiation on the blackened side of one of the discs causes it to be l as explained in (§465). The suspended system conse- r turns through a small angle which may be determined by ng through a telescope the image of a scale reflected in the

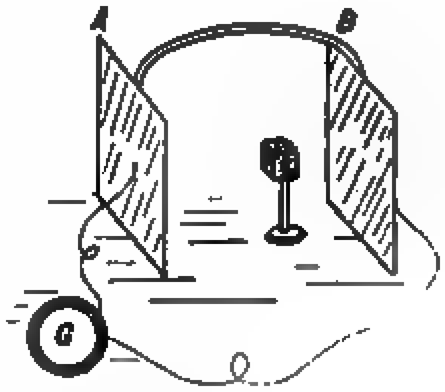


Fig. 248.

Radiating Power.—Bodies at the emperature may differ greatly in ng power. This is shown by cube, which is a brass cubical containing boiling water. One our lateral faces of the cube is of d metal, one is coated with lamp- the third is covered with a layer k or whiting, while the fourth is shed metal that has been varnished or lacquered.

n the radiation from such a cube falls on a thermopile or ter it is observed that the radiation from the lamp-black is most energetic, that from the whitened surface is nearly t, that from the varnished surface is less, while the polished urface gives off the least radiation.

Absorbing Power.—That bodies differ in their absorbing as radiating powers is shown by the following experiment. n plates *A* and *B* connected by a thin strip of tin, are d so as to face each other. The inner face of one is with lamp-black while the other is left with its bright c surface. On the outside of each plate is soldered at its a copper wire by which connection is made to a sensitive ometer of low resistance. The junctions of the copper ith the tin plates form a pair of copper-iron thermo-couples either is *warmed more than* the other a current will be

observed in the galvanometer. On placing midway between the plates a hot piece of iron or other radiating body the galvanometer will at once indicate that the junction attached to the blackened tin plate is more heated than the other. Moving the hot body away from the blackened plate and toward the one having a bright metallic surface a position may be found where there is no current in the galvanometer, showing that the two plates are equally heated. It is clear from this experiment that the polished plate is not so good an absorber of radiation as the blackened plate.

The surface of a tea-kettle is made of bright polished metal which is a poor radiator and consequently loses but little heat by radiation, while the blackened bottom readily absorbs radiation from the fire.

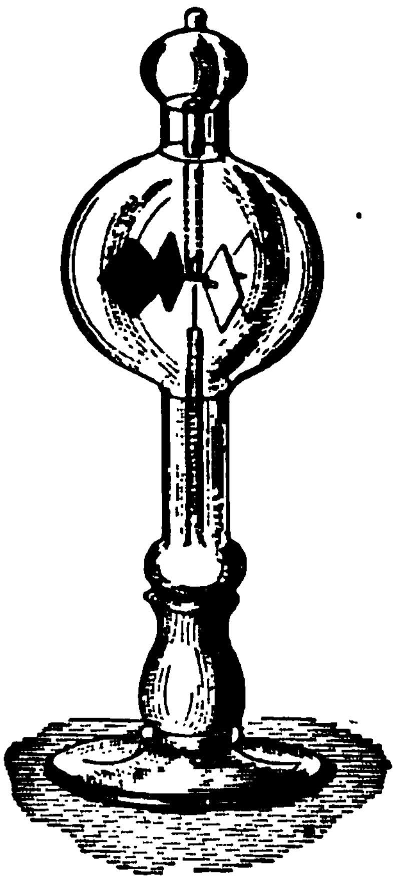


Fig. 249.—Crookes' radiometer.

465. Radiometer.—The radiometer of Crookes depends for its action on the difference in absorbing power between a blackened surface and one of polished metal.

In this device a light cross of aluminum, having vanes of aluminum foil, each coated with lamp-black on one side, is balanced on a pivot in a bulb in which there is a high vacuum; an exhaustion to about the millionth of an atmosphere is required.

When sunlight or the radiation from any hot body falls on the instrument the balanced cross revolves, the blackened surfaces of the vanes facing backward as it turns.

The commonly received explanation of the action is that the blackened surfaces, absorbing the radiation better, are more heated than the polished surfaces and the molecules of air or gas in the bulb are driven off more energetically and consequently with greater velocity from the hotter surfaces than from the colder ones and the consequent reaction causes the vanes to move as though pushed against on the blackened sides.

The action can only go on when the exhaustion is so great that the mean free path of the molecules is of the same order of magnitude as the distance from the vanes to the walls of the

ulb, for if the molecules of gas as they fly off from the blackned surfaces strike against other molecules they will drive nem back against the polished sides and prevent rotation. But they strike against the walls of the bulb the latter *must tend rotate opposite to the vanes.* That this is the case has been erified by floating the bulb in water.

466. Prevost's Theory of Exchanges.—It was urged by Preost, of Geneva, that *the radiation given out by a body must depend ly on its temperature and the nature of its surface,* and not at all n the nature or temperature of other surrounding bodies. onsequently a process of exchange is going on between every ody and those surrounding it, each constantly giving out diation on all sides and also receiving and partly absorbing the diation which falls upon it from surrounding objects. When er therefore the temperature of a body remains constant it ust be receiving and absorbing just as much energy as it is ving out in radiation.

This view is known as *the theory of exchanges.*

467. Apparent Radiation of Cold.—The radiation from a ock of ice may be converged upon a thermopile by means of concave mirror and produces a decided cooling effect.

But whatever radiation goes from the ice to the thermopile ust have energy and hence when absorbed must give *heat to* e thermopile. The explanation of the cooling is found in the eory of exchanges; for while the thermopile is receiving radiaon from the ice, it is itself giving out more energetic radiation d is therefore cooled. The ice gives its feeble radiation to e thermopile but it intercepts the more intense radiation that ould have reached the thermopile from other warmer bodies the ice had not been there.

468. Equality of Radiating and Absorbing Powers.—*The ewart-Kirchhoff Law.* Imagine a body A supported at the nter of a hollow vessel from which the air has been completely hausted, and the interior surface of which is coated with mp-black. If the outer vessel is kept at a constant temperare the inner body will also finally come to that temperature. will then be in a state of equilibrium, *giving out just as much ergy in radiation as it absorbs from the radiation that falls on it.*

If the central body is a good reflector, like a piece of polished metal, it will reflect most of the radiation that falls upon it, absorbing only a small fraction. But its own radiation must exactly make up for what it absorbs, consequently it will radiate but little, and the *total* radiation coming from the body, being made up of what is reflected together with what the body radiates, must be just equal to the total radiation falling upon it.

If, on the other hand, the central body is a good absorber, such as a fragment of carbon, it will absorb nearly all the radiation falling upon it, reflecting very little. In this case it will radiate strongly, the radiation being equal to what it absorbs, and here also the total stream of radiation coming from the body is exactly equal to that which falls upon it, for its own radiation supplies the place of what it absorbs.

In a closed region, then, which is all at one temperature, the total radiation coming from any surface, partly reflected and partly radiated, is the same whatever may be the nature of the surface, whether it is a good reflector or a poor one, and is equal to the radiation which would be given off at that temperature from a *perfectly absorbing body* or an *ideal black body.*

The above conclusion was reached independently by Kirchhoff and Balfour Stewart about 1858, it is illustrated by an experiment performed by Draper in 1847, in which a gun barrel containing fragments of various metals, colored crockery, etc., was heated in a furnace to a red heat. On looking into the gun barrel one substance could not be distinguished from another, the stream of radiation being the same from all, and equal to black-body radiation. Draper drew from his experiment the erroneous conclusion that all bodies became self-luminous at the same temperature, about 525°C.

The radiation which a body gives off at a given temperature is therefore equal to what it can absorb in the same time of black-body radiation for the same temperature.

Consequently bodies which are good reflectors and poor absorbers are also poor radiators, while poor reflectors which absorb strongly are also good radiators.

469. Law of Total Radiation.—From the study of a large number of experimental results Stefan in 1879 concluded that

e total energy of radiation R coming from a body was proportional to the fourth power of the *absolute* temperature, or

$$R = CT^4$$

iere C is a constant.

Boltzmann, in a masterly discussion based on Maxwell's ectromagnetic theory of light, reached the conclusion that efan's law is strictly true for *black-body radiation;* and this nclusion is borne out by the very thorough experimental vestigations of Lummer and others. The law is also found be approximately correct in other cases of purely thermal diation.

The radiation in gram-calories per second from 1 sq. cm. surface of a *black body* at temperature T reckoned from e absolute zero, is found to be $(1.30 \times 10^{-12})T^4$.

470. Law of Cooling.—If a body at a temperature t is placed an enclosure at some lower temperature t', it will cool, and e rate of cooling will be rapid if $t - t'$ is large. It was assumed Newton that in such a case the rate of cooling is proportional $t - t'$, or the heat lost per second equals $K(t - t')$, where K a constant to be determined by experiment. This law is arly true if the difference between the two temperatures is it large, and it is often convenient to use. But the true law cooling is based on the law of exchanges. By the preceding ragraph the heat given out in radiation by a body whose absote temperature is T is equal to CT^4 where C is a constant. If receives radiation from a black body at temperature T_1 it ll absorb CT_1^4, consequently the loss of heat per second is ual to

$$C(T^4 - T_1^4).$$

471. Wave Length of Most Energetic Radiation—Wien's splacement Law.—The radiation from a hot body, as we shall later (§900), is complex in its nature and made up of ether ves of different wave lengths.

In figure 250 are given curves each of which corresponds to ertain temperature and shows how the intensity of the radian of a *black body* at that temperature varies with the wave igth.

It will be *observed that the* hotter the radiating body the *orter is the wave length* of most energetic radiation (cor-

responding to the highest points on the curves). Experi and theory have combined to establish the remarkable that the wave length of most energetic radiation is inve proportional to the absolute temperature, or in symbols:—

λT = a constant, found to be 2940 by Lummer and Pringsl where λ is the wave length in thousandths of a millimete the highest point on the energy curve and T is the corresp ing temperature measured from the absolute zero.

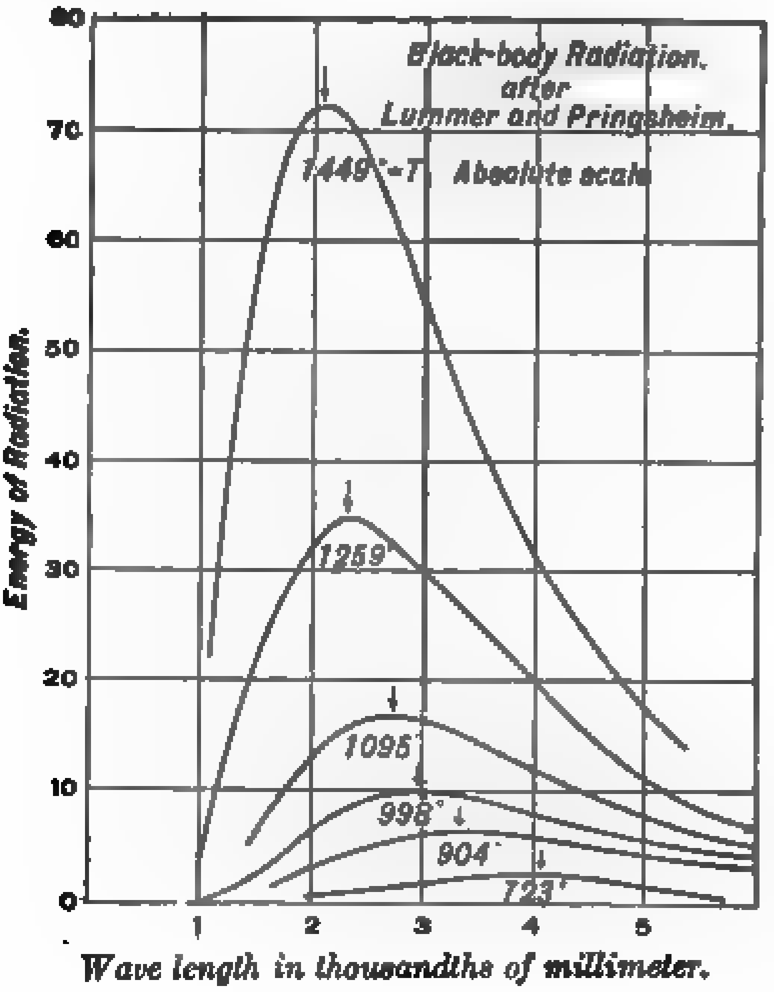


FIG. 250.—Curves showing that the wave length of most energetic radiati shorter in proportion as the temperature of the radiating body is high

Assuming that the radiation from the sun is sufficiently that from a black body for the law to apply, we may determin temperature. For the wave length of maximum energy in sun's radiation is found by Langley to be 0.0005 mm., which g for the sun's temperature $\frac{2940}{0.5} = 5880°$ absolute, or 5607°C.

Since the longest radiation waves that affect the eye give sensation of red, the above law shows why a heated body sh

first become *red* hot and as the temperature rises the shorter waves become relatively more energetic until finally it appears *white hot.*

471a. Quantum Theory.—In order to account for the manner in which the energy of the radiation from a black body is distributed among the different wave lengths, as shown by such curves as those in figure 250, Planck has found it necessary to assume that there is something in the process of radiation, due no doubt to the structure of the atom itself, which causes energy to be radiated in certain small units called *quanta,* and that the elementary unit or quantum for any given wave length is hV/λ, where V is the velocity of light, λ is the wave length of the radiation, and h is an absolute constant known as *Planck's constant.* The value of h in C. G. S. units is found to be 6.55×10^{-27}, and though no explanation is known why radiation should be given out in such units, the constant h is found to enter into so many different phenomena where atomic vibrations are involved, that its great significance can scarcely be doubted.

472. Dew.—Leaves and grass are rather good radiators and on clear nights they radiate strongly toward the sky and receive very little radiation in return. What is received comes for the most part from the air, which like all gases is a very poor radiator. Consequently vegetation is cooled and if there is much moisture in the air it condenses in the form of dew. Cloudy nights are unfavorable for the formation of dew since clouds radiate toward the earth.

When the temperature of the air is near the freezing point the chilling due to radiation causes ice crystals to form and *frost* is deposited instead of dew.

On windy nights there is usually no dew or frost because the rapid movement of the air over leaves and grass acts by *conduction* to keep vegetation at the same temperature as the general mass of air, thus the heat lost in radiation is supplied by convection and conduction; but on still nights the layer of air resting next to a cooled leaf soon becomes chilled below the average air temperature.

MAGNETISM

Properties of Magnets

473. Natural Magnets.—It was known to the ancients that certain iron ores had the power of attracting iron filings and small fragments of the same ore. The first specimens of this ore were obtained at Magnesia in Asia Minor and were on that account known as *magnets*. The mineral exhibiting this quality in the highest degree is a compound oxide of iron now known as *magnetite*. If such a natural magnet or *lodestone* is dipped into a mass of iron filings they cling to it in tufts especially at certain points called poles.

474. Mariner's Compass.—If a lodestone having a strong pole at each end is balanced on a point or suspended by a cord or placed upon a float in water, it will set itself with one pole toward the north and one toward the south. The mariner's compass, which makes use of this property of the lodestone, was known in Europe in the year 1200 and probably earlier among the Chinese.

475. Artificial Magnets.—If a small strip of hardened steel is brought into contact with a lodestone it becomes a magnet, and retains the property even when taken away. Iron filings will cling to it in tufts usually at its ends. If it is balanced on a point, one end will turn toward the north just as in case of the lodestone. Such a piece of steel is said to be *magnetized* and to *exhibit magnetism.* When balanced on a point so that it can freely turn it is called a magnetic needle.

Very powerful magnets are made by causing a current of electricity to flow around a core of soft iron; such *electromagnets*, as they are called, will be discussed later (§681).

476. Magnets Have Two Kinds of Poles.—The fact that a magnetic needle will always set itself with the same pole pointing to the north indicates that the two poles are different. If two magnetic needles are brought near each other it will be found

that the two north seeking poles repel each other; so also the two poles that turn toward the south repel each other; but if the north pole of one is brought near the south pole of the other decided attraction is observed. **Thus like poles repel and unlike attract each other.**

The pole turning toward the north is usually called the north pole in English books, but the French call it the south pole because its polarity must be like that of the south pole of the earth, considering the earth as a magnet.

477. Number of Poles.—If a thin strip of hardened steel, a piece of clock spring for example, be magnetized by drawing a pole of a lodestone or other magnet over it from one end to the other it will probably be found to have two well-marked poles, one at each end. If we break the magnet in the middle and try to isolate one pole, it will be found that poles have appeared where it was broken and that each fragment has two opposite poles. **However small the magnet may be broken up, each piece shows a north pole and a south pole. No one has ever made a magnet with one pole.**

It is possible, however, for a magnet to have any other number of poles and a ring may be magnetized and have no poles at all. **Long thin bars of steel when magnetized often show more than two poles.**

478. Relative Strength of the Poles.—When a magnetic needle is floated in a dish of water it at once sets itself in a north and south direction, but **it shows no tendency to be drawn toward the north or toward the south.** The north pole of the magnet is urged toward the north with a force equal and opposite to that acting on its south pole. The force between the earth and the north pole of the magnet is equal and opposite to that between the earth and the south pole of the magnet. It is concluded, then, that **the two poles of a magnet are equally strong.**

If the floating magnet has more than two poles, the result is the same, it is not drawn either toward the north or south. This indicates **that the combined strength of the north poles in a magnet is equal to that of its south poles.**

479. Nature of Magnetism.—The fact that the fragments of a magnet *always have two poles* indicates that **magnetism is a**

condition which prevails throughout the whole mass of the magnet, and polarity is merely an external manifestation of that condition. A piece of steel or iron is conceived as made up of particles or molecules each one of which is a little magnet. When the steel is not magnetized these particles are thought of as turned under the influence of their mutual attractions so as to form little closed groups in which the north pole of one particle is drawn toward the south pole of a neighboring one. At any point of the surface north poles and south poles thus neutralize each other so far as any external effect is concerned. Such a bar shows no evidence of poles, and we say it is not magnetized.

If, however, the bar of steel is placed between the poles of a powerful magnet, or if the opposite poles of two magnets are placed near together on the middle of the bar and then drawn apart toward its two ends, the particles of the bar are rearranged, being drawn apart from their former association under the

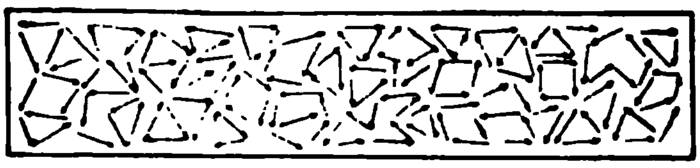


Fig. 251.—Non-magnetized steel bar.

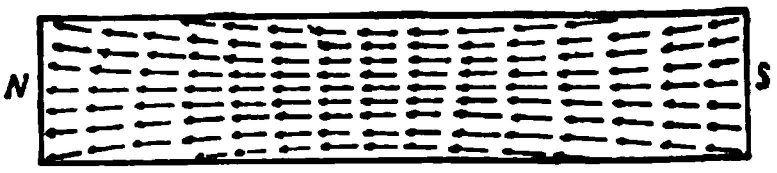


Fig. 252.—Magnetized bar.

influence of the more powerful external attraction. The arrangement is now that shown in figure 252 where the particles are arranged in continuous chains or filaments, running from one end of the magnet toward the other. All the little south poles now have one general direction and all the ends of the filaments terminate on the end and sides of the magnet toward the right, while the north ends (points of the arrows) all terminate toward the left. The bar now exhibits north polarity on the left and south polarity on the right. If all the magnetic filaments were *straight parallel* chains of particles extending from one end to the other then polarity would be found only on the extreme ends, but in consequence of the mutual repulsion of similar poles the arrangement becomes as above shown, and the poles always extend over the sides of the magnet.

480. Magnetic Induction.—When the pole of a strong magnet is placed on the upper end of a short bar of steel which is clamped

in a vertical position (Fig. 253), the steel becomes magnetic and will support a considerable mass of iron filings at its lower end. If the magnet is now separated from the steel bar some of the filings will drop off but enough will remain to show that the steel retains considerable of the magnetism that was induced in it by the presence of the magnet. It is permanently magnetized. If a bar of soft iron of the same size is now substituted for the steel and the experiment repeated, it will be found that the iron becomes more strongly magnetic than the steel when under the influence of the magnet, but when the magnet is withdrawn it loses its magnetism almost entirely. It does not become permanently magnetized. The magnetic particles may be thought of as subject to a kind of frictional resistance in case of steel which makes their arrangement and the consequent magnetization of the steel more difficult than in case of soft iron, but also prevents the arrangement from being so easily broken up by the forces with which the molecular magnets act on each other.

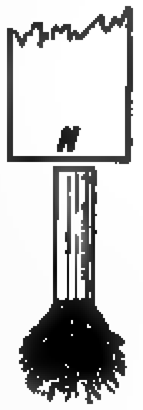


FIG. 253. Magnetic induction.

481. Magnetism Acts through Different Media.—Magnetism acts through most substances just as through air or vacuum. Take a powerful horseshoe magnet or, better still, an electromagnet and place across its poles a thin sheet of cardboard and then bring up a mass of iron filings under the cardboard: they will cling in a great mass under the poles. If a plate of glass or lead or wood or of any other non-magnetic substance be substituted they will cling in the same way. Let the plate now be fixed in position a short distance down from the poles of the magnet, as in figure 254, some iron filings will fall off as it is lowered, but if not too far separated a considerable mass will still cling. Now slip in a plate *A* between the magnet and the lower plate on which the filings rest. There will be no change if the plate *A* is of wood, glass, brass or any other non-magnetic substance, but if an *iron* plate is introduced at *A* immediately some or all of the filings will fall, showing **that an iron plate will screen the region beyond, at least partially, *from the* action of the magnet.**

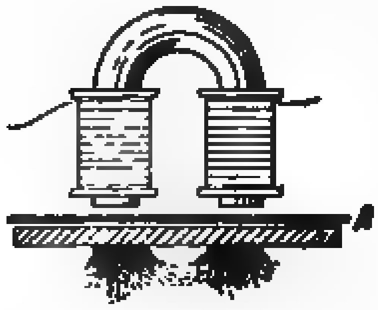


FIG. 254.

Law of Force and Magnetic Field

482. Law of Force Between Two Poles—Coulomb's Law.—The law of force between two magnets was first carefully studied by Coulomb (1736–1806) by means of the *torsion balance*. A magnet was suspended by a fine wire in a horizontal position inside of a glass vessel by which it was screened from air currents. The upper end of the wire was attached to a graduated head by which it could be twisted through any desired number of degrees. A second magnet mA was then fixed in a vertical position with one of its poles near the similar pole of the first magnet. The force of repulsion was measured by the torsion of the wire. By twisting up the wire by the head e the poles were brought closer together, and the increased torsion in the wire gave the increase in repulsive force. The method is complicated by the fact that the magnetic attraction of the earth as well as the torsion of the wire acts on the suspended magnet, and the action of both poles of each magnet must be taken into account. By this investigation Coulomb was led to enunciate the law that **the force between two magnet poles is proportional to the strength of the poles and inversely proportional to the square of the distance between them.** It may be expressed thus

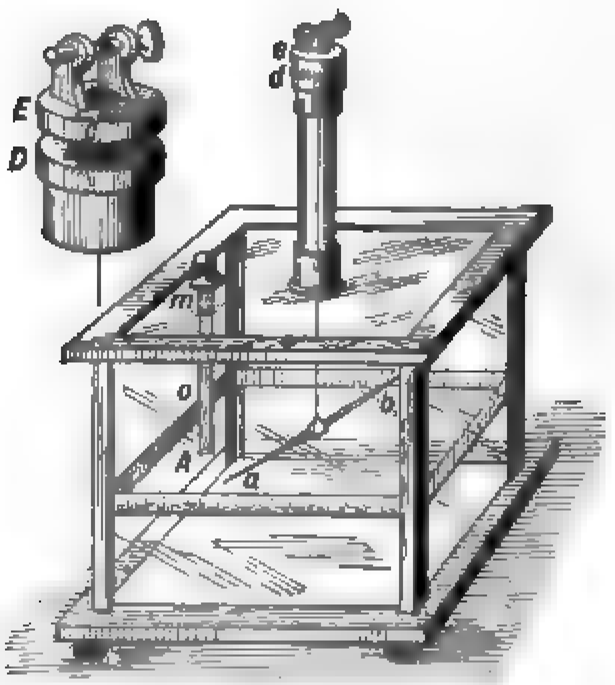


Fig. 255.—Torsion balance.

$$F = K\frac{mm'}{r^2}$$

where F represents the force, m and m' the strengths of the two poles, and r the distance between them; K is a constant which depends on the units in which the various quantities are measured, and, as is now known, on the medium surrounding the magnets. The law assumes that the poles m and m' occupy so little space that they may be regarded as points compared with

the distance r, and when so understood the most refined modern measurements only confirm its truth.

483. Field of Force.—The region around a magnet is said to be a *field of force*. An interesting way of examining the field of force near a magnet is as follows. Lay a sheet of glass on the magnet and dust over it fine iron filings. On gently tapping the plate the filings will gather into lines or filaments as shown

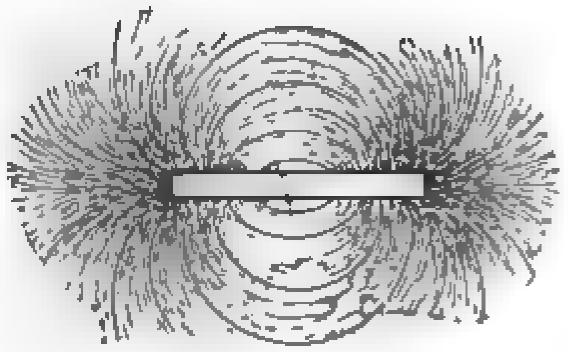


FIG. 256.—Iron filings near a magnet.

in the figure. These lines indicate the direction of the magnetic force in the field. A minute compass needle placed at any point takes the direction of the line at that point. *Lines having at every point the direction in which a compass needle would stand if placed there, are called lines of force.* Of course an infinite number may be imagined drawn from one pole to the other.

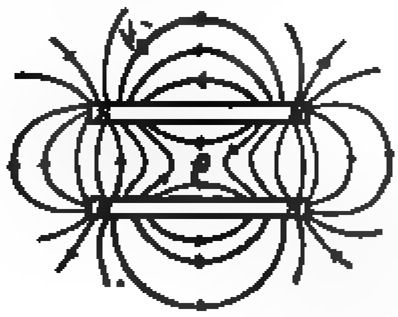


FIG. 257.

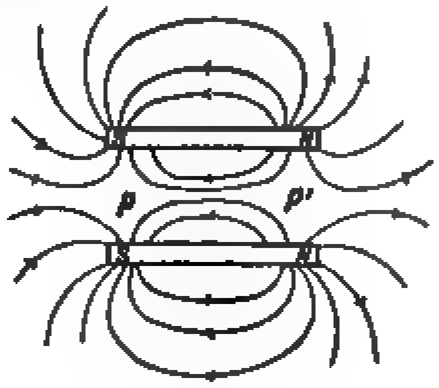


FIG. 258.

In figures 257 and 258 are shown the lines of force in case of two magnets placed near each other. In figure 258 the corresponding poles are near each other and the magnets repel. It will be seen that no lines of force pass from one to the other, and two neutral points are shown at p and p'. In figure 257 the magnets are shown with the north pole of one opposite the south pole of the other. The two attract each other in this

case and lines of force pass directly across from one to the other, leaving a single neutral point at p.

484. Faraday's Theory of the Magnetic Field.—Faraday believed that the medium around a magnet, wherever there is a magnetic field, is in a peculiar physical condition, which we may call **magnetized**; and that in consequence of this condition it has at every point a tendency **to contract or shorten in the direction of the lines of force, and to expand or spread out at right angles to the lines of force**, or that there is a *tension* in the direction of the lines of focre and a *pressure* at right angles to that direction. He saw in these tensions and pressures the explanation of all the forces between magnets. For example, from figure 258 it will be clear that if the medium were to expand at right angles to the lines of force the two magnets would be pushed apart or repelled. While the shortening of the medium in the direction of lines of force would, in the case shown in figure 257, draw the magnets together. James Clerk Maxwell, a Scotch physicist of great genius, showed that Faraday's theory was competent to explain the action of one magnet upon another, and that the tension along the lines of force at any point in the medium is equal to the pressure at right angles to them.

485. Unit Pole.—For the exact study of magnetism it is necessary that certain units should be adopted as a basis for measurements.

A unit pole, or a pole having unit strength, **is one which if placed one centimeter from an equal pole in vacuum will repel it with a force of one dyne.**

It will be observed that this unit is based directly on the C. G. S. units of length and force. If these units are employed the expression for the force in dynes between two magnetic poles of strengths m and m', and r centimeters apart in vacuo, is

$$F = \frac{mm'}{r^2}.$$

For all practical purposes this expression also gives the force in air and in all other media that are not distinctly magnetic.

486. Strength of Field.—When a magnet is placed in a magnetic field, due either to the earth or to some other magnet, each pole is acted on by a force which depends both on the

strength of the pole and on the strength of the field in which t is placed.

The strength or intensity of a magnetic field at any point is :he force in dynes on a unit magnet pole placed at that point.

Thus the earth field has a strength 0.5 at a point where a nagnetic pole of unit strength is acted on with a force of 0.5 lynes.

When a pole of strength m is at a point where the strength of ield is H, it is acted on by a force of Hm dynes.

In any field of force the two poles of a magnetic needle are ırged in opposite directions. **The direction in which the north ›ole tends to move is known as the positive direction of the line ›f force at that point.**

487. Magnetic Moment.—Suppose that in magnetic field of strength H, a magnetic ıeedle is placed in such a position that the ine joining its poles makes an angle a with he lines of force of the field. Let m represent the strength, or number of units in its north pole, and $-m$ the strength of its south pole. Then the north pole is urged with a force Hm in the positive direction of the lines of force of the field and the south pole experiences an equal force in the opposite direction. These equal and parallel forces constitute a couple whose moment is $Hml \sin a$, where l is the distance in centimeters between the two poles of the magnet. The quantities m and l belong to the magnet and their product ml is known as the magnetic moment of the magnet, and is represented by M.

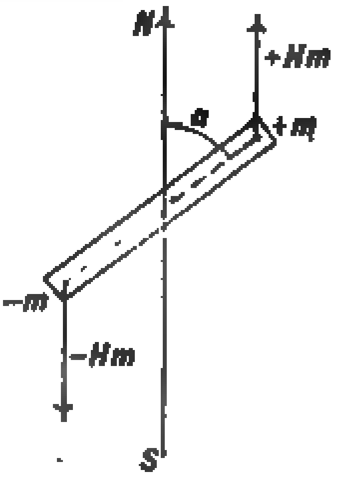


Fig. 259.

Thus **the magnetic moment of a magnet may be defined as the product of the strength of one of its poles by the distance between them.**

The couple which acts on the magnet may then be expressed by the formula,

$$HM \sin a.$$

488. Period of Oscillation of a Magnet in a Magnetic Field.—If a magnetic needle in a field of force is disturbed from its position of rest, it will vibrate to and fro just as a pendulum

oscillates in the field of the earth's attraction. The period of one complete oscillation of a pendulum has been shown to be

$$T = 2\pi\sqrt{\frac{K}{mgh}} \qquad (\S 142)$$

where K is the moment of inertia of the pendulum and mgh is a quantity which when multiplied by the sine of the angle of inclination of the pendulum gives the moment of force which at that instant urges it toward its equilibrium position.

In case of the oscillating needle the mechanical conditions involved are the same, except that the couple causing the motion is due to magnetism instead of to gravitation. The factor HM is the quantity which when multiplied by *sin a* gives the couple acting to turn the magnet; it, therefore, plays the same part in this case as the factor mgh in case of the pendulum.

Hence the period of oscillation of a magnet in a field where the strength is H units, is

$$T = 2\pi\sqrt{\frac{K}{HM}}$$

where K is the moment of inertia of the magnet and M is its magnetic moment. It should be observed that in this case as in that of the pendulum the formula gives the period when the arc is exceedingly small. With large arcs of vibration the period is longer.

From the above formula it is clear that the stronger the field of force at a point where a magnetic needle is placed the more rapidly it will oscillate when set in vibration. This is the explanation of the rapid quivering of a compass needle when brought near the pole of a magnet.

Problems

1. A short compass needle is placed near the side of a straight bar magnet and equidistant from its poles. In what direction does it point? In what direction will it move if floating, and why?
2. Two bar magnets, exactly alike, are placed in line with each other, their north poles toward each other and south poles directed away. What is the strength of field at a point midway between the two and the direction of the lines of force near that point?
3. What is the magnetic moment of a bar magnet having poles of strength

200, and 20 cm. apart, and what would be the moment of the couple required to hold it at right angles to the lines of force in a field of strength 5 in C. G. S. units.

What couple would be required to hold a magnet with pole strength 150, and with 16 cms. between poles, at an angle of 30° with the lines of force in a field of strength 2.

Find the ratio of the strengths of field at two places when a certain magnetic needle oscillates n times per sec. at one place and n' times per sec. at the other.

489. Strength of Field at a Point Near a Magnet.—The direcon and intensity of the force near a magnet may be calculated follows. Let m and $-m$ be the strengths of the two poles the magnet, and let r be the distance from the point P to the orth pole of the magnet and let r' be its distance from the uth pole. Then if a unit pole were at P it would be subject a force $\frac{m}{r^2}$ in the direction a, and to a force $\frac{m}{(r')^2}$ in the direction Laying off distances a and b proportional to the amounts

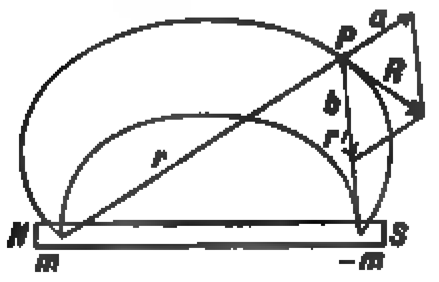


Fig. 260.

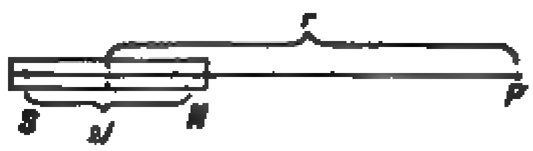


Fig. 261.

these two forces, the resultant force will be represented on e same scale by the diagonal of the parallelogram on a and b. he resultant is, of course, tangent to the line of force at P.

It is sometimes desirable to calculate the force due to a magnet a point P in line with the axis of the magnet as shown in figure 31. Let m be the strength of each of the poles, r the distance P from the center of the magnet, and l the distance of either ole from the center of the magnet. Then by Coulomb's law e force on a unit north pole placed at P due to the north pole $\frac{m}{(r-l)^2}$ and is directed toward the right. That due to the her pole is $\frac{m}{(r+l)^2}$ and is toward the left. The resultant force P is then toward the right and may be written

$$F = \frac{m}{(r-l)^2} - \frac{m}{(r+l)^2}.$$

or

$$F = \frac{4rlm}{r^4 - 2r^2l^2 + l^4}.$$

If l is small compared with r the terms involving l^2 and l^4 in the denominator may be neglected, as they are insignificant compared with r^4, so *approximately*

$$F = \frac{4ml}{r^3} \text{ or } F = \frac{2M}{r^3}$$

where $M = 2ml$, the *magnetic moment* of the magnet.

Problems

1. What is the amount and direction of the magnetic force on a unit pole placed at a point in line with a bar magnet and 20 cm. away from its N pole, if the strength of the magnet's poles is 100, and its length between poles is 20 cm.?
2. The poles of a bar magnet have a strength of 200 units each and are 30 cm. apart. Find the direction and amount of the force due to the N pole at a point 30 cm. from each pole. Find also the force due to the S pole at the same point. Find the resultant strength of field at the point by the vector diagram.
3. Find the amount and direction of the magnetic field strength at a point 30 cm. distant from each of the poles of a bar magnet, in which the poles have strengths +300 and −300 and are 30 cm. apart.
4. Find the amount and direction of the force on a unit north pole placed in line with the magnet described in problem 3 and 30 cm. distant from its north pole.
5. Find the strength of the magnetic field at a point 5 cm. distant from the center of the magnet of problem 3 in a direction at right angles to its axis.
6. Calculate by the method used in §489 the strength of field at a point on a line drawn through the center of the magnet at right angles to its axis.

TERRESTRIAL MAGNETISM

490. Declination of the Magnetic Needle.—The compass needle, mounted so as to rotate in a horizontal plane, does not in general point directly north, but a few degrees east or west of north, and this deviation is called its declination. In observing

the declination it will not do to assume that the magnetic axis of the needle is in the same direction as its axis of figure. If the magnetic axis is as represented by the dotted line in figure 262 then the apparent declination is in the first case too small. If the needle is now turned over and suspended with the opposite side upward, it will give too great an apparent declination. The mean of the two will be the true declination. This direction is called the magnetic meridian.

491. Dip or Inclination.—It was observed by Hartmann (1489–1564) that if a needle was balanced before being magnetized the north end would dip downward after magnetization. The so-called dipping needle was first made by Norman, a London instrument maker, who mounted a needle on a horizontal axis so that it could swing freely in a vertical circle. The needle was then carefully balanced so that it would stand in any position before magnetization. But after it was magnetized it was observed that the north pole pointed downward some 70° below the horizontal if the plane in which it turned was north and south by the compass.

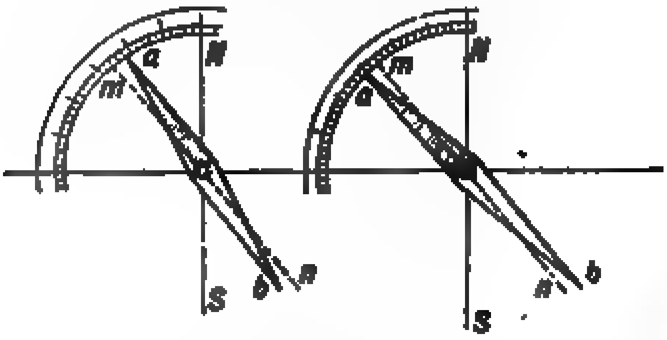


Fig. 262.—Declination of compass.

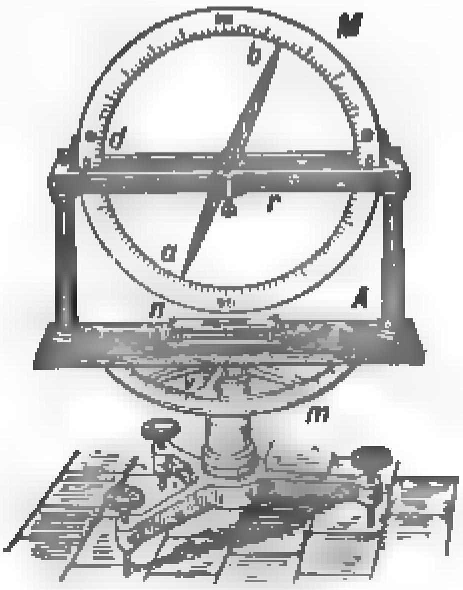


Fig. 263.—Dipping needle.

To diminish friction the cylindrical pinions on the ends of the axis of the dipping needle usually rest on horizontal plates of polished agate, on which they roll as the needle turns.

In this case also the needle must be reversed, the side toward the east being turned toward the west, to guard against error due to the axis of the needle not being in line with the direction of its magnetization. To guard against any want of balance in the needle, it should be magnetized over again with its poles *reversed*, and *the dip again* observed. If the needle is well con-

structured, the mean of these four observations will be the required dip or *inclination.*

492. Resultant Direction of Magnetic Force.—The dipping needle gives the direction of the resultant magnetic force at any point. It is found that near the equator the needle is horizontal, as it is taken north its north pole points downward by an amount which increases steadily till at some point northwest of Hudson Bay it points vertically downward. That point is called the *north magnetic pole,* and near there a horizontal compass needle would have no directive tendency and would be useless. North of that point the north pole of the compass needle would point south.

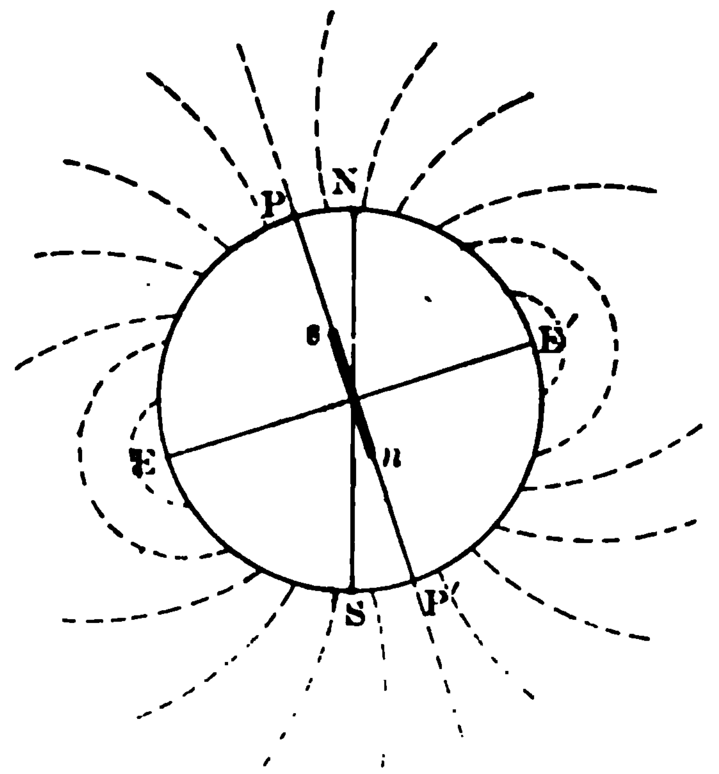


Fig 264.—Lines of force of the earth.

In vessels that change their latitude greatly the compass needle must be provided with a little sliding weight or counterpoise to correct its dipping tendency. South of the equator the south pole of the needle dips downward.

Figure 264 shows the probable form of the lines of magnetic force around the earth; of course, the direction of these lines of force is known only at the earth's surface. The magnetic condition of the interior of the earth is entirely unknown.

The declination or deviation of the resultant force from the geographic north direction also varies from point to point on the earth, at some points being east of north and at others west of north. This fact was first observed by Columbus who, as he advanced in his voyage, was alarmed to see that the compass no longer pointed as it had done when he started.

The chart on the following page shows the declination of the magnetic needle throughout the United States in 1900. Isogonal lines connect places where the declination is the same.

493. Intensity of the Earth's Magnetism.—The magnetic force of the earth at any point may be considered as the resultant of two component forces, the horizontal component H

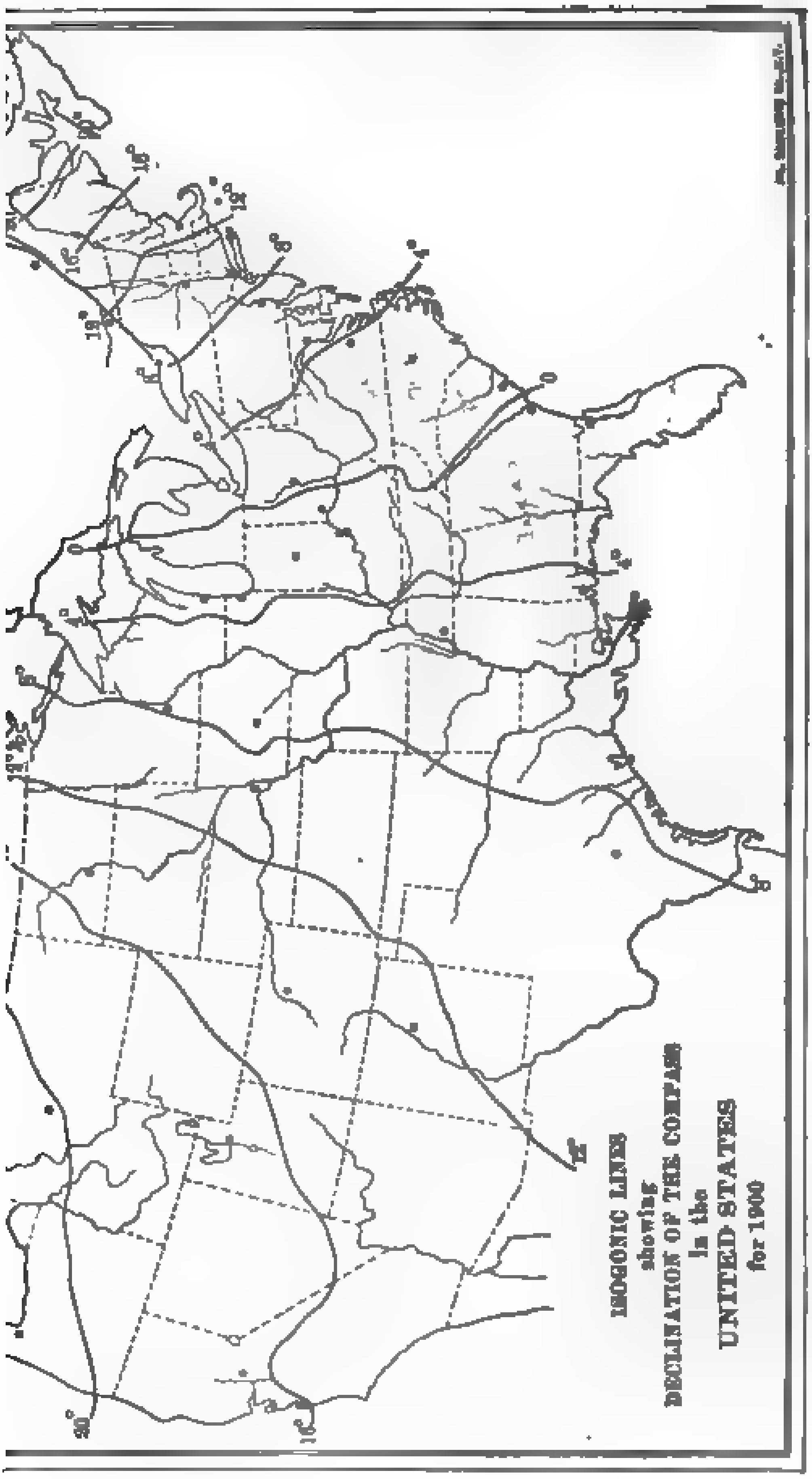


Fig. 265.—Isogonal lines showing the declination of the magnetic needle throughout the United States in 1900.

(Fig. 266), and the vertical component V. The horizontal component H is the force which is effective in directing the compass needle. The smaller this component the more feebly will the needle be affected. When a needle is balanced or suspended in the usual way so as to vibrate in a horizontal plane, its period of oscillation depends on this component only.

As shown in paragraph 488, the intensities at different places may be compared by causing the same magnet to vibrate first at one place and then at the other. The intensities are proportional to the squares of the number of vibrations per second. By this means the horizontal component of the intensity may be determined at any point as compared with that at some standard place.

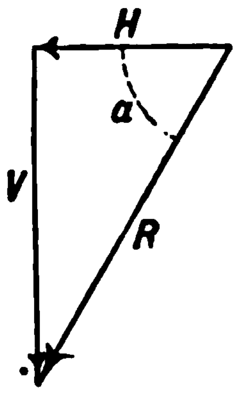


Fig. 266.

When the horizontal intensity and the dip are both known, the resultant intensity may be found by the relation

$$R=\frac{H}{\cos a}$$

when a is the angle of dip.

The following table shows the value of the horizontal component and total force at certain places. Notice how the horizontal force becomes less in higher latitudes. The intensity is given in C. G. S. units, or the force in dynes upon a unit pole.

Strength of the Earth's Magnetic Field (Dynes per Unit Pole)

Places	Dip	Horizontal intensity	Resultant intensity
Central South America	0°	0.30	0.30
North coast of South America	40°	0.32	0.42
Cuba	50°	0.32	0.50
Georgia	60°	0.26	0.52
New York	70°	0.18	0.53

494. Secular Change in the Magnetic Field.—One of the most remarkable features of the earth's magnetism is that it is continually changing. The declination of the needle is slowly changing everywhere; that is, the magnetic poles are slowly shifting their positions. At the same time the dip is changing.

The changes in declination and dip at London are shown by the curve in figure 267. The pole of a freely suspended needle would at that place apparently move through a complete cycle of change in about 470 years. This slow change is called the *secular* change.

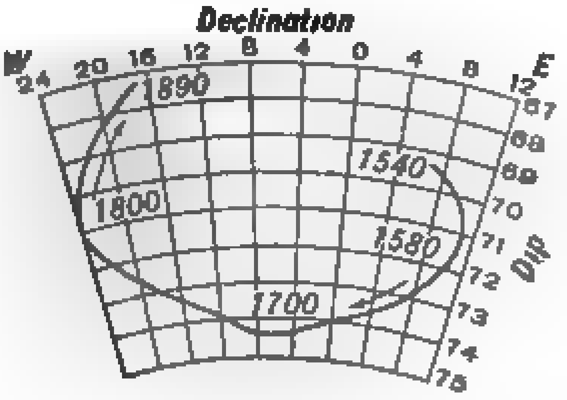


FIG. 267.—Secular change in dip and declination at London. (After L. A. Bauer.)

495. Diurnal Variations.—The careful study of the magnetic conditions at any place by self-recording instruments shows that there is a periodic change in the magnetic elements depending on the time of day.

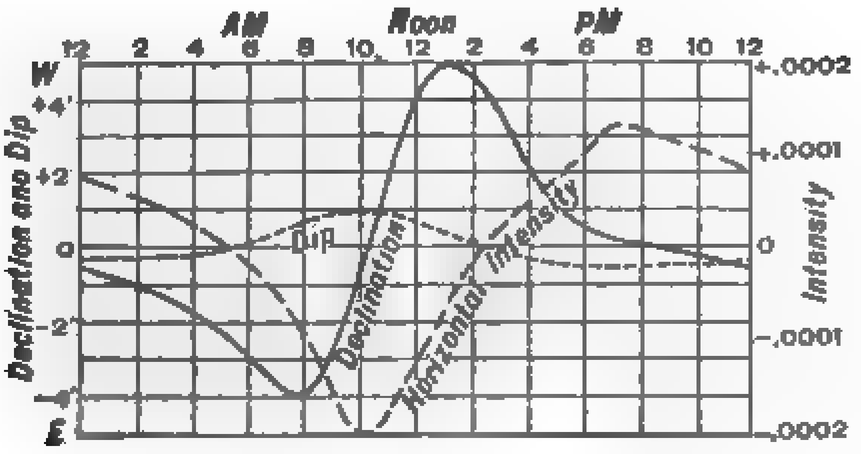


FIG. 268.—Diurnal changes in dip, declination and intensity at Kew.

The curves of figure 268 show the average variations at Kew, near London. It will be observed that the maximum changes take place in the daytime and may be due to variations in temperature of the earth's surface.

496. Irregular Disturbances.—The magnetic needle is also often disturbed by what are called magnetic storms; these disturbances *usually accompany* any marked display of the aurora

borealis, and they also seem to be more prevalent at times of sunspot maxima.

497. The Earth a Magnet.—It was suggested by Dr. William Gilbert (1600), physician in the court of Queen Elizabeth and the first to take up the scientific study of magnetism, that the earth itself was probably a great magnet, and later observations have borne out this idea. Two well-marked magnetic poles being found, one northwest of Hudson Bay in North America and the other south of Australia.

But while there is this general resemblance to a simple magnet, the direction of the magnetic force varies from place to place in a way that cannot be wholly accounted for by the supposition of simply two poles.

The magnetism of the earth seems to be due to a variety of causes, the presence in the earth of magnetic masses is a cause of local variations and may have great influence in the surface layer of the earth, but it seems probable that the temperature in the interior of the earth is too high for it to possess any very strong magnetism. Electric currents flowing in the surface of the earth and due to its varying temperature as first one side and then another is exposed to the sun, as well as currents of electricity in the upper air, probably play an important part in determining its magnetic state. But the complete explanation has not yet been given, and any theory to be satisfactory must account for the remarkable secular changes in its magnetism which go on slowly and progressively year after year.

498. Gauss' Method of Measuring the Horizontal Intensity.—The horizontal component of the earth's magnetic force may be measured by the following method due to Gauss. A small steel bar magnet is suspended horizontally by a fine fiber in a closed box by which it is protected from air currents. It is then set oscillating through a small arc and the period of oscillation carefully determined. This period depends on M the magnetic moment of the magnet and on H the horizontal component of the earth's magnetic force. By §488

$$HM = \frac{4\pi^2 K}{T^2}$$

where K is the moment of inertia of the magnet, a quantity that is determined by its mass, size, and shape, and T is the period of a complete oscillation. The product HM is thus found.

To determine the relation of H to M a second experiment is necessary. Suppose P is the point where the magnetic force H is to be determined and

where the period of oscillation of the magnet NS was observed in the first experiment. Place at P a very short magnetic needle, while the magnet NS is placed exactly east or west of P and with its axis on the east and west line, as shown in Fig. 269. If r is the distance from the center of NS to P, then the force at P due to the magnet is, as shown in §489.

$$F = \frac{2M}{r^3}.$$

Then at P the force H due to the earth and the force F due to the magnet are at right angles to each other, as shown by the arrows in the figure. The needle at P will take the direction of the resultant force R and will therefore be deflected through the angel a, but

$$\tan a = \frac{F}{H} \quad \text{or} \quad \tan a = \frac{2M}{r^3H}$$

whence

$$\frac{H}{M} = \frac{2}{r^3 \tan a}.$$

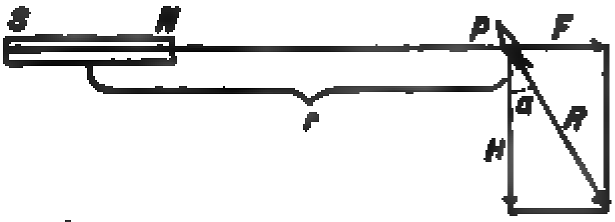


FIG. 269.

This expression shows that to determine the ratio of H to M it is only necessary to measure the distance r and the angle of deflection a.

Having by the first experiment determined the product HM and by the second the ratio $\frac{H}{M}$, it only remains to multiply the two together to find H^2 and so determine H.

Unit Tubes of Force

499. Number of Lines of Force.—Up to this point lines of force have been regarded as simply expressing the direction of the force in the magnetic field. We must now follow Faraday in a very remarkable development of the idea.

In a stream of water flowing steadily lines may be imagined drawn which at every point are in the direction of flow, and which may be called stream lines. An infinite number of such lines may be drawn. The whole stream may then be conceived to be divided up into *tubes of flow* by means of surfaces which everywhere coincide with stream lines. These tubes of flow may be taken of such a size that each will transmit the same

quantity of water per second, say one cubic foot. Then, where the stream is most rapid, the cross sections of the tubes of flow will be smallest and they will widen out as the velocity diminishes. The whole number of such tubes in the stream will be equal to the number of cubic feet of water transmitted per second. These tubes of flow may be called *unit tubes*, and the number of them crossing perpendicularly a surface 1 sq. ft. in area is equal to the number of cubic feet of water crossing that area per second. Thus **the number of unit tubes passing perpendicularly through a unit surface at any point in the stream is equal to the velocity at that point.**

Now in the same way the magnetic field may be conceived as divided up into unit tubes by means of surfaces parallel to the lines of force. And it may be proved that where such a unit tube is smaller the field is more intense, and where it widens out the strength of field is less, just as the velocity varies in case of the stream of water. So that it is possible to take these tubes of such a size that **the number passing perpendicularly through a square centimeter of surface at any point may be equal to the strength of the magnetic field at that point.**

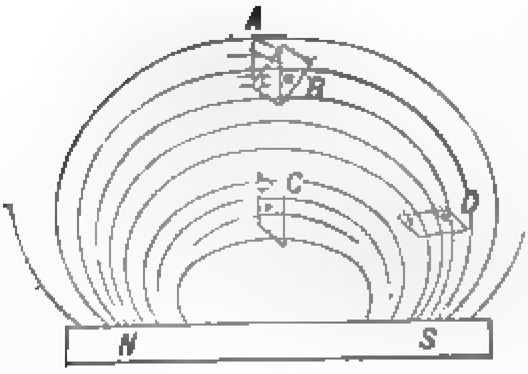


Fig. 270.

We may imagine that **each unit tube is represented by a line of force drawn through its center or axis, and when the phrase number of lines of force is used it refers to such lines.**

Using the term in this way, it is clear that in the case shown in figure 270 more lines of force pass through a card in position C than in position A, as the force is greater at C than at A, and consequently there are more lines of force to the square centimeter. Clearly, also, fewer lines of force pass through the card in position B than in A and most of all in position D. If the card were placed parallel to the lines of force none at all would pass through it.

The number of lines of force through A is found by taking the average strength of the field at the surface A and multiplying this by the area of A in square centimeters, since the number of lines per square centimeter is equal to the strength of the field at that

point. If the surface is oblique to the lines of force as at B, the number of lines of force passing through it will be found by multiplying the number in the perpendicular position A by the *cosine* of the angle a, or, what comes to the same thing, multiply the average strength of the field at the surface B by the projection of that surface on a plane at right angles to the lines of force.

Problems

1. How many lines of force pass through a square meter of floor area where the total strength of the earth's magnetic field is 0.6 and the lines of force are inclined 60° from the horizontal?
2. How many lines of force in this case would pass through an area of 1 square meter on an east and west wall, and how many in case the wall ran north and south?
3. How many lines of force pass through an area of 4 sq. cm. placed as at A in figure 270, with its center 12 cm. from each pole of a magnet 20 cm. long between poles, strength of poles being 288?
4. How many lines of force pass through a circle 1 cm. in diameter placed 8 cm. from the north pole of a bar magnet 16 cm. long, which points directly at it and has poles of strength 200? The plane of the circle is perpendicular to the axis of the magnet.

500. Lines of Force Inside a Magnet.—The lines of force of a magnet are not to be supposed as only outside of it. If we

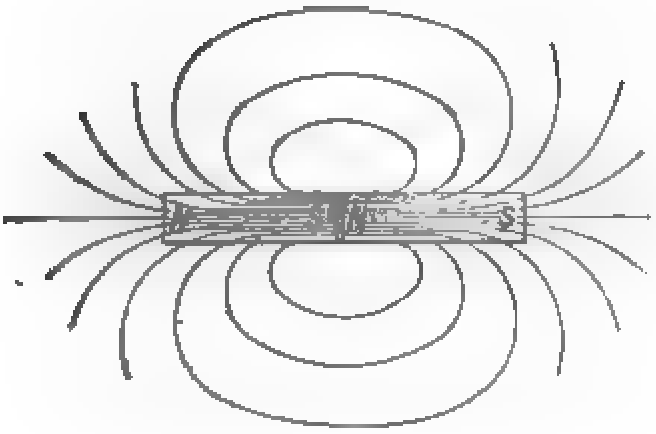


FIG. 271.

imagine a minute magnetic needle placed in a crack extending across the magnet it will be acted on most powerfully by the poles $N'S'$ on each side of the crack, but it will also be affected by the attraction of the end poles N and S of the magnet. In consequence of the superior influence of the poles $N'S'$, it will set its north pole toward *the left or* S'. If we now imagine the cleft

shifted along toward the north end of the magnet, the force inside the cleft will become less because it will be nearer to N, which pole tends to make the needle point in the opposite direction. But still the needle will point from right to left. When the cleft comes infinitely near to the end N, the magnetism of N and of S', which form two opposite and equally magnetized layers, will neutralize each other so that the effect is the same as though the needle were just outside the magnet at N. We see in this way **that the force in the cleft is absolutely continuous with that outside of the magnet**: there is no abrupt change in passing through the surface. **The force in such a cleft is called the magnetic induction and the lines of force outside of a magnet form continuous closed curves with the lines of induction inside of the magnet.** The lines of force outside are also called lines of induction, as there is no distinction between the two except inside of a magnetic medium. What is called the positive direction of these lines is from the north to the south pole *outside* of the magnet. Of course as many lines of force as emerge from the north pole enter at the south pole, and all the lines of force or induction in the magnet pass through its middle section. Looked at in this way, **the poles are seen to be simply those regions where the lines leave or enter the magnet, and the most intensely magnetized portion of the magnet is the center where the lines of induction are closest together.** If a little block could be cut from the center of the magnet without disturbing its magnetism, it would be found a more powerful magnet than a similar block cut from any other part where the lines of induction are not so close.

Warning.—In using the words *entering* and *emerging* with reference to lines of force nothing like *flow* or *motion* must be supposed; when what we arbitrarily call the *positive direction* of the line of force is toward a surface, it is spoken of as entering it; and when that direction is away from a surface the line of force may be said to leave the surface or emerge from it.

501. Influence of the Shape of a Magnet on Its Power and Retentiveness.—A short thick bar of steel is more difficult to magnetize strongly than a long thin one and loses its magnetism more easily. A thick magnet may be thought of as made up of a bundle of thin ones of the same length. But it is clear that in such a bundle each little magnet would tend to set up lines of

force down through its neighbor in such direction as to oppose or weaken the other's magnetism.

Thus there is a demagnetizing tendency which is greatest in a short thick magnet. Horseshoe magnets are long and have their poles close together and consequently there is very little demagnetizing tendency. There is, however, a tendency for the lines of force in this case to pass across on the inside of the poles instead of out at the ends. A soft-iron block placed across the poles, and called an armature or keeper, provides an easy path for the lines of force from one end around to the other and thus tends to keep the poles near the ends.

502. Ring Magnet.—A uniform ring of iron or steel may be magnetized by means of an electric current so that the lines of force are circles entirely within the substance of the ring. In such a case the magnet has no poles as there are no places where the lines of force enter or leave the ring. Such a magnet has no external field of force and would not act on a magnetic needle placed near it, and yet it is magnetized, as will be evident if it is broken, for in that case each half will show two poles.

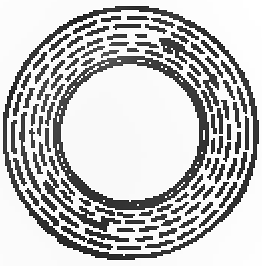


Fig. 272.

Magnetic Induction

503. Induction Studied by Iron Filings.—If the lines of force of a horseshoe magnet are examined by means of iron filings on a plate of glass, as described in §483, and if a bar of soft iron is then placed a short distance in front of the poles of the magnet and the field again examined in the same way, a notable change will be observed. The lines of force are bent toward the two ends of the soft-iron bar as though they could be established in the iron more easily than in the surrounding medium. And the softer the iron and the more easily it is magnetized, the greater the number of lines of force that will pass through it rather than the more resisting medium around it. Thus the presence of the iron makes the field of force weaker beyond it, and the nearer the iron bar is to the poles of the magnet the more lines of force will be drawn into it and the fewer there will be in other parts of the field.

504. Permeability.—The ease with which lines of force may be established in *any medium* as compared with a vacuum has

been called by Lord Kelvin the permeability of the medium. Thus iron has a permeability several hundred times greater than air. Most other substances have a permeability which is sensibly the same as air or vacuum, and, therefore, the magnetic field is practically the same in wood, glass, or water as in air.

A hydraulic analogy may aid in forming a clear conception of this subject. Imagine a stream of water continually flowing out of the north of the horseshoe magnet (Fig. 273) and entering its south pole. Suppose the medium surrounding the magnet was of a uniform porous nature that opposed considerable resistance to the flow from *N* to *S*. The lines along which the flow would take place would be like the lines of force in the field before the soft iron was introduced. Now imagine a cavity to be made in the porous medium having just the size and position of the soft-iron bar. The lines of flow would now tend toward this cavity through which the liquid would flow freely and a correspondingly smaller flow would take place in other regions.

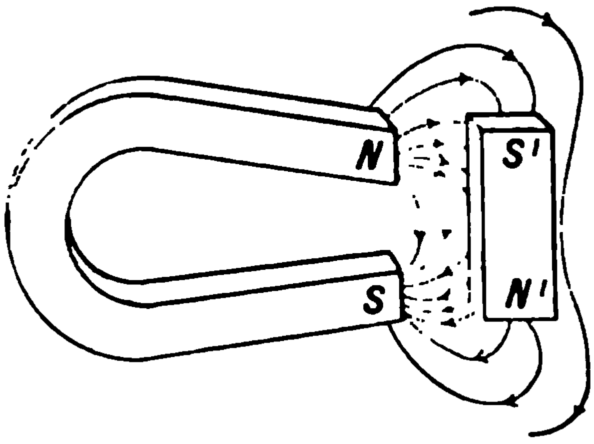


FIG. 273.

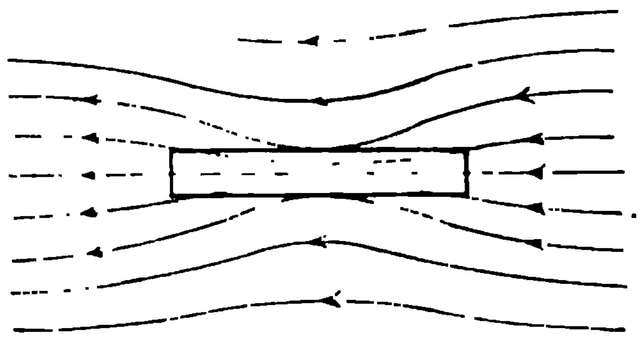


FIG. 274.—Bar of soft iron parallel with lines of force of field.

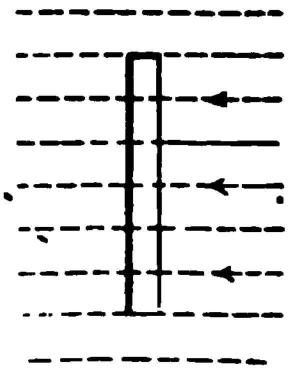


FIG. 275.—Bar of soft iron across the lines of force.

The lines of flow in this case correspond to the lines of force when the soft-iron bar with its great permeability is in the field.

505. Magnets Formed by Induction.—When a soft-iron bar is placed in front of a magnet as shown in figure 273, at the end nearest the north pole of the magnet the lines of force are directed toward the end of the bar as toward the south pole of a magnet and at the other end they are directed away from the bar as from a north pole. The bar of iron thus becomes a *magnet by induction*.

If it were of steel it would retain some of this magnetism when taken out of the field.

Suppose a *long* bar of soft iron to be placed in a magnetic field parallel to the direction of the lines of force. The result will be as shown in figure 274, lines of force will be drawn into the bar in consequence of its great permeability entering it at one end and leaving it at the other, so that one end becomes a south pole and one a north pole. On each side of the bar the field is weakened.

When, however, the bar is placed across the field of force as in figure 275 it will have only a very slight effect on the field of force since the lines of force can pass through only a small thickness of iron. So also a thin flat sheet of iron placed perpendicular to the lines of force of the field would have practically no effect on the field.

506. Effect of Heat and Jarring in Case of Magnetizing by Induction.—The magnetism induced in an iron or steel bar placed in a magnetic field parallel to the lines of force may be increased by striking the bar with a hammer or jarring it while under the influence of the field, also by heating the bar red-hot and allowing it to cool in the magnetic field. These disturbances seem to facilitate the arrangement of the molecules under the influence of the magnetic force and help to overcome the resistance to magnetization which especially characterizes hard steel.

507. Magnetic Induction in the Earth's Field.—If a bar of soft iron having no permanent magnetism is placed in the earth's field parallel to the lines of force, that is, in the direction of the dipping needle, its lower end in north latitudes will become a north pole and its upper end a south pole, as may be shown by a magnetic needle. If jarred by the blow of a hammer while in this position it will be found permanently magnetized. If, however, it is placed at right angles to the lines of force of the earth it is scarcely magnetized at all (§505).

In consequence of induction iron ships are magnetized by the earth differently when pointing in different directions.

In such vessels the standard compass is usually *compensated* by having soft-iron bars so placed near it that the magnetism induced in them will in every position just balance that induced in the ship, while permanent steel magnets may be used to compensate the permanent magnetism of the ship.

508. Hysteresis.—When the magnetic field in which a mass of iron is placed is varied in strength, the changes in the magnetism of the iron lag behind the changes in the field. This is known as hysteresis and is discussed in connection with the magnetization of iron by electric currents, §680.

Permeability, Diamagnetism, and Influence of Medium

509. Magnetic Substances Attracted.—**When a fragment of iron is placed in a magnetic field it experiences a force in that direction in which the strength of the field increases most rapidly.** If at *A* (Fig. 276) it is drawn directly toward the magnet in the direction of the lines of force. If at *B* it is drawn toward the magnet at right angles to the lines of force. If at *C* it will be drawn in a direction oblique to the line of force somewhat as shown. If it is in a uniform field, as in the earth's magnetic field, or is at a point in a magnetic field where the force is a maximum or a minimum, it will be in equilibrium and have no tendency to move in any direction. Such a point of equilibrium would be found midway between two equal poles either like or unlike.

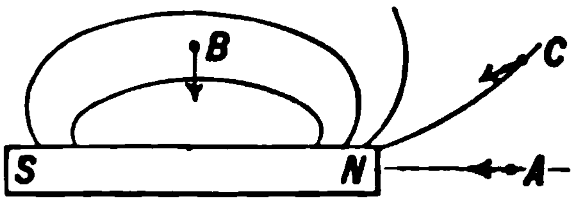


Fig 276.

If the fragment is long in shape it will turn and *point* in the direction of the line of force, but it will not always tend to *move* along that line.

Any substance whose permeability is greater than vacuum will act in this way in a vacuum and such are known as **paramagnetic** or simply **magnetic substances.**

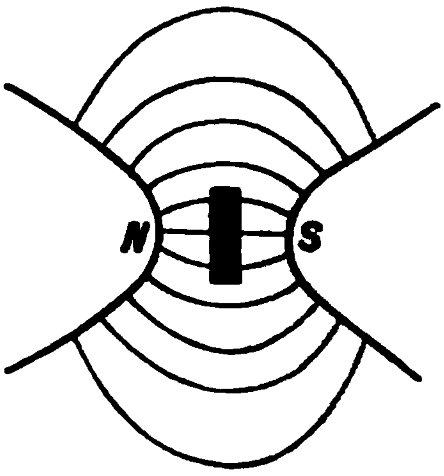


Fig. 277.—Bismuth in magnetic field.

510. Diamagnetic Substances.—Faraday (1845) experimented on the behavior of a great variety of substances in the intense field between the poles of a powerful electromagnet. A little oblong of pure copper when suspended by a fine fiber in this field was found to set itself at right angles to the lines of force, as shown in figure 277. So also fragments of wood, paper, aluminum, bismuth, glass, and many other substances. These

substances Faraday called *diamagnetic.* Substances like nickel, cobalt, and manganese which behave like iron, setting themselves in the direction of the lines of force, he called *paramagnetic* or *magnetic.*

Diamagnetic substances when placed in a magnetic field are driven from a stronger field toward a weaker, **the force acting on a fragment of such a substance being in the direction in which the strength of the field diminishes most rapidly.** This may be well shown in the following way. A ball of bismuth, which is the most strongly diamagnetic substance known, is suspended between the poles of a powerful electromagnet, being hung from one end of a light arm of wood which is itself supported in horizontal position by a delicate bifilar suspension, so that the slightest force will cause the arm to swing around carrying the ball out of the magnetic field. If while the ball hangs between the two poles the current is applied to the electromagnet, the bismuth ball will at once be driven aside out of the intense field.

The setting of the diamagnetic bars *across* the lines of force described at the beginning of this section finds its explanation in the preceding experiment; for the field of force between the magnet poles is most intense next the poles as is shown by the crowding together of the lines of force, and so the ends of the bar are in a much less intense field when the bar stands across the lines of force than if it were to be directed along them; it therefore assumes the former position.

511. Influence of the Medium.—By the following interesting experiment Faraday showed that **the medium surrounding a body in a magnetic field plays an important part in determining the magnetic force upon it.**

When a thin-walled glass capsule, long in shape, is filled with a weak solution of ferric chloride and suspended between the poles of a magnet, it sets itself along the lines of force showing that the ferric chloride is *magnetic.* This happens whether the capsule is hung in air or water. **If, however, it is surrounded by a solution of ferric chloride stronger than that within the capsule it will act as if diamagnetic,** placing its length across the lines of force.

512. Permeability of Magnetic and Diamagnetic Substances.—When the permeability of a substance is greater than that of

the surrounding medium, the lines of force are drawn in toward the substance, as already discussed in §505 and as shown in figure 278 which represents the disturbing effect of a ball of substance whose permeability is greater than that of the medium around it.

If, however, the permeability of the ball is less than that of the medium, the lines of force will be spread, as shown in figure 279. A magnetic needle placed near the ball will point aside instead of toward it.

In the first case if the ball is in a field that is not uniform, as near the pole of a magnet, it will be attracted or drawn toward the stronger field. If, however, the ball has a permeability less than the surrounding medium, it will be driven away from the pole toward a weaker field.

Magnetic or paramagnetic substances may then be defined as those whose permeability is greater than that of vacuum, while those whose permeability is less than vacuum are diamagnetic.

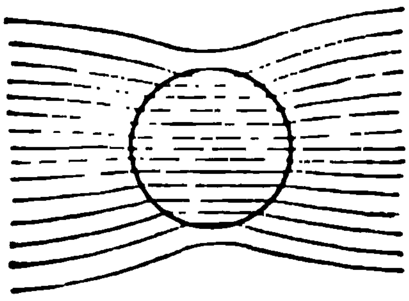


Fig. 278.—Permeability of ball greater than that of medium.

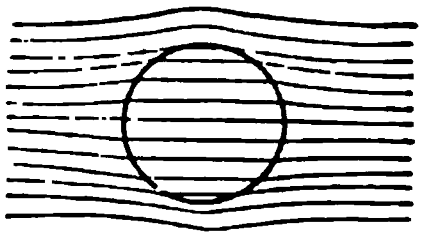


Fig. 279.—Permeability of ball less than that of medium.

513. Magnetism of Gases.—Faraday also studied the magnetic qualities of different gases. Oxygen gas was found to be attracted toward the poles, while hydrogen was repelled. Oxygen was thus shown to be more permeable than air. Later experiments have shown liquid oxygen to be decidedly magnetic.

514. Magnetic Alloy.—In 1903 Heusler made the very interesting discovery that an alloy of 25 parts manganese, 14 aluminum, and 61 copper, had decided magnetic properties, although none of the substances of which it is made is magnetic except in the very slightest degree. It seems to indicate that magnetism depends upon molecular rather than atomic structure. The permeability of this alloy has been found to be nearly 33.

515. Effects of Heat on the Magnetism of Metals and Magnets.—The permeability of iron and nickel diminishes as the temperature rises. At 737°C. iron ceases to be magnetic. A small piece of iron heated to a bright red heat is not attracted

even by a powerful magnet, but as it cools to 700° it again becomes strongly magnetic. A steel magnet when heated to bright red heat loses all trace of magnetism, and if cooled while away from magnetic influence will be found completely demagnetized.

Even when a magnet is slightly heated, say to 100°C., it is not as strong as at lower temperatures.

516. Force with which a Magnet Attracts its Armature.—The force with which a magnet attracts its armature evidently depends on the fact that the permeability of the armature is greater than that of the surrounding medium. *If there were no difference between them there would be no change in the lines of force on withdrawing the armature and consequently no attractive force.*

When the armature is of such a size that most of the lines of force from one pole to the other pass through it, the force of attraction is given very nearly by the formula

$$P = \frac{AB^2}{8\pi}$$

where A is the combined area of the two poles and B is the *induction* or the number of lines of force that pass from a pole into the armature across a square centimeter of surface. If these quantities are taken in C. G. S. units, the attractive force P will be found in dynes.

Problems

1. Find the force 5 cm. away from a pole of strength m, the other pole being so far away in comparison that it may be disregarded. How many lines of force go through the sphere of 5 cm. radius surrounding the pole m?
2. If a magnet having poles of strength 300, and 30 cm. apart is mounted on a pivot in a uniform magnetic field of strength 0.2, how much force, applied 10 cm. from the pivot, will be required to hold it at right angles to the lines of force of the field.
3. What is the magnetic moment of the magnet in problem 2. What torque is required to hold it at an angle of 45° to the lines of force of the earth field of strength 0.2?
4. Where the total intensity of the earth's magnetic field is 0.6 and the dip 70°, how many lines of force pass through a circular hoop 50 cm in diameter lying horizontally on the floor? How many if the plane of the hoop is vertical facing north and south?
5. If a compass needle oscillates 2 times per sec. when 15 cm. distant from the pole of a long magnet, how fast will it vibrate when 8 cm. from the pole, neglecting the influence of the other pole?

ELECTROSTATICS

Electrification

517. Electrification.—If a hard-rubber rod is rubbed with fur or flannel it will attract light fragments of pitch, paper, or gold leaf. A light ball of pith suspended by a thread, as shown in figure 280, is strongly attracted. A rod of sealing wax, or sulphur, or indeed of dry wood, will show the same power. In cold dry weather if a piece of paper is laid on a table and rubbed with flannel, it will be found to cling to the table and a slight crackling may perhaps be heard as it is pulled away; and if in this condition it is held near the wall it will be drawn toward it and cling to it. The shavings which come from a carpenter's plane in winter when the air is very dry will often behave in the same way. In all of these cases the substances are said to be *electrified*, a term which comes from *electron*, the Greek word for *amber*, a substance which was known to the ancients to possess this power.

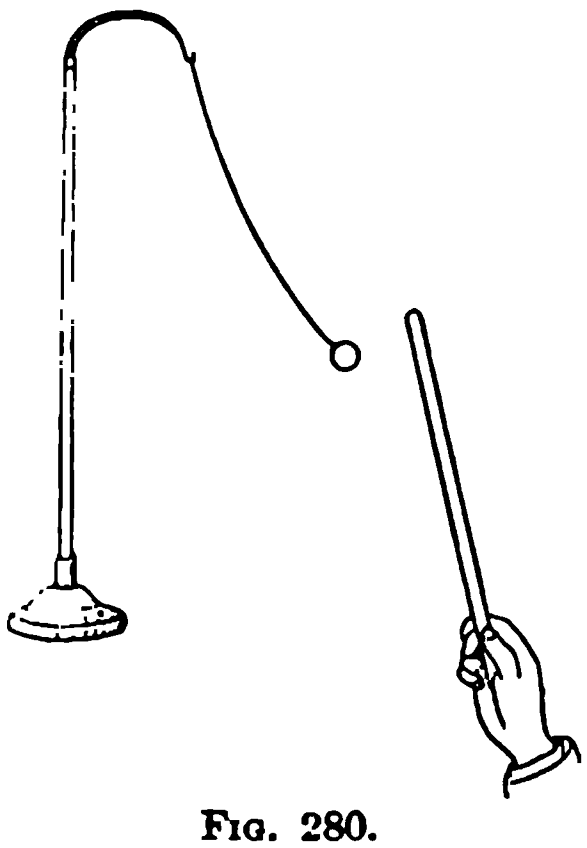


Fig. 280.

About the year 1600, Dr. Gilbert, who was also a pioneer in the study of magnetism, found that a very large number of substances could be electrified by rubbing, though with metals he could get no results. He accordingly classified substances as *electrics* and *non-electrics*, according as they could, or could not, be electrified by rubbing.

518. Conductors and Non-conductors.—In 1729 Stephen Gray discovered that the electrification of a glass rod would leak off from it and could be communicated to a ball through a damp cord. His experiments showed that electrification could be communicated through certain bodies which he called **conductors**, while it could not be communicated through others which were named **non-conductors.**

Metals were found to be the best conductors, wood and damp cord were fairly good, while glass, sulphur, and resin were non-conductors.

Gray then showed that the substances which Gilbert had classed as non-electrics were conductors, and if they were *insulated* or mounted on non-conducting supports they could be electrified as other substances can. The old distinction of electrics and non-electrics was therefore abandoned, and substances were classified as **conductors** and **non-conductors or insulators.**

The insulating power of bodies may be compared by the times required for a given amount of electrification to leak through similar rods of the different substances.

In the following table bodies are classified according to their *resistances* or insulating powers.

Insulators	*Poor Conductors*	*Good Conductors*
Amber	Dry wood	Metals
Sulphur	Paper	Gas carbon and graphite
Fused quartz	Alcohol	Aqueous solutions of salts and acids.
Glass	Turpentine	
Hard rubber	Distilled water	
Air and gases		

519. Electricity.—Take two metal pails, each mounted on an insulating support (Fig. 281), electrify one of them and then connect the two by a conductor, such as a cord or a wire. Both will show electrification when tested by the suspended pith ball, though the electrification of the one first charged will be less than before the two were connected.

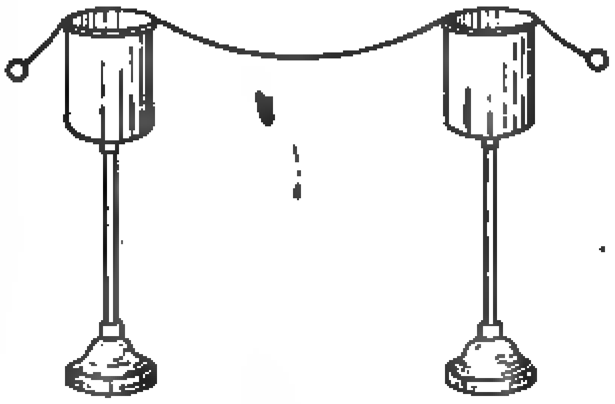


Fig. 281.

(The connecting conductor must be supported on glass rods or from loops of silk thread or otherwise insulated when placed on the pails.)

If the pails are connected by a metallic wire the redistribution of the electrification is instantaneous; if by a rod of wood or by a cord a perceptible time is required.

The communication of electrification from one body to another,

one always losing as the other gains, suggests a transfer of *something* of which electrification is the external evidence. **This something is called electricity, and when electrification is communicated from one body to another, there is said to be a flow of electricity.**

520. Two Kinds of Electrification.—Rub a rod of hard rubber or sealing wax with fur, and when strongly electrified present it to a suspended pith ball. The ball will be attracted at first, but if allowed to touch the electrified rod it may cling for a moment and then spring away, strongly repelled.

If a rod of glass, electrified by rubbing with silk, is now brought near the pith ball, it will fly to the glass, but after contact it will be repelled as it had been from the rubber rod.

While repelled by the glass it will be attracted by the electrified rubber, and *vice versa*. It is clear that the electrical states of the glass and rubber are different.

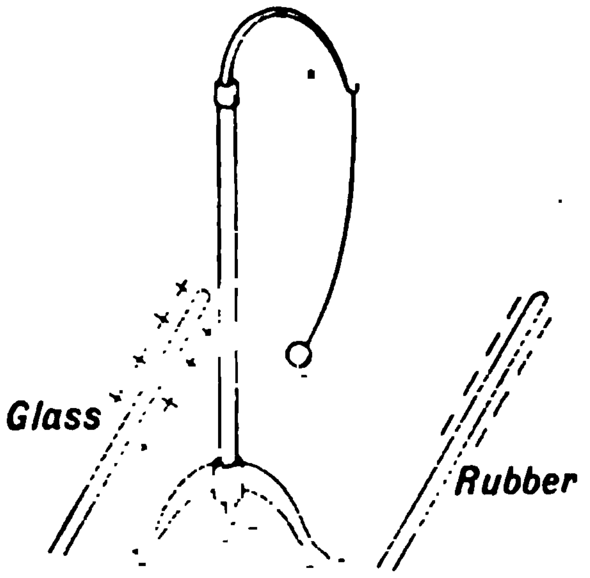


FIG. 282.

This discovery was made by Du Fay, a French investigator, in 1733. He found that all electrified substances behave either like glass or rubber, and the two kinds of electrification were accordingly called *vitreous* and *resinous*. Franklin named the electrical state of the glass **positive** and that of the rubber **negative**, and these names have been universally adopted.

521. Similarly Electrified Bodies Repel.—Two strips of hard rubber electrified by fur and suspended near together repel each other. Two strips of paper if drawn through a fold of flannel in dry cold weather, or even when drawn between the fingers, will repel each other and stand apart. On the other hand, a strip of rubber negatively electrified by fur is attracted by a rod of positively electrified glass.

Similarly electrified bodies repel, while oppositely electrified bodies attract each other.

The pith ball attracted by the rod of glass is repelled after contact because it receives from the glass a charge of *positive electricity* by conduction.

522. Gold-leaf Electroscope.—An instrument, such as the suspended pith ball, used to detect electrification is called *an electroscope*. A much more sensitive electroscope is that shown in figure 283.

Two strips of thin gold leaf about ½ in. wide and 3 in. long are attached to the end of an insulated brass rod so that they hang side by side in a glass jar which screens them from air currents. The brass rod passes through the insulating stopper of the jar, and terminates above in a plate or knob.

Two strips of tinfoil *a a* on the inside of the jar are in metallic connection with a metal base or tinfoil coating over the outside of the bottom.

If a charge is given to the upper plate of the electroscope it at once distributes itself by conduction over the rod and gold leaves, causing the latter to repel each other and diverge as shown in the figure. If the charge should be too great the leaves will diverge enough to touch the side strips *a* through which the whole charge will escape.

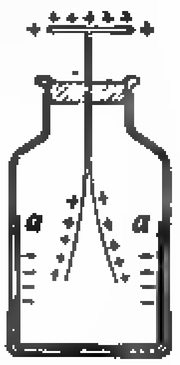


Fig. 283.

523. Electric Series.—**In every case of electrification by friction the substances rubbed together become oppositely electrified,** as though something was taken from the one and added to the other.

When glass is rubbed with silk the glass becomes positively charged and the silk negatively, but when hard rubber is rubbed with silk the rubber becomes negative and the silk positive. The silk thus becomes negative in one case and positive in the other. And in general, **any substance may become either positive or negative, depending on what it is rubbed with.** It is possible to arrange substances in a series such as the following in which any substance is more positive than those below it in the list, but is negative to those that precede it.

Glass (surface rubbed clean and polished).
Fur.
Flannel.
Glass (passed through a Bunsen flame).
Silk.
Wood.
Sealing wax.
Hard rubber.
Sulphur.

Law of Force and Distribution of Charge

524. Coulomb's Law.—The French physicist Coulomb (1784) investigated the law of force between two electrified bodies using a torsion wire balance, illustrated in figure 284. By means of a very fine wire a light horizontal bar of shellac, glass, or other insulating material is suspended inside of a glass jar by which it is screened from air currents. On the end of the suspended bar is a light pith ball *n* which is covered with gold foil. A metal ball *m* mounted on the end of an insulating glass rod can be introduced into the glass jar through a hole in the cover.

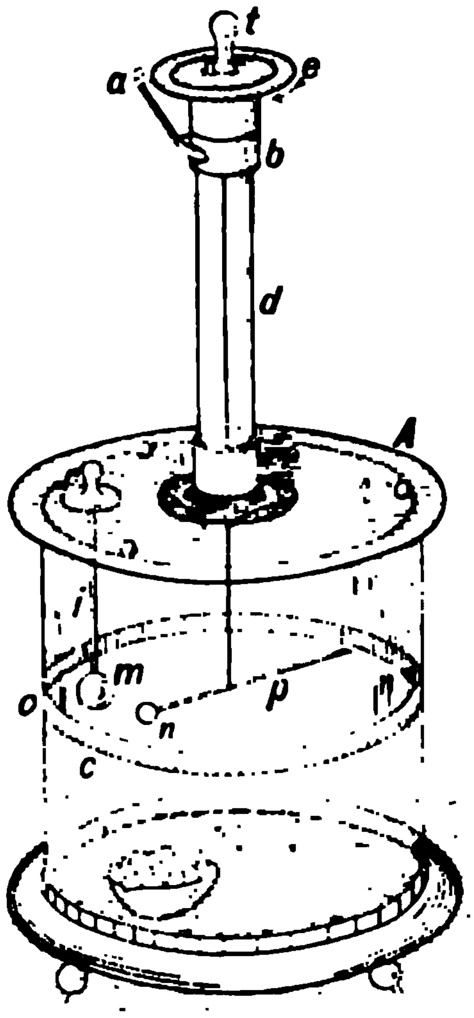


Fig. 284.—Coulomb's balance.

To use the instrument remove the ball *m* and by means of the graduated head from which the wire is suspended turn the wire until the ball *n* hangs exactly in the place of *m*. Now give a charge to *m* and introduce it into the jar. At first *n* is attracted and touches *m*, the charge then divides between the two since both balls are conductors, and immediately *n* is repelled to such a distance that the twist in the wire balances the force of repulsion. The distance between the balls is observed and also the number of degrees through which the wire is twisted.

Now increase the twist in the wire by means of the graduated head, thus forcing *n* toward *m*. It will be found that when the two are at one-half the first distance the force of repulsion as measured by the twist in the wire is four times as great.

To study the effect of changing the quantity of charge, a second insulated brass ball is taken of the same size as *m* and mounted on a glass rod in the same way. The ball *m* with its charge is now withdrawn from the jar and touched to the other similar ball which has no charge. Immediately the charge divides equally between the two (since they are alike) and *m* now has only half the charge which it had at first. If it is carefully introduced without permitting it to touch *n*, the charge on the latter will not be changed, and if the force is observed when the balls are the same distance apart as in the first experiment it will be found that the force is only one-half as great.

From many such experiments Coulomb concluded that the force between two given charged bodies, provided they are small

compared with the distance between them, is inversely proportional to the square of the distance and directly proportional to the amounts of their charges.

This law may be expressed algebraically thus

$$F = \frac{qq'}{Kr^2}$$

where F represents the force; K is a constant, and r is the distance between the centers of the two bodies whose charges are represented by q and q'.

The constant K depends on the units that may be used, and also, as was shown by Faraday, on the medium between the two charged bodies.

525. Unit Charge.—**Unit charge or unit quantity of electricity,** ***in the electrostatic system of units,*** **is defined as that quantity which when placed one centimeter from an equal charge in vacuum repels it with a force of one dyne.**

The force in dynes between two electric charges in vacuum may therefore be expressed by the formula

$$F = \frac{qq'}{r^2}$$

where the quantities q and q' are measured in electrostatic units and where r is the distance between the charges, measured in centimeters.

When the charges are in any other medium the force is usually less and the formula is

$$F = \frac{qq'}{Kr^2}$$

where K is a constant usually greater than one, known as the *specific inductive capacity* or *dielectric constant* of the medium.

The force between two charges in air is appreciably the same as in vacuum, for the specific inductive capacity of air is greater than that of vacuum by only about one part in 2000.

526. Distribution.—The distribution of an electric charge may be examined by means of a little metal disc mounted on an insulating handle and known as a *proof plane.* If the disc is placed flat against the surface of the charged and insulated pail shown in figure 285 and then removed, it will carry away

a charge which may be tested by the gold-leaf electroscope. When examined in this way it is found that the greatest charge is obtained from the outer surface of the pail near its upper edge and on the outer corner at the bottom, less is found on the middle of the side and none at all in the interior except near the upper edge.

When there is a metal cover on the pail, absolutely no charge can be found on any part of the interior.

Other irregular bodies may be examined in the same way and it will be found that the greatest density of charge is at corners and knobs projecting outward. For example, in a conductor shaped as in figure 286 the greatest density will be found

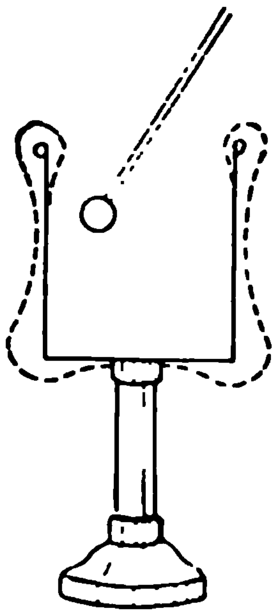


FIG. 285.—Distribution on pail.

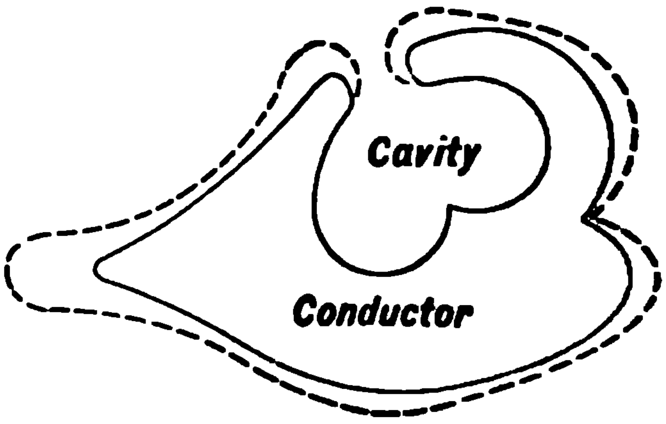


FIG. 286.—Density of distribution indicated roughly by dotted line.

on the projecting point on the left, no charge will be obtained in the cavity even though there is a sharp point there, and very little will be found toward the bottom of the dimple on the right end.

527. Charge Entirely on the Surface of a Conductor.—The following experiment was carried out independently by Cavendish and Biot.

A metal ball having two closely fitting hemispherical metal cups which were provided with insulating handles, was insulated and then charged strongly with electricity. **When the cups were simultaneously removed they were found to have the entire charge, the ball being left without any trace of electrification, showing that the whole charge was on the surface.**

528. Discharge from Points.—The density of charge on sharp *projecting* points of conductors may be so great that the charge

will escape to the air. This discharge is accompanied by a stream of air which, if the point is connected with an electrical machine, may be strong enough to blow out a candle or turn a

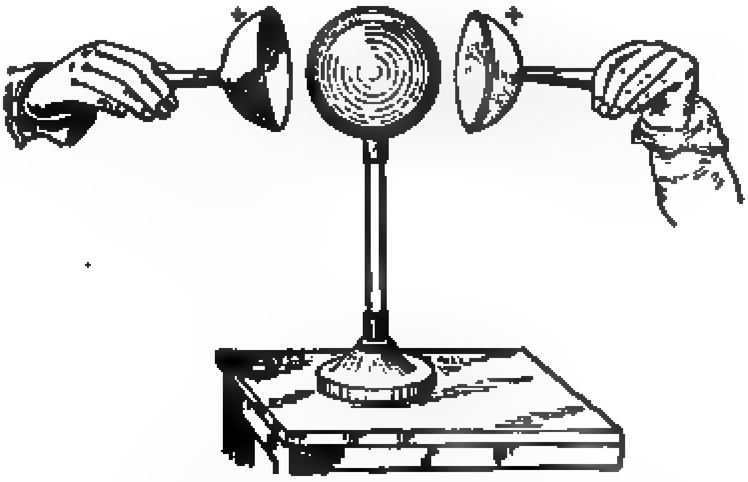


Fig. 287.

little wheel with light vanes, or if the point from which the discharge takes place is movable it will be driven backward as illustrated in figure 288. Conductors which are designed to hold electrical charges should therefore have all projecting parts or corners carefully rounded, otherwise they will be rapidly discharged.

Fig. 288.—Electric wind.

529. Frictional Electric Machine.—The early forms of electrical machines were frictional; the one illustrated in figure 289 is a good type of this class. A circular glass plate is mounted firmly on an axle so that it can be turned between leather covered rubbers, which are pressed against the glass by springs. The charge from the glass is received by a metal conductor which is on an insulating support of glass or of hard rubber. From this conductor there are two branches which reach out, one on each side of the glass plate, and on the inside of each is a row of sharp metal points projecting toward the glass plate, like the teeth of a comb. The electric charge excited on the glass by the rubbers is carried under the combs by the turning of the plate, and through them it readily passes from the glass into the conductor.

At the same time that the plate is positively electrified the rubbers become negatively charged and should be connected

by a chain or wire with the ground to permit this negative charge to escape. It is usual also to have the lower half of the

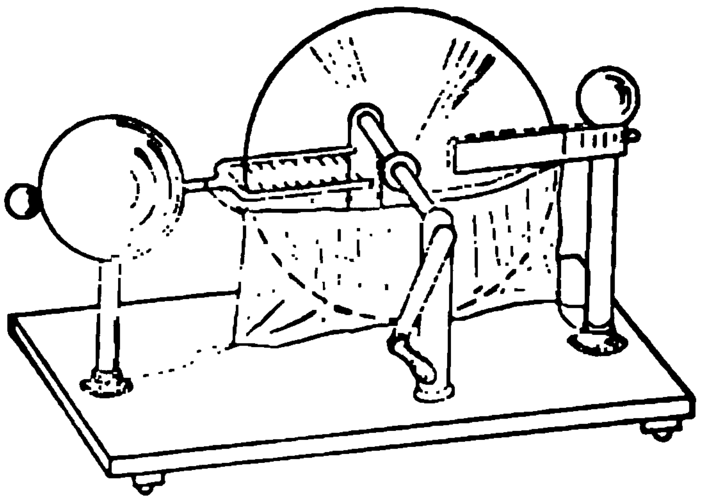


FIG. 289.—Electrical machine.

glass plate covered with a silk bag which prevents the escape of electricity from the glass as it turns.

Problems

1. If the charges on two small conducting pith balls are +8 and −8, will they attract or repel and with what force when 4 cm. apart? What if they are allowed to touch?
2. Two small conducting balls of the same size and 6 cm. apart have charges +36 and −4, respectively. What is the force between them? Also what will the force become if they are touched together and then placed as before?
3. Two pith balls 3 cm. apart and equally charged repel each other with a force of 16 dynes; find the charge on each.
4. Two pith balls hung from the same point and weighing 1/10 grm. each are equally charged and repel so that they diverge until the threads are at right angles to each other. What is the force of repulsion in grams and in dynes? If they are 8 cm. apart, what is the charge on each?
5. Two pith balls, each weighing 1/10 grm. and suspended from the same point by threads 30 cm. long, are equally charged and repelling each other, hang 8 cm. apart. What is the charge on each ball?

INDUCTION

530. Induction.—When a conductor having no charge is insulated and then brought near a positively charged body, such as A, figure 290, it is found to be negatively electrified on the side next to A and positively electrified on the farther side. This can readily be tested by means of the proof plane and electroscope.

In this case no charge has passed from A to B. The conductor B is insulated and no flow of electricity either into it or out of it is possible. When B is taken away from the charged body A it is found to have no charge, just as before it was brought up. Also no charge is lost or gained by the body A in the operation. The charges produced in the conductor B by the proximity of A are said to be **induced.**

531. The Induced Charges are Equal.—If, instead of the one conductor B of the previous experiment, we take *two* insulated conductors B and C, having no charge, and bring them near the charged body A while in contact with each other, as shown in figure 291, a negative charge will be induced in the nearer one and a positive charge in the farther one. If they are now separated and then removed the positive charge is trapped in

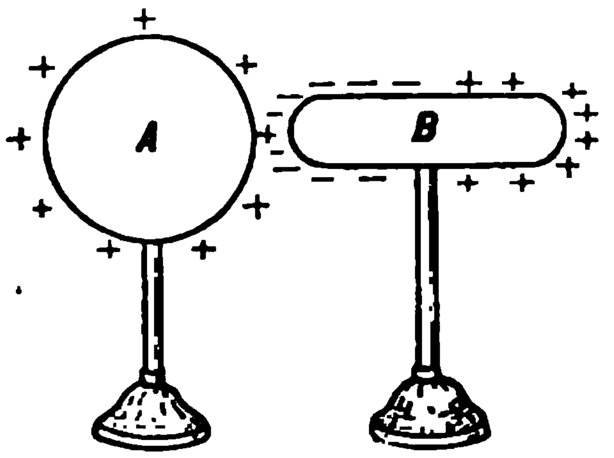


FIG. 290.

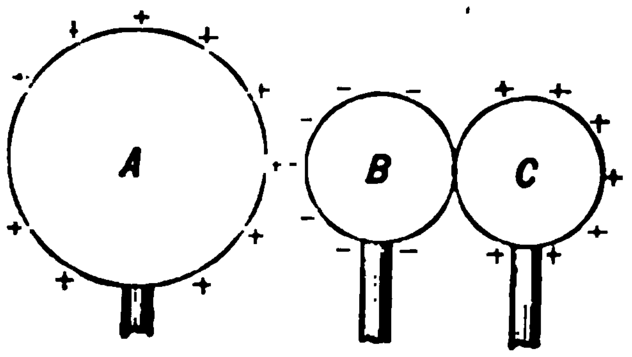


FIG. 291.

one and the negative in the other. On bringing them near an electroscope they are found oppositely electrified; and if while away from the vicinity of A they are touched together, the charges totally disappear, showing that the positive charge of the one was just sufficient to neutralize the negative charge of the other. **The charges are therefore equal and opposite.**

The same result is reached whatever the shape or size of the conductors B and C. Thus B may be large and C small, or *vice versa*, and still the charges on each when removed from the influence of A will be found to be equal and opposite.

For if we consider the single conductor B used in the first experiment (Fig. 290) it is clear that if it is imagined cut in two at any point, near the positively charged end for example, the positive charge on the smaller part will be equal to the excess of negative over positive on the greater part.

Thus **the positive and negative charges produced by induction are always equal.**

532. Induction when the Conductor is Already Charged.—When the conductor has a charge to begin with the inductive action takes place in the same way, but the original charge is combined with the induced charge. Thus if *B* (Fig. 292) is given a positive charge and brought toward the positively charged body *A*, it will be found that the positive charge is denser on the side away from *A*.

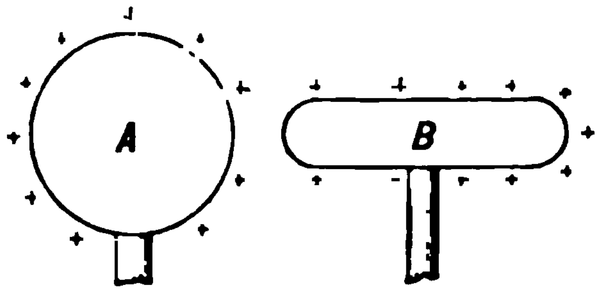


FIG. 292.

At a certain distance there will be no charge at all on the end toward *A*. If brought still nearer a negative charge will be found on that end, while the positive charge on the rest of the conductor will be correspondingly increased.

If *A* and *B* are both conductors and similarly charged, say positively, they will react on each other, and the greatest density will be found on the outer side of each.

If, however, they are oppositely charged the greatest densities are on the adjacent sides.

533. Effect of Connecting with the Earth.—If, while in the presence of the positively charged body *A*, the conductor *B* is connected with the earth by a wire, or by touching it with the finger, the positive charge will escape to the earth, and at the same time the negative charge will increase somewhat, so that the conductor will be left with a negative charge greater than when it had both positive and negative charges together.

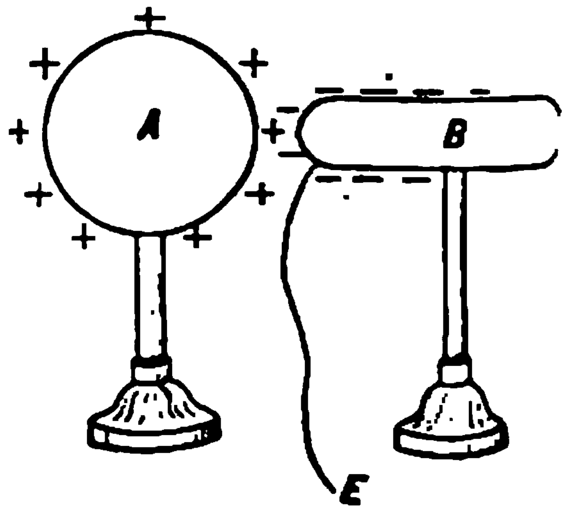
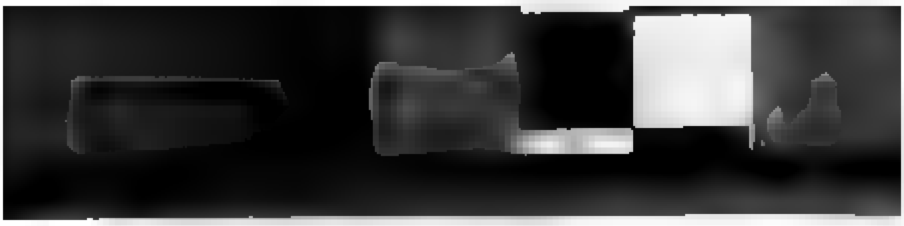


FIG. 293.

It makes no difference what part of *B* may be connected to the earth, whether the nearer end, as shown, or the farther end, the result is exactly the same.

The induced charge that remains is sometimes called a *bound charge* because it does not flow off or disappear when *B* is touched, but is held by the presence of the charged body *A*.

534. Charging Electroscope by Induction.—When a rod of



rubber, negatively electrified by rubbing with fur, is brought ·d a gold-leaf electroscope, the leaves will be observed to ge strongly while the rod may be several inches from the ıment.

sitive electrification is induced in the top of the electro- and an equal negative charge is given to the leaves pre- · as in the case discussed in §530.

the top of the instrument is touched for an instant by the · while the electrified rod is still held near, the negative ·ification of the leaves will escape and they will hang straight as in *b* (Fig. 294). If the finger is now removed, and then ɔd, the positive charge will distribute itself over the top and s of the electroscope and they will diverge as shown in *c*.

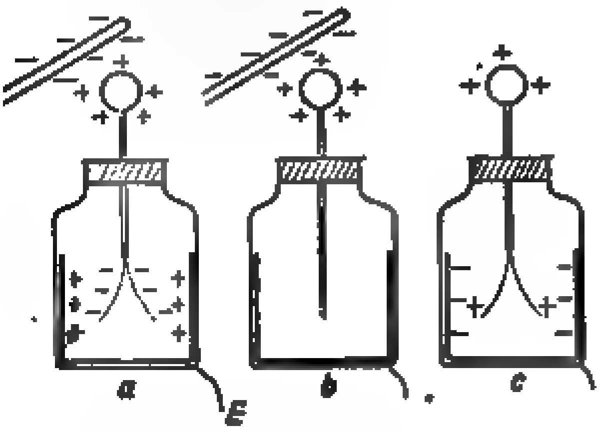


FIG. 294.

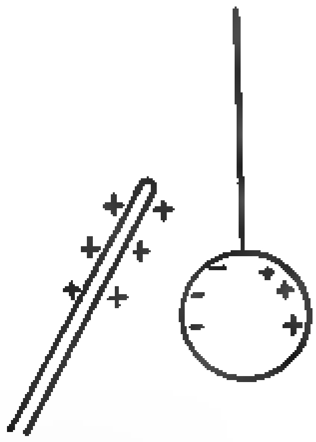


FIG. 295.

hen the electroscope is positively charged the approach of a ively charged rod increases the divergence of the leaves, : a negatively charged body will draw them together unless brought too near, in which case they will again diverge with ȝative induced charge.

will be noticed that there are shown in figure 294 induced ȝes on the metal side strips inside the electroscope. These ȝes are opposite to the charge on the leaves and increase divergence.

5. Attraction of Pith Balls Explained by Induction.— n an electrified rod is brought near a pith ball (Fig. 295) ıductive action takes place as shown, and the attraction een the positively electrified rod and the negatively elec- d side of the ball is greater than the repulsion of the posi- y electrified side, since the negative side of the ball is nearer e rod.

If the ball had an initial positive charge it would be repelled, though even in this case if the rod is much more strongly electrified than the ball and is brought very near to it there may be attraction.

536. Induction Takes Place through Non-conductors.—The interposition of a sheet of glass or hard rubber or a cake of beeswax or any other insulator between an electrified body and an electroscope does not interfere with the inductive action. Indeed induction takes place more readily through these substances than through air, though but slight evidence of change would be observed in such a rough experiment.

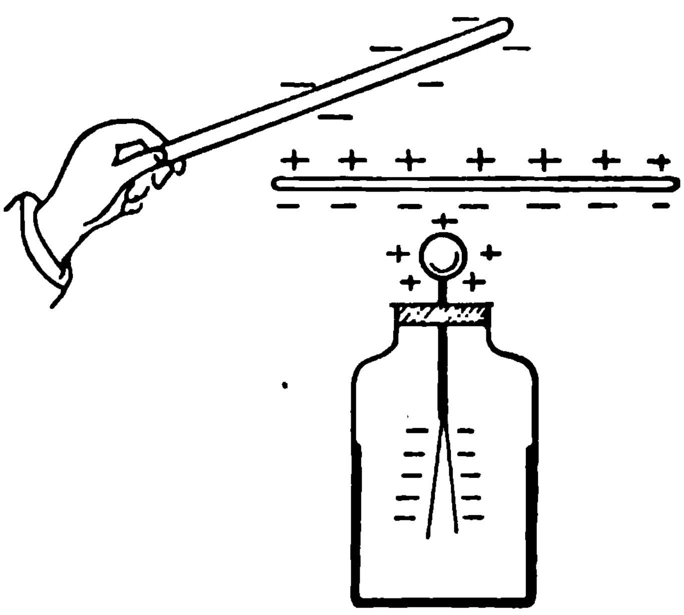


Fig. 296.

537. Induction through Conductors.—If a charged rod (Fig. 296) is held over an electroscope and a small *insulated* sheet of metal is interposed between the two, the gold leaves will diverge as though the plate were not there, for if the rod is negative a positive charge will be induced on the upper side of the plate and an equal negative charge on the lower side which in turn will act on the electroscope.

If, however, the plate is connected with the earth the charge on its lower surface will escape and the electroscope will be screened almost entirely from the effect of the rod.

538. Conductor Surrounding an Electroscope.—When an electroscope is entirely surrounded by a conducting sheet it is absolutely protected from all *outside* inductive action. It has already been shown (§526) that there is no electrification on the interior of a closed conductor, so also there is no induction from the outside. If a delicate electroscope is enclosed in a cage of wire gauze which is underneath as well as around and above it, the cage may be strongly electrified by a machine and there will be no disturbance of the electroscope, except such as may be due to electrified air passing into the interior. Faraday constructed a small room about 6 ft. each way and covered with tinfoil and found that within it he was unable to detect any disturbance of his most delicate electroscope though an assistant

electrifying the outside by a machine so that long sparks
ıped from it.

89. Electrophorus.—A simple form of induction apparatus
ised by Volta is known
the electrophorus. It con-
s of a cake or plate of
ı-conducting material,
h as resin, sulphur, or
d-rubber, supported on a
tal base and having a
tal cover provided with an
ılating handle of glass or
d rubber. The upper sur-
; of the resinous plate is
atively electrified by rub-
g it with fur and the
er is then placed upon it.
ositive charge is induced
the lower side of the
er and a negative charge
its upper side, and on
ching it with the finger or
necting it by a metal
in or wire with the base plate the negative charge escapes,
ing the positive charge held by the negatively charged
n. If the cover is now lifted from the resin, on presenting

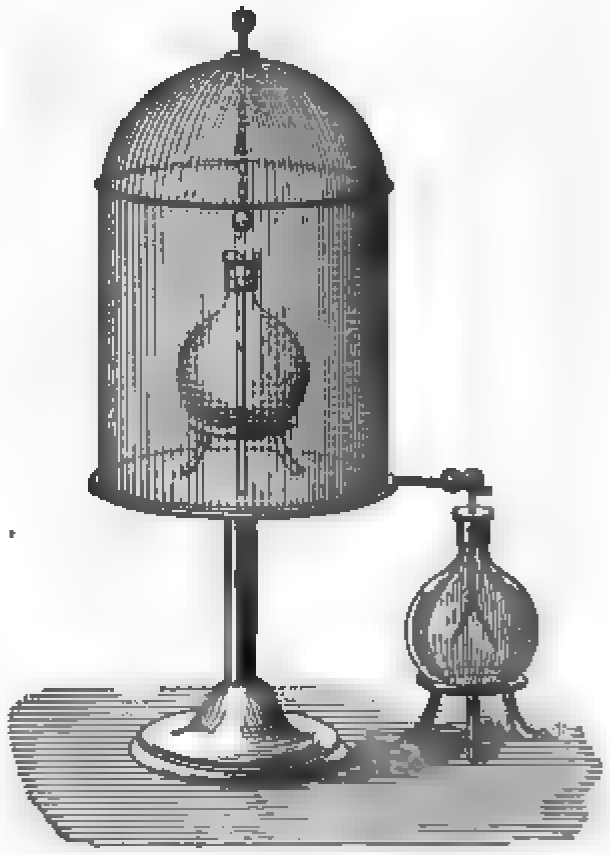


Fig. 297.—No electric force within enclosing conductor.

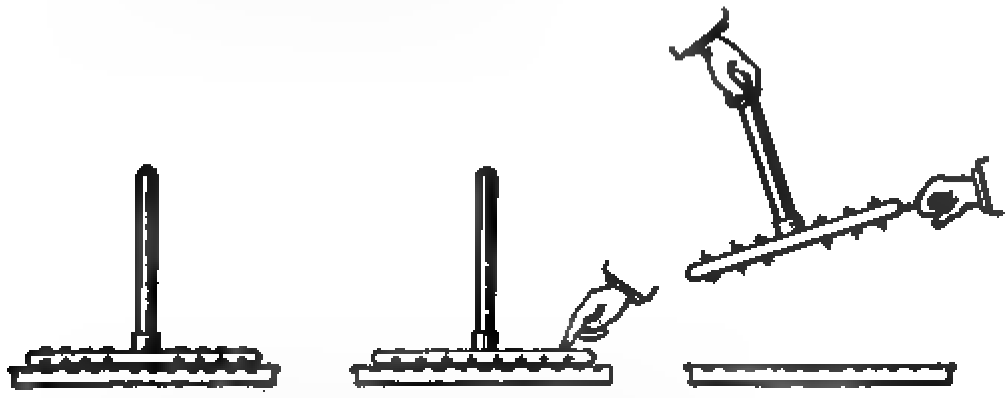


Fig. 298.—Electrophorus.

knuckle the positive charge escapes in a bright spark. The
er may then be again placed on the resin, touched, and with-
wn and a second *positive charge* obtained, and so on indefinitely.

In this way a great number of charges may be obtained without renewing the electrification of the resinous sole plate.

The resin is a good insulator and the cover touches it at so few points that there is very little direct loss by conduction.

540. Source of the Energy of the Charges.—Each of the charges obtained in this way has energy, as shown by the noise and light given out by the spark. This energy does *not* come from the energy spent in electrifying the plate of rubber or resin for a spark is obtained every time the insulated cover is touched and withdrawn without any appreciable loss of electrification by the resin.

The energy must be supplied in the operation of withdrawing the plate. This will be made evident by the following experiment.

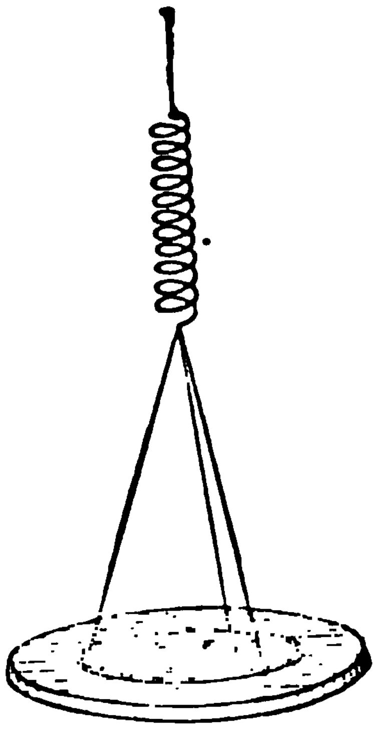


Fig. 299.

Suspend the cover by silk cords from a spring, and after having discharged it let it be lowered upon the resin and then withdrawn *without being touched.* The spring is scarcely stretched more when the plate is withdrawn than when it was lowered. But if when the cover is on the resin *it is touched*, the negative charge escapes and the attraction between the positive charge in the cover and the negatively charged resin causes the spring to be greatly stretched when the plate is raised, showing that more work has to be done to raise the plate after it has been touched than before.

It is this work done by the person lifting the plate that is the source of the energy of the charge that is obtained.

If the cover is raised only an inch from the resin the spark will be much less energetic than if it had been raised 10 in., for less work has been done.

541. Faraday's Ice-pail Experiment.—A very important case of induction is where the charged body is surrounded by a conductor. This was first investigated by Faraday as follows:

Taking an insulated metal pail having a metal cover and connected with an electroscope, it was observed that when a charged metal ball was brought up toward the pail the divergence of the leaves of the electroscope increased until the ball was entirely

rithin the pail, after which no change was observed whatever the oosition of the ball might be, whether it was close to the bottom r to one side or in the middle.

The ball was now permitted to touch the inside of the pail, ut not the slightest change in the gold leaves was observed. Vhen the ball was withdrawn it was found completely discharged vhile the leaves remained diverging.

The same observations may be made using a deep open pail, s in figure 300, provided the ball is not too near the open top.

When the positively charged ball is introduced into the pail here is induced a negative charge on he inside of the pail and a positive harge on the outside, as may be hown by a proof plane.

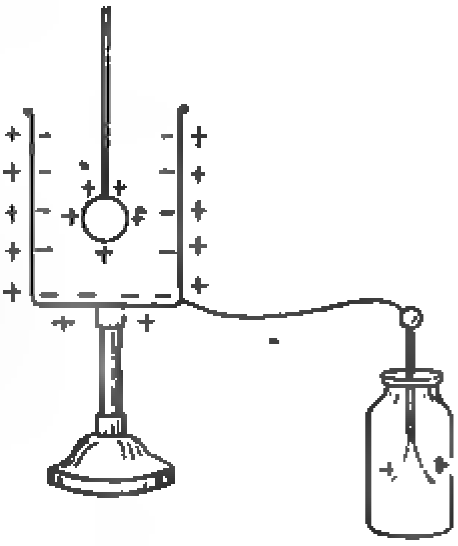


Fig. 300.

When the ball touches the interior of the pail the charges on the ball and on the interior of the pail disappear, for the ball and pail then become one conductor and there is no charge on a cavity in a conductor.

If these charges were not exactly equal there would be some excess of either positive or negative charge which would pass to the outside and cause a change in the electroscope.

The experiment then leads to the following conclusion: **When a charged body is surrounded by a conductor a charge is induced on the inside of the conductor equal and opposite to that on the body.**

The walls, ceiling, and floors of ordinary rooms are fairly good conductors so that when we have a positively charged body in a room we may be sure that an exactly equal negative charge is distributed over the walls and neighboring objects.

542. Positive and Negative Electrifications Always Equal.— Hold a rod of sealing wax in an insulated pail connected with an electroscope and rub it with a pad of flannel which is insulated on another rod of sealing wax.

They may be rubbed quite vigorously but no sign of electrification *is shown by* the electroscope, but if either one by

itself is drawn out of the pail there is decided divergence of the gold leaves. It follows that the electrifications developed on the sealing wax and flannel, respectively, are equal and opposite.

In every case of electrification, whether by induction or friction, equal positive and negative charges are produced.

543. Electric Charges Multiples of a Certain Unit.—Very important results have recently been obtained by the exact measurement of extremely small electric charges. A method which has proved most fruitful was developed in 1910 by R. A. Millikan, by which the minute electric charges on microscopic drops of oil spray from an atomizer were accurately measured.

A drop was isolated and observed through a low power microscope as it slowly settled down through air in the space between two horizontal metal plates which were connected together so that there was no electric force in the region between them. When the drops had nearly reached the lower plate the two plates were electrified, one positive and the other negative, in such a way that the electric force on the charged drop carried it upward. As it neared the top the plates were once more connected and discharged permitting the drop to settle again—and so on indefinitely.

It was possible in this way to observe a single drop for hours at a time, and to measure accurately the velocity with which it settled downward and also the velocity with which it was urged upward. From the former of these two measurements the size of the drop could be determined, and then from its upward velocity in the electric field its charge could be calculated. **In several thousand such experiments the charges upon the drops, whether positive or negative, were always found to be exact multiples of a small charge *e*, which had the value 4.77×10^{-10} in electrostatic units.**

It is believed that all electric charges, whether large or small are a whole number of times this elementary charge, so that it is impossible to increase or diminish an electric charge by a fractional part of e.

544. Induction Machines.—Various forms of electrical machines have been devised in which charges developed by induction from small initial charges are continually added to the original charges until powerful effects are obtained. The first powerful

and successful machine of this kind was made by Holtz about 1864.

A modification of this machine, due to the labors of Voss and Toepler and known as the Voss-Holtz or Toepler-Holtz machine, is shown in figure 301 with a diagram illustrating its action.

A circular plate of glass carrying on its front surface six small discs of tin foil, marked a_1, a_2, . . . a_6, is rotated rapidly in front of a fixed plate of glass, on the back of which are attached

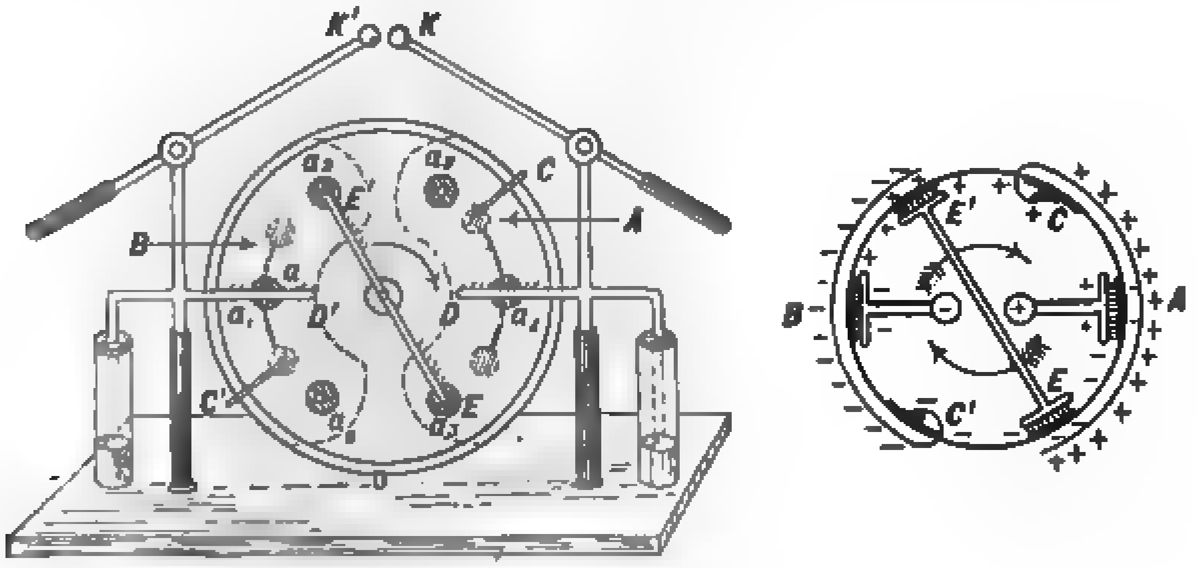


FIG. 301.—Toepler-Holtz machine and diagram.

two conductors of paper *A* and *B*, called armatures (outlined by the dotted lines). In front of the rotating plate are mounted on insulating supports the two conducting combs *DD'* with sharp points close to the plate and directed toward it; these conductors are connected to the knobs *K* and *K'*. A conducting bar, called the equalizing bar, *EE'*, crosses diagonally in front of the rotating plate and is also provided with combs directed toward the plate.

At *C* and *C'* there are metallic arms which are connected with the armatures on the back of the fixed plate and carry little tinsel brushes that touch the tinfoil discs on the revolving plate as they pass.

Suppose, now, that *A* is slightly more positive than *B*, owing to the remains of a previous charge or to the influence of some neighboring charged body or to the brushes at *C* and *C'* rubbing a little differently on the discs as the plate is turned. And suppose that the discs a_2 and a_5 are under the combs of the equalizing bar *EE'* and are connected with it by the tinsel

brushes carried by that bar. The bar with the two discs thus forms one continuous conductor with the two inductors A and B opposite its ends. A negative charge will therefore be induced in the end toward A and an equal positive charge in the end toward B. If the plate is now turned, a_5 carries its negative charge past the position a_6 until it is between the brush C' and the armature B with which that brush is connected, and is therefore situated almost as if inside of a conductor; it accordingly gives up almost its entire charge through the brush C' to the armature B which thus becomes more negative. At the same time the disc a_2 has moved past the position a_1 and given up its positive charge to the armature A through the brush C.

The armatures with their increased charges act more powerfully on the next pair of discs that pass under the bar EE', and so these discs carry still larger charges to the armatures and thus the effect rapidly increases till the armatures are so highly charged that they lose by leakage as rapidly as they gain.

When the armature A is positively charged it acts inductively on the comb D through the two layers of glass, attracting a negative charge on the points of the comb and repelling positive to the knob K, but the negative charge induced on D is discharged upon the surface of the revolving glass plate from the sharp points of the comb, and is carried away by the motion of the plate, leaving the conductor and knob K strongly positively charged. At the same time the positive charge induced on D' is discharged on the revolving plate, leaving K' negatively charged, and if the gap between K and K' is not too great, spark discharges will take place between them. Two small Leyden jars (§567) are connected with the conductors D and D' and act as reservoirs in which the charge accumulates between discharges.

545. Wimshurst Machine.—In the Wimshurst machine two circular plates of glass are revolved in opposite directions, one in front of the other.

On the outer surface of each are a number of radial strips or sectors of tinfoil, on each of which is a little metal knob or button. As the plates rotate, the buttons strike the tinsel brushes of a pair of equalizing bars, one of which is fixed in front

of each plate, one inclined to the right and one to the left, so that the two are nearly at right angles to each other.

In the diagram the inner and outer circles represent the two plates while the heavy lines indicate the positions of the conducting sectors.

Suppose that the plates turn in the directions of the arrows and that the sector *a* is slightly positive and *b* negative.

Then *c* and *d* which are connected by the equalizing bar will become oppositely charged by induction, *c* negatively and *d* positively. The rotation of the plates carries *c* with its negative

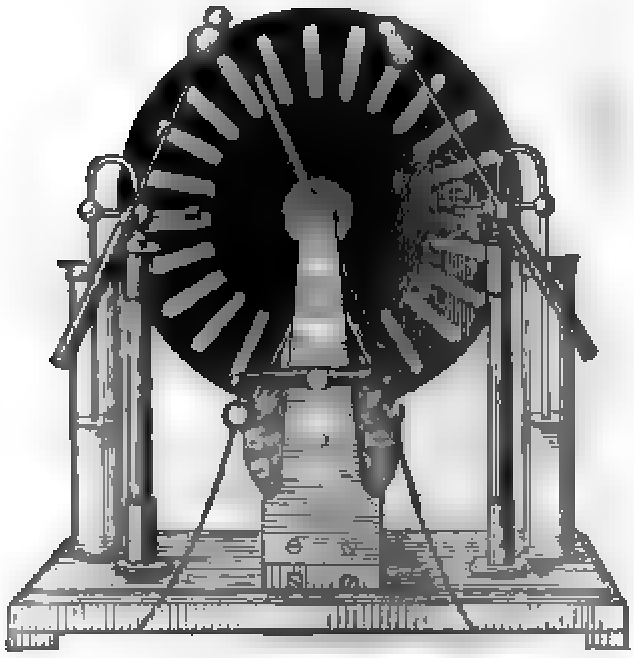


FIG. 302.—Wimshurst machine.

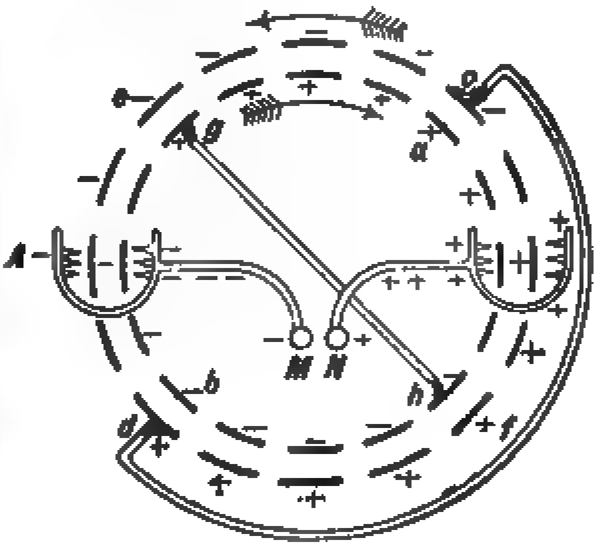


FIG. 303.—Diagram.

charge to *e*, and *d* with its positive charge to *f*, where they are opposite the ends of the second equalizing bar and by induction attract positive charge into *g* and negative into *h*.

At *A* and *B* are combs between which the plates turn, the rotation of the plates carries the positively charged sectors toward *B* and the negatively charged ones toward *A*, making the knobs *N* and *M* positive and negative respectively.

546. Summary.—The following is a summary of the main preceding facts relating to electric charges.

1. There are two kinds of charges. Bodies with like charges repel and with unlike charges attract each other.

2. The force between two small charged bodies is directly proportional to their charges and inversely proportional to the square of the *distance between* them.

3. The force between charged bodies also depends on the medium between them.

4. It is impossible to produce one kind of charge without at the same time producing an equal charge of the other kind.

5. Whenever a positive charge disappears an equal negative charge also disappears.

6. The *total charge* in a body or the sum of the positive and negative charges which it may exhibit does not change so long as the body is truly insulated.

7. The distribution of charge in a conductor is influenced by neighboring charged bodies. (Induction.)

8. Charges are always multiples of an elementary unit *e* taken a whole number of times.

Potential and Electrometers

547. Potential or Electrical Pressure.—Suppose an electroscope connected with a charged pail by a wire. It makes no difference in the indication of the electroscope whether it is connected to the inside of the pail where no charge can be obtained by the proof plane or to an edge where the density is greatest. There is perfect equilibrium between the electroscope and the charged pail and no tendency of the charge to flow from one to the other. When two conductors are in this relation they are said *to be at the same potential* or *to have the same electrical pressure.*

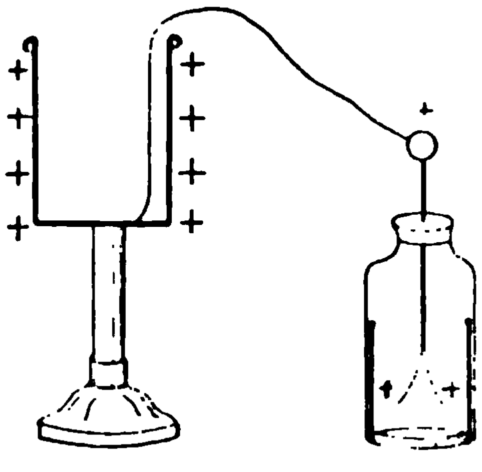


Fig. 304.

The potential of a conductor is that electrical condition which determines the flow of electricity.

Potential determines the flow of electricity just as pressure determines the flow of fluids and temperature the flow of heat.

When any conductor is connected with the earth flow takes place until the conductor comes to the potential of the earth.

When two charged conductors are connected by a wire, the one that loses positive charge is said to have been at a higher potential than the other.

548. Effect of Increased Capacity.—If an insulated conductor having no electric charge is brought up and touched to the

charged pail shown in figure 304, part of the charge will flow into the conductor and the whole system comes to a new state of equilibrium in which the potential is less than before. This is shown by the fact that the gold leaves of the electroscope do not diverge so strongly.

In this case **there has been a decrease in potential though there has been no change in the total amount of the charge.** The enlarged system of conductors is said to have a greater electrical *capacity* than the original pail and electroscope.

Change in potential due to a change in capacity of the charged conductor is well shown by Faraday's apparatus, figure 305, in which a roller suspended by insulating silk cords carries a conducting ribbon of tin-foil. When rolled up and charged the pith balls diverge widely, but as the ribbon is unrolled, thus increasing the surface and capacity of the conductor, the pith balls approach each other. That this is not due to any loss of charge is shown by the fact that when the ribbon is again rolled up the pith balls diverge as at first.

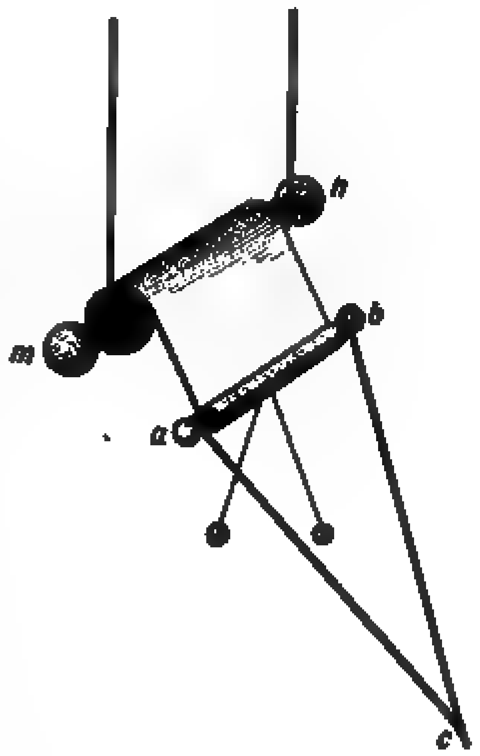


Fig. 305.

549. Potential of a Conductor.—In non-conductors the potential may be different at different points, but in a conductor or in connected conductors, when the electrical charge is at rest, all parts are at the same potential, since otherwise flow would take place from one part to another.

Even when a conductor is hollow the potential in its interior, due to charges at rest, is everywhere the same as at its surface, provided it does not contain any insulated charged bodies; for Faraday showed (§538) that there was no electric force inside of a hollow conductor in such a case and consequently it must be a region of uniform potential.

Thus **the surface** of a conductor is an *equipotential surface*, since all *points of it* are at the same potential, and the interior

of a conductor, provided it does not contain insulated charged bodies, is an *equipotential region.*

550. Zero Potential.—Bodies are usually discharged by connecting them with the earth, and its potential is accordingly taken as the *zero.* Bodies which give up a positive charge when connected to the earth are said to have a positive potential, while bodies which receive a positive charge from the earth have a negative potential.

The walls and floors of wood and plaster which enclose ordinary rooms are conductors, though they conduct rather slowly, the interiors of rooms are therefore to be regarded as cavities in conductors which are at the earth potential. *When there are no insulated charged bodies inside of such a room it is an equipotential region, all at zero potential, even though there may be electrified clouds floating overhead.*

551. Potential Without Charge.—It was shown by Faraday that when an insulated conductor is touched to the inside of a hollow conductor which completely surrounds it, it receives absolutely no charge. It follows that in such a case there is no flow of electricity from one to the other and therefore *both must have been at the same potential before they touched.* We see, then, that merely putting a conductor without charge inside of a hollow conductor brings it to the potential of that conductor; that is, **a conductor which has no charge takes the potential of the region where it is placed.**

552. Potential Affected by Neighboring Charges.—The case just discussed is a special instance of the general principle that **the potential of a conductor depends not only on its own charge, but on that of all neighboring objects.**

This is well shown in case of the electrophorus (§539), for when the metal cover of that instrument is removed and discharged by touching it, it comes to the earth potential. But when it is placed on the negatively charged base its potential is lowered, as is shown by the fact that if it is now connected with the earth *positive electricity flows from the earth to the cover.* It thus comes to the earth potential and has a positive charge.

If it is now removed from the negatively charged base plate its potential is raised, for on touching it *positive electricity flows from it to the earth.*

It thus appears that **the potential of a conductor depends not only upon its own charge, but also upon all other charges near enough to affect it.**

If a conductor were removed from all other charged bodies its potential would depend only on its own charge and would be proportional to that charge. But if a positively charged body is brought near the conductor its potential is raised though its charge is not changed.

A positive charge not only raises the potential of the body to which it is given, but it raises the potential of the whole neighboring region and of any bodies that may be near. So also a negative charge lowers the potential of all points in its vicinity.

553. What Determines the Potential of a Conductor.—From what precedes it will be seen that the potential of a conductor depends upon the following three conditions:

1. *Its capacity,—determined by its size and shape.*
2. *Its charge.*
3. *The charges on surrounding bodies.*

554. Measure of Difference of Potential.—When a small electric charge is moved along an equipotential surface or from one part of an equipotential region to another, *no work is done, for there can be no electric force acting on the charge since there is no tendency to flow.*

If two conductors are at the same potential no work is done in transferring a small charge from one to the other. But if they are not at the same potential work must be done to carry a small positive charge from the one at the lower potential to the one at the higher, just as in case of two vessels, each containing a fluid under pressure, work must be done by a pump to force any fluid from the vessel in which the pressure is less into the one in which it is greater.

It may be proved that if a little charge is transferred from one conductor to another the work done will be the same by whatever path the transfer may be made, and accordingly the work done may be used as a measure of the difference of potential of the two conductors.

Thus **the difference of potential of two conductors is measured by the number of ergs of work required to transfer unit charge from one to the other.** Difference of potential determined in this

way is in electrostatic units and one *electrostatic unit of potential is very nearly equal to* 300 *volts*, the *volt* being the unit of potential in what is called the *practical system* of units.

555. Instruments to Measure Potential.—*Electroscopes and electrometers are potential measuring instruments.* For instance, the deviation of the gold leaves of a gold-leaf electroscope depends on the difference of potential between the leaves and the side conducting strips that are connected with the earth. The leaves have the same potential as any conductor that may be connected with them. But this instrument, in the ordinary

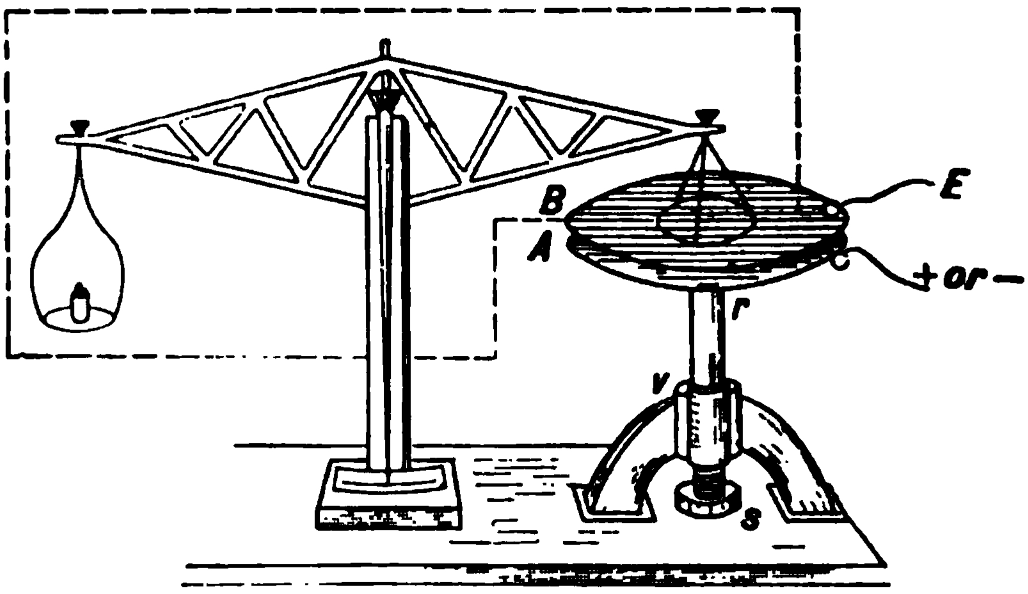


FIG. 306.—Attracted-disc electrometer.

form, is not well adapted for exact measurements, though a modified form in which the deflection of a single narrow strip of gold leaf is measured by a low power microscope, is valuable for some investigations.

556. Attracted-disc Electrometer.—The instrument shown in figure 306 is known as the *attracted-disc electrometer*, or *the Kelvin absolute electrometer*, and may be used to measure large differences of potential.

Two circular flat plates of metal, *A* and *B*, are mounted parallel to each other. By means of the screw *s* the plate *A* may be raised or lowered and the distance between the two plates may be determined by the scale and vernier *v*. The upper plate is made in two parts, a central disc and a surrounding ring. The disc is suspended from one arm of a balance and hangs so that its lower surface is exactly flush with that of the ring, which is separately supported, though the two are in conducting communication.

The plate B is connected with the earth, while A, which is insulated by hard rubber or glass at r, is connected with the charged conductor whose potential is to be determined. There is a charge on A and an opposite induced charge on B and consequently an attraction between them. By means of the balance the force with which the disc is attracted is exactly measured.

The difference of potential V between the plates A and B may then be found in electrostatic units by the formula

$$V = d\sqrt{\frac{8\pi F}{S}}$$

where d is the distance between the plates in centimeters, F is the force of attraction in dynes, and S is the area of the disc in square centimeters.

The instrument is called an *absolute* electrometer, because its determinations depend directly on measurements of length and force and it may be used to standardize other instruments.

The *guard ring*, as it is called, which surrounds the attracted disc was introduced by Lord Kelvin to cause a uniform distribution over the central disc, without which the difference of potential could not be calculated by the above simple formula. For in case of two parallel plates the distribution is denser toward the edges, but is extremely uniform near the center if the plates are not too far apart.

The balance must be enclosed in a metal case, as shown by the dotted lines, to screen it from all outside disturbing electrical attractions.

557. Quadrant Electrometer.—The quadrant electrometer, also designed by Lord Kelvin, is shown in figure 307. A small round brass box is cut into four quadrants which are slightly separated from each other, and mounted on insulating supports as shown in the figure. The needle consists of a thin flat plate of aluminum, broad at the two ends as shown in the plan, and mounted on a light vertical wire of aluminum which passes through its center and carries on its upper end a small mirror by which the motions of the needle are observed.

The flat needle is suspended by a fine quartz fiber or by two parallel fine fibers of silk constituting a bifilar suspension, so that it hangs horizontally in the middle of the box formed by the four *quadrants* and in the position shown.

The diagonally opposite quadrants A A are connected by wire conductors to the pole P', while the quadrants B B are connected to the pole P. The needle is given a positive charge so that if the A quadrants are connected with a positively charged body while the B quadrants are joined to earth, it will turn toward the B quadrants; while if the A quadrants are negative it will turn toward them. The deflection of the needle is read by the motion of a narrow beam of light, reflected from the attached mirror upon a graduated scale, and is nearly proportional to the difference of potential between the A and B quadrants.

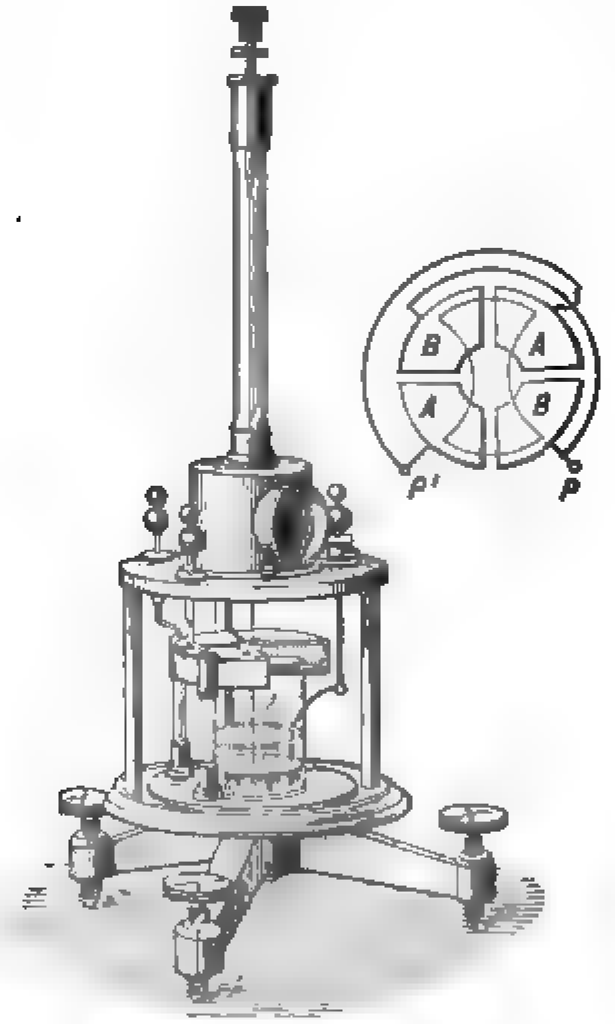


FIG. 307. – Quadrant electrometer.

The sensitiveness of this instrument may be many times greater than that of the gold-leaf electroscope. To secure the greatest sensitiveness a very light paper needle is used, hung by an exceedingly fine quartz fiber.

Another method of using the instrument is to connect the B quadrants to the body to be tested, while the needle and A quadrants are connected together and to the earth. In this case the needle will turn toward the B quadrants whether the charged body is positive or negative, and the deflection is nearly proportional to *the square* of the difference of potential measured.

ELECTRON THEORY

558. Theories of Electricity.—The early investigators thought of electrified bodies as containing something which they called an *imponderable fluid*, because it could flow from one body to another and yet did not seem to possess weight or inertia.

Symmer conceived two such fluids, positive and negative electricities, which neutralized each other when mingled.

Franklin, however, advocated the view that there was but a single electricity and that for every body there was a normal amount when it showed no electrification; if there was an excess it showed one kind of electrification, while if there was a deficiency of the electric fluid the body was electrified in the opposite way.

The strong points of this theory are that it explains how opposite charges neutralize each other and how it is impossible to develop a positive charge anywhere without at the same time causing an equal negative charge to appear somewhere else.

But Franklin's theory assumed that each portion of the electric fluid repelled every other portion directly.

559. Faraday's Theory.—Faraday, however, conceived that electrical forces were communicated by the insulating medium between electrified bodies and showed that, while the force between two charged conductors does not depend on the kind of metal used for the conductors or whether they are solid or hollow, it does depend on the kind of insulating medium that separates them.

560. Electron Theory.—Maxwell, following out Faraday's idea of the importance of the dielectric, showed how electric phenomena might be explained by what may be called the displacement theory. He conceived all substances, conductors and insulators alike, as full of electricity which could flow freely through conductors, but in insulators experienced an elastic resistance which prevented it from being more than slightly displaced. A form of this theory based on the modern conception of electrons, is known as *the electron theory*.

The experiments of Millikan, (§543) together with recent researches relating to electric discharge in gases, and in the field of radioactivity, have led to the belief that all electric charges whether positive or negative are exact multiples of a unit charge e having the value 4.70×10^{-10} in electrostatic units, which is so small that there are more than 2000 million of them in the electrostatic unit as defined in §525.

The *positive* elementary unit charge is never found separate from atoms of matter, but the *negative* unit, as was shown by Sir J. J. Thomson, is carried by the small particles or corpuscles that make up the cathode-rays in a vacuum tube (§774) and have only $1/1800$ the mass of the hydrogen atom. These negative particles or *electrons*, as they are called, exist in all kinds of matter, can pass through conductors, and may be transferred from one body to another.

The electron theory supposes that every atom of matter in the neutral state is made up of a certain number of the elementary positive units and an equal number of electrons held in equilibrium by electric forces.

When an atom loses an electron it becomes positive, when it gains an extra one it becomes negative.

561. Conductors and Insulators.—In conductors the slightest *external electric force causes* electrons to pass continuously from atom to

atom through the body, thus constituting a flow or current of electricity. In this motion the electrons are constantly checked by their impacts against atoms of matter or other electrons and in this way they are retarded by a sort of *frictional* resistance, just as shot are retarded in moving through a mass of molasses; but there is nothing like an *elastic* resistance to make them spring back when the displacing force is removed.

In insulators, on the other hand, if electric force is applied, there is a certain yielding or displacement of the electrons, if the force is increased the electrons are displaced more, but there is no continuous flow as in a conductor, and as soon as the external force is removed they spring back to their original positions, behaving as shot would if imbedded in a mass of rubber.

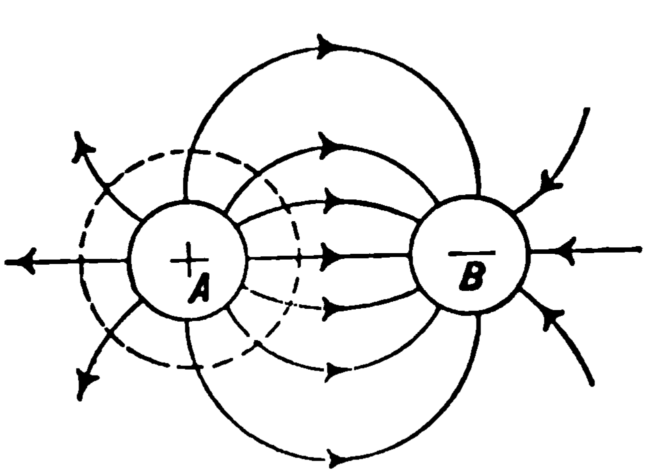


Fig. 308.

Now suppose that the process of charging two conductors *A* and *B* by an electric machine consists in forcing some electrons out of *A* into *B*, thereby making *A* positive and *B* negative. This will cause a crowding outward of the electrons in the dielectric immediately surrounding *B*, while those around *A* will be displaced inward to make up for its deficiency, and so in all the dielectric surrounding *A* and *B* the electrons will everywhere be displaced in the opposite direction to the arrows which indicate the positive direction of the lines of force.

But since in a dielectric electrons are not free to move, their displacement at any point is only through a very small distance and is opposed by the internal electric forces between the positive and negative elementary charges in the dielectric which urge all the electrons back toward their original unstrained positions. There is therefore produced a back pressure on the electrons in *B* and a negative pressure on that in *A*, so that if *A* and *B* are now connected by a conducting wire there will be a flow of electrons from *B* to *A*, until the displaced electrons in the dielectric have sprung back into their original positions. The discharge is thus conceived as forced from *B* to *A* by the springing back of the electrons in the dielectric in consequence of the internal electric forces in the dielectric.

This difference in pressure between *A* and *B* due to the reaction of the displaced dielectric is the difference between their potentials. Suppose that after *A* and *B* are charged they are moved nearer together. The strain will now take place through a less thickness of dielectric and the difference in pressure between *A* and *B* will accordingly be less. The work required to produce a given charge will therefore be less when they are nearer together; that is, the energy of the charge will be less. They will therefore tend to move together; that is, there is an *attraction* between *A* and *B*. For if they are moved apart they will have more energy, but they can only get this additional energy from the work done in separating them; therefore there must be a force opposing the separation or a force of attraction.

562. Tubes of Force.—The electric field may be conceived as divided up into tubes by means of surfaces in the direction of the lines of force. (Compare §499). These tubes of force will always have at *one end a positive charge and at the other an equal negative charge.*

On the electron theory there will be as many electrons displaced inward across one end of a tube of force, as will be displaced outward across the other end.

563. Induction as Explained on Electron Theory.—Suppose *A* and *B* are conductors near each other (Fig. 309) and having no charge at first. Let a negative charge be given to *A*.

In doing this we may suppose that electrons are transferred from the ground so that the walls of the surrounding room become positive. Tubes of

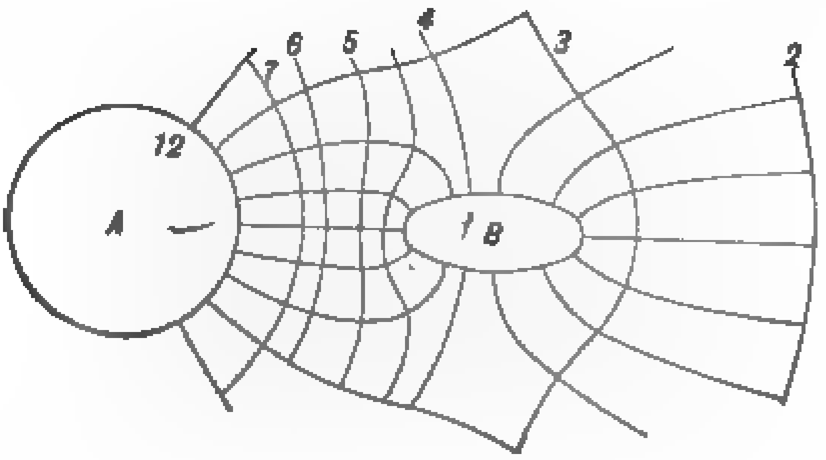


FIG. 309.—Induction.

force in which electrons are displaced outward will extend outward from *A* in every direction toward the walls of the room. But since *B* is a conductor there is no force resisting the displacement of the electrons through it, whereas in every other direction there is the active elastic resistance of the dielectric to be overcome. Displacement can therefore take place more readily on the side of *A* toward *B* than in other directions and a number of tubes of displacement will terminate on *B*, and the electrons where the tubes terminate will be displaced toward *B*'s interior, while on the farther side of *B* there will be an equal outward displacement of electrons. These constitute equal positive and negative charges respectively.

564. Why an Electrostatic Charge Appears Only on the Surface of a Conductor.—If a charge is given to a hollow conductor (Fig. 310) all parts of the metal shell come to the same potential and there is a displacement of electrons in the dielectric surrounding it, and this displacement is either away from the conductor or toward it depending on the kind of charge given to the conductor.

But the dielectric *A* in the interior *is entirely surrounded by the conductor and is therefore pressed upon equally in every direction* consequently there can be no displacement of its electrons. The pressure or potential in the interior is, however, *everywhere the same as that in the conductor which surrounds it. If a small conductor B is* touched to the interior of the shell it comes to

the same potential as the shell, but the electrons in the dielectric around it being equally urged in every direction are not displaced and accordingly B neither gains nor loses electrons; that is, it does not receive a charge.

It is much as though a bottle with flexible rubber walls was filled with water and then put inside of a vessel full of water under considerable pressure. If the stopcock is then opened no water will flow either into the bottle or out of it. For the pressure is the same inside of the bottle as it is outside. If the stopcock is closed and the bottle is removed from the region of pressure it will be found neither to have gained nor lost charge.

So it is also with the conductor B; when inside of A it is at the same potential as A, but it does not receive any charge because no displacement in the dielectric at its surface is possible, and so when removed from A its potential changes to that of the region where it is placed, but it shows no trace of charge.

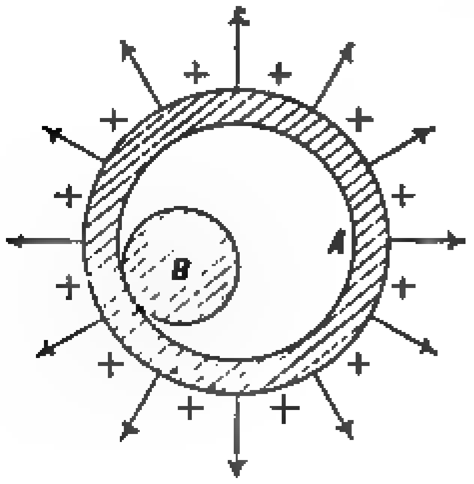


FIG 310.—Hollow conductor.

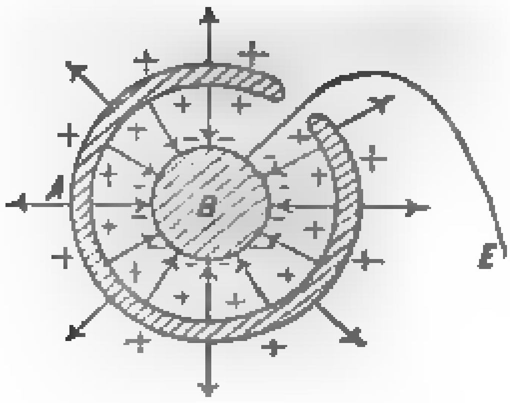


FIG. 311.

565. A Case of Induction.—Suppose, however, that the conductor B while inside the charged body A and insulated from it, is connected with the earth by a wire as shown in figure 311. The electrons in the dielectric will then be displaced not only in the dielectric between A and the walls of the room, but also between A and B, and if A is positive they are displaced toward A and away from B causing a corresponding flow of electrons into B from the earth.

If B is now insulated and removed from the interior of A it will be found negatively charged, for the displacement in the dielectric around it cannot disappear until the electrons that flowed into B are permitted to escape.

Since the outward displacement on B must be equal to the inward displacement over the inner surface of A, the charge on B must be equal and opposite to that on the interior of A; this has previously been shown to be the case in Faraday's ice-pail experiment (§541).

Condensers and Capacity

566. Condenser Experiment.—Take a tin plate, mount it *bottom* upward on an insulating stand and connect it with a

gold-leaf electroscope. Cover the plate with a sheet of glass and then give it a sufficient charge to cause the gold leaves to diverge strongly. Now take another tin plate, connected to earth by the hand or by a wire, and lower it upon the glass. The gold leaves will be observed to come together as the plates approach each other, showing that the potential of the charged conductor is diminished by the approach of the grounded conducting plate. The closer the two plates are brought together, the greater will be the decrease in potential. On removing the upper plate the leaves diverge as at first, showing that there has been no loss of charge.

In this experiment evidently the *capacity* of the first plate has been increased by the proximity of the second uninsulated plate. Such a combination is known as a *condenser*, because it can take a large charge at a small potential.

The decrease in potential is due to the presence of an induced charge on the second plate opposite in kind to that on the first.

567. Leyden Jar.—The earliest form of condenser was devised in 1746 by Musschenbroek, of Leyden. That experimenter, in attempting to charge a glass of water with electricity, held the glass in his hand while one pole of the electrical machine was connected with the water through a nail resting in the glass. After charging it well, the knuckle of the other hand was touched to the nail and a smart electric shock was obtained.

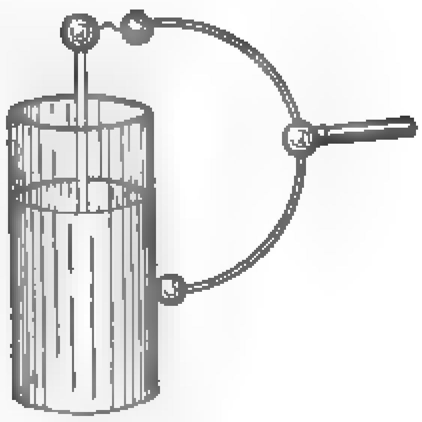


Fig. 312 — Discharge of Leyden jar.

Further experimentation showed that all that was necessary was that there should be two conducting coatings separated by the glass.

Accordingly, a *Leyden jar*, as it is called, is made by coating a glass jar or bottle inside and outside with tinfoil for about two-thirds the height of the jar. Connection is made with the inner coating through a metal rod terminating in a knob.

To charge such a jar one coating must be connected to one pole of the electrical machine while the other coating is connected to the other pole of the machine either directly or through the earth, so that *the two coatings simultaneously receive opposite charges.*

The jar is discharged by connecting the knob and outer coating by a conductor.

If the outer coating is touched with one hand while the knuckle of the other hand is brought to the knob, the jar is discharged through the body and the sensation of shock is experienced. Slight shocks are felt in the hand and arms while stronger shocks are felt in the body.

A Leyden jar is said to have greater *capacity* for charge than an ordinary insulated conductor, because it will receive a much greater charge from a given electrical machine than will be taken by the simple conductor.

568. Condensers.—Any contrivance in which two conductors are separated by a thin dielectric which has sufficient dielectric strength to prevent discharge between them, is a condenser and has the same properties as a Leyden jar.

A convenient form of condenser, due to Franklin, may be made by coating the opposite sides of a plate of glass with tinfoil, which, however, must not reach too near the edges of the plate.

569. Insulated Leyden Jar.—**If a Leyden jar has either of its coatings insulated, no more charge can be given to the other coating than to a simple metal conductor of the same shape and size.** If a Leyden jar is charged and then placed on an insulating stand, it cannot be discharged by simply touching the knob. But if the finger is touched first to the knob and then withdrawn and touched to the outer coating and so on *alternately* a small spark will be obtained every time and the jar may thus be very slowly discharged.

570. Explanation of Action of a Condenser.—The older way of explaining the action of a condenser is as follows: The plate *A* receives, say, a positive charge from the electrical machine. This charge acts inductively through the glass dielectric and attracts a negative charge from the earth, which in turn reacts on the positive charge in *A* attracting it, and so enabling a much larger charge to be given to *A* by the machine. If the plate *B* were not connected to the earth the positive charge which would be induced on its outside surface could not escape and would by its repulsive action on the charge in *A* neutralize the attractive action of the negative induced charge, so that no more charge could be given to A than if the plate B were not there.

But it is better to look at the action in the following way, from the standpoint of the displacement theory. Let A (Fig. 314) represent the inner coating of a Leyden jar which is stripped of its outer coating. Connect A to the positive pole of an electrical machine and connect the negative pole to the floor or walls of the room. The conductor A will become positively charged and an equal and opposite negative charge will be found on the walls of the room. The tubes of force or displacement extend from A to the walls, but the difference of potential produced by the machine can cause only a small strain or displacement of the electrons in so thick a dielectric, and, therefore, only a small charge will be given to A. But if the outer coating is now put upon the jar and connected with the negative pole of the machine, as in figure 315, the whole strain will take place in the thin layer of glass and so a great displacement of electrons in the glass will take place involving a large flow of electrons into one coating and out of the other, leaving the jar strongly charged.

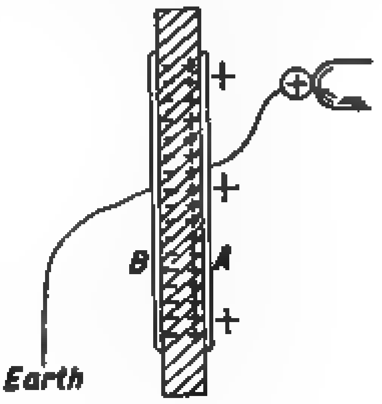


FIG. 313.—Charging a condenser.

If the outer coating were *insulated* and the negative pole of

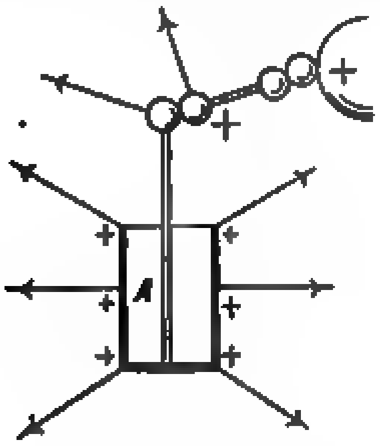


FIG. 314.

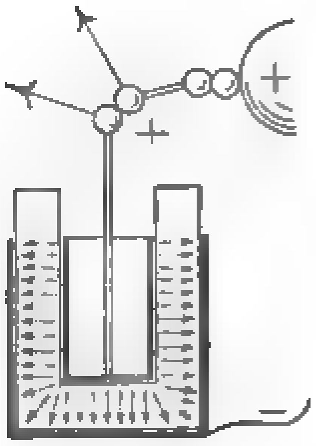


FIG. 315.

the charging machine were connected to the walls of the room, no displacement could take place through the glass toward the outer coating without an equal displacement taking place outward from the outer coating toward the walls, so that the jar would take no greater charge than if there were no outer coating.

571. Capacity of a Conductor.—From the above discussion it will be clear that **the charging of every conductor is analogous to the charging of a Leyden jar**; the conductor itself corresponds to the inner coating of the jar; while the surrounding walls or conductors on which the tubes of force terminate correspond to the outer coating; **it differs from a Leyden jar only in the greater thickness of the dielectric.**

572. Capacity.—The larger the charge given to a Leyden jar or condenser, the greater is the difference of potential between its coatings, so that we have

$$Q = VC$$

where Q is the charge, V is the difference of potential between the coatings of the jar, and C is a constant called the *capacity* of the jar.

When $V = 1$, $C = Q$, and, therefore, **the capacity of a condenser is the quantity of charge required to make unit difference of potential between its coatings.**

Capacity depends on that area S of one surface which is opposed by the other and varies inversely as the thickness of the dielectric d which separates them. It may be computed from the formula

$$C = \frac{SK}{4\pi d}$$

where K is a constant which depends on the nature of the dielectric and is called its *specific inductive capacity* or *dielectric constant.*

The derivation of this formula is given in §583.

Caution.—The student is warned against thinking that the capacity of a condenser is the greatest charge which it can hold. The maximum possible charge of a condenser depends upon its insulation and the strength of the dielectric between its coatings to resist disruptive discharge. One condenser may be charged to many times its capacity before it discharges, while another may break down or discharge before it is charged to one-tenth of its capacity.

Some Specific Inductive Capacities, or Dielectric Constants

Hard rubber	2.5	Paraffin	2.0	Air (normal pressure)	1.00059
Glass	6 to 8	Turpentine	2.2	Carbon dioxide	1.00090
Mica	8.0	Petroleum	3.1	Hydrogen	1.00028

573. Hydraulic Analogy.—It is instructive to consider the following hydraulic model of a condenser. A metal box is divided into two parts *A* and *B* by a partition of thick sheet rubber. Each side is provided with a tubular opening controlled by a stopcock, the whole is then filled with water and immersed in a pond. While the stopcocks are open the pressure is the same in *A* and *B* and the rubber is not strained. It is like a Leyden jar uninsulated and discharged. Now attach a pump to *B* and force water in while the stopcock of *A* is left open or, what amounts to the same thing, connect the pump both to *A* and *B* so that it pumps water out of *A* and into *B*. The rubber will be strained as shown in the figure, the side *A* will be at the pressure of the pond which may be called zero, while the other side is at a higher pressure *p*. This difference in pressure *p* between the two sides is due to the strain of the rubber. If *A* and *B* are now connected by a pipe and the stopcocks are opened there will be a flow from one side to the other as the rubber springs back into the unstrained condition.

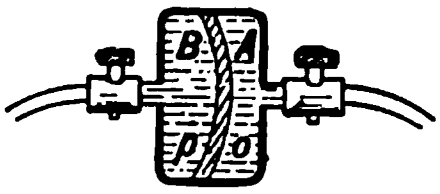


FIG. 316. — Hydraulic model of Leyden jar.

So when a Leyden jar is discharged electricity may be thought of as forced from one coating to the other by the springing back of the displaced electrons in the dielectric.

If the rubber diaphragm were thicker more difference in pressure would be required to force in a given charge. So in a Leyden jar, the thicker the dielectric the greater the difference of potential between its coatings when it has a given charge.

If the diaphragm were made of a substance that was more *yielding* than rubber, it would correspond to a dielectric of *greater specific inductive capacity;* for a given pressure would then force in a greater charge.

Also suppose the stopcock *A* is closed and the pump connected to *B*, pressure will be produced in *B* and perhaps a slight amount of water forced in due to the elastic yielding of the box itself, but the rubber diaphragm will not be appreciably strained and the pressure will be the same on both sides. This is the case of trying to charge an insulated jar. The stiff and but slightly yielding walls of the box represent the insulating dielectric that surrounds the Leyden jar and extends to the walls of the room, while the rubber diaphragm represents the thin glass dielectric between the coatings of the jar.

Remembering that the dielectric surrounding the jar is slightly yielding will enable the student to explain the succession of small sparks obtained from the insulated jar as described in §569.

574. Energy of Charge.—When we begin to charge a Leyden jar or condenser the two coatings are at the same potential and, therefore no work is required to transfer the first little portion of charge from one coating to the other. But as the charging goes on the difference of potential between the two coatings increases *and more work is required* to produce a given increase in charge.

Suppose the final potential to which the jar is charged is V, and suppose that in charging it Q units of electricity are transferred from one coating to the other, giving one a charge $+Q$ and the other a charge $-Q$. If during this transfer the difference of potential between the coatings were to remain constant and equal to V the work done in charging would be QV ergs. But since the difference of potential is zero at the start and increases in proportion to the charge, **the average potential during charging is $\frac{1}{2}V$ and the work actually done in charging is $\frac{1}{2}QV$, which is therefore the energy of the charge.**

The case is analogous to the filling of a cylindrical water-tower, the pressure is zero when the tower is empty, and increases as the water rises until the final pressure p is reached. The work done is, therefore, $\frac{1}{2}pv$ where v is the total volume of water pumped in.

The energy of the condenser exists as electrical strain in the dielectric.

575. Dissected Leyden Jar.—That the energy of the charge is in the dielectric and not in the conducting surfaces is shown by the following experiment.

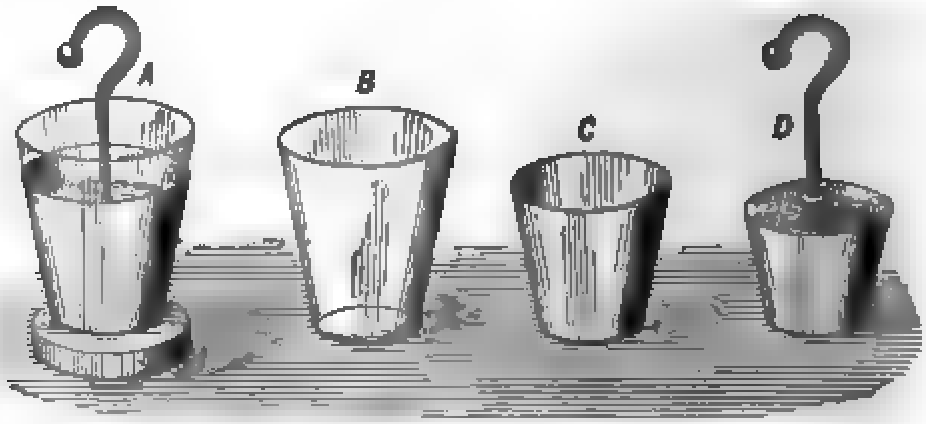


Fig. 317.—Leyden jar with removable coatings.

Take a Leyden jar, such as is shown in figure 317, in which the metal coatings can be removed from the glass. Charge it strongly and first remove one coating while the other is insulated, and then remove the other also. They are found to have only slight charges, but when they are again fitted upon the glass a vigorous discharge may be obtained.

576. Leyden Battery.—The Leyden jars in the combination shown in figure 318 have their inner coatings connected together and are mounted in a box lined with tinfoil by which their outer

ıoatings are also joined. Such an arrangement is known as a ,eyden battery, the jars are also said to be connected in *parallel* ɪr *multiple*, and the combination is equivalent to a single large ar having a ***capacity equal to the sum of the capacities of the sepa-ate jars.***

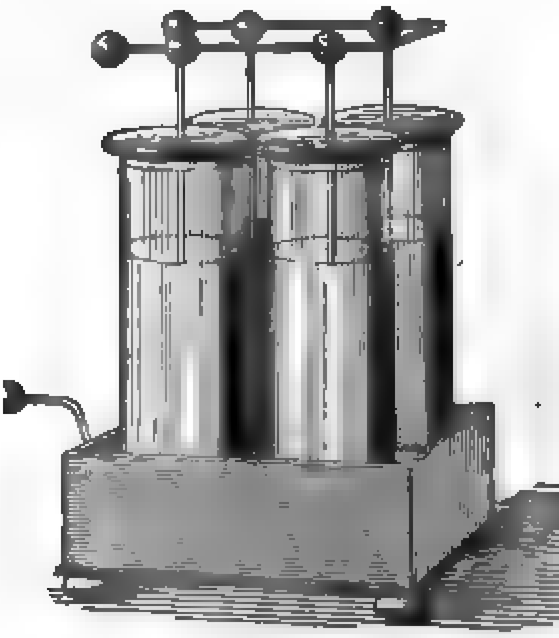


Fig. 318.

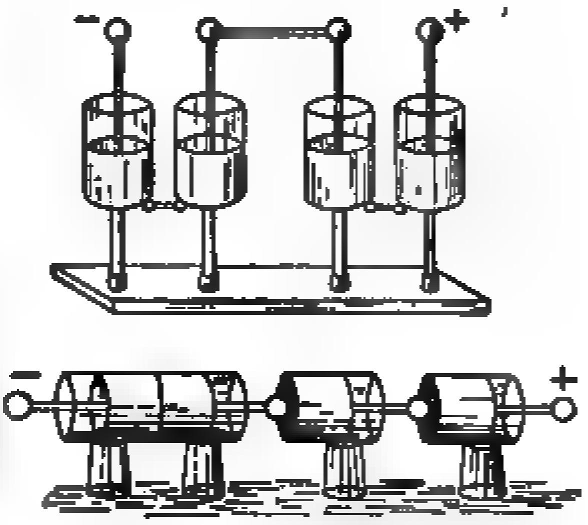


Fig. 319.

577. Leyden Jars Connected in Cascade or Series.—In each of .he two arrangements shown in figure 319 four jars on insulating ıtands are connected in such a way that if the discharge were ,o burst through the glass of the jars it would have to pierce all 'our jars to pass from one end to the other, as four layers of glass

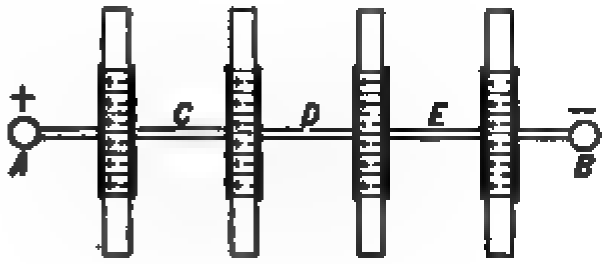


Fig. 320.—Condensers in series.

ntervene between the terminal conductors. In such a case the ars are said to be joined in cascade or in series. The diagram [Fig. 320) shows the state of electrification, the regions between .he charged plates representing the layers of glass.

Four similar jars joined in this way are like a single jar having a lielectric four times as thick, and the capacity of the combination is me-fourth that of a single jar.

This case is well illustrated by the hydrostatic analogue (Fig. 321) in which four models such as are described in §573 are connected in series. Clearly when water is pumped in at A and out at B the rubber diaphragms are all strained and an equal quantity of water is displaced from each into the next succeeding, thus representing the equality of the charges in each. The pressures represent the potentials. Evidently the pressure p_1 is greater than p_0, and p_4 is the greatest of all, and to force in a given quantity of water four times as much pressure must be used as to force it into a single one of the cells.

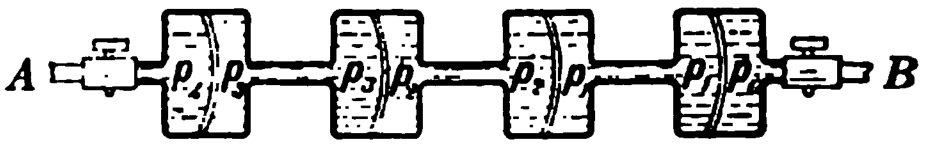


Fig. 321.

The chief practical advantage of the cascade arrangement is that it has *great dielectric strength* and sparks do not easily burst through the glass; for this reason the small jars used on induction electrical machines are usually connected two in series, one being connected to one pole of the machine and one to the other, while their outer coatings are joined by a wire.

When Leyden jars of different capacities C_1 C_2 C_3 are joined in series, the capacity C of the combination is found from the relation

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

To prove this formula let Q represent the charge, which will be the same for each jar when they are charged in series. The potentials of the jars will be

$$V_1 = \frac{Q}{C_1}, \quad V_2 = \frac{Q}{C_2}, \quad V_3 = \frac{Q}{C_3}.$$

The total difference of potential between the end coatings of the series will therefore be

$$V = V_1 + V_2 + V_3 = Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

and if C is the capacity of the combination we have

$$V = \frac{Q}{C}; \text{ therefore, } \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Problems

1. A Leyden jar 14 cm. in diameter and made of glass 3 mm. thick is coated on the bottom and sides up to a height of 20 cm. What is its capacity and what charge is required to bring it to potential 30? Take dielectric constant of glass = 6.
2. Two Leyden jars, one of capacity 300 and charged to potential 20, the other brought to potential 30 by a charge of 7200 units, are connected

[redacted]

together in parallel, positive coating being connected to positive and negative to negative. Find the resulting potential and the charge in each jar after being connected.

3. If in the preceding problem the positive coating of each jar is connected to the negative of the other, what will be the resulting difference in potential and charge in each jar?

4. A jar of capacity 1000 is charged to potential 50; find the heat developed in gram-calories when it is discharged through a long fine wire.

5. Three jars each of capacity 500 are charged each to potential 12 and then joined in series and finally discharged by connecting the end coatings of the combination. Find the difference of potential between the end coatings and the quantity of charge that passes through the discharge, and thence calculate the energy expended in the discharge.

6. Let the three Leyden jars of the previous problem be joined in parallel and then discharged. Find the difference of potential between coatings, and the quantity of discharge, and thence determine the energy of the discharge.

7. A Leyden jar of capacity 500 is joined in series with another of capacity 200, and the combination is given a charge 3000. Find the difference of potential between the coatings in each jar. Thence find the difference of potential between the end coatings of the combination. What is the capacity of a single jar which when given the charge 3000 would have the same difference of potential as the combination?

8. A Leyden jar of capacity 600 is joined in series with one of capacity 400. Find the capacity of the combination.

9. Two Leyden jars of capacities C_1 and C_2 are joined in series and given a charge Q. Find the capacity of a single jar equivalent to the combination, by following the method of problem 7.

10. A Leyden jar of capacity 800 is joined in series with another of capacity 200, and the combination charged to potential 20. Find the charge in each jar and the difference of potential between the coatings of each jar.

Calculation of Potential and Capacity

578. Potential at a point.—Up to this point we have thought of electrical potential simply as a certain condition which determines the flow of electricity; and we have shown that the difference of potential between two conductors may be measured by the work done in transferring unit charge from one to the other (§554).

But potential is not a property of conductors only. When a little charge is brought up to any point whatever in space, work must in general be expended in bringing it to that point, on account of the attractions or repulsions of neighboring charges; and this work, per unit charge, is used as the measure of the potential *at that point.*

Definition.—*The potential at any point is measured by the w done against electrostatic forces in bringing a unit positive ch up to that point from an infinite distance.*

This work may be calculated as follows:

Suppose there is a charge of q units of electricity at A (] 322), and it is required to find the work done in carrying ı charge from C to B in the same straight line with A when ai the medium between the charges.

Conceive the distance BC divided into n small parts at the po a_1 a_2, a_3,.etc., and let r be the distance from A to B and r_1, the tance from A to a_1, etc. Then the force with which q repels ı charge at B in air is $\frac{q}{r^2}$ (§525), while the force at a_1 is $\frac{q}{r_1^2}$. get the work w_1 done when unit charge is moved from a_1 to B,

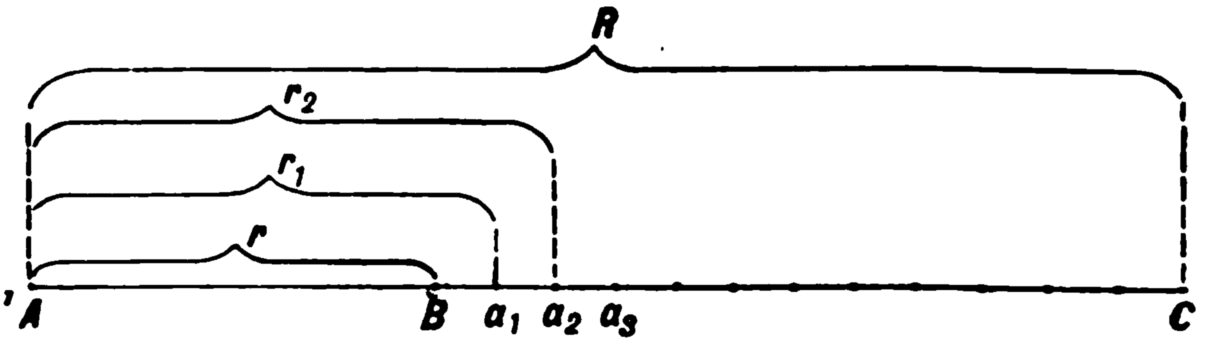


FIG. 322.

average force must be multiplied by the distance from B tı which is $r_1 - r$.

The geometrical mean of $\frac{q}{r^2}$ and $\frac{q}{r_1^2}$ is $\frac{q}{rr_1}$; and this may be ta as the average force between a_1 and B if these points are cl together. The work done in this element of the distance ' therefore be

$$w_1 = \frac{q}{rr_1}(r_1 - r) = \frac{q}{r} - \frac{q}{r_1}$$

and similarly the work done when unit charge is moved from a: a_1 is

$$w_2 = \frac{q}{r_1} - \frac{q}{r_2}$$

so also

$$w_3 = \frac{q}{r_2} - \frac{q}{r_3}$$

and finally

$$w_n = \frac{q}{r_{n-1}} - \frac{q}{R}$$

Adding, we find

$$w_1 + w_2 +, \text{etc.}, + w_n = \frac{q}{r} - \frac{q}{R}$$

where $w_1 + w_2 +$, etc., $+ w_n$ is the whole work done in moving the unit charge from C to B. In the final result all intermediate terms have disappeared, the result is therefore the same however great the number of parts into which CB may be divided; it is therefore clear that no error was introduced by taking the geometrical mean of the forces at B and a_1 as the average between those points.

It may be shown that the work will be the same along any path whatever between B and C, even though these points may not lie in the same direction from A.

Thus the work done in carrying unit positive charge from C to B (Fig. 323) against the repulsive force of a charge q at A is

$$\frac{q}{r} - \frac{q}{R}.$$

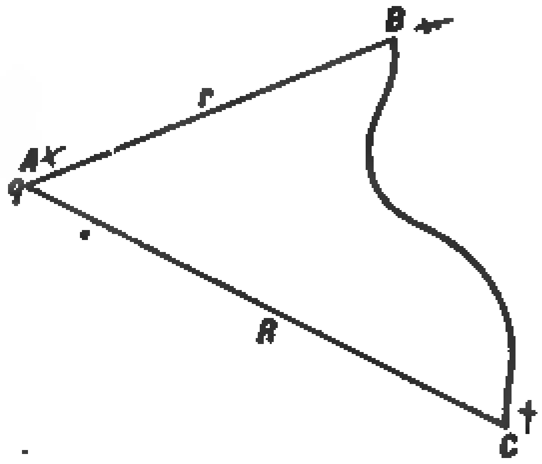


Fig. 323.

Now if the point C is at an infinite distance from A then $\frac{q}{R} = 0$, and the work done against the repulsion of q, in bringing a unit charge up to B from an infinite distance in air or vacuum, is simply $\frac{q}{r}$, and this, by definition, is the potential at B due to the charge q. Representing this potential by V we have, $V = \frac{q}{r}$.

If there are a number of charges $q_1 q_2 q_3$, etc., at distances $r_1 r_2 r_3$, respectively, from the point B and in any directions whatever, the potential at that point becomes, when air is the medium,

$$V = \frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} +, \text{ etc.}$$

since the potential at a point depends only on its distance from charges and not on their directions. The signs of the terms depend on whether the charges are positive or negative.

In any other medium than air the potential V may be computed from the formula

$$V = \frac{1}{K}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} +, \text{etc.}\right)$$

where *K is the specific inductive* capacity of the medium (§525).

579. Zero Potential.—According to the definition just given, those points are at zero potential which are at an infinite distance from all electrified bodies. But the earth's potential has also been defined (§550) as zero potential. These two definitions are inconsistent if the earth has a charge, and there are reasons for thinking that it has.

But any charge which the earth may have will change the potential of the earth and of all bodies in our laboratory rooms by the same amount, so that *differences of potential will be unchanged,* and it is only differences of potential that are measured by our instruments.

In discussing problems that involve the electrical state of the heavenly bodies or of regions remote from the earth, of course it will not do to assume that the earth potential is zero. The zero must then be taken as defined in the preceding paragraph.

580. Equipotential Surfaces.—Suppose there is a charge of 12 units at A (Fig. 324) which is not near any other charged body. Then the potential due to A may be calculated from the formula

$$V = \frac{q}{r}$$

where $q = 12$. At 1 cm. from A in any direction the potential will be 12. The sphere of radius *one* having A as center is therefore an equipotential surface of potential 12. The sphere whose radius is 2 cms. is the surface of potential 6, the surface of potential *one* would have a radius of 12 cms., while zero potential would be at an infinitely great distance. If the charged point A is inside of a room the surface of the room will be at zero potential, for there will be an induced negative charge at each point of the surface sufficient to counteract the action of the charge A.

The figure shows the position of the successive equipotential surfaces, differing by unity, from 2 to 12. It will be noticed that they are closer together the nearer they are to A. The same amount of work must be done to move a unit charge from the surface 2 to 3, as from 3 to 4 or 11 to 12; in each case one erg of work is done. But the shorter the distance in which a given amount of work is done the greater the force that must be exerted, hence the surfaces are closer together near A where the electric *force is greater.*

No work at all is done when an electric charge is moved along an equipotential surface, hence at every point the direction of the resultant force must be at right angles to the equipotential surfaces.

Lines of force, or lines which at each point have the direction of the resultant force, must therefore cut equipotential surfaces at right angles, and in the above case are a set of radial straight lines.

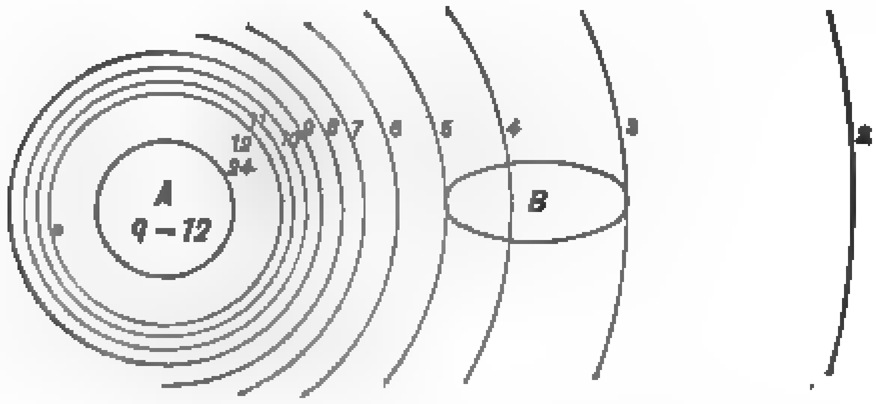


FIG. 324.—Equipotential surface due to a charge 12 at *A*.

581. Induction from the Point of View of Equipotential Surfaces.—In figure 324 notice that the region *B* surrounded by the elliptical line reaches from a point where the potential is 3 to where the potential is 5. If *B* is a non-conductor this distribution

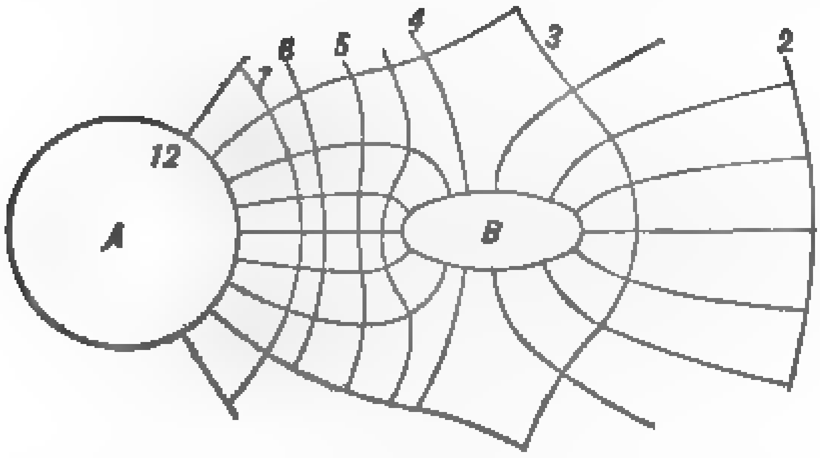


FIG. 325.—Equipotential surfaces where *B* is a conductor.

of potential is possible, but if *B* is a conductor flow must take place until it is all at the same potential. The left-hand end will receive a negative charge which will lower its potential, while the right-hand end will have its potential raised by a positive charge till all parts come to some potential intermediate between 3 and 5. The lines of force and equipotential surfaces in this case are shown in figure 325. In this diagram the conductor is supposed to come

to potential 4, all other equipotential surfaces are bent outward or inward away from B. Some lines of force from A terminate on the negatively charged left end of B, while lines of force go out from the positively charged end of B to the right. This case is analogous to the formation of a level spot or pond on the side of a mountain. The ground must be cut away on the side toward the mountain, and built up on the outside.

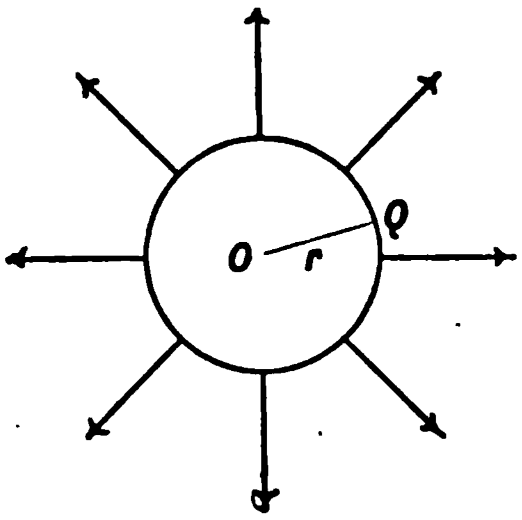


Fig. 326.—Isolated sphere.

582. Capacity of an Isolated Sphere.—Suppose an insulated sphere in the center of a large room. If it has a positive charge Q, its lines of force terminate on an equal quantity of negative electricity induced on the walls of the room. To find the potential at the center of the sphere we have the formula,

$$V = \frac{q_1}{r_1} + \frac{q_2}{r_2} +, \text{ etc.} \qquad (\S 578)$$

In the present case the only charge which is near enough to produce any appreciable effect at O is the charge $+Q$. Although this charge is distributed over the sphere, it is all at the same distance r from O. Therefore the potential at the center of the sphere is

$$V = \frac{Q}{r}$$

But in case of a charged conductor all parts of it, inside and outside, are at the same potential, the sphere is, therefore, all at the potential V of its center.

But by §572

$$Q = VC$$

therefore

$$C = r$$

or the capacity in electrostatic units of an isolated sphere surrounded by air is numerically equal to its radius.

If the medium surrounding the sphere has specific inductive capacity K, its capacity becomes (§572)

$$C = Kr.$$

583. Capacity of a Condenser Made of Two Concentric Spheres.—Suppose we have a condenser such as shown in figure 327, consisting of two concentric metal spheres with air between them. Let r_1 be the outer radius of the inner sphere and r_2 be the inner radius of the outer sphere. If a charge $+Q$ is given to the inner sphere, an induced charge $-Q$ will be found on the outer sphere. If the outer sphere is connected to earth it comes to zero potential and all charge disappears from its *outer* surface.

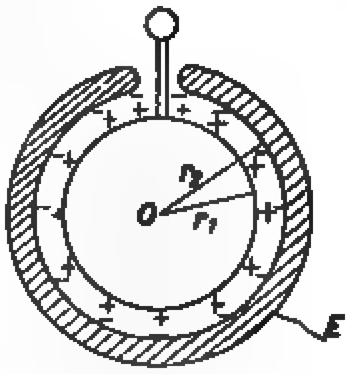


Fig. 327.—Spherical condenser.

The potential at O the center of the small sphere is therefore

$$V = +\frac{Q}{r_1} - \frac{Q}{r_2}$$

since the charge $+Q$ is at a distance r_1 from the center, and the charge $-Q$ is at a distance r_2 from the center.

But the potential everywhere inside of a closed conductor is the same as at its surface. Hence the potential of the inner sphere is

$$V = +\frac{Q}{r_1} - \frac{Q}{r_2} = Q\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

and since the outer sphere is at zero potential, V is the difference of potential between the two.

But by the definition of capacity $Q = CV$ therefore

$$C = \frac{1}{\frac{1}{r_1} - \frac{1}{r_2}} = \frac{r_1 r_2}{r_2 - r_1}.$$

If the medium between the spheres has a specific inductive capacity K, the capacity of the condenser will be

$$C = \frac{K r_1 r_2}{r_2 - r_1}.$$

If the spheres are close together we may write $r_1 r_2 = r^2$ and $r_2 - r_1 = d$ where r is the mean radius of the spheres and d is the thickness of the space between them.

Then

$$C = \frac{Kr^2}{d} = \frac{K4\pi r^2}{4\pi d}$$

but $4\pi r^2$ is the area of surface of a sphere of radius r; therefore

$$C = \frac{KS}{4\pi d}.$$

In this form the formula can be used for any condensers where the two surfaces are close together, as in a Leyden jar or in a condenser made of two flat parallel plates.

Problems

1. How much work must be done to carry a unit positive charge from a point 1 meter distant from a charge $+100$ to a point 2 cm. from it?
2. What is the potential at a point half-way between two equal spherical conductors having charges $+100$ and -100, respectively?
3. What is the potential at one corner of a rectangle which measures 40×30 cm. when there is a charge -300 at the diagonally opposite corner and $+120$ at each of the adjacent ones?
4. A spherical conductor 10 cm. in diameter has a charge of $+200$ units and a small body having an equal plus charge is situated 1 meter from the center of the sphere. What is the potential at the center of the sphere? What is the potential at its surface? Is the charge distributed uniformly over the sphere?
5. How much work would be done in moving the small charged body of the preceding question up to 50 cm. from the center of the sphere?

Electric Discharge

584. Electric Discharge through Air at Ordinary Pressure.— Three forms of discharge are recognized through air at ordinary

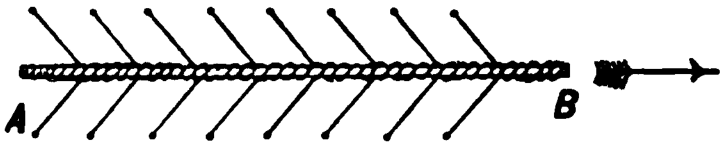


Fig. 328.

pressures, the *electric spark or disruptive discharge, brush discharge*, and *glow discharge.*

In the ordinary spark discharge there is a flash of light accompanied by heat and sound and the medium is mechanically rent. The energy that was in the strained dielectric is dissipated in these various ways.

The discharge must not be thought of as "jumping across" from one body to the other, it cannot be said to leap from positive pole to negative or from negative to positive, but *takes place simultaneously at every point along the path of discharge.* Imag-

ine a piece of rope AB held in position by a set of elastic bands which are attached to nails on each side of it, as shown in figure 328. If we pull the rope toward B the elastic bands are stretched and resist; but if enough force is exerted they will break, and the breaking will begin not necessarily at one end or the other, but wherever the weakest one is found. But when breaking occurs all parts of the rope move forward at once. This illustrates very crudely what probably takes place in disruptive discharge; the strained medium begins to break down at the weakest point, wherever that may be, but the electric discharge takes place simultaneously at all points along the line of discharge.

The *brush discharge* is seen when in a darkened room the hand is brought near the positive conductor of a highly active electrical machine. If it is not held near enough for the spark discharge a luminous brush, like a little tree with branches of light ramifying from a short stem, extends out toward the hand from some point on the positively charged conductor. It seems to be caused by an almost continuous succession of extremely small discharges.

Sometimes in the dark when an electric machine is highly excited, but when the conductors are separated too far for sparks to pass, a faint velvety glow of violet light known as the *glow discharge* is seen on the knob of the negative conductor.

585. Oscillatory Discharge.—When a spring is bent and let fly it oscillates back and forth, coming to rest when its energy is

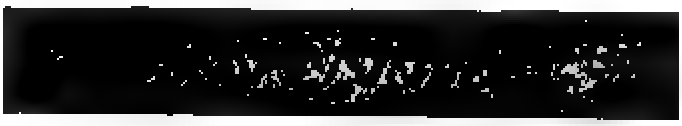


Fig. 329.—Oscillatory discharge.

finally spent in heat, sound, and air waves. So when a charged Leyden jar is discharged through a circuit of small resistance the energy of the charge cannot be dissipated in the first rush and consequently there is a back-and-forth rush of current from one coating to the other until the energy is finally spent in sound, heat, light, and electric waves. This is known as the oscillatory discharge. If there is sufficient resistance in the discharge circuit there is no oscillation, just as a pendulum hung in molasses will sink to its lowest position without oscillation.

The oscillatory discharge was examined by Feddersen in 1863 by means of a rapidly rotating mirror. Seen in this way, each discharge showed as a group of sparks at regular intervals and rapidly dying out, as shown in figure 329; see also §758.

586. Mechanical and Heating Effects of Disruptive Discharge.—When the discharge takes place through a sheet of glass it is pulverized at the point of discharge. Pasteboard is perforated by the discharge, the edges of the hole being raised in a burr on each side as if by the sudden expansion and bursting out of the contained air or moisture. When trees are struck by lightning they are apt to be splintered, large slivers being flung violently out sidewise, perhaps due to sudden vaporization of moisture. When a living tree is struck the discharge usually takes the sap layer and frequently follows the grain. A glass tube having a fine bore filled with water and with a wire thrust a short distance in each end may be burst by the discharge of a Leyden jar.

The electric discharge is accompanied by heat. Ether and bisulphide of carbon are readily ignited by it. Buildings containing inflammable material are occasionally set on fire when struck by lightning. A mixture of one volume of oxygen with two volumes of hydrogen explodes with violence if even a minute electric spark passes in it.

A little gunpowder placed between the ends of two wires through which a discharge is sent will usually be scattered unless the discharge is retarded by causing it to pass through a wet string or other poor conductor, in which case the powder may be ignited.

Narrow strips of gold foil, 1 or 2 mm. in width, gummed to a sheet of paper so that they form a conducting strip, may be deflagrated or volatilized by the discharge of a Leyden battery. The purple stain which is left is wider than the gold-foil strips and is streaked at right angles to its length as though the volatilized metal had been driven violently out from the path of discharge.

587. Lightning.—The resemblance between lightning flashes and electric sparks was early noticed. Franklin, in 1752, performed the celebrated experiment of obtaining electric charges by means of a kite as a thunder storm was approachi[illegible] The

kite was provided with metal points and the linen kite cord was a fairly good conductor when wet. To the lower end of the kite cord was fastened a metal key to which a silk cord was attached which was held in the hand and acted as an insulator Sparks were obtained from the key and Leyden jars were charged, and the familiar phenomena of electric charges were observed.

The so-called *globe lightning*, described by different observers as a ball of fire slowly moving along and then suddenly exploding with terrific violence, has never been imitated by any electrical discharges obtained in the laboratory and is so different from the ordinary phenomena of discharge that many physicists consider such observations illusory and due to a subjective effect of the discharge on the eye of the observer.

588. Atmospheric Electricity.—The electrical separation in thunder storms according to the theory of Simpson, is due to the disruption of rain drops in the uprushing current of air; for laboratory experiments show that when a drop is broken up by falling on a vertical jet of air the resulting drops are positively charged while the current of air carries off negative charge. Rain from the lower part of the cloud will carry down positive charge while rain from higher regions of condensation will be negatively charged.

Another circumstance that very possibly plays a part in the development of thunder storms is that condensation of moisture in the atmosphere takes place more easily around negative nuclei or electrons than it does around positive nuclei, and the fall of such drops to the earth will give it a negative charge. In fair weather the earth is usually negative, the potential being higher at points above the earth's surface, increasing at the rate of from 75 to 150 volts per meter above level ground, while in thunderstorms the atmospheric potential fluctuates greatly and may even be negative to the earth.

589. Lightning Rods.—It was shown (§538) that when an electroscope was surrounded by a conducting surface or even enclosed in a wire cage it was screened from outside electrical disturbances, and this suggests how buildings should be protected.

Buildings with metal outer sheathing need no other protection, though care should be taken that the metal walls are at least as well conne[illegible] [illegible]amp earth as the gas and water pipes within.

Wooden structures should have low metal points on the chimneys and gables and other projecting portions, these points should be connected together by heavy wires or other conductors which run down the main corners of the building to the ground. At or near the ground it is well to have them connected together by a wire passing entirely around the building, and at two points on opposite sides of the building good ground connections should be made by connecting to pipes driven down to water or to a metal plate bedded in coke in damp earth.

Insulation from the building is not needed, metal roofs and gutters and rain-water pipes should be connected together and may serve for lightning conductors if given good ground connections. Ordinary heavy galvanized iron telegraph wire will serve well for the conductor or, still better, a flat ribbon of sheet copper.

ELECTRIC CURRENTS

OR

ELECTRODYNAMICS

The Electric Current and Voltaic Cell

590. The Electric Current.—When a Leyden jar is discharged or when a series of sparks from an electrical machine pass through a conductor, in fact whenever a charge is communicated from one point to another, there is what is called a flow of electricity, or an electric current.

The current is said to flow from the positive to the negative conductor. *This is a convention;* for vitreous electrification was called positive, and resinous was called negative, long before there was any idea of the direction of flow. Recent investigations have led to the belief that *in an electric current there is a flow or transfer of elementary negative charges or electrons from the negative to the positive conductor*, thus what is believed to be the actual direction of flow is exactly opposite to the ordinary convention.

In all the cases hitherto considered the flow has been so transitory as to be almost instantaneous. We now come to a series of discoveries which made possible the production of currents of electricity lasting for a considerable time.

591. Galvani's Discovery.—In 1786, Galvani, professor of anatomy at Bologna, in experimenting on the muscular contractions produced by discharges from an electric machine, noticed that frogs' legs, hung on metal hooks in such a way that they rested in contact with a strip of another metal through which the hooks had been driven, were thrown into convulsive movements such as were produced by electric discharges. Following up the observation, he found that if strips of two unlike metals, such as zinc and copper were taken and one put in contact with the main nerve of the frog's leg while the other was touched to

the thigh muscles, spasmodic muscular contractions took place, provided the other ends of the metal strips were in contact with each other.

592. Volta's Discovery.—Volta, who was professor of physics in the University of Pavia, believed that the source of the electrical effects observed by Galvani was to be found in the contact of the dissimilar metals. But if there was any difference of potential produced in such a case it was far too small to be detected by the gold-leaf electroscope as ordinarily used. This difficulty was most ingeniously overcome by Volta's device of the condensing electroscope.

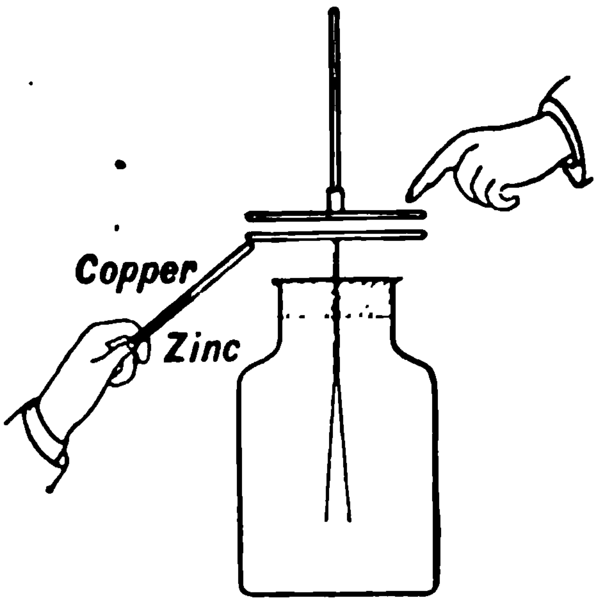
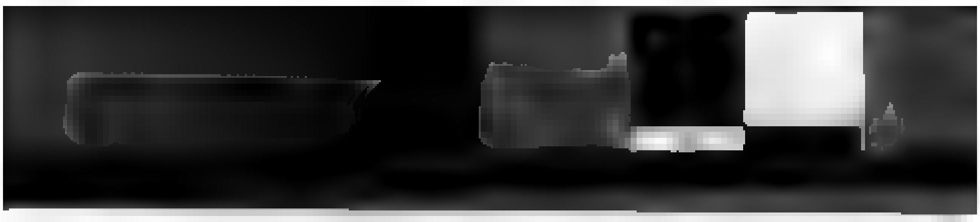


Fig. 330.—Volta's discovery.

A gold-leaf electroscope was constructed having a flat brass plate instead of a knob, as shown in figure 330, on which rested a second brass plate of the same size having an insulating handle of glass by which it could be raised. Both plates were given a thin coating of shellac varnish by which they were insulated from each other and thus formed a condenser of large capacity, since the separating dielectric was thin.

The lower plate was then touched by a strip of copper soldered to the end of a zinc strip held in one hand while at the same time the upper plate was touched with the other hand. When the upper plate was raised after breaking these contacts, the gold leaves diverged with negative electricity, showing that the upper plate of the condenser had been charged positively and the lower negatively by the operation. The advantage of the condenser was that although the difference of potential between the plates was exceedingly small, a considerable charge was accumulated which was set free when the plates were separated.

When the two condenser plates were of brass and *directly connected* by the copper-zinc circuit, as in figure 331, no charge was obtained since the end metals were alike, being the two brass condenser plates; but if at any point in the circuit two dissimilar metals were connected by a dilute acid or salt solution, as shown in figure 332, the condenser plates were charged. In this case



the solution takes the place of the body of the experimenter in the original experiment.

It is now believed that the differences of potential obtained by Volta were mainly due not, as he supposed, to the contact of

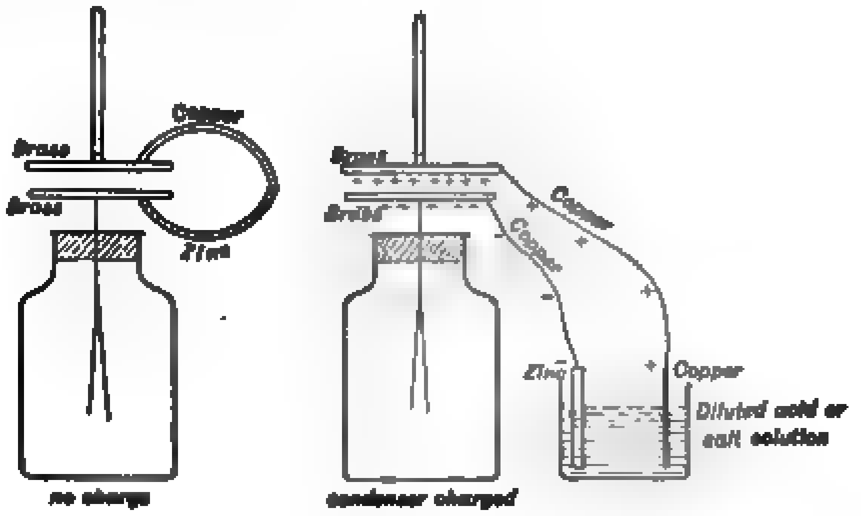


FIG. 331. FIG. 332.

dissimilar metals, but to the contacts between these metals and the hands of the experimenter or the acid or salt solution.

593. Voltaic Pile.—In seeking to obtain a larger effect Volta found that when he took two cells in which strips of zinc and copper dipped into dilute acid, and joined them in *series*, as shown in figure 333, he obtained in the electroscope double the charge given by one cell. **The effect was found to depend only on the kind of metals and acid used and not at all on the size of the plates.**

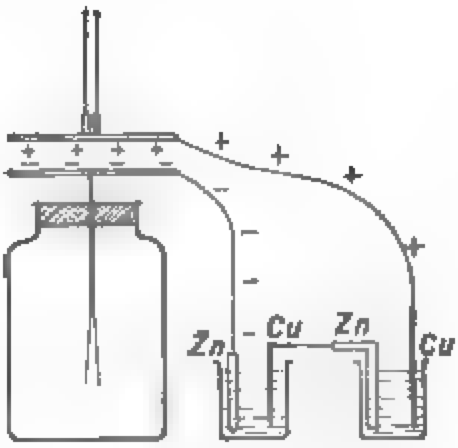


FIG. 333.—Charge by two cells.

The *Voltaic pile,* based on this discovery, consists of discs of copper, zinc, and cloth or paper saturated with acid or salt solution, piled one upon another, first a disc of copper, then acidulated cloth, then zinc, then again copper, cloth, and zinc, and so on. A pile having 50 such combinations will produce 50 times the difference of potential that can be obtained from a single element consisting of zinc-acid-copper.

What are known as *dry piles* are made by taking discs of gilt paper and so-called silver paper, placing them in pairs, the gilt face of one against the silver face of the other, and then

making a pile of such pairs, the same kind of paper being uppermost in each pair. In a moist climate the natural dampness of the paper enables it to play the part of the acidulated cloth layers in Volta's pile.

594. Voltaic Cell.—A cell having a plate of zinc and a plate of copper dipping in dilute sulphuric acid is known as a simple Voltaic cell, and several cells combined constitute a Voltaic or Galvanic *battery*. Since Volta's day many improved kinds of battery cells have been devised, some of which will be considered later (§§626–635).

The two plates of a Voltaic cell are called the *electrodes*, and the terminals of the plates where the external wires are connected are called the *poles* of the cell. The copper terminal is at a higher potential than the zinc terminal and gives a positive charge, it is therefore called the *positive pole*, while the zinc terminal is the *negative pole*.

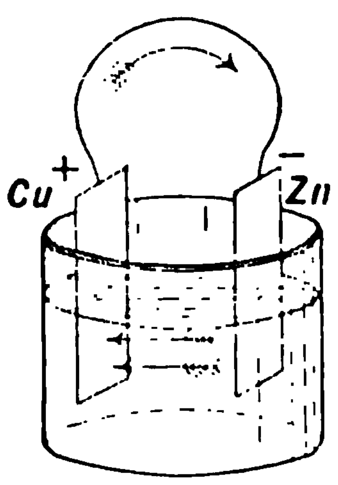


Fig. 334.—Electric circuit.

On the other hand, the copper plate is often spoken of as the *negative electrode* or *electronegative element* in the cell and the zinc as the *positive electrode* or *electropositive element*, because positive charge is transmitted through the acid of the cell from the zinc to the copper plate as though repelled by the zinc and attracted by the copper.

Electric Current in a Cell.—Since the two poles of a Voltaic cell are at different potentials, an *electric current* is established when they are connected by a metallic wire just as when the two coats of a charged Leyden jar are connected. This current, however, flows steadily instead of lasting only for an instant. The metallic wire together with the plates and liquid between them form a conducting *circuit* in which **the positive direction of the current or that in which positive electricity flows is from copper to zinc through the outside wire and from zinc back to copper inside the liquid of the cell.**

There are *three principal evidences* of the existence of the current:

1. Heat is developed in all parts of the circuit.

2. Every part of the circuit affects a magnetic needle brought *near it*.

3. Chemical action takes place at the surfaces of contact between the metal electrodes and the liquid. If the copper and zinc plates are in dilute sulphuric acid, bubbles of hydrogen gas appear at the surface of the copper plate, while the zinc plate is eaten away by the acid, and zinc sulphate is formed.

All these phenomena cease at once when the current is interrupted, either by breaking the metallic connection between the plates or by separating the acid around one plate from that around the other by a non-conducting partition.

595. Contact Potentials in a Voltaic Cell.—When zinc is immersed in the acid there is what may be called a solution pressure, or tendency for the zinc to be dissolved and form zinc sulphate in solution, each atom of zinc carrying into the solution a positive charge.

As the positively charged atoms of zinc pass into solution, the plate, losing positive charge with each one, becomes negative, while the solution becomes positive, in consequence of which there is an electrostatic force tending to prevent the positively charged zinc atoms from going into solution. Therefore when a certain difference of potential between the zinc and acid solution is reached there will be equilibrium between the electrostatic force and the solution tendency, and the zinc will cease to be dissolved.

There is thus definite difference of potential due to the contact of zinc and acid when there is equilibrium between them, and another due to the contact of copper and acid which is less than the former since the solution pressure of copper in the acid is less than that of zinc.

596. Electromotive Force.—The diagram, figure 335, represents the relative potentials of the elements in a Voltaic cell. The acid has the highest potential and is positive both to zinc and copper. The difference between acid and copper is, however, less than between it and zinc, and the copper is therefore at a higher potential than the zinc as shown.

If the copper pole of the cell is connected to the earth, it comes to the earth potential or zero, and the zinc pole as tested by a quadrant electrometer is found to have a negative potential. On the other hand if the zinc pole is connected to the earth it will be at *zero potential* while the copper pole will be found to

be positive; but *the difference of potential* **between them will be** the same in each case.

Every cell can produce a certain maximum **difference in** potential between its two electrodes, **and when this is reached** there is equilibrium and the chemical action stops.

The maximum difference of potential which a cell can produce is called its electromotive force; it is measured by the difference of potential between the electrodes when there is no current and the chemical action has ceased.

The electromotive force of a ***cell depends only on the chemical relations of the constituents of the cell and is therefore the same whether the plates are large or small.***

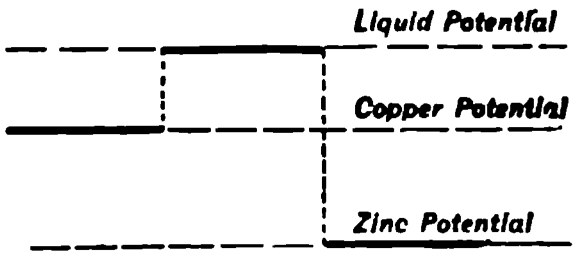


FIG. 335.

A small cell formed by dipping the tips of a zinc and of a copper wire into a single drop of acid will cause as great a deflection of a quadrant electrometer as a cell of the same kind with plates a foot square.

A convenient abbreviation for electromotive force is E.M.F., or in equations the symbol E is commonly used.

597. Hydraulic Analogy to Voltaic Cell.—The following analogy given by Lodge is instructive. Two tall open vessels con-

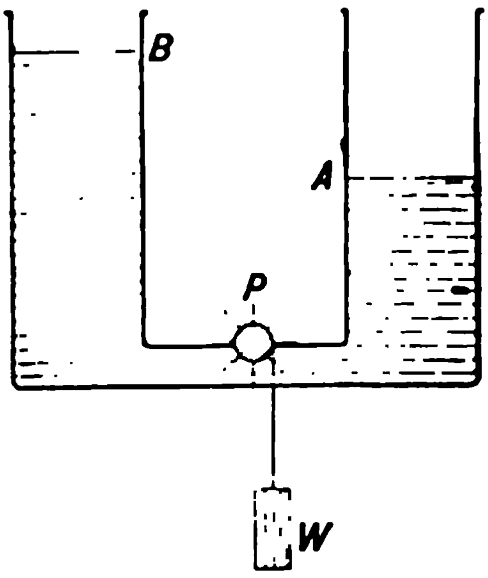


FIG. 336.

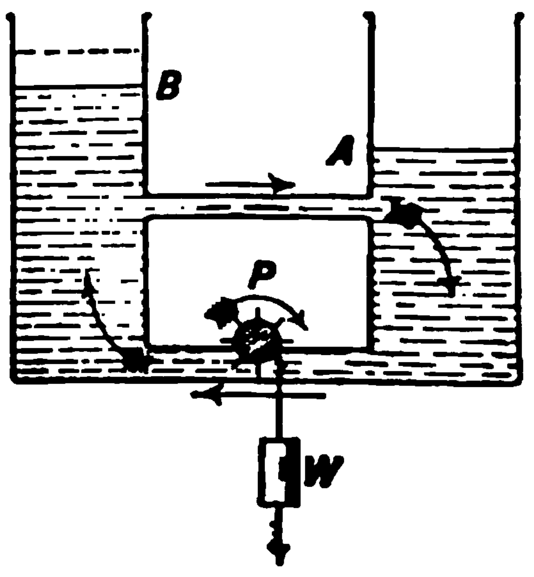


FIG. 337.

taining water are connected by a pipe in which is a pump *P* driven by a weight *W* (Fig. 336). The water **will flow from one** vessel to the other until the back pressure **on the pump due to the** higher level of *B* just balances the force **of the weight. The** difference in level will be the same whether **the vessels are large** or small. The difference of level represents the difference of

potential between the zinc and copper which is independent of the size of the cell, the pump with its driving weight is the electromotive force of the cell, which through chemical action can produce a certain definite difference of potential and no more.

Figure 337 represents the state of things when the zinc and copper plates are connected by a wire, represented by the tube shown. The difference of pressure causes a flow through the tube from *B* to *A*, at the same time the level sinks in *B* and rises in *A* so that the difference in pressure on the two sides diminishes and is no longer able to balance the pressure of the pump, which therefore begins to act, forcing water from *A* to *B;* at the same time the weight *W* descends, supplying energy for the circulation, which will be maintained so long as the weight can move downward.

Here it is seen that the electromotive force, represented by the power of the pump to produce pressure, is the same as before, but the difference of potential between the plates, shown by the difference between the levels of *A* and *B* is less than before. The work done by the pump in circulating the water is obtained from the weight, which loses potential energy as it descends. So in the Voltaic cell, **the energy expended by the electric current is supplied by the chemical changes which take place at the electrodes.**

598. Magnetic Effect of Current.—In 1819 Oersted discovered that when a wire connecting the poles of a Voltaic cell was held *over* a balanced magnetic needle and parallel to it, the needle was deflected, the north pole of the needle moving toward the west when the current was from south to north, as in the diagram, while if the current was reversed the north pole of the needle moved toward the east. The effect was reversed when the wire was placed *under* the needle.

This discovery aroused the greatest interest, as *it was the first evidence of a connection between magnetism and electricity.*

599. Electric Circuit.—It was also found that the action was the same whatever part of the wire connecting the plates was brought near the needle, the deflection produced by the current in the middle of the wire being just as great as that near its ends.

By this experiment also the *direction of the* current in the *electrolyte may be shown to be opposite* to that in the wire; for if

two vessels are used connected by a short tube containing the acid, and if a zinc plate is placed in one vessel and copper in the other, as shown in figure 339, a magnetic needle will be deflected toward the west when placed under the wire connecting the plates, but toward the east when under the tube. The experiment shows that the current in the electrolyte is just as strong as that in the wire, but in the opposite direction.

From experiments such as the above it is inferred that **steady electric currents always flow in closed circuits and are equally strong at every point, and if the circuit is interrupted at any point, whether in the electrolyte or the wire, the magnetic action and all other current effects cease everywhere at almost the same instant.** It is very much as when an incompressible liquid circulates in a closed tube, just as much liquid must pass any one section of the tube as any other during the same time.

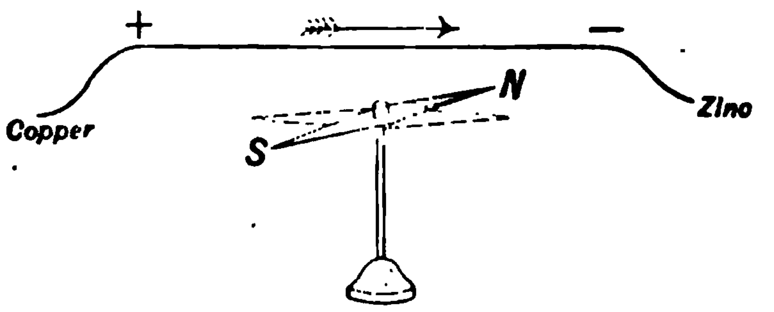


FIG. 338.—Current and magnetic needle.

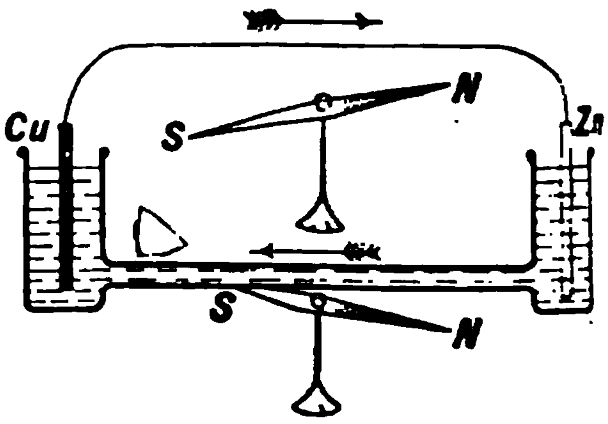


FIG. 339.—Current in electrolyte.

600. Galvanometers.—When a wire is bent into a vertical circle having its plane parallel to the direction of a magnetic needle balanced at its center, if a current is established in the wire all parts of it act together to deflect the needle, turning the north pole to one side or the other, depending on the direction of the current. An instrument which measures electric currents by the deflection of a magnetic needle is known as a *galvanometer*.

601. Galvanometers Measure Current.—**Faraday showed that a galvanometer measures the quantity of charge transmitted per second, or what is called the current strength.** For he found that when a Leyden jar was discharged through a sensitive galvanometer there was an instantaneous swing of the needle to one side, the amount of which depended only on the *quantity* of the charge; that is, the swing produced by forty turns of his

electrical machine was the same whether the charge was held in a small jar at high potential or in a large Leyden battery at low potential, and whether the wet string through which the discharge was sent was long or short.

It was also established by Faraday that when *a constant current* flowed through a galvanometer producing *a steady deflection* of the needle, **the magnetic force on the needle due to the current was proportional to the quantity of charge transmitted per second.**

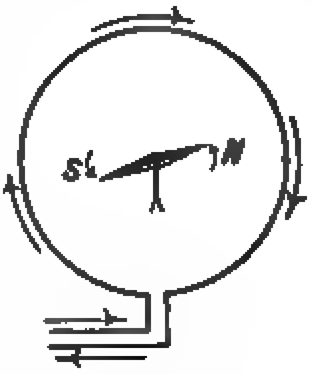


FIG. 340.—Galvanometer.

602. Unit Current.—Instead of measuring electric currents by the quantity of charge in electrostatic units transmitted per second, it is found better to adopt a new system of units based on the magnetic effect of a current and using magnetic units as already defined. This system is known as the *C. G. S.* or *absolute electromagnetic system,* since it also is based on the centimeter, gram and second.

In this system *a unit current is one which, flowing in a circular coil of one centimeter radius, will act on a unit magnetic pole at its center with a force of one dyne for every centimeter of wire in the coil.*

The Ampère or Practical Unit of Current.—The unit of current in the practical system is called the **Ampère** in honor of the French physicist who first investigated the laws of the magnetic effects of currents. It is defined as **one-tenth of the absolute or C. G. S. unit current,** being chosen smaller than the absolute unit for reasons of convenience.

The quantity of charge transmitted by one ampère in one second is called a *coulomb.* One coulomb is equal to 3,000,000,000 electrostatic unit charges as defined in §525.

603. Unit of Electromotive Force.—In our studies of electrostatics it was shown (§554) that the difference between the potentials of two conductors might be measured by the work required to transfer unit charge from one conductor to the other. Just so in the *absolute electromagnetic system of units* **two points in a conductor are said to have unit difference of potential when one erg of work is required to transfer the C. G. S. unit quantity of electricity from one point to the other.**

Unit quantity of electricity in the C. G. S. system is of course *the charge transmitted* per second by unit current in that system.

The unit of potential in the absolute system is found to be so small compared with the electromotive forces of ordinary battery cells, that it was decided to adopt for ordinary use a unit one-hundred million times as great, called the *volt* in honor of Volta.

The Volt is the unit of electromotive force in the practical system and is 10^8 times as great as the C. G. S. electromagnetic unit of potential. It is much smaller that the electrostatic unit of potential defined in §554, the latter being almost exactly equal to 300 volts.

The electromotive force of the Voltaic cell is nearly 1 volt.

604. Resistance.—Let a circuit be made up of two battery cells A and B joined in series with some other conductors and a galvanometer, the two cells being so connected that their electromotive forces act in the same direction. After observing the current strength as shown by the galvanometer, let the circuit be rearranged, taking the same components in any other order whatever. If the two electromotive forces still act together the current will be found the same as before, showing that the current strength is not affected by the particular order of the parts in an electric circuit. But if one cell is turned around so that its electromotive force opposes that of the other cell, then the effective electromotive force in the circuit will be the *difference* between the electromotive forces of the two cells instead of their *sum* as in the former case, and the current in this case will be smaller than before, just in proportion as the electromotive force is smaller.

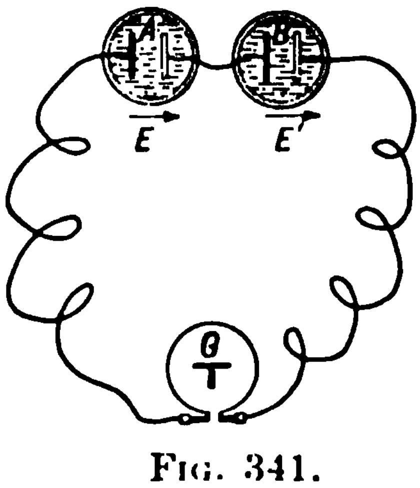


Fig. 341.

That is, the current strength is proportional to the effective electromotive force; or, in other words, **the ratio of the electromotive force to the current strength in a given circuit is a constant,** which depends only on the make-up and physical condition (temperature, stress, etc.) of the circuit. *This constant is called the resistance of the circuit and is not affected by the order in which the various conductors, cells, etc., are connected, nor by the direction in which the current flows through them.*

This relation, established by the German physicist G. S. Ohm, is known by his name and may be stated as follows:

Ohm's Law: The ratio of electromotive force to current in a given circuit is a constant which may be called the resistance of the circuit.

Or, in symbols,

$$\frac{E}{I} = R = a\ constant$$

where E represents the electromotive force, I the current, and R the resistance of the circuit.

It is also established by experiment that *each battery cell and piece of wire or other conductor has a definite resistance which belongs to it individually and depends only on its temperature and state of stress* (provided that the same two points on the conductor are always used in making connection with the rest of the circuit); and when the several parts of a circuit are joined together one after another, in *series* as it is called, the resistance of the whole is the sum of the resistances of the several parts.

605. Unit of Resistance.—In honor of the discoverer of this law the unit of resistance in the practical system is called the *ohm; it is the resistance of a circuit in which an electromotive force of one volt will produce a current of one ampère.*

Ohm's law may then be expressed in units of the practical system, thus:

$$\textit{Current in ampères} = \frac{\textit{Electromotive force in volts}}{\textit{Resistance in ohms}}.$$

The electrical resistance of a conductor is analogous to the frictional resistance which a pipe offers to the flow of liquid through it. In both cases work done against the resistance appears as heat, and in neither case does the resistance have any tendency to produce a back current.

606. Exception to Ohm's Law.—In gaseous conductors the ratio of the electromotive force to the current is not constant as in other conductors, but depends on the strength of the current.

Chemical Effects of Current X

607. Decomposition of Water.—When a current of electricity is passed through dilute sulphuric acid (1 part acid to 10 of water), using platinum electrodes immersed in the acid, gas is given off at each electrode. The gases may be separately collected in tubes filled with the *dilute acid* and inverted over the electrodes as shown

in figure 342. The gas liberated at the positive electrode is found to be oxygen while that at the negative electrode is hydrogen, and the volume of hydrogen is just twice the volume of the oxygen. These volumes are exactly in the ratio in which the gases combine to form water, and on this account it was at first supposed that the current directly decomposed water.

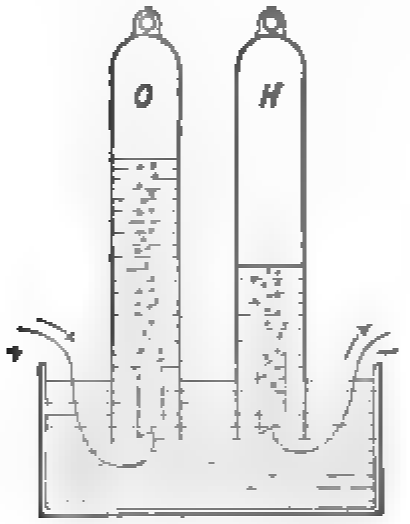


Fig. 342.—Electrolysis of water.

The decomposition of water in this way by the electric current was first accomplished in 1800 by Carlisle and Nicholson.

608. Discovery of Potassium and Sodium.—Sir Humphrey Davy, in 1807 by the use of a powerful battery of 250 cells, decomposed caustic potash, obtaining metallic potassium at the negative electrode. A fragment of caustic potash slightly moistened was laid on a platinum plate which was connected to the positive pole of the battery; on touching the potash with a platinum wire connected with the negative pole, minute globules appeared at the negative electrode which rapidly oxidized in air or took fire; these he recognized as a new metal which he named potassium. In a similar manner metallic sodium was obtained from caustic soda.

609. Faraday's Researches.—About the year 1833 Faraday began the systematic investigation of the chemical effects of the electric current.

Substances which are decomposed by the passage of the electric current he called *electrolytes;* the electrode connected with the positive pole of the battery or that through which (according to ordinary convention) current enters the electrolyte was named the *anode* (Greek, *inward path*), while the electrode through which the current leaves the electrolyte was named the *kathode* (Greek, *outward path*). The two constituents into which a molecule of the electrolyte is broken up were called *ions* (Greek, *wanderers*) that which is set free at the kathode being the *kation,* while that which appears at the anode was named the *anion.*

610. Faraday's Laws.—The following are some of the most important results of Faraday's investigations:

1. By introducing a number of electrolytic cells in different

parts of a circuit it was shown that the amount of substance decomposed is the same in each cell through which the whole current passes, and in case of a divided circuit the sum of the amounts decomposed in the branches is equal to the amount in the undivided parts of the circuit.

2. The quantity of a given substance electrolyzed in a cell is proportional to the amount of charge or quantity of electricity which passes.

3. If several electrolytic cells containing different substances are connected in series in the same circuit, the quantities of the ions set free at the electrodes are proportional to their chemical combining equivalents.

611. Electrochemical Equivalents.—The electrochemical equivalent of a substance is the quantity that is set free per second by a current of one ampère or by the passage of one coulomb of electricity. The following table gives the electrochemical equivalents of some well-known substances. It will be noticed that they are proportional to the combining equivalents.

Electrochemical Equivalents

Substance	Atomic weight	Valence	Combining equivalent	Electrochemical equivalent. Gms. per coulomb
Kations				
Hydrogen..........	1	1	1	0.000010357
Copper............	63.18	2	31.59	0.00032840
Silver.............	107.7	1	107.7	0.00111800
Anions				
Oxygen............	16	2	8	0.00008283
Chlorine..........	35.37	1	35.37	0.0003671

96,550 coulombs are transmitted when the number of grams liberated equals the combining equivalent of the substance.

612. Primary and Secondary Actions.—It is important to distinguish between the direct or primary effect of the current in electrolysis and the secondary chemical reactions that take place when the ions are set free. In illustration of this difference take the electrolytic apparatus containing dilute sulphuric acid, as described in paragraph 607, and connect it in series with a precisely similar apparatus containing a solution of sodium sulphate

in water, colored by an infusion of purple cabbage. On sending a current through, both hydrogen and oxygen gases are set free in one cell exactly as in the other, and at the same time the coloring matter in the sodium sulphate solution turns red around the positive electrode, or *anode,* and green around the negative electrode, or *kathode,* showing that the originally neutral salt has become acid at the anode and alkaline at the kathode. Analysis shows that sodium hydroxide (NaOH) has appeared at the one electrode and sulphuric acid (H_2SO_4) at the other.

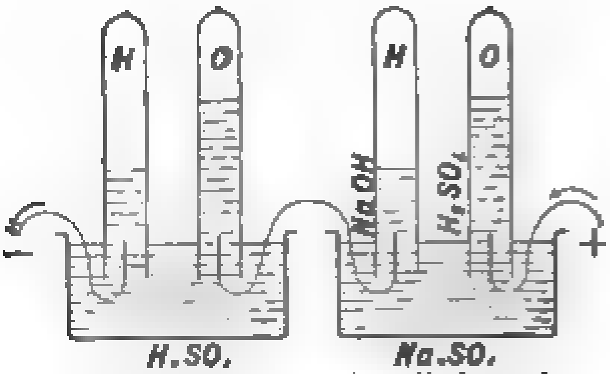


FIG. 343.—Electrolytic cells in series.

It might seem at first that more decomposition was effected by the current in one cell than in the other, in violation of Faraday's law, for equal amounts of gas are set free in both cells, and in addition the sodium sulphate in the second cell is decomposed, while the sulphuric acid in the first cell remains unchanged.

But it is believed that the *primary* effect of the current is to separate precisely equivalent quantities of H_2SO_4 and N_2SO_4 in accordance with Faraday's law, the other changes being *secondary* chemical actions.

Thus in the sulphuric acid cell the primary action of the current is to separate the H_2 and SO_4 ions; oxygen (O) is set free at the anode as the result of a secondary reaction in which the SO_4 ion displaces the oxygen from a water molecule (H_2O) and forms sulphuric acid (H_2SO_4), which remains in solution.

In sodium sulphate the primary action of the current is to separate the Na_2 and SO_4 ions. Then secondary reactions take place at both electrodes, the SO_4 ions effect the liberation of oxygen at the anode exactly as in the other cell, while the positively charged sodium (Na_2) ions pass to the kathode, where each combines with two molecules of water $2(H_2O)$, forming sodium hydroxide 2(NaOH), which remains in solution, and setting free hydrogen (H_2), which gives up its positive charge and escapes at the kathode.

These secondary reactions may be expressed symbolically thus:

$$SO_4 + H_2O = H_2SO_4 + O$$
$$Na_2 + 2(H_2O) = 2\,(NaOH) + H_2$$

613. Theory of Electrolysis.—The earlier explanations of electrolysis supposed the decomposition of the electrolyte to be effected by the electric current, but it is now believed that a large per cent. of the electrolyte is *ionized*, or broken into positively and negatively charged ions, as a result of going into solution, and that the electric force in the electrolyte is simply *directive*, causing the positively charged ions to move with the current and the negatively charged ions to move in the opposite direction through the solution until they reach the electrodes where their charges are given up and the molecules are set free in the neutral state. The current is supposed to be made up of the charges which are thus carried convectively by the moving ions.

Hittorf showed that different kinds of ions moved through the electrolyte with widely different velocities, and measured the relative velocities of anions and kations in aqueous solutions of many different salts and acids. While Kohlrausch, by measurement of the electric charges transmitted per second through these solutions, and assuming that the electric current is transmitted through the electrolyte wholly by the charges carried by the moving ions, has determined the actual velocities with which different kinds of ions move in aqueous solutions, and finds them proportional to the electric force, that is, to the fall of potential per centimeter in the solution.

Some values found by Kohlrausch are given in the following table.

Ionic Velocities for a potential gradient of one volt per centimeter

Kations		Anions	
Na	$45. \times 10^{-5}$ cm./sec.	Cl	$69. \times 10^{-5}$ cm./sec.
H	320. " "	NO^3	64. " "
Ag	57. " "	OH	182. " "

614. Ionic Charges.—Faraday's laws show that every univalent ion carries a certain charge $\pm e$ which is either positive or negative, depending on whether the ion is an anion or a kation; while bivalent and trivalent ions carry charges $\pm 2e$ and $\pm 3e$, respectively.

It was suggested by Helmholtz that the charge e may be the *atom of electricity* from which all other charges are made up and of which they are therefore multiples. This is borne out by the experiments of Millikan as we have already seen (§543).

In the electrolysis of 1 gram of hydrogen 96550 coulombs of electricity are transmitted; and assuming that each atom of hydrogen carries the charge e as found by Millikan, we find that in 1 gram of hydrogen there are 5.91×10^{23} atoms.

615. Polarization.—At the electrodes where the ions are set free or enter into new combinations there are generally electromotive forces, because at those points electric energy has to be spent to effect chemical changes. The resultant of these electromotive forces is called the polarization of the cell.

In case of the electrolysis of copper sulphate between copper electrodes the chemical change which takes place at the kathode is opposite to that at the anode: Cu and SO_4, are separated at the one and united at the other. Therefore, in such a cell there is on the whole no electromotive force of polarization.

But if dilute sulphuric acid is electrolyzed between platinum electrodes there is an electromotive force developed *against* the current, or a back electromotive force of about 1.7 volts, and unless the battery employed has an electromotive force greater than this the current cannot be maintained.

While the electrodes are thus polarized the cell is in reality a *battery cell*, and if it is disconnected from the main circuit and its electrodes joined by a conducting wire, a current is obtained opposite to that which caused the polarization. This current flows until the gaseous layers on the electrodes disappear. The cell is thus really a storage battery cell of very small capacity.

All storage battery cells or accumulators depend on the electromotive force of polarization.

When dilute sulphuric acid is electrolyzed with a zinc anode and copper kathode, as in the simple Voltaic cell, more chemical energy is given out at the anode where zinc sulphate is formed than is absorbed at the kathode where hydrogen is liberated from the solution, and consequently, on the whole, energy is given out by the chemical changes instead of being required to bring them about, hence the electromotive force of polarization is *with the current* instead of against it, and the combination is called a *battery cell*.

Measurement of Current.—Currents of electricity are con-
/ measured by their electrolytic effect. In the instrument shown in
4, known as a *voltameter*, dilute sulphuric acid is
zed between platinum electrodes, and the es-
ases are caught mingled together in the graduated
ove the electrodes. From the temperature,
and pressure of the collected gas its weight
etermined, and if the time during which the
vas flowing is known the current can be calcu-
ıce 1 ampère will set free in 1 minute 0.00559
ıbout 12.2 c.c. of the mixed gases.

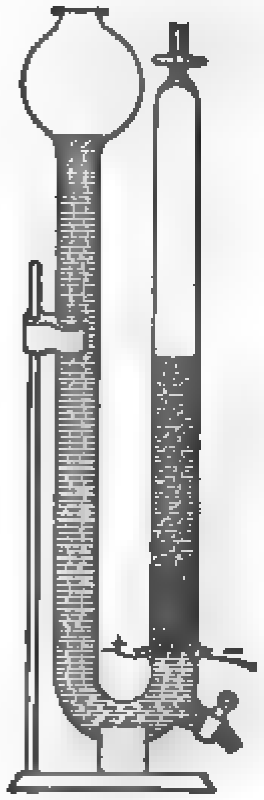
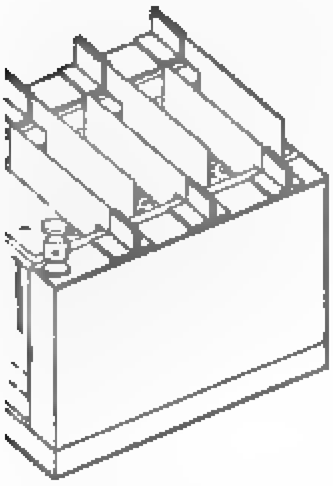


Fig. 344.—Voltameter or Coulomb-meter.

Copper Voltameter.—A more accurate in-
, for the measurement of current is the copper
er, in which copper sulphate is electrolyzed
copper electrodes. From the gain in weight
athode while the current is flowing the amount
r deposited per second is determined, and so
nt is found from the electrochemical equivalent
r. The form shown in figure 345 is con-
The alternate plates are connected into one
form the anode, while the intermediate plates
kathode, so that each kathode plate is between
le plates. The number of plates used depends
trength of the current to be measured. To
e best results something like 40 sq. cms. surface
re is required in the kathode.

Silver Voltameter.—For standard determinations it is found
most reliable results are obtained from a form of *silver voltameter* in which the liquid is a standard solution of nitrate of silver contained in a platinum cup which also serves as the kathode, while the anode is a rod of pure silver which dips into the liquid. The anode must be surrounded by a covering of filter paper to prevent any particles of silver that may become loosened from the anode from falling into the platinum cup.



—Copper voltameter.

619. Electroplating.—By means of the electric current metallic objects may be plated with gold or silver or other metals. Figure 346 shows a form of electroplating bath. The objects to be plated
on metal rods which are all connected with the negative pole of
ry or dynamo which supplies the current. If silver is to be de-
plates of silver connected with the positive pole are hung in the
ween the objects which are being plated, so that while silver is
posited *the anode plates* lose an equal amount and the strength of

the solution is maintained constant. The thickness of the deposit will generally be greater on projecting parts of the object plated and on parts that are nearer to the anode plate.

620. Capillary Electrometer.—Let a drop of mercury rest in a level tube turned up at the ends and full of dilute sulphuric acid (Fig. 347). If a current of electricity is passed through the acid the mercury will move

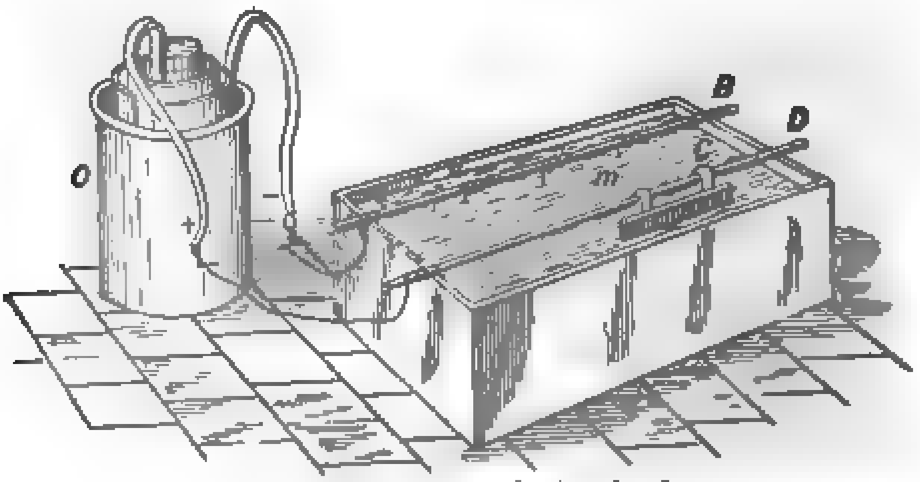


Fig. 346.—Electroplating bath.

along in the direction of the current; *i.e.*, from the positive toward the negative terminal. The surface tension where the current passes from acid to mercury is increased and that where it passes from mercury to acid is decreased, hence the motion takes place (§259).

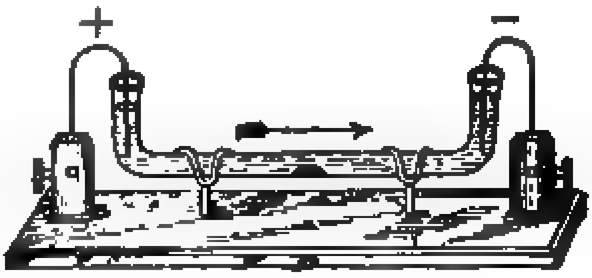
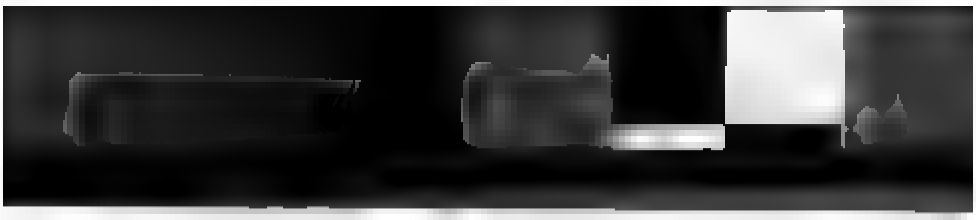


Fig. 347.—Drop of mercury in acid.

Lippmann has devised an electrometer based on this principle, known as the *capillary electrometer*, which is useful in measuring electromotive forces of less than a volt.

Problems

1. If in a copper voltameter the kathode plates gain 1.50 grm. of copper in ten minutes, find the average current strength in ampères.
2. How many grams of hydrogen and of oxygen are set free when 1 grm. of water is decomposed by electrolysis; and how many cubic centimeters of each of these gases will there be at 0°C. and 76 cm. pressure?
3. How many coulombs of electricity will be required to effect the decomposition in the previous problem, and how long a time must a current of 0.5 ampères flow to accomplish it?



Battery Cells

621. Battery Cells.—A battery cell is a combination in which electromotive force is produced by chemical action. The simple cell of Volta is the earliest type, but it has important practical defects.

An ideal cell will have:

1. Small resistance.
2. Large electromotive force.
3. A constant electromotive force whatever the current.
4. No local action or wasteful chemical action.

622. Resistance of Battery Cells.—When the electrode plates are large and close together the resistance of the cell is small. While if the plates are very small the resistance of the cell may be so great that even when the poles are *short-circuited* or connected by a short copper wire offering very little resistance, the current will be extremely small.

Cells from which large currents are to be obtained must, therefore, have large plates separated by a comparatively thin layer of electrolyte.

623. Local Action.—If commercial zinc is used in a Voltaic cell hydrogen gas will be given off at the surface of the plate as soon as it is placed in the acid and before it is connected with the copper plate. This is accompanied by a corresponding wearing away of the zinc and formation of zinc sulphate, which goes into solution. This wasting of the zinc is called *local action* and is due to impurities. Suppose that a particle of iron or carbon imbedded in the surface of the zinc is in contact both with the zinc and acid; it forms a minute Voltaic cell, in which the current flows from the iron or carbon to the zinc and through the acid from zinc to iron again, as indicated in the figure, and zinc is eaten away near the impurity and hydrogen set free at its surface.

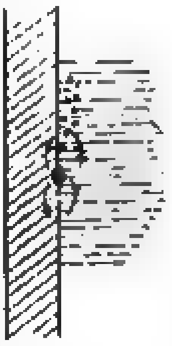


Fig. 348. Local action.

To prevent local action the zinc surface is freshly amalgamated with mercury, which dissolves the zinc, covers up the impurities, and presents a homogeneous surface to the acid.

624. Polarization.—When the poles of a simple Voltaic cell are connected by a wire, the current does not remain constant but rapidly decreases in strength.

This weakening of the current is due to *polarization*. The hydrogen set free at the copper electrode forms a sort of gaseous

layer over the plate which interferes with the action of the cell in two ways. In the first place, **the resistance of the cell is increased,** for the flow of electricity is interfered with by the bubbles of gas. In the second place, **the electromotive force of the cell is diminished,** for the hydrogen layer is much more like zinc in its relation to the acid than is the copper which it covers.

This difficulty is most effectively met by the use of two electrolytes.

625. Primary and Secondary Battery Cells.—Cells such as the Voltaic cell in which the current is obtained from the chemicals of which the cell was originally constructed are known as *primary cells,* while cells in which the chemical state necessary for the production of a current is produced by sending through the cell a current from some outside source for a certain length of time, are known as *secondary batteries, storage cells,* or *accumulators.*

A few of the cells most commonly used in practice will now be considered.

Primary Battery Cells

626. The Daniell Cell.—One of the first and most useful two fluid cells was devised by Daniell in 1836. It consists of a copper electrode immersed in a solution of copper sulphate and an electrode of amalgamated zinc immersed in dilute sulphuric acid, the two being separated by a partition of porous earthenware. In figure 349 the copper electrode with its solution is represented as contained in a cup of porous earthenware surrounded by the zinc and dilute acid.

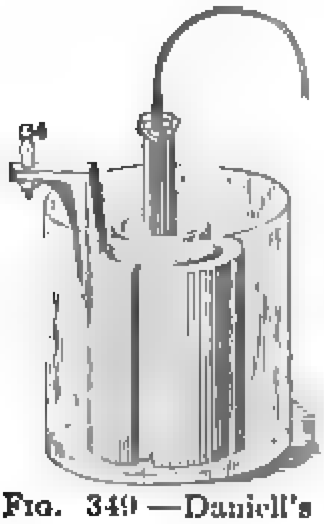


FIG. 349.—Daniell's cell

When the circuit is closed, the positively charged zinc atoms pass into solution forming zinc sulphate with the negative SO_4 ions, while the positively charged hydrogen ions (H_2) in the acid move toward the copper plate, passing through the porous cup by diffusion and forming sulphuric acid (H_2SO_4) with the negative SO_4 ions from the copper sulphate, and displacing the positive copper ions (Cu)

which give up their charges and are deposited on the copper plate.

In the dilute acid	$Zn + H_2SO_4 = ZnSO_4 + H_2$.
In the copper sulphate	$H_2 + CuSO_4 = H_2SO_4 + Cu$.

Thus zinc is dissolved and zinc sulphate formed, copper sulphate is used up and copper deposited on the copper electrode. There is no hydrogen layer formed on the copper and consequently no polarization. The electromotive force of this cell is about 1.08 volts.

627. Gravity Cell.—A form of Daniell cell which has been extensively used in telegraphy and is still much used where a *small constant current* of electricity is required is the *gravity cell*, so called because the liquids are kept separate by gravity alone, the denser copper sulphate solution resting at the bottom of the cell, while the lighter acid or zinc sulphate solution floats above it.

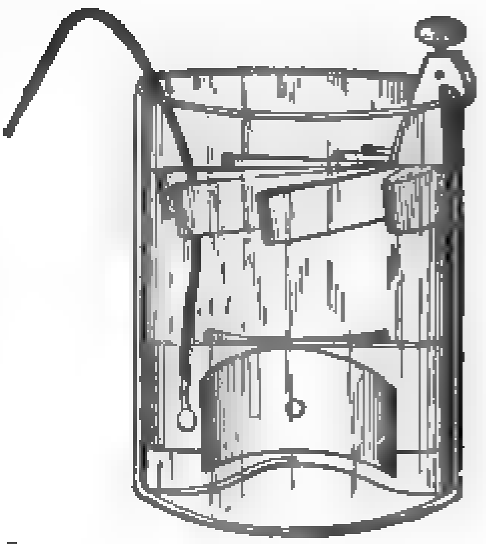


FIG. 350.—Gravity or crowfoot cell.

If the gravity cell stands without being used the copper sulphate diffuses gradually up into the acid above and copper is deposited on the zinc, causing extensive local action. A small current, sufficient to balance the diffusion, should always be kept flowing while the cell is set up.

628. The Bichromate Cell.—In this cell the positive pole is a plate of gas carbon in a solution of bichromate of potassium. The negative pole is of zinc in dilute sulphuric acid and the two solutions are separated by a cup of porous earthenware which holds one of them. The electromotive force of the cell is about 2.0 volts.

In the ordinary *plunge* battery the carbon and zinc plates both dip into the same bichromate solution to which a little sulphuric acid has been added. The zinc plate is lifted out of the solution when not in use.

629. Leclanché Cell.—This very useful form of cell has a zinc and a carbon electrode. The carbon is packed in a porous cup with a mixture of fragments of carbon and black oxide of manganese; the zinc electrode is in a strong solution of ammonium chloride (sal ammoniac) which surrounds the porous cup.

The hydrogen which would polarize the carbon electrode combines with oxygen from the manganese dioxide and forms water. But as the depolarizing agent is in the solid form its action is slow, and the cell polarizes temporarily. It is extensively used, however, for open-circuit work, such as for

bells, annunciators, and clocks, where a steady current is not required. It is entirely free from injurious or disagreeable fumes, there is but little local action, and no trouble from diffusion, so that the cells may stand set up for a year or two without attention, and ready for use at any instant. Its electromotive force is about 1.40 volts.

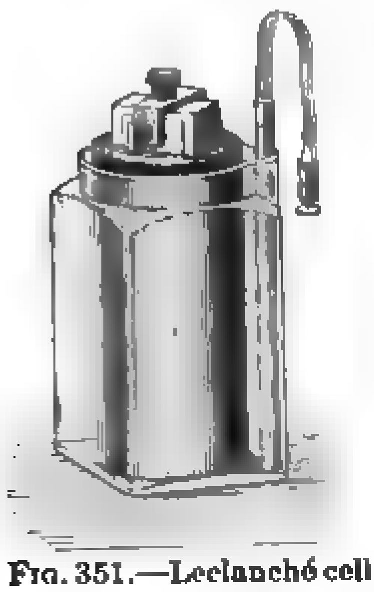


FIG. 351.—Leclanché cell.

The chemical changes in this cell are:

$$2NH_4 + MnO_2 = 2NH_3 + H_2O + MnO$$
$$2Cl + Zn = ZnCl_2$$

630. Dry Cells.—The so-called dry cells are ordinarily a form of Leclanché cell. The outer cylindrical cup forms the zinc electrode which is lined with thick absorbent paper and packed with the pulverized manganese dioxide and carbon mixture surrounding the central carbon rod. The whole is saturated with ammonium chloride solution and sealed with pitch to keep it from drying out.

631. Edison-Lalande Cell.—In this form of cell, devised by Lalande and improved by Edison, the positive plate is a tablet of compressed black oxide of copper, while the negative plate is of zinc. These are immersed in a strong solution of caustic potash, which is covered with a thick layer of heavy oil to prevent evaporation and the creeping up of the solution on the sides and plates. The plate of copper oxide acts both as electrode and depolarizer, the hydrogen which is set free at that pole reducing the copper oxide to metallic copper.

When the cell is exhausted both plates as well as the liquid must be renewed.

This cell does not polarize and may be used where a steady current is required and where a Daniell or gravity cell would have too much resistance. It has the further advantage that it is quite free from local action and may be left standing without deterioration when no current flows. Its electromotive force is low, being about 0.75 volt.

Its chemical changes may be expressed as follows:

$$2K + CuO + H_2O = Cu + 2KOH$$
$$2OH + Zn + 2KOH = K_2ZnO_3 + 2H_2O.$$

Secondary Cells

632. Grove's Gas Battery.—The English physicist Grove showed that when in the decomposition of water long electrodes were used, extending to the tops of the tubes in which the gases were collected, as in figure 352, on changing the switches ss' to the dotted positions, thus disconnecting the batter[y] supply

joining the two electrodes together through a galvanometer G, a current was obtained which was in the opposite direction to the decomposing current. At the same time a gradual recombination of the hydrogen and oxygen took place until these gases had entirely disappeared.

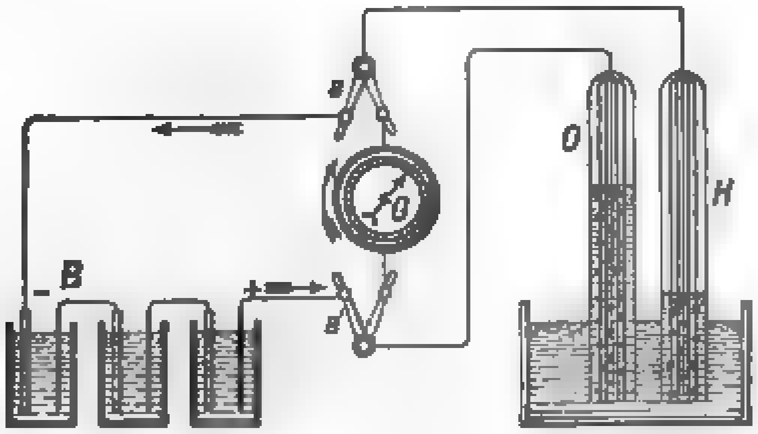


FIG. 352.

Long electrodes were necessary since each electrode must pass through the surface where the gas and electrolyte meet.

633. Planté Cell.—If a current of electricity is sent through a cell consisting of two plates of sheet lead in dilute sulphuric acid it becomes polarized, one plate becoming oxidized while hydrogen is set free at the surface of the other, reducing any oxide that may be there.

In the year 1860 Planté, a French physicist, found that secondary cells of large capacity could be made in this way. His method was to send a current through the cell in one direction until one plate was well oxidized, after which the cell was discharged and then charged by a current in the opposite direction, thus oxidising the other plate and reducing the oxide on the first to metallic lead in a spongy form. The cell was then again discharged and charged with a current in the same direction as at first, and so by alternately charging and discharging, first making one plate positive and then the other, a deep layer of active material was formed on each plate. The plates were then said to be *formed.*

634. Storage Cells—Accumulators or Secondary Batteries.—Secondary battery cells, or storage cells as they are frequently called, have become extensively used in electric motor vehicles, in electric power plants, and in telegraphy. The plates are usually heavy lead grids full of holes or grooves containing the active material, which is either packed in them mechanically or formed in them by some such process as that used by Planté. The positive plates contain a high oxide of lead, PbO_2, while the active parts of the negati[illegible]f spongy lead. A cell is formed of a number of such

plates, alternately negative and positive, as shown in the figure, set in a suitable vessel containing dilute sulphuric acid. The negative plates are connected together and form one set and the positive plates form another, there being one more negative than positive plate, so that each positive plate is between two negative ones. This is to prevent buckling or bending of the positive plate, for the formation of oxide in charging is accompanied by an increase in volume or swelling of the plate, which would warp it badly if it took place only on one side.

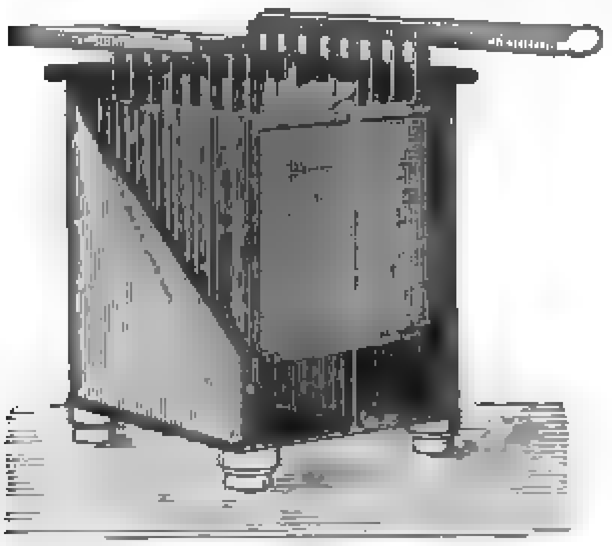


FIG. 353.—Storage cells.

The nearness of the plates to each other and the large surface obtained by using a number of plates cause the resistance of the cell to be very small. The greater the number and size of the plates in a cell the larger the current that can be sent through it without injury to the cell. About 1 ampère per 12 sq. in. of opposed surface is usually a safe rate of discharge.

The commercial importance of such storage cells is due in part to their *extremely small resistance* and to the fact that they are renewed not by means of costly chemicals, but by a current obtained from a dynamo machine driven by an engine or by water power. They can be used, therefore, to store the superfluous energy of a power plant at times when but little power is used and give it back again in times of need.

The electromotive force of this type of storage cell is about 2.10 volts.

The chemical changes in such a cell are as follows:

Discharging

Positive plate $PbO_2 + H_2 + H_2SO_4 = PbSO_4 + 2H_2O$

Negative plate $Pb + SO_4 = PbSO_4$

Lead sulphate is thus formed at each plate.

Charging

Positive plate $PbSO_4 + SO_4 + 2H_2O = PbO_2 + 2H_2SO_4$

Negative plate $PbSO_4 + H_2 = Pb + H_2SO_4$

635. Edison Storage Cell.—A new form of storage cell has been devised by Edison in which the active materials of the electrodes are the oxides of nickel and iron, respectively, the electrolyte being a solution of caustic potash in water. A battery of these cells is said to weigh one-half as much as the equivalent lead cells. The cells are very durable and are not so easily injured as lead cells by overcharging or leaving uncharged.

Modes of Connecting Cells

636. Battery Cells in Series.—When battery cells are connected as shown in the figure, the positive pole of one being joined to the negative pole of the next, they are said to be joined in **series, and the electromotive force of the combination is the sum of the electromotive forces of the several cells.** As the whole current must pass through each cell **the resistance of cells joined in series is the sum of the resistances of the separate cells.**

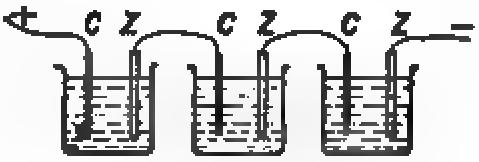


FIG. 354.

Suppose that three cells, each with electromotive force e and resistance r, are connected in this way with an external resistance R. The total electromotive force is $3e$ and the resistance is $3r + R$ so that by Ohm's law,

$$I = \frac{3e}{3r + R} \qquad \text{(for 3 cells in series)}$$

where I is the current.

This arrangement is advantageous when the external resistance R is large and a large electromotive force is required.

Battery cells in series may be likened to a series of pumps, the first of which lifts water to a certain level where the second takes it and lifts it to the next higher level and then the third raises it again to a still higher level, etc.

637. Battery Cells in Parallel.—If cells are joined together as shown in figure 355, all the copper poles being connected together for the positive pole and all the zincs for the negative, they are said to be joined *in parallel.*

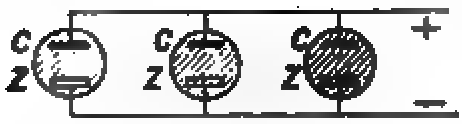


FIG. 355.

Such a combination has precisely the advantage that a large cell has over a small one. **Its electromotive force is the same as that of one cell, while its resistance is less**—a combination of four similar cells joined in this way having only one-fourth the resistance of a single cell.

Only similar cells should be joined in parallel, otherwise a cell of smaller elec otive force may have a reverse current sent through it : cell.

This m nent is useful when the external resist-

ance in the circuit is much smaller than that of a single battery cell or where the current to be obtained is more than can advantageously be transmitted through a single cell.

Cells in parallel may be likened to a set of pumps which are all lifting water from the same lower canal to another at a higher level.

638. Combined Series and Parallel Arrangement of Cells.—Several similar series of cells may be combined in parallel as shown in figure 356, where two series of three cells each are connected in parallel.

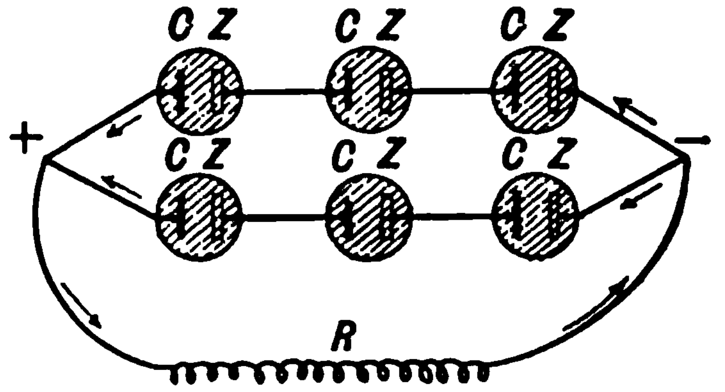


FIG. 356.

It will be observed that the resistance of each of the rows is $3r$ and the electromotive force of each row is $3e$. The resistance of the two rows in parallel is then $\frac{3r}{2}$ or one-half that of the row, while the electromotive force of the combination is the same as that of a single row. The expression for current is then, for this particular arrangement,

$$I = \frac{3e}{\frac{3r}{2} + R}.$$

A square arrangement, where the number of cells in each row is the same as the number of rows, will have the same resistance as a single cell.

If a given number of cells are to be combined so as to give the maximum current through a given outside resistance, use that arrangement which will make the battery resistance most nearly equal to the external resistance.

Commercial types of storage battery cells have such small resistances that except where very large currents are required they are connected in series. If large currents are to be obtained, several series of cells are arranged in parallel so that the current through each series will only be such as the cells are adapted to transmit without injury, and in each series as many cells are used as are necessary to give the required electromotive force.

Problems

1. If two gravity cells and one Leclanché cell are joined in series with a coil of wire having a resistance of 5 ohms, what current is obtained, when the gravity cells have a resistance of 2 ohms each, while the Leclanché cell has resistance 0.4 ohms?
2. Find the current when the zinc pole of a gravity cell of 2 ohms resistance is connected to the zinc pole of a Leclanché cell of 0.5 ohms resistance while their other poles are connected by a wire of 1 ohm resistance.
3. Reverse the gravity cell in problem 2 so that its copper pole is connected to the zinc pole of the Leclanché cell, the other poles being connected by the 1-ohm wire, and find the current as before.
4. Make a diagram of two gravity cells of 2 ohms resistance each, connected in parallel to a coil of wire having 1 ohm resistance, and show what is the current in the wire and what current through each cell.
5. What is the electromotive force and internal resistance of a combination of 12 gravity cells, consisting of three series of four cells each, the three being connected in parallel? Take resistance of each cell 2 ohms and its E.M.F. 1 volt.
6. When the terminals of the battery described in question 5 are connected by a wire having a resistance of 3 ohms, find the current in the wire and also in each cell.
7. If a single storage cell has an electromotive force of 2 volts and a maximum permissible discharge rate of 10 ampères, how many such cells will be required and how arranged to give a current of 30 ampères and have an electromotive force of 50 volts?
8. If the cells in the last problem each have a resistance of 0.01 ohm, find what is the smallest resistance that can be permitted in the outside circuit.
9. How many gravity cells having a resistance of 2 ohms and E.M.F. 1 volt each, will be required to light a 50-volt incandescent 16-candle-power lamp which has a resistance of 50 ohms and requires 1 ampère of current, and what arrangement will require the smallest number of cells?

   Note.—The smallest number will be required when the battery resistance is equal to the external resistance.

Fall of Potential and Resistance

639. Fall of Potential along a Circuit.—In every electric circuit there is a gradual decrease of potential along the external circuit from the positive to the negative pole of the battery.

Faraday showed that *in all parts of a simple undivided circuit the current is of the same strength.* Therefore, except at points where electromotive forces are introduced, the potential must everywhere gradually change from point to point, for there is

no current between points at the same potential. In figure 357 is shown a circuit with a corresponding diagram of potentials, the potential at each point of the circuit being represented on the diagram by the vertical distance from the base line to the upper curve. Starting at C, it will be observed that the potentials steadily diminish till the zinc pole is reached at Z, where the lowest value is found. But passing from the zinc into the acid the potential is raised to its maximum at A by the chemical action; here there is an electromotive force and consequently a sudden change in potential. In the acid the potential again steadily falls as we pass from zinc to copper, until at

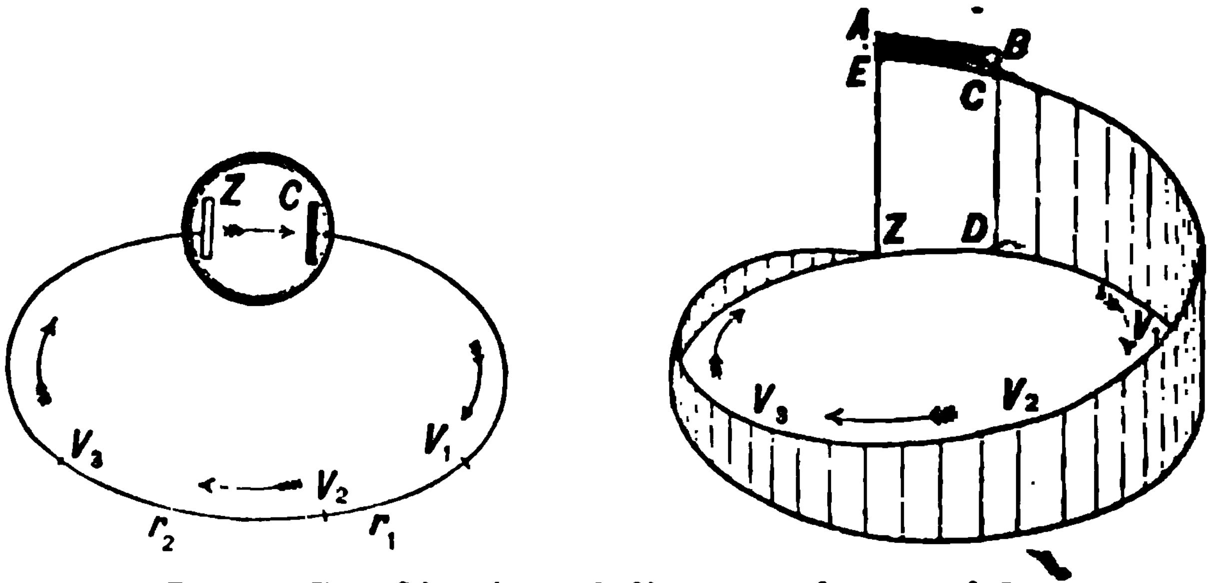


FIG. 357.—Circuit and diagram of potentials.

the surface of the copper there is another electromotive force and a consequent sudden drop in potential bringing us again to the starting point at C.

If in the diagram AE is laid off equal to BC, then EZ is the difference of potential which represents the total electromotive force of the cell, for it is the difference between the two oppositely directed electromotive forces AZ and BC, and the fall of potential from A to B is equal to that from E to C. It is therefore clear that **the total fall of potential around the circuit, including that which takes place within the acid as well as that in the outside circuit, is equal to the total electromotive force of the cell.**

It will be noticed that CD, *the difference of potential between the two poles, is equal to the fall of potential in the external circuit,* and *is therefore less than the total electromotive force of the cell whenever there is any current flowing,* for there is then a fall of potential within the cell itself from A to B.

640. Ohm's Law Applied to Part of a Circuit.—It has been seen (§604) that in any whole circuit

$$I = \frac{E}{R} \tag{1}$$

where E is the electromotive force in the circuit and R is a constant known as the resistance of the circuit. Similarly in any *part* of a circuit such as that between A and B (Fig. 358) the current I is proportional to the difference of potential between those points and may be written

$$I = \frac{P}{r} \quad \text{or} \quad Ir = P \tag{2}$$

where P is the drop in potential between A and B and r is the resistance of that part of the circuit.

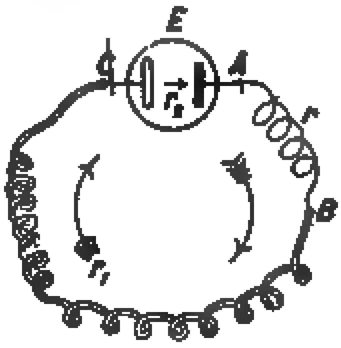


FIG. 358.

When there is no source of electromotive force, such as a battery cell, in a given portion of a circuit, the difference of potential between its ends in volts is equal to the product of the current in ampères by the resistance of that portion in ohms.

When an electromotive force E is included in any part of the circuit considered, the difference of potential between the ends of that part may be written

$$P = Ir - E \qquad (E.M.F. \textit{ in same direction as current}) \tag{3}$$

or

$$P = Ir + E \qquad (E.M.F. \textit{ acting against the current}) \tag{4}$$

For Ir measures the *drop* in potential in the direction of the current, but when E is *with* the current it lifts the potential, as seen in the preceding paragraph. The sign of E is therefore opposite to that of Ir in that case.

641. Resistances in Series.—In a complete circuit made up of several conductors, having resistances $r\,r_1\,r_2$, and including an electromotive force E, as in figure 358, the successive steps in potential may be written thus

from A to B	resistance $= r$	$P = Ir$	(5)
from B to C	resistance $= r_1$	$P_1 = Ir_1$	(6)
from C to A	resistance $= r_2$	$P_2 = Ir_2 - E$	(7)

The sum $P + P_1 + P_2$ is evidently the total change of potential around the circuit from A around to A again, but this must be zero for it ends at the same potential as it began. Therefore, adding 5, 6, and 7 we have

$$Ir + Ir_1 + Ir_2 - E = 0$$

but this may be written

$$I = \frac{E}{r + r_1 + r_2}$$

and by Ohm's Law

$$I = \frac{E}{R}$$

hence $R = r + r_1 + r_2$; *i.e.*, **the resistance of several conductors connected in series is the sum of their separate resistances.**

642. Combining Resistances in Parallel.—Let three conductors having resistances r_1, r_2, r_3 be joined in parallel in a battery circuit as shown in figure 359. It was shown by Faraday that the sum of the currents in the branches is equal to the total current I before it divides,

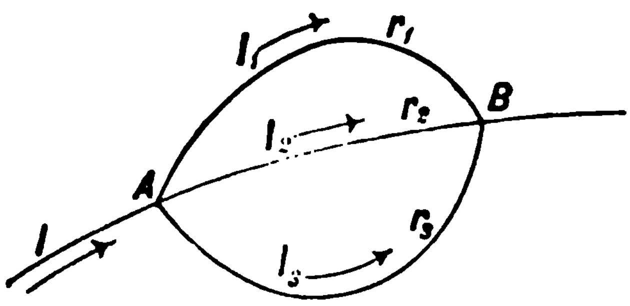


Fig. 359.

$$I = I_1 + I_2 + I_3 \tag{1}$$

But the drop in potential from A to B must be the same along either branch. Letting P represent this drop we have

$$I_1 = \frac{P}{r_1} \quad I_2 = \frac{P}{r_2} \quad I_3 = \frac{P}{r_3}.$$

Therefore by (1)

$$I = \frac{P}{r_1} + \frac{P}{r_2} + \frac{P}{r_3} = P\left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}\right).$$

But if R is the effective resistance of the three branches combined

$$I = \frac{P}{R}.$$

Therefore

$$\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}.$$

The reciprocal of the resistance of a conductor is called its

conductance, hence **the sum of the conductances of several conductors joined in parallel is the conductance of the combination.**

If the three resistances above considered are equal the combination will have one-third the resistance of one alone.

643. Galvanometer Shunt.—It frequently happens that a current is to be measured by a galvanometer adapted to smaller currents. In such a case a wire S of suitable resistance, called a *shunt* (Fig. 360), may be connected across from one galvanometer terminal to the other. The current then divides between the shunt and the galvanometer. If the resistance of the galvanometer is just 9 times that of the shunt used, the current will divide in the ratio 1 : 9, so that one-tenth of it flows through the galvanometer and nine-tenths through the shunt.

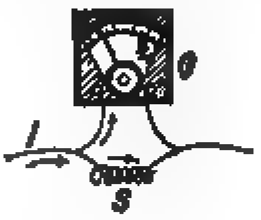


Fig. 360.

644. Resistance of Wires and Specific Resistance.—The resistance of a wire or of any conductor of uniform cross-section increases with its length and is inversely proportional to its cross section. The results of the last two articles show that this is so. For if the cross-section of a conductor is doubled it is equivalent to two of the original conductors side by side in parallel, and hence by §642 the resistance is one-half as much as before.

The resistance of a cylindrical conductor of a given substance one centimeter long and one square centimeter in cross section is called the specific resistance of that substance, or its resistivity.

When a wire of length l and cross section s is made of a substance having *resistivity* ρ, its resistance R is given by the formula

$$R = \frac{\rho l}{s}.$$

645. Resistivities.—The curves given in figure 361 show the specific resistances of certain pure metals and alloys and also the variation of the resistances with temperature.

If the curves for the pure metals are produced it will be found that they intersect the base line in the region of the absolute zero ($-273°$). The experiments of Onnes on the resistance of gold, silver, mercury, lead and tin at very low temperatures show that as the temperature is lowered they approach zero resistance at a point *a few degrees above the absolute zero*, the change being sud-

den at the last. The resistance of mercury, for example, creases slowly from 4.41° to 4.21° above the absolute zer then rapidly diminishes and practically disappears at 4.19°. following table shows some specific resistances at 0°C. in

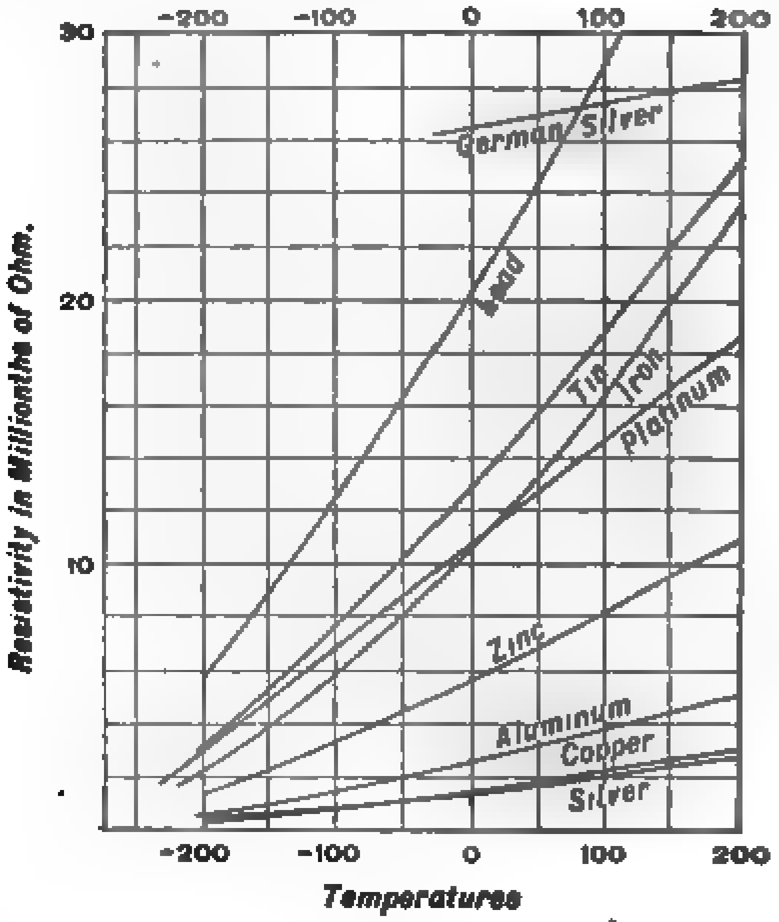


Fig. 361.

lionths of an ohm with the corresponding increase in resista per ohm when the temperature is raised from 0° to 100°C. (Fi Dewar and Fleming.)

Specific Resistances at 0°C

Substance	Millionths of an ohm	Increase per ohm fr 0° to 100°C
Platinum	10.917	0.367 ohm
Silver	1.468	0.400
Copper	1.561	0.428
Iron	9.065	0.625
Nickel	12.323	0.622
Lead	20.380	0.411

The resistance of carbon decreases with rise of temperature instead of increasing, so that the filament of an incandescent lamp may have only one-third as much resistance when hot as when cold.

The resistivities of alloys cannot in general be calculated from those of their constituents, but are often much greater than would be expected. The temperature coefficients of German silver, platinoid, and manganin are much less than those of pure metals; for this reason as well as for their large specific resistances these substances have been used extensively in making resistance coils.

Alloys	*Temperature Coefficients*
German-silver (Cu 50, Ni 26, Zn 24)......	0.00040
Platinoid (Cu 60, Ni 14, Zn 24, Tg 2)......	0.00022
Manganin (Cu 84, Ni 12, Mn 4)..........	0.000001

646. Standard Resistance.—Standard resistances are made of wire having a small temperature coefficient and not otherwise subject to change. The best coils are made of *manganin*. The

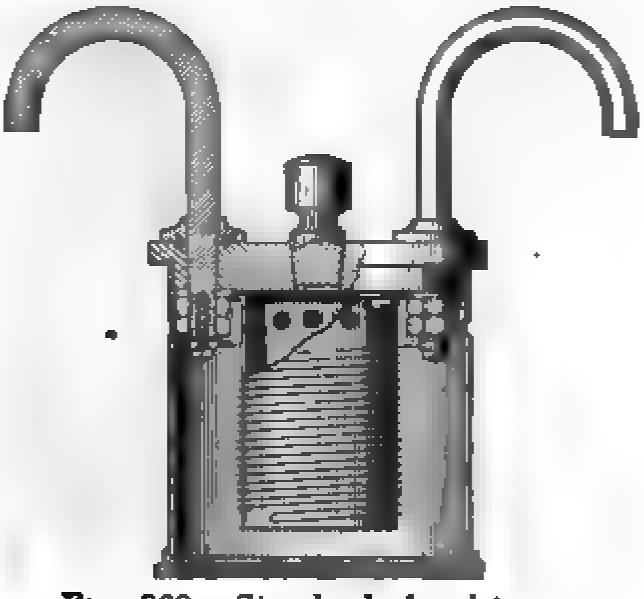


Fig. 362.—Standard of resistance.

coil is provided with heavy copper terminals of almost negligible resistance, and is so mounted that it will quickly take the temperature of the oil bath in which it is immersed, and by which its temperature is maintained constant.

647. Resistance Boxes.—Boxes of coils having different re-

sistances are made so as to be conveniently used in measurements, as shown in figure 363. On the hard-rubber top of the box are mounted a number of blocks of brass which can be connected by brass plugs fitting between them. Within the box are the resistance coils wound on spools, one end of a coil being soldered to one block and the other end to the next one so that

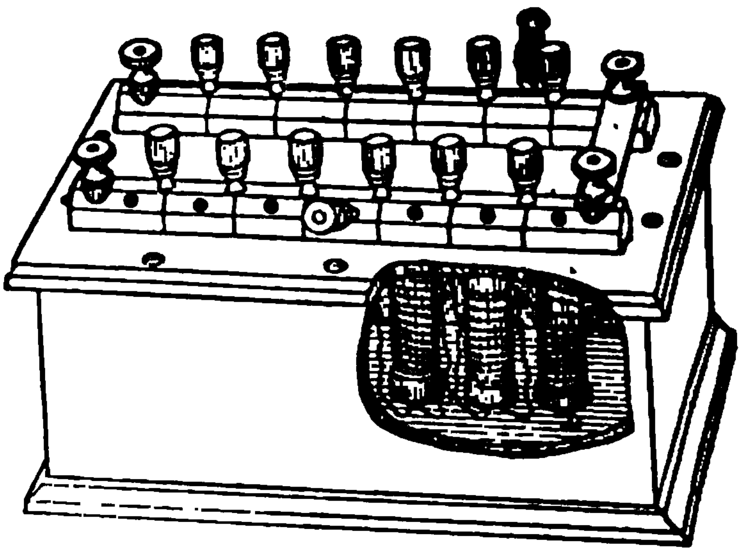


FIG. 363.—Resistance box.

one coil bridges each gap. The external circuit is connected at the terminal binding screws, and when all the plugs are in, the only resistance is that of the brass blocks and plugs themselves. But if a plug is pulled out the current must then flow through the coil joining the blocks, and accordingly that resistance is introduced.

648. Rheostats.—Coils of wire so mounted that they can easily be thrown into or out of a circuit to regulate the strength of current without particular reference to measurement are known as *rheostats*. A convenient form is shown in figure 364, where, as the radial arm is moved around the dial from block to block, one coil after another is added to the circuit until as many as may be desired are thrown in.

649. Wheatstone's Bridge.—If a current from a battery E (Fig. 365) divides between the two conductors ACB and ADB, the resistances of these branches may be very different and consequently the current in one may be much larger than in the other, but as they both start at the same point A and end together at B, the *fall in potential* must be the same in each, and corresponding to any point, such as C in the one, there must be a point D in the other where the potential is the same.

If p, q, r, s are the resistances of the four segments, AC, CB AD, DB then it may be shown that $p:q::r:s$.

Let I_1 be the current in the upper branch and I_2 that in the lower branch, then I_1p is the drop in potential from A to C and is equal to I_2r the drop from A to D, thus

$$I_1p = I_2r \tag{1}$$

so also

$$I_1q = I_2s \tag{2}$$

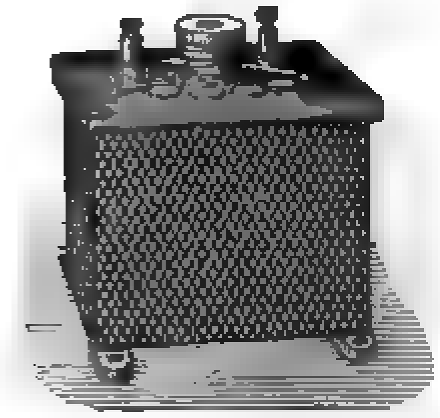


Fig. 364.

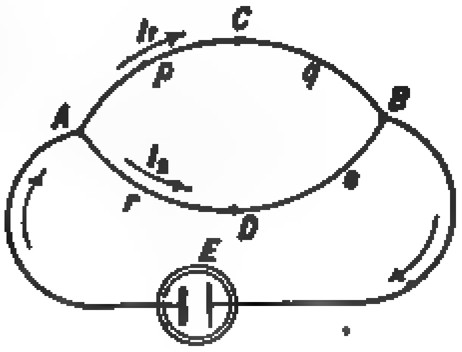


Fig. 365.

and dividing (1) by (2) we find

$$\frac{p}{q} = \frac{r}{s}.$$

650. Slide Wire Bridge.—The relation just demonstrated is made use of in the comparison of resistances, a convenient device for the purpose being the slide wire bridge shown in figure 366.

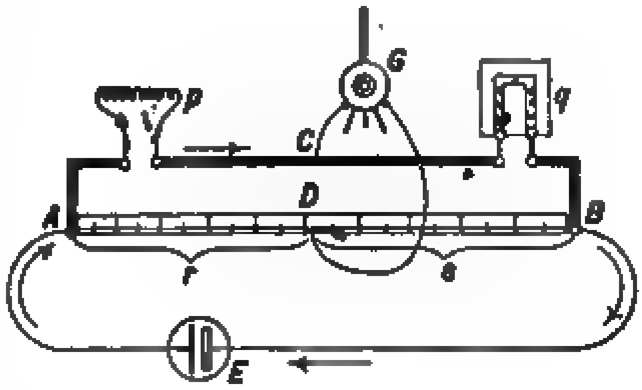


Fig. 366.—Slide wire bridge.

Suppose p is some coil of wire whose resistance is to be measured by comparison with a standard resistance box q. The current from the battery E divides at A, part flowing through the branch ACB which consists of the two resistances to be compared, p and q, connected by thick copper strips of extremely small resistance, while part flows through the branch ADB which is a uniform wire stretched along a graduated scale.

A sensitive galvanometer has one terminal connected at C midway between p and q while the other is attached to a slider which is *moved along* the stretched wire until the point D is

found where there is *no deflection of the galvanometer*, showing that C and D are at the same potential. Then by the previous paragraph the resistance of p must be to that of q as the resistance of r is to that of s, where r and s are the segments of the bridge wire on each side of D. But since the wire is uniform the *resistances* of r and s are in the same ratio as their *lengths*, so that the resistance of p is to that of q as the length of r is to the length of s, and therefore p may be calculated by proportion when q is known.

The resistances of the heavy copper connecting strips are so small compared with the resistances of p and q that they may ordinarily be neglected.

651. Platinum Resistance Thermometer.—The increase in resistance in a coil of platinum wire when its temperature is raised has been used by Callendar in the accurate measurement of temperature.

Problems

1. An electric car line has a resistance of 0.4 ohm per mile. What is the drop in potential in the line if a car 3 miles from the station is using 50 ampères?
2. If one car 1 mile from station and another 2 miles from the station are each using 50 ampères, what is the drop in potential to the more distant car, resistance being as in preceding problem?
3. What external resistance when joined to a gravity cell having a resistance of 2 ohms will make the potential difference between the terminals of the cell 0.7 of its electromotive force?
4. A gravity cell of E.M.F. 1 volt and resistance 2 ohms, is connected with another battery by which a current of 1 ampère is made to flow through the cell from the copper to the zinc pole inside the cell. Find the drop in potential in the cell due to resistance, also the difference in potential between the two poles and which is at the higher potential.
5. When the conditions are as in problem 4 except that the current flows through the cell in the opposite direction, find the potential difference between the poles and which is at the higher potential.
6. When the current through the cell of problem 4 is ½ ampère, and flows through the cell from the zinc toward the copper pole, what is the potential difference between the two poles?
7. A gravity cell of resistance 2 ohms and E.M.F. 1 volt, a dry cell having a resistance of 0.5 ohm and E.M.F. 1.4 volts and a wire of resistance 2.3 ohms are joined in series. Find the drop in potential due to resistance in each part of the circuit, also the potential difference between the terminals of each cell.

8. If the cells in the preceding problem are reversed so that one acts against the other, find the drop in potential in each cell and in the external resistance and the potential differences as before.
9. What is the resistance of two conductors connected in parallel, one of 3 and the other of 10 ohms resistance?
10. When a current of 31 ampères divides between three parallel conductors whose resistances are 2, 3, and 5 ohms, respectively, find the current in each branch, also the drop in potential in the parallel combination.
11. What part of the whole current will flow through a galvanometer having a resistance of 5 ohms if shunted by a wire of 0.1 ohm resistance?

Energy and Heating Effect of Current

652. Energy of a Current.—When a current flows from a point where the potential is V_1 to another point where it is V_2, each unit charge that passes has less energy at the lower potential than at the higher, **and the difference between the two must be the energy which in some form or other is spent between the two points; it may be in heat in the conductor, or in chemical action, or in doing mechanical work.** When unit charge passes from V_1 to V_2 (Fig. 367) the work expended is $V_1 - V_2$ (§554). If Q units pass in t seconds the work is

$$w = Q(V_1 - V_2)$$

and the energy spent per second is

$$\frac{w}{t} = \frac{Q}{t}(V_1 - V_2)$$

But $\frac{Q}{t}$ equals I, the *current*, and the potential difference $V_1 - V_2$ is represented by P.

Therefore

$$\frac{w}{t} = I(V_1 - V_2) = IP$$

Or, the energy spent per second in any part of a circuit is the product of the current strength by the fall in potential in that part.

If the current and potentials are measured in C. G. S. units then the product will give the energy spent in ergs per second. When the current is in ampères and the potentials are in volts the energy per second is given in units called *watts*.* Since the

*The watt is named in recognition of the researches of James Watt on the power of engines.

volt is 10^8 times the C. G. S. unit of potential, while the ampère is one-tenth the C. G. S. unit of current, it follows that *one watt = 10^7 ergs per second.*

Thus a watt represents a certain rate of spending energy per second, it is therefore a unit of power, and bears a definite ratio to other units of power.

$$1 \textit{ Horse-power} = 746 \textit{ watts} = 550 \textit{ foot-lbs. per second.}$$

653. Where Energy is Absorbed and Where Given Out.— From the diagram (Fig. 367) it is seen that everywhere in the external circuit from C to Z the current flows from points of

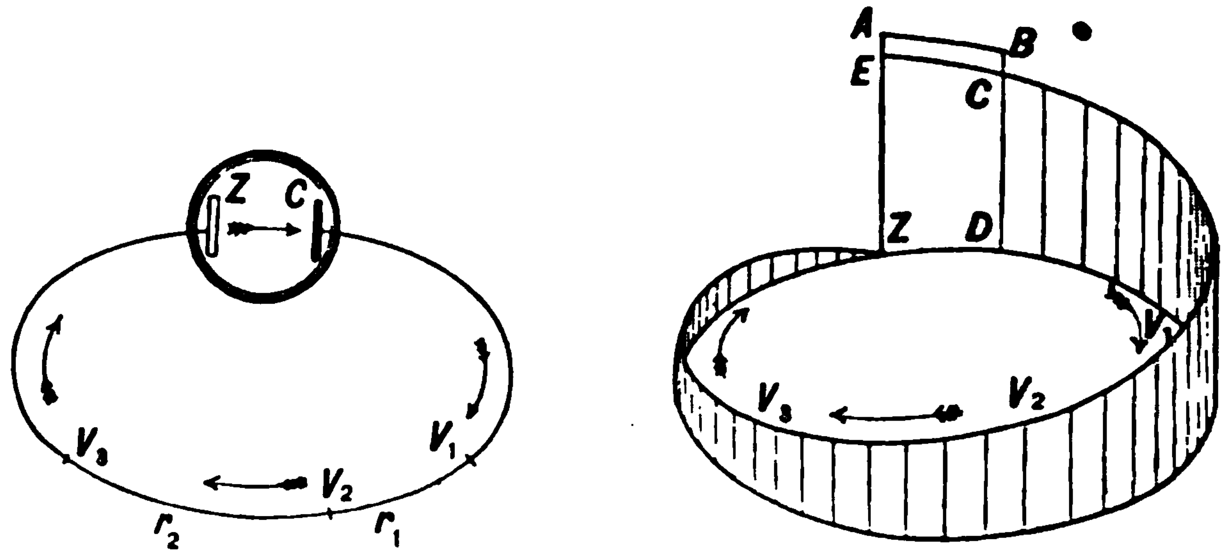


Fig. 367.

higher to lower potential, and is therefore spending energy. But there is a great *rise* in potential from the zinc to the acid at A; at this point the current must therefore *receive* energy from the chemical action which effects the transfer in the face of the opposing difference of potential. From A to B there is again a fall in potential and spending of energy within the battery cell, then at B there is a sudden drop of potential which shows that at that point also energy must be spent, but in this case against the chemical forces which at this point exert an electromotive force *against* the current.

The only place where energy is absorbed by the current is at the surface of the zinc plate, and therefore the energy of the chemical action at the zinc plate supplies that which is spent in all other parts of the circuit. This conclusion is based on the law of the conservation of energy.

At any point in the circuit where there is an electromotive

rce E, the energy taken in or given out per second, is IE, where is the current strength. The energy is absorbed if the current is with the electromotive force, and given out if the current against it.

654. Heating Effect of Current.—Suppose a circuit, such as own in figure 368, contains a battery B, a coil of wire of restance r, a motor M, and a cell N ntaining some electrolyte. Let P, P_1, and P_2 be the potential differences etween the terminals of the coil, the otor and the cell, respectively, when ere is a current of I ampères.

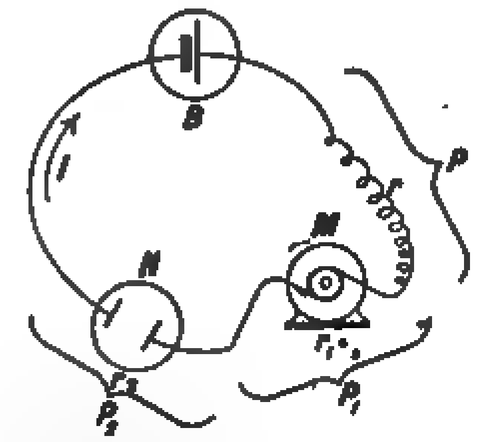


Fig. 368.—Composite circuit including resistance, motor, and electrolytic cell.

Then IP, IP_1, and IP_2 are the watts pent in the corresponding parts of e circuit.

In the coil there is no electromotive orce and therefore

$$P = Ir \qquad \text{and} \qquad W = I^2r.$$

n this case there is no mechanical or chemical work done and ll the energy is spent in heat.

In the motor there is an electromotive force against the urrent as will be shown later (§742) and therefore

$$P_1 = Ir_1 + E_1 \qquad \text{and} \qquad W_1 = I^2r_1 + IE_1.$$

The term I^2r_1 represents the power spent in heat while IE_1 the power spent in driving the motor.

In the third case, where there are chemical changes, there is sually also an electromotive force which may be either with the urrent, as in case of a battery cell, or against it, as in charging a torage cell; therefore, $P_2 = Ir_2 \pm E_2$

$$\text{and } W_2 = I^2r_2 \pm IE_2.$$

Here, again, I^2r_2 is the watts spent in heat, while IE_2 is the atts spent in chemical work or received from chemical work s the case may be. (In which case is the sign to be taken lus?)

The *heating effect* of a c nt of I amperes in a resistance of ohms may always be e tts by the formula

But 1 watt = 10^7 ergs per second, and 1 gram-calorie of heat is equivalent to 4.187×10^7 ergs, therefore, 4.187 watts = 1 gram-calorie of heat per second, and we have,

$$\textit{heat in gram-calories per sec.} = \frac{\textit{watts}}{4.187}.$$

Summary

The *total* watts spent in any portion of a circuit in which P is the fall in potential in volts and I is the current in ampères, is given by the formula

$$W = IP$$

the watts spent in *heat*, by

$$W = I^2r$$

while the heat in gram-calories per second is expressed by

$$H = \frac{I^2r}{4.187}.$$

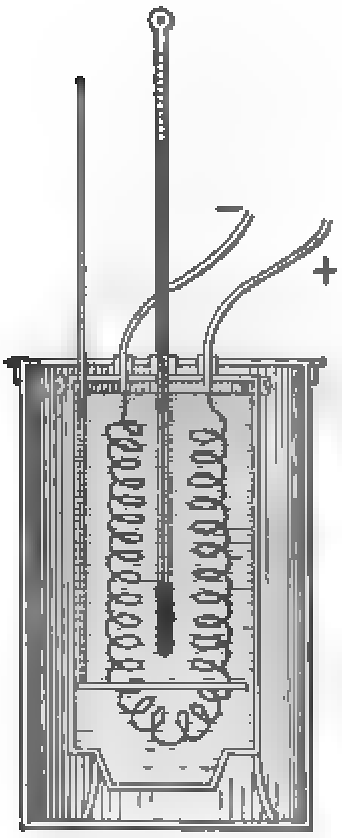


Fig. 369.—Electrical calorimeter.

655. Electrical Calorimeter.—The heat developed in a conductor may be readily measured by a calorimeter such as shown in the figure. A coil of wire is immersed in a non-conducting liquid (distilled water may be used) contained in a calorimeter which is screened from outside radiation. The current passing through the coil is measured, and also the difference in potential of the two terminals, and so the electrical work can be calculated and compared with the heat developed, which is determined by the rise in temperature of the liquid in the calorimeter when its mass and specific heat are known.

656. Incandescent Electric Lamps.—In the ordinary incandescent lamp used in electric lighting the current passes through a fine filament of carbon or tungsten enclosed in a glass bulb from which the air is thoroughly exhausted. The ends of the filament are joined to wires which must be of platinum where they are sealed into the glass, because that metal has very nearly the same coefficient of expansion as glass and therefore the joint does

not crack with changes in temperature. The carbon filament possesses the advantage of being extremely infusible and an excellent radiator besides being cheap. Ordinary carbon lamps have an efficiency of about 3 watts per candle-power, and though by raising the temperature of the filament the candle-power per watt is greatly increased, the filament rapidly volatilizes, blackening the bulb, and finally breaks. The tungsten filament in vacuum is ordinarily operated at a temperature of 2400°C. and gives about 1 candle-power per watt, but if its temperature is raised to 2800°C. it gives more than 2 candle-power per watt in vacuum, though it volatilizes so rapidly that the life of the lamp is very short. Where the bulb is filled with nitrogen, however, the evaporation of the filament is only about 0.01 of what it is in vacuum, and so the lamp may be run profitably at the higher temperature. Such a nitrogen filled lamp may have a temperature of 2800°C. and an efficiency of 1.5 candle-power per watt. It will be observed that filling with nitrogen causes some loss of luminous efficiency due to the conduction of heat away from the filament. The filament is therefore made in the form of a rather closely wound helix, for the cooling effect of the gas is then much less than when the turns of the filament are widely separated.

Incandescent lamps are usually connected in parallel between two conductors as A and B (Fig. 370) which are maintained at a constant difference of potential by a dynamo or battery of *low resistance*. They are called 50-volt lamps when they are brought to their proper luminosity by a difference of potential of 50 volts between their terminals.

The candle-power depends on the power supplied to the lamp and therefore both on the current and voltage. Good carbon lamps as ordinarily used have an efficiency of about 3 watts per candle-power, so that a 16-candle-power lamp will consume 48 watts.

Comparison of a 50-volt and 100-volt lamp of 16 candle-power, and equal efficiencies.

Voltage	*Current*	*Resistance*	*Power*
50 volts	1 ampère	50 ohms	50 watts
100 volts	½ ampère	200 ohms	50 watts

As the current in the 100-volt lamp is only one-half as great as in the 50-volt lamp, there is only one-fourth as much loss in heat in wires of a given resistance leading to the lamps.

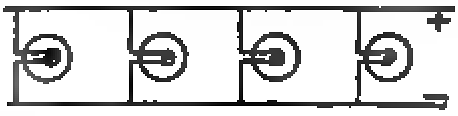


FIG. 370.—Lamps in parallel.

FIG. 371.—Lamps in series.

657. Incandescent Lamps in Series.—The mode of connecting incandescent lamps in parallel shown in figure 370 has the advantage that any one lamp can be turned on or off without particularly disturbing the others. But the current in the main wires is the sum of the currents in the lamps and, as the energy spent in the circuit is proportional to the square of

the current, the loss of energy in the main wires will be serious unless they are large and of low resistance. For street lighting, lamps are commonly connected in series so that however many lights there may be, the current in the conducting wires is no greater than for one lamp. There are two considerations which prevent this system from being used in house lighting. First, the potentials required are dangerous. If 30 lamps are connected in series and each requires 20 volts, the dynamo must have an electromotive force of 600 volts, and if a break or interruption of the circuit occurs at any point, the full difference of 600 volts will be experienced there. Second, if the filament of one lamp breaks it stops the current and all the lamps go out. This latter difficulty is overcome most ingeniously in street lighting by arranging a little side circuit or *by pass* in each lamp which is complete except at one point where a slip of paper is interposed. When the lamp is acting the current passes wholly through the filament; but if this breaks, the current is interrupted and immediately the whole electromotive force of the dynamo is brought to bear on the paper which is thereby punctured, permitting the current to pass through the side circuit.

658. Nernst Lamp.—Another form of incandescent lamp, devised by the German chemist Nernst, is also in use, in which the glower is a little rod made of a mixture of infusible earths, which though almost a non-conductor when cold becomes a suitable conductor when hot. A heating coil of platinum which is automatically cut out when the current begins to pass through the glower, is therefore used to start the lamp.

659. Electric Arc.—In the year 1801, Sir Humphrey Davy, who had constructed an immense battery of 1000 cells, observed that when the terminal wires were touched together for an instant and then drawn apart the discharge took place through the air like a stream of fire from one pole to the other, and at the same time the tips of the wires were intensely heated. The effect was most marked when carbon rods were used for the terminals. When the discharge took place horizontally it was bent upward like a bow (on account of the heated air rising) and so Davy called it the *arc* discharge. This tendency to curve out to one side is noticed in every long arc whatever its position. In arc lamps the carbons are only slightly separated, both tips are intensely heated, particles of carbon are carried across from the positive to the negative carbon causing a crater-like cup on the end of the positive carbon while the negative carbon is pointed; the positive carbon also is used up about twice as fast as the negative. The point of most intense luminosity is in the crater of the positive carbon where the temperature is found to be about 3500°C. The difference in luminous power of

large and small carbon arcs seems to be due to the greater extent of luminous surface in one than in the other, the actual brightness of the glowing surface being the same in all.

An electromotive force of 40 volts is required to maintain an arc between carbons. The temperature of the arc is the highest that has been produced by artificial means. Copper, iron, gold, silver, and platinum, if placed in it, are melted and volatilized.

660. Street arc lamps have two regulating coils. The whole current flows through one, which by its magnetic action tends to draw the carbons apart if the current is too strong. A second coil is arranged through which a branch or shunt current flows, which acts to push the carbons together, and the further apart the carbons are the stronger this current will be. A sort of balance is thus maintained which keeps the lamp in adjustment. There is also for series lamps a device by which if the carbons become caught and do not make the arc at all the current can still flow through the lamp.

Arc lamps are usually connected in series for street lighting because a current of only about 7 ampères is then needed; but as 50 volts must be allowed for each lamp, to operate 50 lamps on the one circuit an electromotive force of 2500 volts is required; hence arc light circuits are dangerous.

What are known as *enclosed arcs*, have surrounding the arc a small closely fitted glass globe, which soon becomes filled with carbon dioxide gas so that there is no ordinary combustion of the carbons. The carbons last much longer in these lamps, they require a higher electromotive force, about 70 volts, and give a softer light.

661. Electric Furnace.—In the production of aluminum and carborundum electric furnaces are employed. In these furnaces great carbons 2 in. in diameter are mounted horizontally and embedded in the materials that are to be heated, the whole is surrounded by walls of brick or fire clay, and the electric arc is established between the carbons. Under the combined influence of the enormous heat and the electrolytic action of the current the desired transformations are wrought.

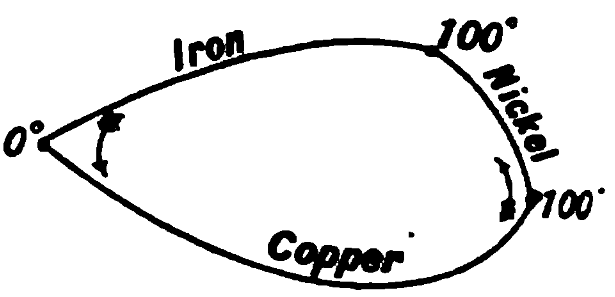
Problems

1. A current of 14 ampères divides between two branches, one of 2 ohms and one of 5 ohms resistance. Find the current in each branch, and the watts spent in each. In which resistance is the greater amount of heat developed per second?
2. The terminals of a gravity cell of 2 ohms resistance and 1 volt E.M.F. are connected with a coil of resistance 3 ohms. Find the watts spent in heat in the coil and also in the cell, also the total watts supplied by the cell.
3. What must be the resistance of a coil of wire in order that a current of 2 ampères flowing through the coil may give out 1200 gram-calories of heat per minute?

4. If the difference in potential of the ends of a coil is 50 volts, what must be its resistance that 500 gram-calories of heat may be developed in it per second?
5. Find the gram-calories per second developed in each of two coils; one having resistance 3 ohms and current 6 ampères, the other a resistance of 4 ohms and a difference of potential of 20 volts between its ends.
6. How many horse-power must be expended to maintain 200 100-volt lamps in operation, each lamp taking ½ ampère of current and having a potential difference of 100 volts between its terminals?
7. How many horse-power are required to operate a series of 60 incandescent street lamps in series, the current in each lamp being 3 ampères and the resistance per lamp being 7 ohms?
8. In an electric railway having a total line resistance of 0.4 ohm per mile, what is the loss in horse-power in two miles of line when a current of 50 ampères is being supplied to a distant car?
9. At 10 cents a kilowatt-hour what is the cost of heating 1000 liters or a cubic meter of water from 20°C. up to 90°C. by electricity?

Thermoelectricity

662. **Seebeck's Discovery.**—In 1821, Seebeck, of Berlin, discovered that **in a circuit made of two different metals if one junction is hotter than the other there is an electromotive force which causes an electric current.** This electromotive force is generally very small compared with ordinary battery cells, and conse-



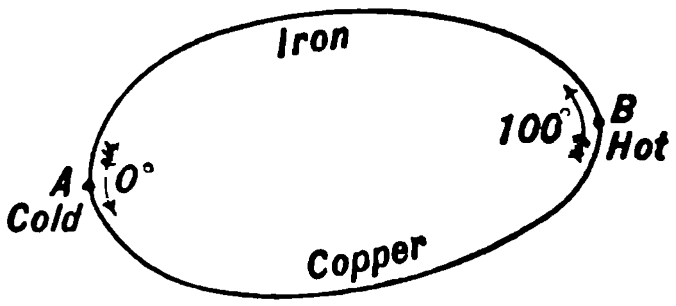


Fig. 372.

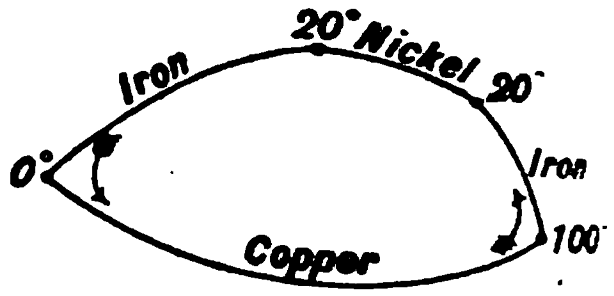


Fig. 373.

quently to obtain much current the circuit must have very low resistance. For example, in the copper-iron circuit shown in figure 372 when one junction is at 100° and the other at 0°, the electromotive force is about 0.001 of a volt and causes an electric current from copper to iron at the hot junction and from iron to copper at the cold one. *The introduction of another metal does not make any difference provided the two junctions of the new metal are at the same temperature.*

For example, the electromotive force is the same in the three circuits shown in figures 372 and 373.

663. Thermopile.—In order to obtain larger electromotive forces pairs of metals are combined in series to form *thermopiles.* The form devised by Nobili and used by Melloni in his researches on heat radiation consists of alternate strips of antimony and bismuth connected as shown in the figure, and carefully insulated from each other except at the junctions, where they are soldered together. These metals were chosen because they give a large electromotive force which acts from bismuth to antimony at the hot junctions and from antimony to bismuth at the cold.

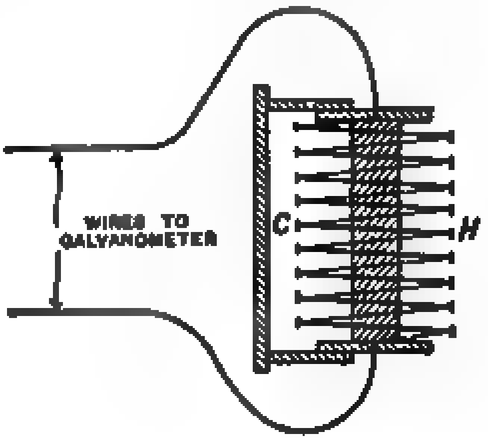


Fig. 374.—Thermopile diagram.

Rubens has improved the thermopile by using fine wires of iron and constantan (a nickel alloy) in place of antimony and bismuth. The mass to be heated in this case is very small so that it warms quickly when exposed to radiation.

The thermopile is usually mounted in a metal case so that only one set of ends is exposed to the source of heat to be investigated. If its terminals are connected to a sensitive galvanometer of low resistance, it becomes an exceedingly delicate means of measuring heat radiation.

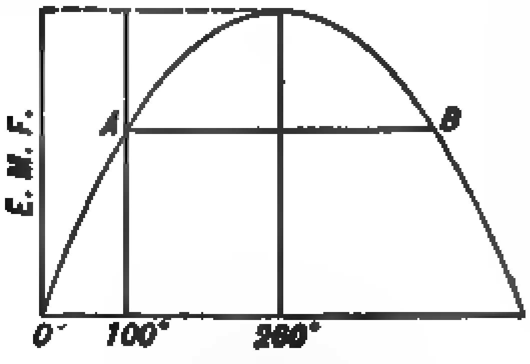


Fig 375.—Thermoelectric curve of e. m. f. of copper and iron.

664. Change of Thermoelectric Force with Temperature.—If one junction of a copper-iron circuit is kept at 0°C. while the other is steadily raised in temperature, the electromotive force is found to increase rapidly at first, then more gradually, reaching a maximum when the hot junction is at 260°C., after which the electromotive force falls off, becoming zero at 520°C. If the junction is heated still hotter the electromotive force reverses and the *current flows* from iron to copper at the hot

junction. If the observations are plotted with the temperature of the hot junction as abscissas and electromotive forces as ordinates a curve such as shown in figure 375 is obtained. It is a parabola and is perfectly symmetrical about the vertical line through its vertex, which corresponds to the temperature of maximum electromotive force.

This reversal of the thermoelectric current was discovered by Cumming in 1823.

If the cold junction is kept at 100° instead of zero the curve will be exactly the same except that the origin of coördinates will be moved from O to A, and electromotive forces will now be measured from the base line AB.

665. Thermoelectric Powers.—It is clear from the foregoing that the inclination of the curve at any point, or the rate of change of electromotive force per degree change in temperature, depends only on the temperature of the junction which is being warmed or cooled and not at all on the temperature of the other junction, provided it is constant.

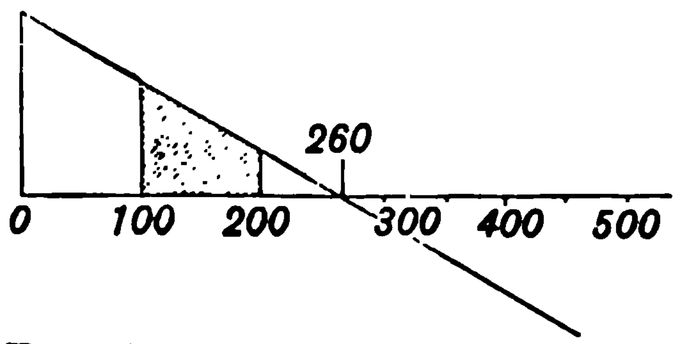


FIG. 376.—Thermoelectric powers of iron and copper.

This change of electromotive force per degree change of temperature of a junction is known as the relative thermoelectric power of the substances involved.

If the thermoelectric powers of iron and copper are plotted as ordinates along a scale of temperatures we shall obtain the diagram shown in figure 376. The curve is a straight line intersecting the axes at 260°C.; for at that temperature the relative thermoelectric power is zero, as is seen also from the curve in figure 375, where the maximum point is at 260°, showing that a small change in temperature produces *no* change in electromotive force. This is called the *neutral temperature* for these two metals.

666. Thermoelectric Diagram.—In the thermoelectric diagram devised by Tait, the thermoelectric powers of the metals referred to lead are plotted as ordinates along a scale of temperatures; lead being taken as standard because in it the Thomson effect (§669) is zero. Such a diagram is shown in figure 377. It will be observed that within the limits of the diagram the

[redacted]

variations with temperature of the thermoelectric powers of the metals are represented by straight lines.

The electromotive force of a couple made of any two metals is expressed by the area included between the lines of the two metals and the ordinates of the temperatures of the junctions.

The diagram is so constructed that the *direction* of the resultant electromotive force is *clockwise;* that is, in case of iron and

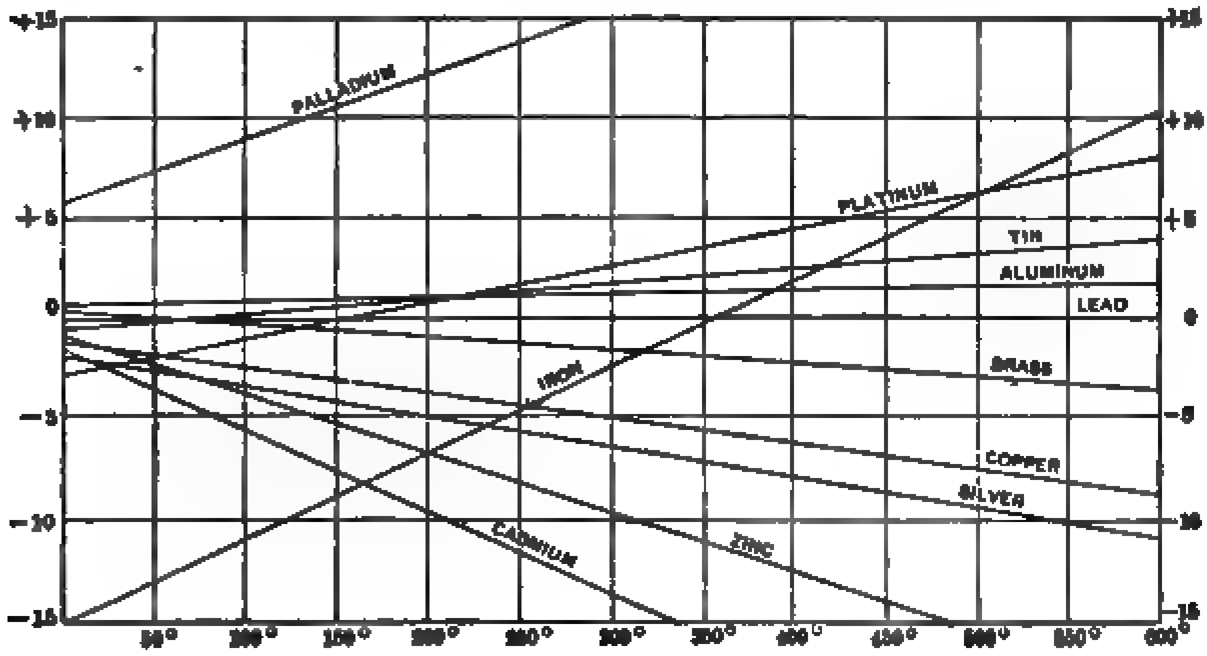


FIG. 377.—Thermoelectric diagram.
Thermoelectric powers are given in micro-volts per degree.

copper between 0° and 100°, the current will be from copper to iron at the hot junction.

667. Peltier Effect.—It was discovered by Peltier in 1834, that if a current of electricity flows around a circuit made up of two metals heat will be given out at one junction and absorbed at the other.

A beautiful demonstration of the Peltier effect was given by Tyndall by means of an ordinary thermopile. A thermopile is taken in which all parts are at the same temperature, so that it gives no current. On connecting it for a few seconds to a battery and then disconnecting it and joining it to a galvanometer a decided current is observed, showing that one set of junctions must have been more heated by the current than the other set. The current obtained is opposite to the first and tends to restore the equality of temperature disturbed by the first, for one stored

up heat energy in the thermopile and the other transforms that energy back again into energy of current.

By a thermopile there is a direct transformation of heat energy into electrical energy, but it is not efficient because there is a serious loss of heat by conduction from the hot junctions to the cold.

668. The Conservation of Energy in Thermoelectricity.—The Peltier effect affords a beautiful illustration of the principle that energy is absorbed at those points in a circuit where there is an electromotive force acting *with* the current and is given out at those points where there is an electromotive force acting *against* the current (§653).

In a circuit of two metals all at one temperature there may be electromotive forces at the two junctions, but since the temperature is the same at both, these electromotive forces are equal and opposite and consequently there is no current. If a current is now caused to flow by means of a battery, energy is given out at the junction where the electromotive force is against the current and that junction is heated, while the other is cooled. The two junctions no longer balance each other, and it is clear that *the resultant electromotive force* which arises from the change in temperature *must be against the current which brought it about.* Otherwise in a simple closed circuit of two metals if one junction were heated a little to begin with, a current would be set up which would still further increase the difference in temperature of the junctions and would so become continually stronger and might be used to run a motor and do mechanical work until all the heat energy in the thermopile was used up and it was reduced to the absolute zero of temperature.

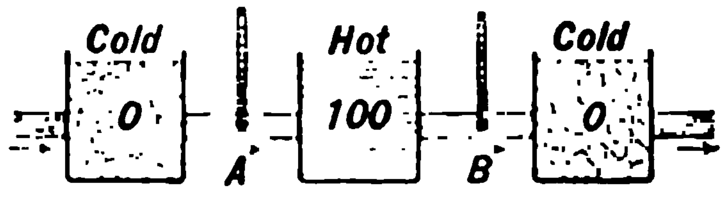


Fig. 378.

669. Thomson Effect.—In 1854, Lord Kelvin (Sir William Thomson) showed that in a thermoelectric circuit there must in general be electromotive forces not only at the junctions, but also in the homogenous conductors between the junctions, as they are not at the same temperature throughout.

This effect was predicted by Lord Kelvin as a consequence of the law of energy and was then verified by the following experiment.

A bar of iron was set up as shown in figure 378 so that the center was heated by boiling water while the ends were cooled with ice. When a current was established all parts of the bar were warmed, but a thermometer at A was observed to stand higher when the current was from left to right than when it was reversed, while the opposite was true at B.

670. Applications.—*The thermopile* as a delicate means of observing the intensity of heat radiation has already been described (§663).

A particularly sensitive instrument for the same purpose was devised by Boys and is known as the *radio-micrometer.* In this instrument a simple circuit of bismuth and antimony is suspended between the poles of a powerful magnet by a fine quartz fiber. One of the two junctions is protected from outside radiation by the surrounding instrument, while the other hangs in an opening so that radiation may be directed upon it. The slightest difference of temperature causes an electromotive force and since the resistance of so short a circuit is very small a comparatively large current is produced, which, reacting on the magnetic field, causes the suspended circuit to turn. A light mirror mounted on the suspended system turns with it so that the angular deflection may be read by a telescope and scale.

For the measurement of *high temperatures* a thermal couple consisting of a wire of pure platinum joined to another of an alloy of platinum and rhodium may be used. In the Le Chatelier pyrometer such a couple, mounted in a protecting sheath of porcelain, is thrust into the furnace or oven of which the temperature is to be determined; wires from the couple lead to a suitable galvanometer graduated to read temperatures directly up to 1500°C.

For the measurement of ordinary temperatures a thermal couple of iron and German silver is often convenient.

Problems

1. Find the thermal electromotive force of an iron-copper circuit in which one junction is at 0° and the other at 200°C.
2. Find the increase in electromotive force in a lead-iron circuit when the temperature of the hot junction is changed from 150° to 151°.
3. What relation must the lines of two metals on the thermo-electric diagram bear to each other in order that the increase in electromotive force per degree rise in temperature of the hot junction may be a constant?
4. When one junction of zinc-iron circuit is at 50°C., at what different temperature may the other junction be without causing any current in the circuit?

Magnetic Effects of Currents

671. Oersted's Experiment.—The first evidence of the magnetic action of an electric current was obtained in 1819 by the Danish physicist *Oersted,* who discovered that when a wire

carrying a current is held in a north and south direction over or under a balanced magnetic needle the needle is deflected as shown in figure 379; and if the directive force of the earth's magnetism is neutralized by means of a magnet, the needle sets itself at right angles to the current.

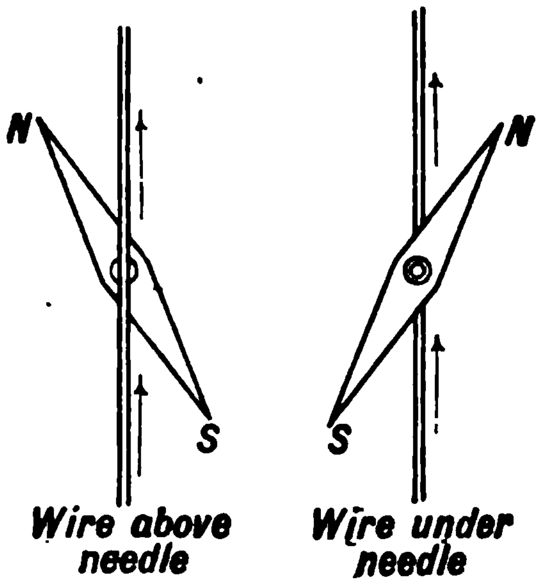


FIG. 379.—Oersted's experiment.

672. Magnetic Field Around a Straight Conductor.—The experiment of Oersted indicates that the magnetic force due to a current is in a plane at right angles to the current. To investigate its direction more fully cause a strong current to flow in a wire which passes vertically through a card on which some fine iron filings are scattered; on tapping the card the filings arrange themselves in circles about the wire as shown in figure 380. If the current is *down* as shown by the arrows, a small compass needle near the wire, at any point such as P, will point with its north pole in the direction of the arrow at

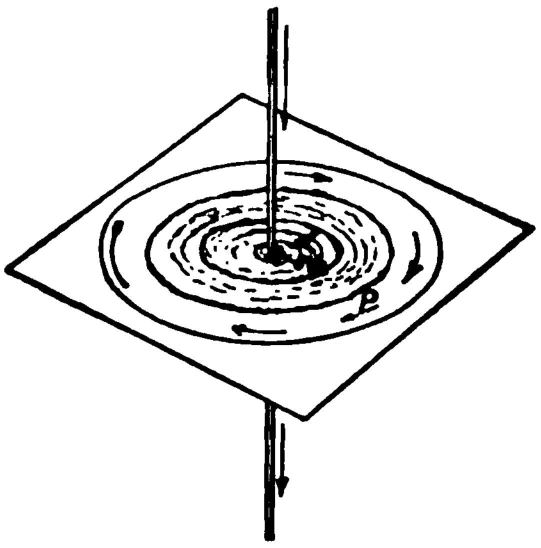


FIG. 380.—Field around current.

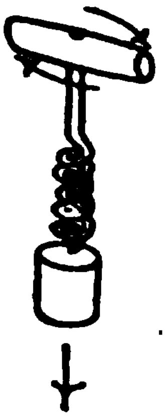


FIG. 381.—Right-handed screw.

that point, tangent to the circle. If the current is reversed the compass needle will point in the opposite direction.

The lines of magnetic force about a straight conductor carrying a current are circles of which the conductor is the axis.

By a comparison of figures 380 and 381 it will be seen that *the positive direction of the lines of force bears the same relation to the direction of the current as the direction of rotation of a right-handed corkscrew bears to the direction in which it advances.*

Another rule that may be given is that if an observer looks

along a conductor in the positive direction of the current, the positive direction of the lines of force as he sees them is clockwise.

673. Strength of the Field.—The strength of the magnetic field near a straight conductor is greatest next to the conductor and diminishes as the distance increases.

The strength of field H, at a distance r from the axis of a long straight wire carrying a current of strength I, is given by the expression

$$H = \frac{2I}{r} \qquad \text{(all quantities in C. G. S. units)}$$

or

$$H = \frac{2I}{10r} \qquad (I \text{ in amperes, } H \text{ in C. G. S. units})$$

since the ampère is one-tenth the C. G. S. unit of current (§602). This formula assumes that the return circuit is so far off that its magnetic effect at the point considered may be neglected.

As the card is tapped on which the iron filings rest, in the experiment described in the last article, the filings work toward the center, the circles gradually getting smaller, for the filings are drawn toward the stronger part of the field.

If a fine copper wire carrying a rather strong current is dipped into some fine iron filings they will cling together in little circular filaments, forming a mass around the wire.

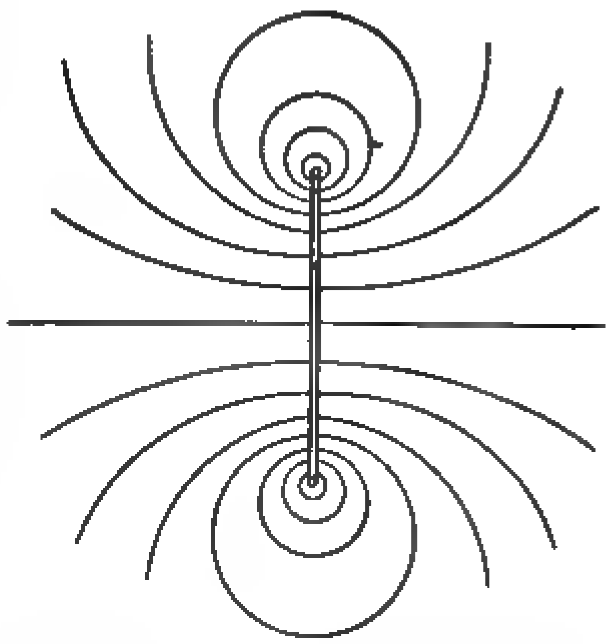


Fig. 382.—Field of circular current.

674. Field of a Circular Current.—When a conductor carrying a current is bent into a circle the lines of force are crowded together within the circle and spread out outside. In this case, shown in figure 382, all parts of the circuit conspire to cause magnetic lines of force of which the direction is through the circuit perpendicular to its plane on the inside, and back again on the outside, as shown in the diagram. The lines of force very near the wire are nearly circles about the wire, while at the center they are nearly straight and perpendicular to the *plane of the coil.*

If the wire carrying the current makes two turns around the circle instead of one, the magnetic force will everywhere be doubled, and so on for any number of turns.

The strength of the magnetic field at the center of a circular current is proportional to the total length of the conductor wound in the circle and to the strength of the current and inversely proportional to the square of the distance of the conductor from the center.

Thus if r is the radius of the coil and if n is the total number of turns, the length of the wire in the coil is $2\pi rn$, and when the current I is measured in C. G. S. units as defined in §602, the strength of the magnetic field H at the center in C. G. S. units is given by the formula:

$$H = \frac{2\pi rnI}{r^2} = \frac{2\pi nI}{r} \quad (I \text{ in C. G. S. units})$$

or, since the ampere is one-tenth the C. G. S. unit current,

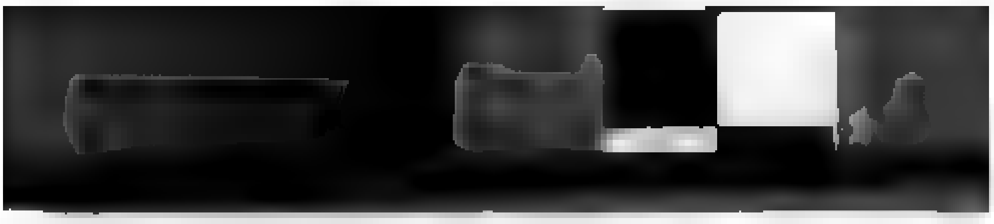
$$H = \frac{2\pi nI}{10r} \quad (I \text{ measured in amperes}).$$

The formula assumes that the cross section of the coil is negligibly small compared with r.

By measuring the magnetic force at the center of a coil of known dimensions, the number of ampères of current may be determined.

675. Rowland's Discovery.—Rowland discovered that a disc of ebonite, charged with electricity and rotating at high speed, acted upon a magnetic needle placed near it, just as a circular current would. The magnetic effect was found to be proportional to the speed of the disc. This remarkable experiment was carried out by him in 1875.

676. Solenoid.—A long helix, such as shown in figure 383, is known as a solenoid, and may be wound with one or several layers. When a current passes through such a coil all the turns act together to cause a field of magnetic force in which lines of force pass lengthwise through the interior looping back around the outside. *Looking through the solenoid in the positive direction of the lines of force, the direction of the current is clockwise.* Inside the solenoid a strong magnetic field of great uniformity is produced.



The system of lines of force of a solenoid is thus like that of a bar magnet, the south pole corresponding to that end of the solenoid about which the current flows clockwise, as seen by an observer facing that end.

If such a solenoid is mounted so that it can turn, it behaves like a suspended bar magnet when another solenoid or a bar magnet is presented to it.

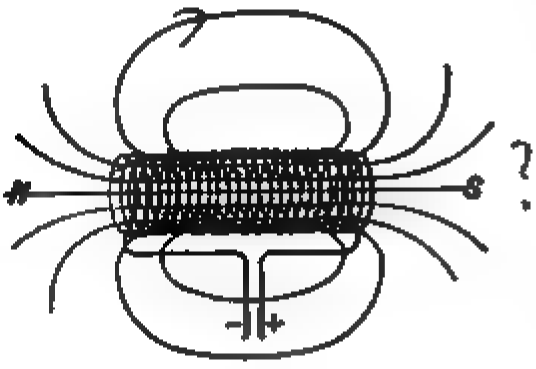


FIG. 383.—Solenoid.

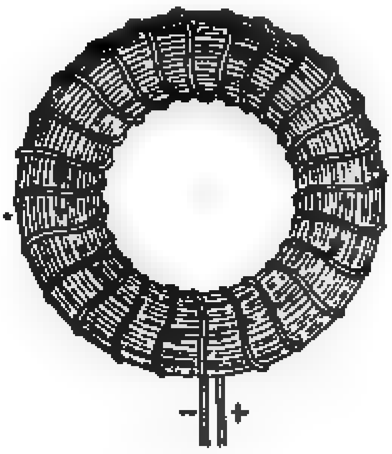


FIG. 384.—Ring solenoid.

677. Iron in a Solenoid.—If a soft-iron core is introduced into a solenoid the number of lines of force is greatly increased, it may be several hundred times, so that it acts as a strong magnet. A hard-steel core does not have so great an effect in increasing the number of lines of force, though it largely retains its magnetism after the current stops.

The precise effect in such a case depends on the relative proportions of the solenoid and its core. In a short broad solenoid an iron core which fills it will not increase the number of lines of force so much as if the core and solenoid were longer in proportion, for the strong poles exert a magnetic force which in the interior of the solenoid is opposite to that of the coil, as explained in §501.

678. Ring Solenoid.—If a long solenoid is bent into a ring so that its two ends come together as shown in figure 384, a ring solenoid is obtained, and when a current flows in such a solenoid it produces a very nearly uniform field in the interior, though since the lines of force are not straight but circles, the force must really be slightly stronger toward the inside where the lines of force are smaller circles. There is no magnetic force outside of such a solenoid.

If the interior of the solenoid is filled with a ring of iron, all parts of the iron experience the same magnetizing force and there are no poles to complicate matters, so that the *permeability* (§504) of the iron can be immediately determined from the increase in the number of lines of force due to its presence. If, for example, the total number of lines of force in the iron is 1000 times what it would have been in the same space if the iron had not been there, the permeability of the iron is said to be 1000.

It is easy to measure by electromagnetic induction (§710) the changes that take place in the number of lines of force through the ring and in this way the permeability of iron was studied by Rowland.

679. Magnetic Induction, Permeability and Magnetizing Force.—The strength of the field inside a ring solenoid when no iron is present may be called the magnetizing force. Let it be represented by H which will thus express *the number of lines of force per square centimeter of cross section* before the iron is introduced.

The number of lines of force in the iron core per square centimeter of section is called its magnetic induction or flux density and may be represented by B (see §500).

We then have

$$\mu = \frac{B}{H} \quad \text{or} \quad B = \mu H$$

where μ represents the permeability of the iron.

The relation between the induction B in iron and the magnetizing force H as the latter is increased, starting at zero, is shown in the curve ab of figure 385, in which abscissas represent values of H and ordinates the corresponding values of B. From this curve it appears that at first B increases slowly, then rapidly, and finally at b as the iron approaches what is called *saturation* a considerable increase in H causes only a small increase in B.

The curve shows that the permeability of iron is not a constant but increases with increase in magnetizing force up to a maximum, after which it rapidly diminishes. Some values are given below.

Permeability of a Sample of Soft Iron

H	B	μ	H	B	μ
1	1,000	1,000	4	9,700	2,425
2	6,000	3,000	5	11,800	1,966
3	8,200	2,733	17	13,000	765

80. Hysteresis.—The changes which take place in iron when magnetism is *reversed* were first thoroughly studied by Pro- sor Ewing of Cambridge University. The curve of figure 385 ws the changes in induction in soft iron when the magnetizing

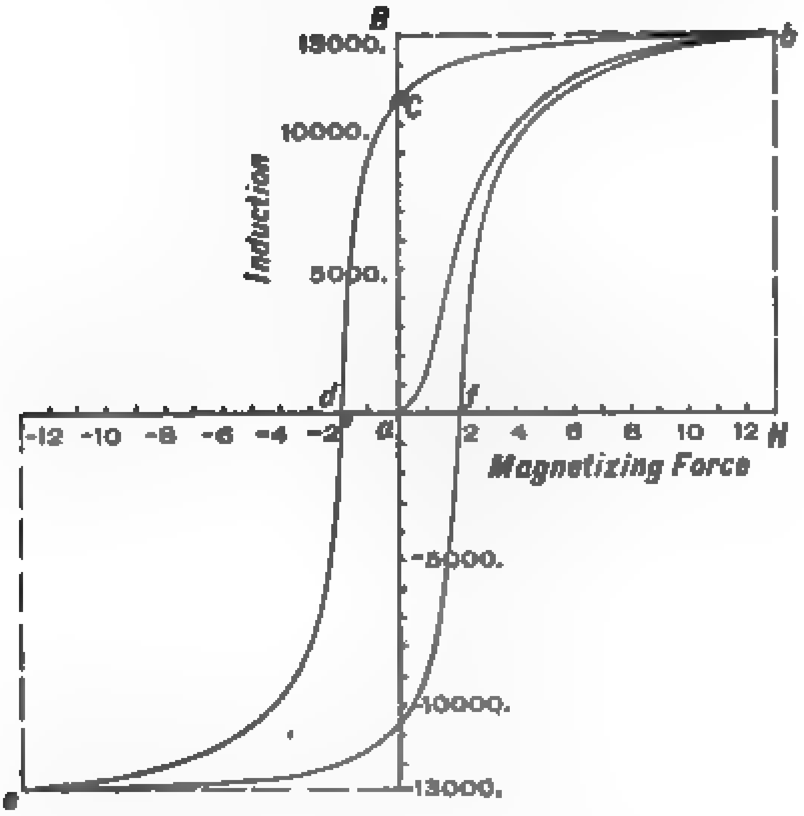


FIG. 385.—Hysteresis curve.

ce is changed from $+13$ to -13 and back again. The rise induction when the iron is first magnetized is shown by the ve from *a* to *b*. The induction is here 13,000 while H is 13. reducing the magnetizing force to zero the induction only ls to 10,800 following the curve *bc*. By means of a gradually reasing reversed current the magnetizing force is made nega- e until when $H = -2.0$ the induction is zero and there are lines of force in the iron. As the force is made still more

negative the iron becomes oppositely magnetized, reaching the value $B = -13{,}000$ when $H = -13$. Then as the force is again reduced to zero the induction drops to $-10{,}800$ following the lower curve, and does not become zero till H is $+2.0$. If the magnetizing force is carried up to the former maximum and then again diminished the curve rises to b and then falls back to c exactly as before.

This *lag* of the induction in iron and steel behind the magnetizing force was named by Ewing *hysteresis*. In consequence of it, when a mass of iron is put through such a complete cycle of changes, more energy is spent in magnetizing than is given back when it is demagnetized, the difference being a certain amount lost in heat. The amount lost in this way per cubic centimeter of iron is proportional to the area of the loop of the hysteresis curve, and with a maximum induction of 5000 it may amount to as much as 2500 ergs per cubic centimeter in each cycle of magnetic change.

Every time the magnetism in a mass of iron is reversed it is put through such changes and since in the iron cores of transformers and dynamo armatures the reversals take place many times in a second it is important in such cases that soft iron should be used in which the hysteresis loss is small. The loss of energy due to this cause may amount to 2 per cent. in a transformer made of fairly good iron.

681. Electromagnets.—Powerful magnets are made by surrounding soft iron cores with magnetizing coils, as was first shown by the French physicist Arago in 1820. A typical form of electromagnet is shown in figure 386.

On each of the two arms of a U-shaped piece of iron is fitted a cylindrical coil made of wire wound with silk or cotton insulation to separate one turn from another. The coils are so connected that the current flows in opposite directions around the two legs of the magnet, making one end a north pole and the other a south pole. When the soft-iron armature is placed across the two poles a closed circuit of iron is formed so that the magnet with its armature resembles somewhat the ring solenoid with iron core described in §678. If the armature is sufficiently large most of the magnetic induction will be in the iron, the lines of force being closed curves. The whole number of lines of force

established in the core of an electromagnet may be considered as due to the relation of two factors, *the magnetizing power of the current in the magnet coils,* called **the magnetomotive force,** and *the resistance to magnetization offered by the iron core,* called its **reluctance.**

$$\textit{Number of lines of force established} = \frac{\textit{Magnetomotive force}}{\textit{Reluctance of core}}.$$

In case of a uniformly wound ring solenoid the magnetomotive force may be shown to be $4\pi nI$ where n is the number of turns of wire around the core and I is the magnetizing current, and we have

$$N = \frac{4\pi nI}{R}$$

where N is the number of lines of force through the core and R is its reluctance, all the quantities being in C. G. S. units.

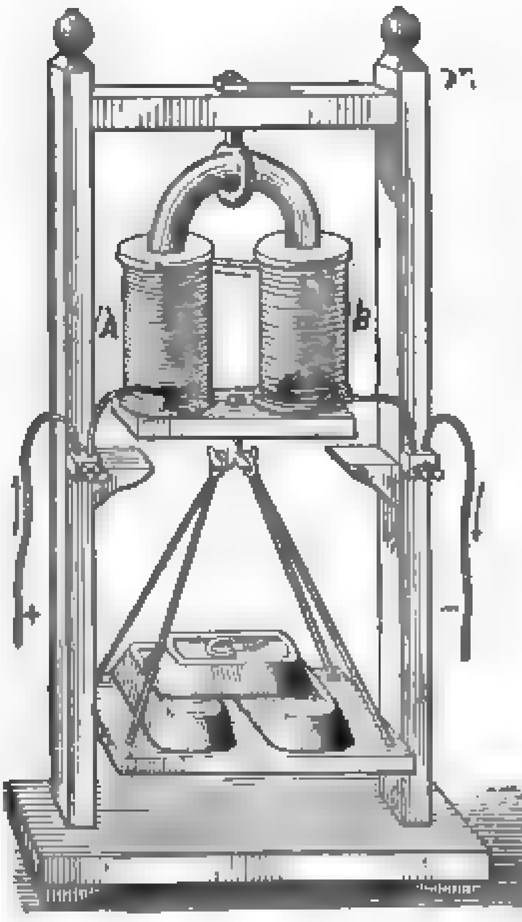


FIG. 386.—Electromagnet.

The above formula applies also approximately to ordinary electromagnets having nearly continuous iron cores.

If the current I' is given in ampères while R and N are in C. G. S. units the formula becomes

$$N = \frac{4\pi nI'}{10R}$$

since the C. G. S. unit of current is equal to 10 ampères.

The product nI' is called the ampère-turns and **the strength of a magnet excited by a small current making many turns is the same as with a large current making few turns, provided the ampere-turns are the same in both cases.**

The shorter the iron circuit and the greater its cross-section the less will be the reluctance and the more lines of force will be established by a given number of ampère turns.

The reluctance of an iron ring may be calculated from the formula

$$R = \frac{l}{\mu A}$$

where l is the mean length of the ring, A is its cross section, and μ is the permeability of the iron. If a circuit is made up of parts that have different permeabilities their reluctances must be calculated separately and added together when the parts are in series.

When the armature is not across the poles the reluctance is greatly increased because of the small permeability of the air through which the lines of force must pass. Therefore the number of lines of force established in that case is very much less than with the armature across the poles.

The force with which such a magnet holds its armature is proportional to the area of its poles, and is expressed by the approximate formula

$$\text{Attraction in dynes} = \frac{B^2 A}{8\pi}$$

where B represents the induction or number of lines of force per square centimeter of the surface between pole and armature, and A is the combined areas of the two poles.

Problems

1. What is the strength of the magnetic field 15 cm. from a straight wire carrying a current of 6 ampères?
2. A wire 3 meters long is made into a circular coil with a mean radius of 6 cm. Find the strength of field produced at the center of the coil by a current of 0.1 ampère in the wire.
3. How much current is flowing in one rail of an electric railway which runs in a north and south direction and causes a deflection of 45° in a compass needle held 30 cm. above the center of the rail; taking the strength of the horizontal component of the earth's magnetic field as 0.20?
4. Find the strength of the magnetic field at the center of a circular coil of 7 turns of wire 18 cm. in diameter when carrying a current of 3 ampères.
5. What will be the deflection of a magnetic needle at the center of the coil in the last problem if the coil is placed with its plane vertical and in the magnetic meridian at a point where the earth's horizontal force is 0.16?
6. A ring solenoid has a cross section of 9 sq. cm. There are 8 turns of wire per cm. of length of the solenoid and its whole length is 30 cm. measured along its axis. What magnetomotive force is produced by a

current of 1 ampère in the coil? How many lines of force will there be in the solenoid if it does not contain iron? And how many if filled with an iron core of permeability 100?

7. How many lines of force will be set up in a horseshoe magnet with iron armature, the iron circuit having an average cross section of 36 sq. cm., each leg being 15 cm. long and the two legs 12 cm. apart between centers? On each leg is a coil of 400 turns of wire carrying a current of 5 ampères. The permeability of the iron may be taken as 100.

8. Find the force in kilograms which an electromagnet can sustain when it is magnetized so that there are 600 lines of force per sq. cm. in the core, each pole piece having an area of 36 sq. cm.

Interaction of Currents and Magnets

682. Mutual Action of Parallel Currents.—*Two parallel conductors carrying currents in the same direction attract each other, while if the currents are in opposite directions they repel.* This may be shown by means of Ampère's frame, a light rectangular frame of wire connected to a battery through two mercury cups so that it can freely revolve, as shown in figure 387. If a

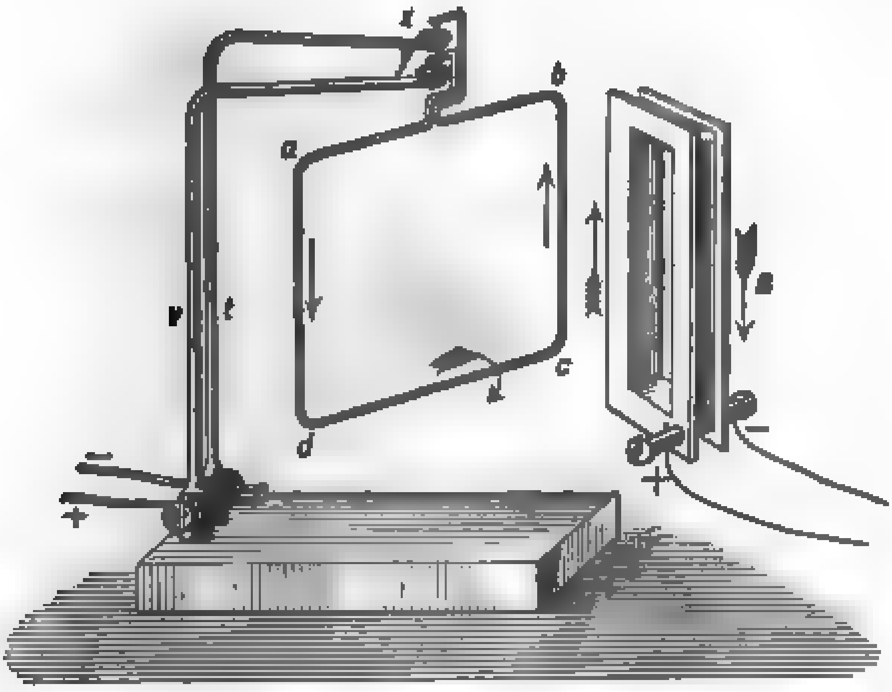


Fig. 387.

second frame having a number of turns of wire through which a current passes is brought up so that one of its edges is parallel and near to one of the vertical wires of the pivoted frame, the attraction or repulsion of parallel currents is easily demonstrated.

Also if the frame *B* is held under the pivoted frame so that its

upper edge is at right angles to the lower wire of the movable frame the latter will then turn until the two are parallel and with the adjacent currents in the same direction.

683. Magnetic Field Around Parallel Currents.—If the lines of force of two parallel currents are studied by means of iron filings or a compass needle, they will be found as in figure 388 when the currents are both in the same direction. While if the currents are in opposite directions the resultant lines of magnetic force are as shown in figure 389.

According to Faraday's conception, the attraction in the first case may be explained by a tension in the magnetized medium or a tendency for it to shorten up in the direction of lines of force; on the other hand, the repulsion in the second case is also in

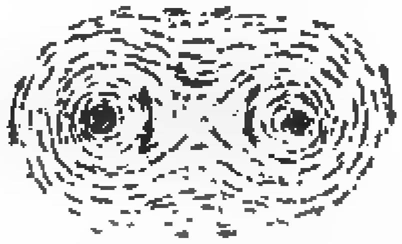


Fig. 388.
Lines of magnetic force, currents perpendicular to paper and both down

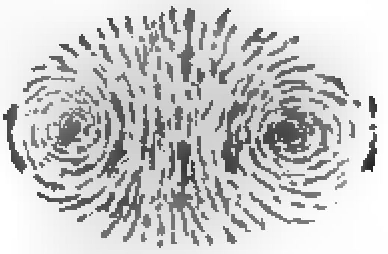


Fig. 389.
Magnetic field of two currents perpendicular to the paper; one down and one up.

accordance with Faraday's idea that there is a pressure or tendency for a magnetized medium to expand at right angles to the lines of force.

It is also to be noticed that *in the first case the field of force is stronger just outside of the conductors than it is between them,* for between the two the magnetic effect of the one is opposed by the other; while *in the second case the two act together to produce a strong magnetic field between them and a weaker field outside.*

684. Action Between Current and Magnetic Field.—In the case just considered each conductor may be thought of as acted on by the magnetic field due to the other. That there is such a reaction between a magnetic field and a conductor carrying a current may be demonstrated by presenting one pole of a bar magnet to one of the vertical branches of Ampère's frame (§682) when the wire will move across the lines of force of the magnet.

Or if a current of electricity is established in a light flexible conductor of tinsel cord hanging between the poles of a horseshoe

magnet the cord is repelled outward from between the poles when the current is downward and the poles are situated as shown in figure 390. If the current is reversed or if the magnet is turned over so that the poles are interchanged the cord is drawn inward. The field of force due to a current flowing across a uniform magnetic field is shown in figure 391, where the current is supposed to flow downward in a wire which intersects the paper perpendicularly at O. The broken lines are the lines of force of the uniform field, the circles are those of the current, and the full lines are the resultant lines of magnetic force. Clearly a tension in the medium along lines of force and pressure at right angles will urge

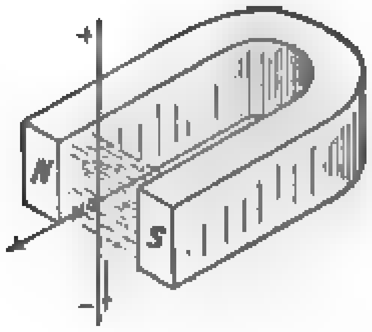


FIG. 390.—Current in magnetic field.

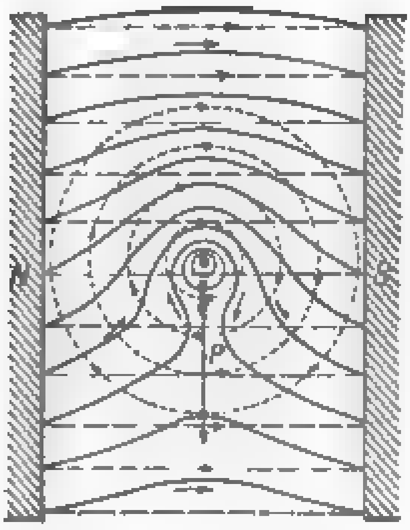


FIG. 391.

the conductor in the direction shown by the arrow. At points nearer the top of the diagram than O, the force due to the currents acts *with* the original field, while below O, the two are in opposition, hence the field above O is strengthened by the current while it is weakened below the conductor, there being a neutral point P where there is no magnetic force at all.

These experiments lead to the following general rule:

When the magnetic field immediately adjoining a conductor carrying a current is strengthened on one side and weakened on the other by the effect of the current, the conductor is urged toward that side where the field is weakened.

If the current is not at right angles to the lines of force of the field, only *that component of the magnetic force which is perpendicular to the conductor* is effective, so that the effective magnetic force, *the current, and the* force acting on the conductor to move

it are in three directions mutually at right angles to each other, and their relation can always be determined by the rule just given.

The amount of the force F experienced by the conductor is

$$F = lHI$$

where l is the length of the conductor in the field, H is the strength of the component of the magnetic field at right angles to the conductor, and I is the current strength, all being measured in C. G. S. units.

685. Magnet and Current in a Coil.—The mutual action of a magnet and a current in a coil may be studied by a little light circular coil of wire connected to zinc and copper terminals which dip into a test-tube containing dilute sulphuric acid, the whole system being floated in a tank by means of a cork.* On presenting the pole of a bar magnet the coil will set itself so that the lines of force of the magnet are in the same direction through the coil as the lines of force of the coil itself, thus strengthening the field within the coil. It will then approach the pole and slip over it to the middle of the magnet.

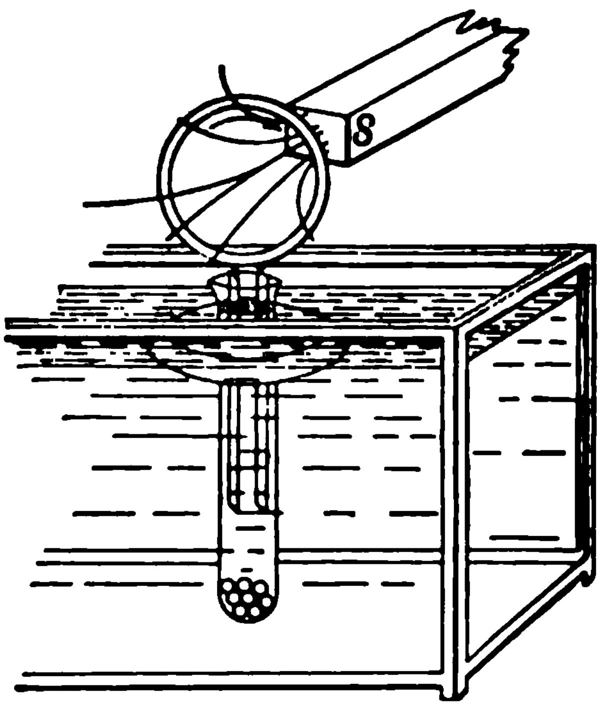


FIG. 392.—Magnet and floating current.

If the magnet is now pulled out and quickly thrust through the coil in the opposite direction, the coil will slip off from the magnet, revolve so as to present the opposite face, and then again approach it.

It may be seen that these actions result from the general rule of §684. For in each portion of the circular circuit the magnetic field is strengthened on one side of the conductor and weakened on the other and each part strives to move from the stronger toward the weaker field. On the whole, therefore, **the coil always turns and moves so as to increase the resultant number of lines of force through it.** It slips to the middle of the magnet and then sets itself obliquely as shown in figure 393, for in that position it

* In this experiment the test-tube should be weighted with shot until the cork is entirely *submerged*, only the upper part of the test-tube projecting above the surface, otherwise the motions will be greatly impeded by the surface viscosity of the water.

embraces the whole number of lines of force through the magnet and avoids also including those that turn back at the sides, which are in the opposite direction. up to here

686. Barlow's Wheel.—In the apparatus shown in figure 394 a copper disc is balanced on an axle so that it can turn freely between the poles of a horseshoe magnet which produces a strong field perpendicular to the disc. The lower edge of the disc dips in a trough of mercury. If one pole of a battery is connected to the axle of the disc and one to the mercury trough, a current will flow through the disc between its center and the trough. This current, being perpendicular to the lines of force

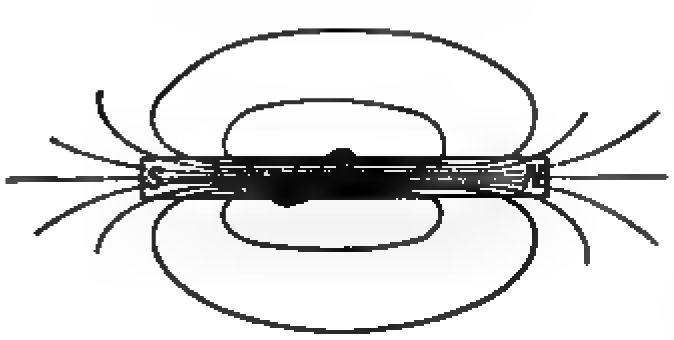


FIG. 393.

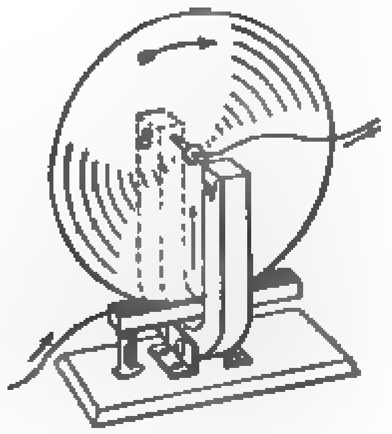


FIG. 394.—Barlow's wheel.

of the magnet, is urged to the right or left, depending on its direction, and accordingly the disc itself is set in continuous rotation.

This experiment appears to show that the displacing force acts on the conductor which transmits the current and not simply on the current itself.

687. Rotation of a Magnet.—The following interesting experiment is due to Faraday. A strong straight steel magnet is mounted vertically between pivots. To one side of it is attached a short arm of copper which reaches out and dips into a circular trough of mercury surrounding the magnet, as shown in figure 395. If one pole of a battery is connected to the magnet and the other pole to the mercury trough, the magnet will rotate about its axis in a direction which depends on the direction of the current and on whether the north or south pole of the magnet is uppermost.

In this case the current in the projecting arm crosses the lines of force of the magnet at right angles and therefore tends to move across them, thus causing the rotation.

The experiment is remarkable because *the motion is caused by a reaction between the cross arm attached to the magnet and the magnetic field of the moving magnet itself.* It shows that in a certain sense the magnetic field of the magnet does not rotate with it when it turns.

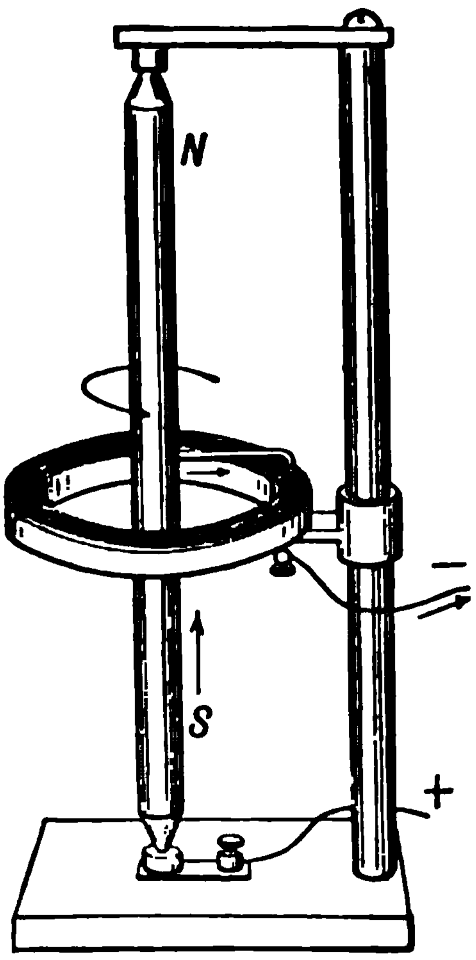


FIG. 395.

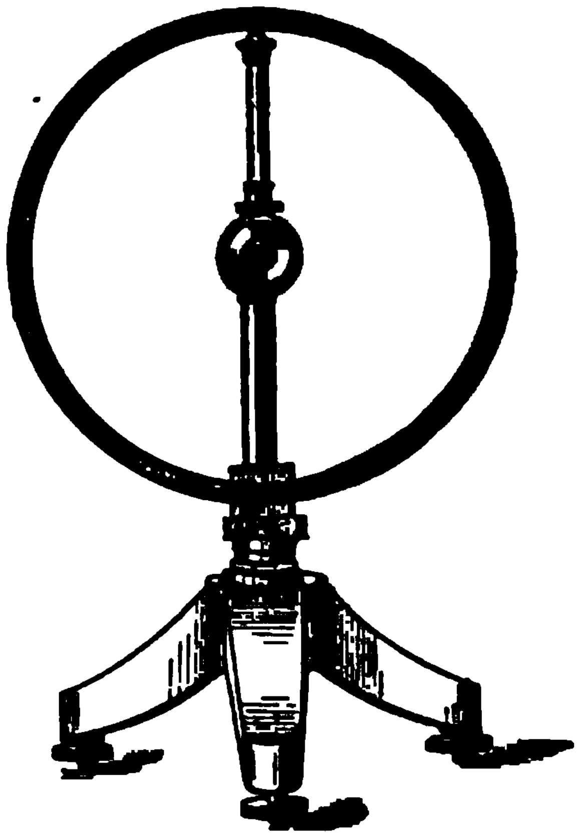


FIG. 396.—Tangent galvanometer.

INSTRUMENTS FOR MEASURING CURRENT AND POTENTIAL

688. Measurement of Current.—The strength of an electric current may be measured by its magnetic effect or by its heating or chemical action. Instruments which measure a current by its action on a magnetic needle are known as *galvanometers.*

689. Tangent Galvanometer.—In the tangent galvanometer there is a circular coil having one or more turns of wire, at the center of which a magnetic needle is either balanced on a point or suspended by a fine fiber of silk or quartz. The instrument is placed so that the plane of the coil is vertical and in the magnetic north and south plane. When a current is sent through the coil the needle turns to one side or the other, **and the strength of the current is proportional to the tangent of the angle of deflection,** as may be shown as follows:

force due to the current in the coil is at right angles to
ıne of the coil at its center (§674) and the strength of the
; that point in a given coil is proportional to the strength
current. Let G represent *the strength of field at the center
the coil when unit current is flowing,*
G will be the strength of field when
ırrent strength is I. Let OA in
397 represent the plane of the coil
the point where the needle is placed,
'hen no current is flowing the needle
in the direction OA, being acted on
y the horizontal component H of the
magnetic force. The magnetic force
to the current in the coil is IG and at

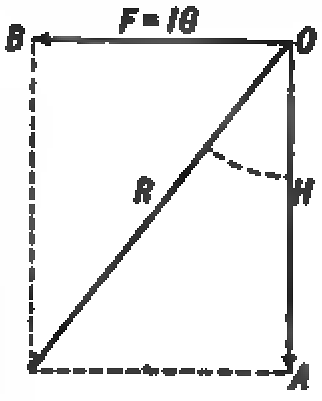


FIG. 397.

ngles to H, therefore, the resultant force R is the diagonal
rectangle whose sides are IG and H, and

$$\tan x = \frac{IG}{H}$$

x is the angle which the resultant force makes with H.
e needle must point in the direction of the resultant force,
x is the angle through which the needle turns. Therefore

$$I = \frac{H}{G} \tan x$$

H and G are known the current may be determined by
ring the angle x.

Coil Constant of a Tangent Galvanometer.—In case of a
galvanometer the magnetic force F due to the coil is expressed

[the current is measured in electromagnetic units,

$$F = \frac{Il}{r^2} \quad \therefore \quad G = \frac{l}{r^2} \qquad (§674)$$

ce the length of n turns of wire of radius r is $2\pi rn$,

$$G = \frac{2\pi nr}{r^2} = \frac{2\pi n}{r}$$

vanometer *coil constant* G can be calculated from this formula when
of the galvanometer has so large a radius compared with the length of
lle that the poles of the needle may be regarded as at the center, and
e cross section of the coil is so small that all the turns bear nearly the
lation to the needle.

If G is determined in this way, r being measured in centimeters, and if H is found by the method described in §498, the current will be found in *C. G. S. electromagnetic units* by the use of the formula $I = \frac{H}{G} \tan x$.

To obtain the current strength in *ampères*, we must take as the value of the coil constant

$$G = \frac{2\pi n}{10 . r}.$$

By this method the strength of a current is determined in amperes directly from the fundamental units of length, mass, and time, for we have already seen how the measurement of H is based on these units. A tangent galvanometer in which the constant is determined in this way directly from measurements of the coil is known as a *standard galvanometer.*

691. Sensitive Astatic Galvanometer.—For the measurement or detection of extremely small currents of electricity the coil of wire must contain a great number of turns as close as possible to the needle, and because the turns nearest to the needle are most effective it is customary to use finer wire for these turns so that a greater number can be placed in a given space.

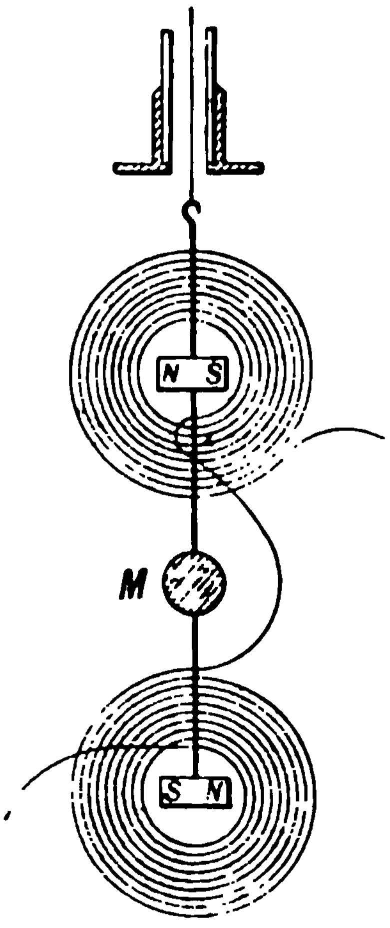


Fig. 398.—Astatic galvanometer diagram.

The sensitiveness of the instrument is further increased by using an *astatic needle.* This is a system of two magnetic needles, as nearly as possible of the same strength, connected together by a light aluminum wire so that the poles of the two needles are oppositely directed, as shown in the figure.

The combination is then suspended by a fine silk or quartz fiber so that the galvanometer coil surrounds only one needle, or the second needle may be surrounded by another coil around which the current flows in the opposite direction to the first so that any current in the coils will tend to turn both needles in the same direction.

If the two needles have equal magnetic moments the earth will have no directive action on the combination. But as no system of needles will remain perfectly astatic, a directive magnet above or below the instrument serves to balance the effect of the earth on the combined system. The influence of this magnet

[redacted]

and the torsion of the suspension fiber serves to give the needle a definite position of rest.

A small mirror attached to the needle enables its angular deflection to be measured by the usual telescope and scale method or by the reflection of light upon a scale.

692. Moving-coil Galvanometer.—In this type of instrument, known also as the *D'Arsonval* form of galvanometer, the suspended system is a coil of fine wire which hangs in a strong magnetic field due to a permanent steel horseshoe magnet. In figure 399 is shown a vertically placed horseshoe magnet, between the poles of which is hung a light rectangular coil of many turns of fine wire, the plane of the coil being parallel to the direction of the lines of force. The coil is suspended by a fine ribbon of phosphor-bronze which also serves to connect one end of the suspended coil to the outer circuit while the other connection is made through a spiral wound strip of the phosphor-bronze ribbon attached to the lower end of the coil.

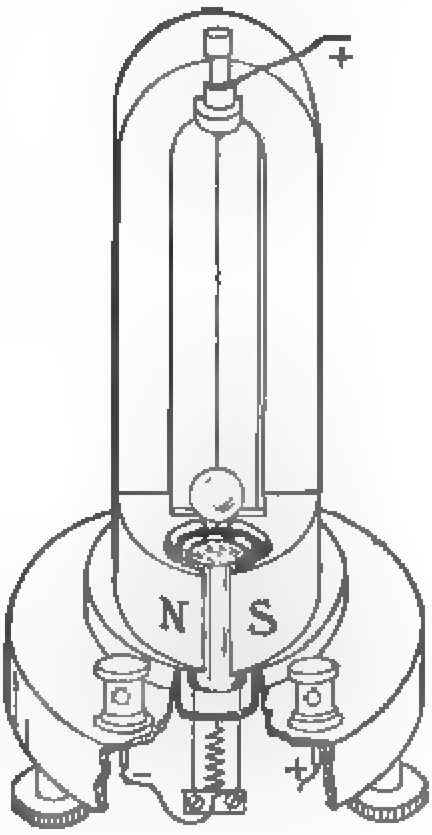


Fig. 399.—D'Arsonval galvanometer.

A cylindrical mass of soft iron is fixed midway between the poles of the magnet so that as the suspended coil turns its vertical branches move in the gaps between the core and pole pieces. This arrangement secures a strong uniform field across which the wires of the coil pass, and when a current is sent through it, it is deflected.

A small mirror mounted just above the coil and moving with it, enables the deflection to be determined by the telescope and scale or reflected spot of light method.

The moving-coil galvanometer has the advantage that it is not affected by changes in the earth's magnetic field, and can be used near dynamo machines and where there is considerable magnetic disturbance. Also the coil damps strongly or comes almost immediately to rest when the wires leading to it are touched

together, forming a *short circuit*, as it is called. This damping is due to electromagnetic induction (§720).

693. Electrodynamometers.—Instruments in which no iron or magnetic substance is used, but where the measurement depends on the mutual action of two coils carrying currents, are known as *electrodynamometers.*

The Siemens electrodynamometer, shown in figure 400, is a

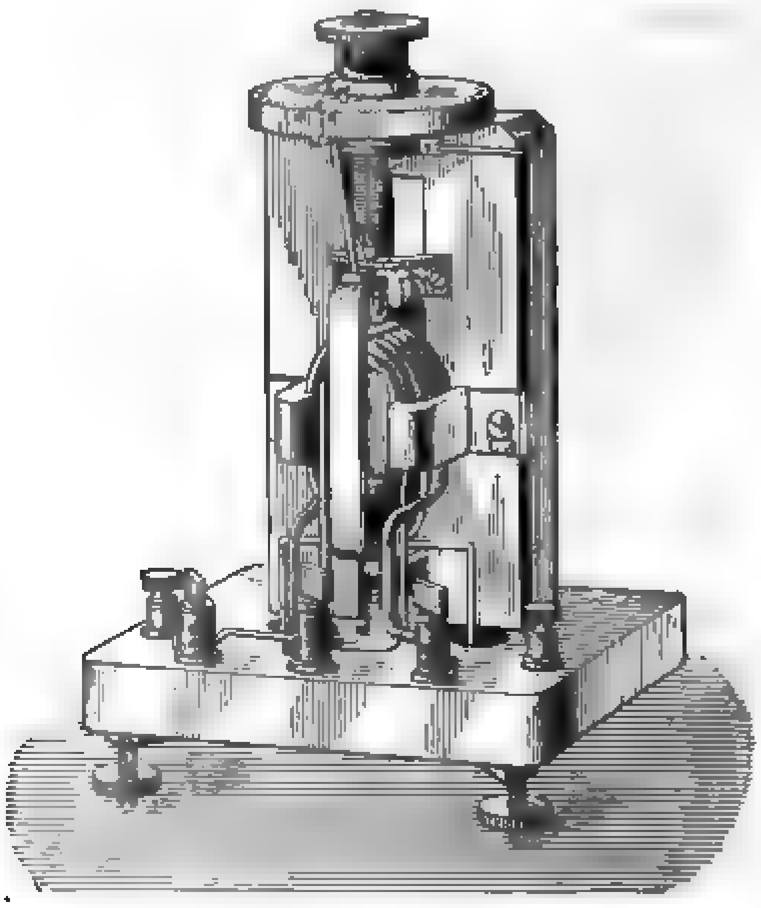


Fig. 400.—Siemens electrodynamometer.

good example. An oblong coil is fixed in a vertical position and surrounding it closely at right angles, but not touching it, is a rectangular suspended coil. The current passes through the fixed coil and is led into the suspended coil through two mercury cups into which its ends dip. The magnetic action of the coils upon each other causes the suspended coil to turn, but its top is attached to a helical spring the upper end of which is fastened to a knob which is turned till the torsion of the spring forces the suspended coil back again into its zero position. The strength of the current is determined from the amount of torsion required, as shown by a circular scale.

In such an instrument the force of torsion T depends on the current strength in each coil or T is proportional to II', but when the current is the same in each coil T is proportional to I^2, whence

$$I = k\sqrt{T}$$

where k is a constant for the instrument which depends on the size, shape, and number of turns in the coils and the scale by which the torsion is measured. It is determined by experiment, by measuring the torsion produced by a known current.

When the current is reversed in both coils the deflection is in the same direction as before; for this reason an electrodynamometer can be used to measure a rapidly alternating current which would give no deflection in a galvanometer.

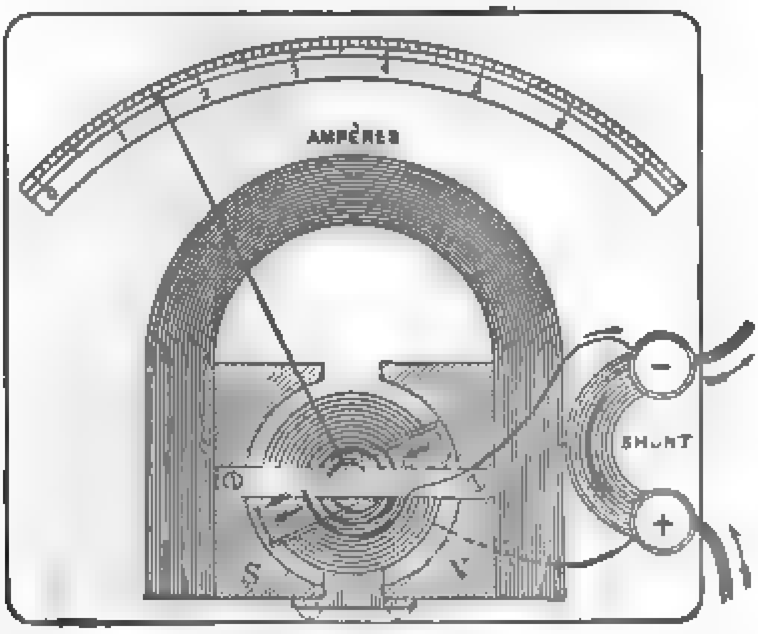


FIG. 401.—Ammeter.

694. Ammeters.—An *ammeter* is some form of galvanometer or electrodynamometer graduated so that the current strength in ampères may be directly read from the scale. A form of ammeter much used for direct currents is shown in figure 401. It consists of a sensitive moving-coil galvanometer in which the coil instead of being suspended is mounted in jeweled bearings and is held in equilibrium by two non-magnetic spiral springs which also serve as conductors for the current. The main current passes through a strip of metal (called a shunt) having very small resistance, only a minute portion of the current passing through the delicate movable coil. But the current in the movable coil is always the same proportional part of the whole

current, and therefore the scale over which the pointer m
may be so graduated as to show directly the number of am
in the *total* current.

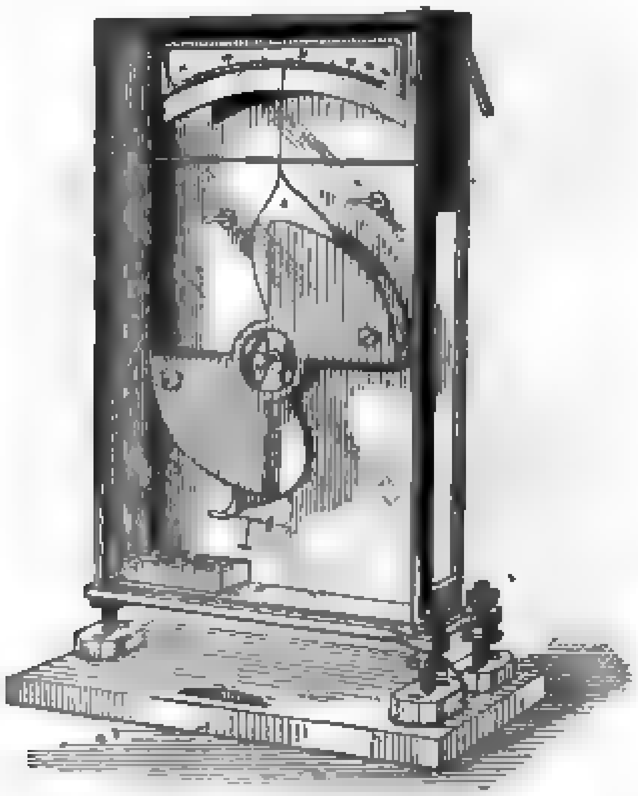


Fig. 402.—Electrostatic voltmeter.

An instrument of this type has the advantage of ha
very small resistance.

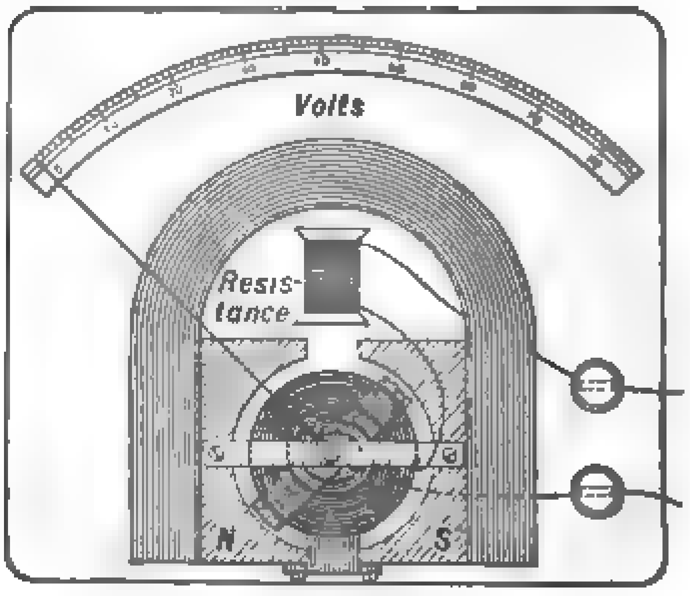
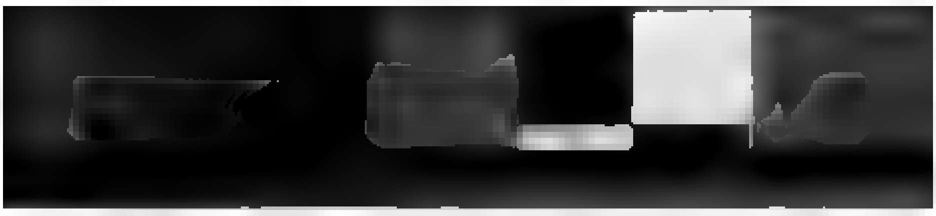


Fig. 403.—Voltmeter.

695. Voltmeters.—A *voltmeter* is an instrument desig
measure differences in potential, and gives the readings



volts. There are two principal types, *electrostatic* voltmeters ıd *those that depend on the flow of current.*

696. Electrostatic Voltmeters.—These instruments are elec-ometers adapted to meet the requirements of ordinary engineer-g practice. Of this type is the instrument shown in figure)2.

697. Current Voltmeters.—A voltmeter using current is a gh-resistance galvanometer with a scale graduated to give rectly the number of volts difference in potential between its rminals.

The voltmeter shown in figure 403 is a moving-coil galva-ometer such as is used in the ammeter shown in figure 401, but ıere is no shunt across between the terminals as in the ammeter, ıd a considerable resistance is inserted in the circuit so that ıly a small current passes through the instrument.

Voltmeters using current give correct values only in circum-tances where the current through the instrument is so small ıat it does not appreciably change the potentials to be measured.

For instance, the difference of potential of two statically ıarged bodies could not be determined by such an instrument, ır they would be instantly discharged ırough it. And if we attempt to measure ıe difference of potential of the terminals f a battery cell whose internal resistance as great as that of the voltmeter itself, ıe deflection will indicate only one-half ıe total electromotive force of the cell, ır the current is such that half the fall ı potential takes place in the cell itself §639).

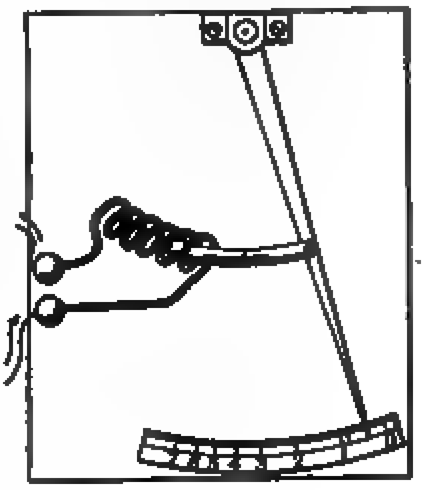


Fig. 404.—Ammeter with iron core.

In ordinary commercial work the other esistances in the circuit are so small ompared with that of a well-constructed voltmeter that there ı no difficulty on this score.

Such a voltmeter cannot be used for alternating currents.

698. Ammeter with Iron Core.—A simple form of ammeter ı that shown in figure 404 in which a soft-iron core is drawn into helical coil through which the current flows. Both ammeters nd voltmeters are constructed on this principle, and as the soft-

iron core is drawn inward when the current is in either direction they may be used either for direct or alternating currents, though the graduation must be different in the two cases.

699. Hot-wire Instruments.—In some instruments the current passes through a fine wire and the elongation resulting from its heating causes a pointer to move over a scale. The scale may be graduated to show either the current in ampères or the difference in potential between the terminals in volts. The wire is mounted in a metal case to screen it from air currents and keep it under as uniform conditions as possible.

The heating effect of a current is irrespective of its direction, and therefore such an instrument may be used either for direct or alternating currents.

700. Wattmeter.—If it is desired to know the *energy per second* or *watts* spent in any part of a circuit, as in the lamps between A and B in the left diagram of figure 405, the current

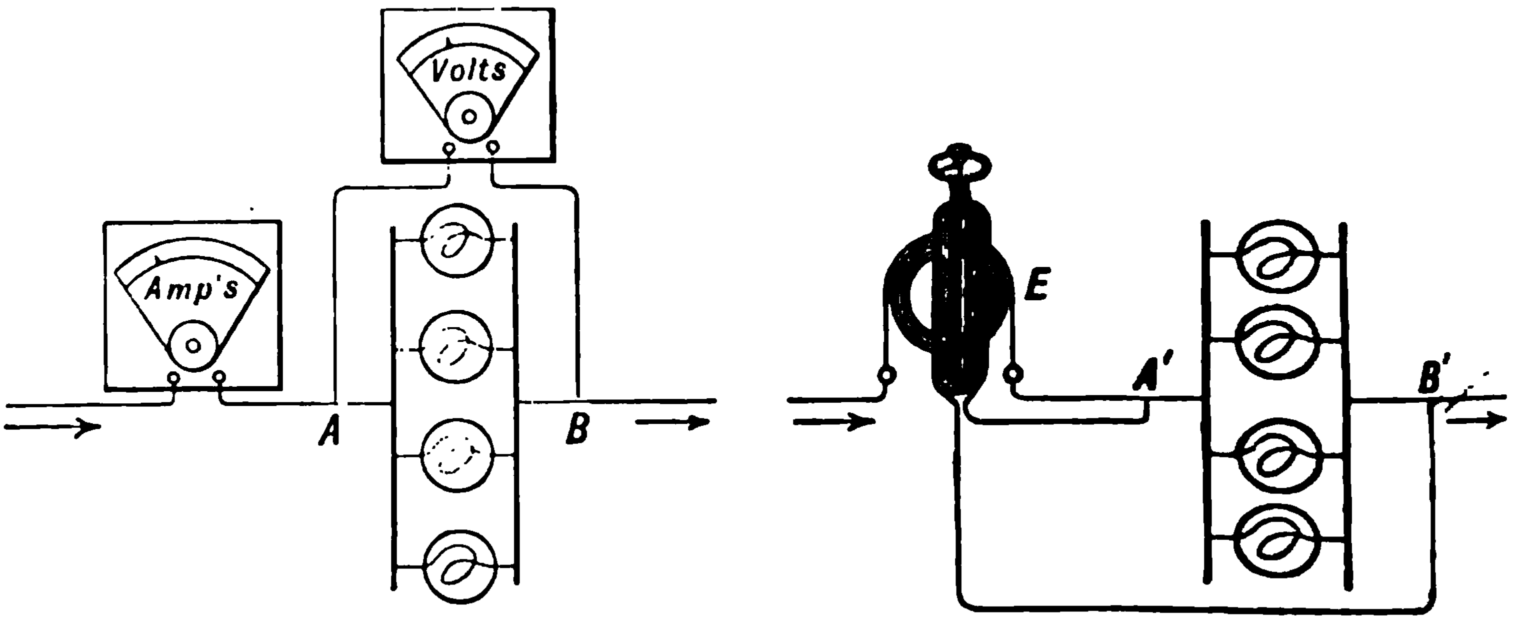


Fig. 405.

may be measured by the ammeter and the difference of potential between A and B by the voltmeter. The watts expended are given by the product of the current in ampères by the volts.

The result may, however, be obtained directly by using a *wattmeter*. This may be an instrument like the Siemens electrodynamometer connected so that the main current flows through the fixed coil E (right diagram figure 405) while the suspended coil has a great many turns of fine wire and is connected at A' and B' to the main circuit, so that the current in the suspended coil will be proportional to the difference of potential in volts between A' and B'. The torsion produced by the mutual action

of the two coils is proportional to the product of the currents in each, and is, therefore, proportional to IP where I is the main current and P is the potential difference between A' and B'. The instrument may therefore be graduated to give directly the watts expended between A' and B'. The suspended coil in this case is known as the potential or pressure coil, while the fixed one is the current coil.

Problems

1. What is the force of attraction between two straight parallel wires 30 cm. long and 1 cm. apart each carrying 3 ampères of current?
2. What must be the diameter of a coil of 3 turns of wire in order that a current of 5 ampères may produce a strength of field at its center of 0.20 dynes per unit pole?
3. A sensitive galvanometer having a resistance of 25 ohms is deflected one scale division by a current of $\frac{2}{1500}$ of an ampère. What resistance is required and how connected to change it into a voltmeter reading 1 volt per scale division, and what resistance and how connected to change it into an ammeter reading 1 ampère per scale division?
4. Given a voltmeter having a resistance of 800 ohms and reading 1 volt per scale division. How can it be made to read 10 volts per division?
5. How can the voltmeter described in problem 4 be used to find the current flowing through a conductor having a resistance 0.01 ohm per foot in length?

Bells and Telegraph

701. Electric Bells.—Bells are rung by electricity by the method shown in the figure. When the key at k is pressed, making a connected circuit, the current flows around the electro-magnet M, causing it to attract the soft-iron armature a to which is attached the hammer which strikes the bell. But as the armature a is drawn toward the magnet a metallic contact at b is separated, thus interrupting the circuit and causing the magnet to lose its magnetism. The armature being mounted on a spring flies back, makes contact again at b, and is then again attracted by the magnet as at first.

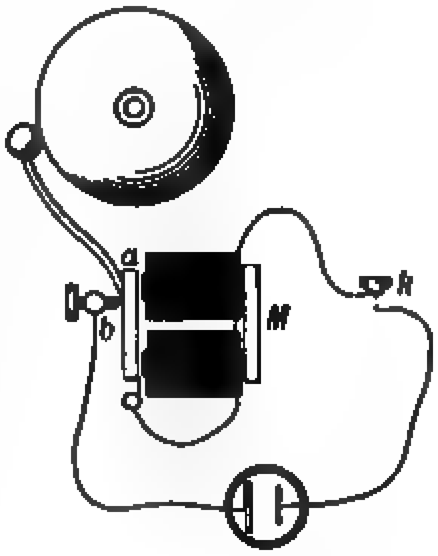


Fig. 406.—Electric bell.*

702. Electric Telegraph.—In the Morse telegraph, as originally used, a recording instrument made a dot or dash when the key was pressed in the distant station. In this instrument a strip of paper was drawn steadily over a roller by clock work, and when the key was pressed an electromagnet drew up a lever provided with a sharp steel point which pressed against the paper making a dot or dash, depending on whether the key made an instantaneous or more prolonged contact.

It was soon discovered that operators read the messages by sound, and therefore the elaborate recording instrument was replaced for the most part by the sounder, a simple electromagnet and armature arranged so that a vigorous *click* is heard when the circuit is closed or broken. In consequence of the resistance of long lines the current is very small and is therefore used to operate a *relay*, which merely closes the connection in a local battery circuit in which the sounder is included. The relay has a magnet wound with a great many turns of wire and in front of its poles is a nicely balanced armature controlled by a delicate spring so that a very small force will attract it. The armature is connected to the binding post *a* and the stop against which it is drawn is connected to *b*, so that when it is attracted by the influence of the main-line current, connection is made between *a* and *b*, thus closing the local circuit which includes the battery *B* and sounder *S*. The feeble motions of the relay armature are thus reproduced by the vigorous clicks of the sounder.

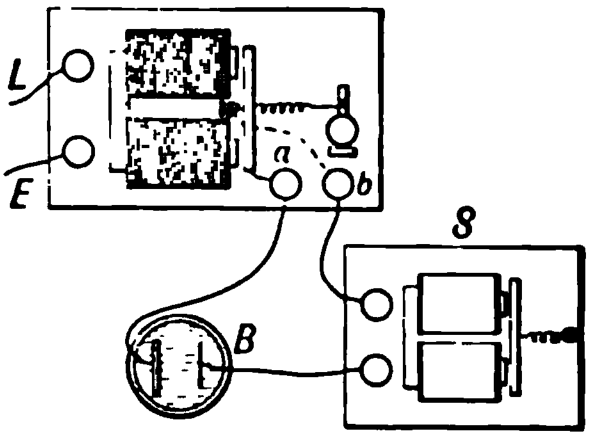


FIG. 407.—Relay and sounder.

Since the magnet of the relay must have a great many turns of wire, it must be wound with fine wire and will therefore have a large resistance; but since the resistance in the main line is already large, the additional resistance of the relay will have a comparatively slight effect on the current.

In the local circuit, the sounder and battery are all included in the same station and the resistance of the circuit may therefore be very small, hence the resistance of the sounder should be small, and accordingly it is wound with fewer turns of coarser wire.

In the main-line circuit a single wire of galvanized iron or hard drawn copper is used, the return circuit being through the earth. The following diagram shows the arrangement of a main line including three stations.

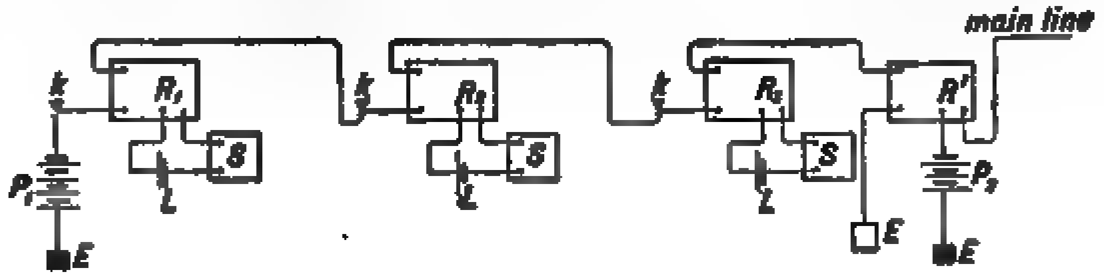


FIG. 408.—Diagram of telegraph line.

Each station has a key, relay, sounder, and local battery indicated respectively, by k, R, S, L.

The main-line battery P_1 operates all the relays for a certain length of the line. At the last station shown in the diagram there is a relay R' which transmits the signals to a second section of the main line which is operated by the battery P_2.

The keys are all provided with switches by which the circuit is kept closed everywhere except in the station where the operator is sending a message.

703. Duplex Telegraphy.—By the duplex system of telegraphy the efficiency of a telegraph line is doubled as it enables messages to be transmitted simultaneously in both directions One arrangement is shown in the diagram, figure 409. When the operator at A presses the key, contact is

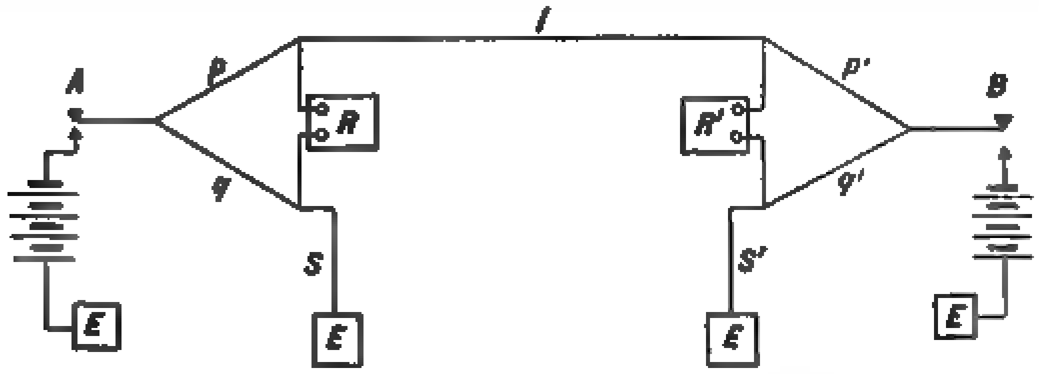


FIG. 409.

made with the battery and the current flows, but it divides between p and q in such a way that there is no flow across through R. This is accomplished, as in Wheatstone's bridge, by a suitable adjustment of the resistances p, q, and s. At the second station, however, part of the current will flow through the relay R', causing it to give the signal.

The relay R acts only in response to the key B, just as that at R' is affected only when A is pressed, and the two may therefore be operated quite independently of each other.

704. Quadruplex Telegraphy.—By the use of polarized relays or relays which act only when the current is in one direction, Edison was able to modify the old duplex method so that two messages could be simultaneously transmitted in each direction, or four altogether.

705. Cable Telegraphy.—Ocean telegraphy presents some serious difficulties from which land lines are comparatively free. The cable acts like an enormous Leyden jar, for it consists of a central conducting core made of a bundle of copper wires twisted together surrounded by a thick coating of rubber insulating material, outside of which is a protecting sheath of hemp and steel wires.

The copper core is the inner coating of the jar and the steel sheathing is the outer coating. The capacity of an Atlantic cable is about equal to 600,000 gallon Leyden jars. When one end of the cable is connected to the battery the current at the other end rises to its full strength only very slowly, as the cable is being charged at the same time. And when the current is broken the whole charge has to escape before the current dies out. In a typical Atlantic cable the current rises to $\frac{1}{10}$ of its maximum value in 0.2 second, and would require 2 seconds to come to $\frac{9}{10}$ of its maximum; therefore, in order to save time, exceedingly sensitive instruments must be used which will give an indication as soon as the current begins to rise at the farther end. In giving a signal, connection is made to the battery for an instant and then the end is grounded, thus sending a sort of wave into the cable which is sufficient to affect the instrument at the other end without fully charging the cable.

A double transmitting key is used by which the cable may be connected either to the positive or negative pole of a battery, and thus a series of waves may be transmitted, positive corresponding to dots, and negative to dashes of the telegraphic code.

The receiving instrument is a sensitive galvanometer which swings to the right or left as the waves of current pass through it.

On a line connecting points so far apart on the earth there is a tendency for earth currents to flow which would powerfully affect the delicate galvanometers used and completely overpower the desired signals. To obviate this difficulty Varley devised the plan of connecting the cable at each end

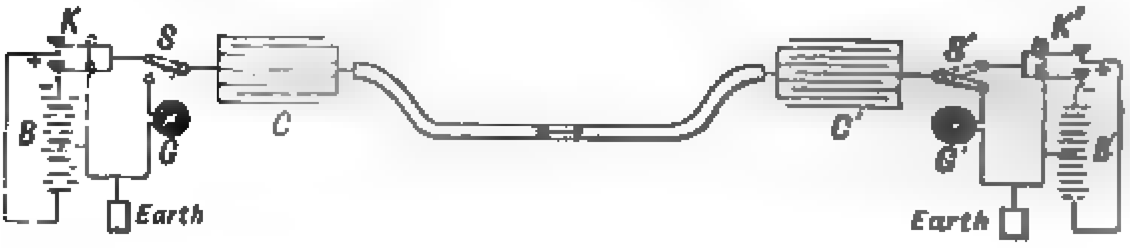


FIG. 410.—Diagram of cable connections.

to a condenser of large capacity which entirely prevents any steady flow through it due to earth potentials, but does not interfere with sending the signal waves.

A simple arrangement of a cable is shown in the above diagram.

The switches SS' are shown in position for sending by the key K and receiving by the galvanometer G'. Pressing the upper key at K gives a positive charge to the condenser C, while the other key gives it a negative charge. One terminal of the galvanometer G' is connected to the condenser C' while the other terminal is connected to earth.

706. Siphon Recorder.—The instrument now commonly used for receiving cable messages is the *siphon recorder* devised for the purpose by Lord Kelvin. It is a galvanometer of the type which later became known as the D'Arsonval form. A coil of wire hangs between the poles of a powerful magnet, and through this coil the cable currents pass, causing it to turn. Attached to the suspended coil is a fine capillary tube of glass shaped like a siphon, one end of which dips into a little cup of ink. The other end of the siphon tube just touches a strip of paper which is carried along by clockwork. As the coil turns the siphon moves to and fro across the paper, tracing a wavy line as the paper moves along. An automatic jarring apparatus prevents the friction between the paper and point of the siphon from interfering with the free motion of the coil.

Electromagnetic Induction

707. Faraday's Discovery.—The year 1831 was made memorable by the discovery of electromagnetic induction by Michael Faraday, then professor in the Royal Institution in London. In seeking to find some action of an electric current on a neighboring conductor Faraday, having placed a coil of wire carrying an electric current upon another coil which was connected to a galvanometer, found that if the electric current was interrupted or broken there was a sudden deflection of the galvanometer lasting only for an instant, and when the battery connection was made again there was an equal deflection but in the opposite direction. But the steady flow of current in one coil had no effect whatever upon the other.

These momentary currents are called *induced* or *secondary* currents, while the battery current by which they are produced is called the *primary* current. The corresponding coils of wire are known as the primary and secondary coils.

708. Induction by a Moving or Varying Current.—Faraday also showed that when a coil carrying a current is moved either toward or away from another coil connected to a galvanometer, an induced current is set up.

Such an arrangement as shown in figure 411 may be used, where the primary coil *A* has a current flowing through it from the battery and the secondary coil *B* is joined to the galvanometer. If the coil *A* is either pushed down inside of the coil *B* or withdrawn from it, an induced current is obtained which flows *around B in the* opposite direction to the current in A

when the two are pushed together, but in the same direction as in A when the coils are drawn apart.

If while the coil A is inside coil B the current in A is made weaker, an induced current is set up the same as though A were

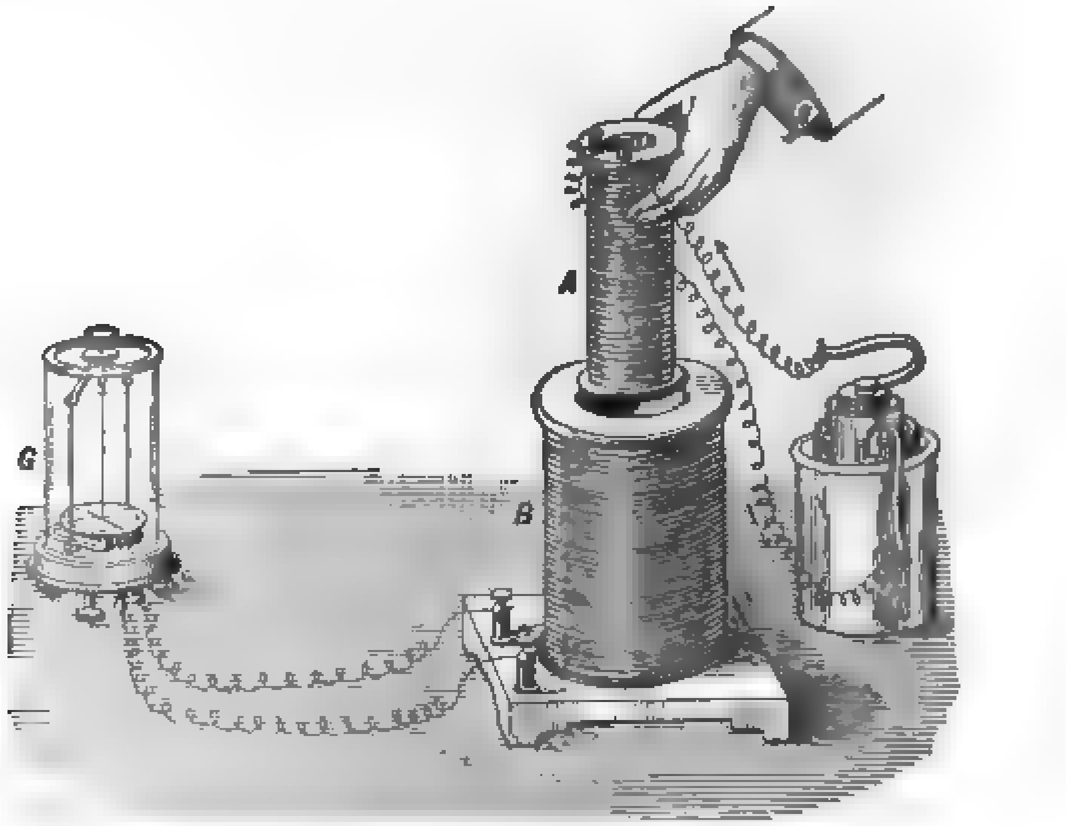


FIG. 411.—Induction by a moving current.

being withdrawn. But when the current in A is strengthened the effect is as though the coils were moved closer together.

709. Induction by Magnets.—Since a coil of wire carrying a current is surrounded by a magnetic field, it may be supposed that a magnet will produce a similar effect, and experiment shows this to be the case. When a bar magnet is thrust into a coil of wire connected in circuit with a galvanometer there is an instantaneous swing of the needle of the galvanometer, but *the needle at once returns to its zero position and remains there so long as the magnet is held at rest;* when it is withdrawn from the coil there is another instantaneous deflection opposite to the first. If the experiment is repeated with the magnet reversed, the deflections are opposite to those previously obtained.

710. General Condition of Induction.—**In general an induced current is set up in a coil whenever there is a change in the number of lines of magnetic force passing through the coil.**

This condition is illustrated in each of the three modes of producing induced currents just described. When the two coils of Faraday's first experiment are placed in the relation shown in figure 412 so that the lines of force due to the primary coil *P* instead of passing through the secondary coil pass on each side of it, there is no induced current in the secondary coil. So also there is no induction when a magnet is brought up to the coil in

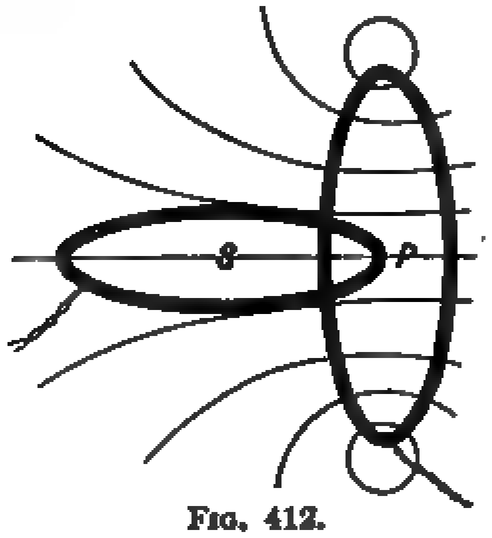


FIG. 412.
Coils with no mutual induction.

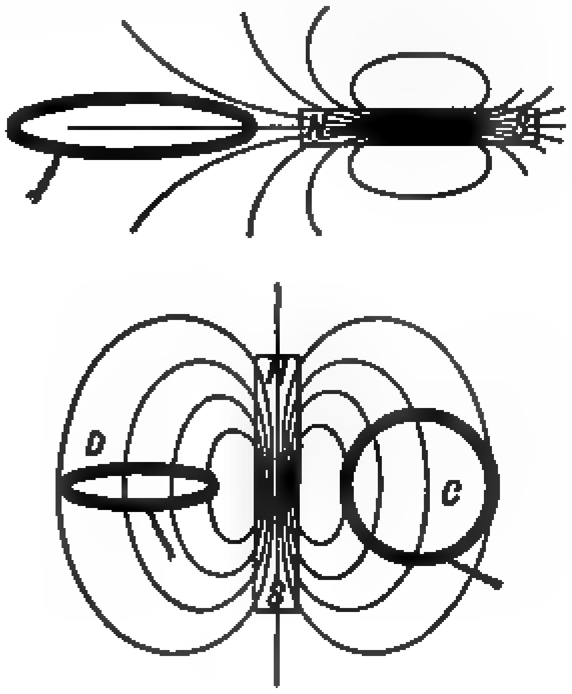


FIG. 413.

the position shown in the upper diagram of figure 413 or when the plane of the coil is parallel to the magnet as shown in the coil *C* on the right of the magnet in the lower diagram, but when the coil is at right angles to the magnet as in the left-hand coil *D* there will be an induced current when the magnet is brought up or taken away, because more lines of force of the magnet pass downward through the coil when it is near the magnet than when it is at a distance. (See §499 on number of lines of force.)

711. Induction by Earth's Field.—The inductive effect of the earth's magnetism may be easily observed by means of a coil of large area and many turns of wire connected with a suitable galvanometer.

If such a coil is held with its plane perpendicular to the lines of the earth's magnetic force as at *A*, figure 414, the maximum number *of lines of force* will pass through it. If it is now turned

quickly into the position B parallel to the lines of force, where none pass through it, there is an induced current because of the change in the number of lines of force through the coil. If the coil, instead of being turned half-way, is turned completely over, its position relative to the lines of force is exactly reversed and the inductive effect is twice as great as when it was turned half-way over.

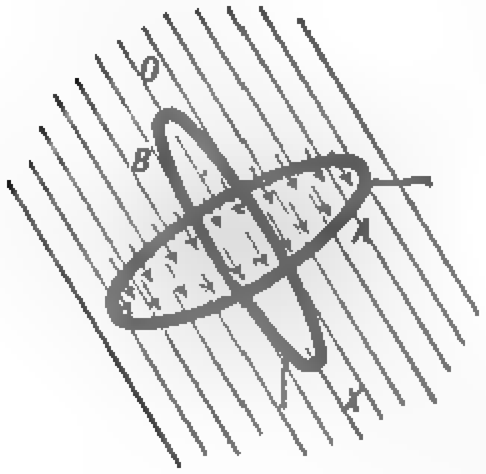


Fig. 414.—Coil in earth field.

When the coil in any position is rotated about an axis OX parallel to the lines of force of the field, there is no induction since no change takes place in the number of lines of force passing through it.

When the coil is laid flat on a table and slipped about from one place to another there is no induction, even if the table is tipped so that its top is at right angles to the lines of force, because the same number of lines of force pass through the coil wherever it is, since the field is uniform.

712. Faraday's Disc.—The following experiment due to Faraday shows that **when a conductor moves across the lines of force of a magnetic field an induced electromotive force is developed.**

A copper disc is mounted on an axis so that it can rotate between the poles of a horseshoe magnet, the axis of the disc being parallel to the lines of force. The edge of the disc dips into a mercury trough connected to one end of a low-resistance galvanometer circuit, the other end of which is put in contact with the axle of the disc.

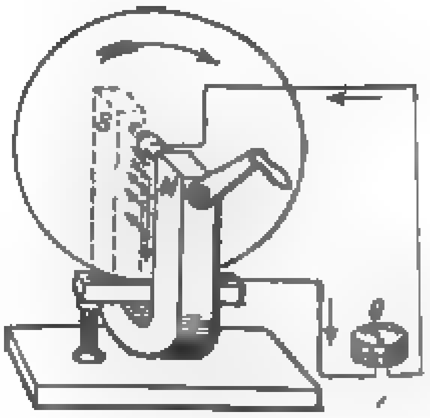


Fig. 415.—Faraday's disc.

On rotating the disc in the direction of the arrow a current is set up in the direction shown in the figure, the strength of which is proportional to the speed of revolution of the disc. If the disc is rotated in the opposite direction the current is reversed.

This experiment shows that each radial strip of the disc, as it cuts across the lines of force of the magnetic field, is the seat of an electromotive force which is found to be proportional to the number of lines of force cut across per second, for it is proportional both to the speed of rotation of the disc and to the strength of the magnetic field.

713. Electromotive Force of Induction.—When the C. G. S. electromagnetic system of units is used (§602–603) **the electromotive force of induction is numerically equal to the number of lines of force, or unit tubes, cut across per second by the conductor**; that is,

$$E = \frac{N}{t} \qquad (E \text{ in C. G. S. units})$$

where E is the electromotive force induced in a conductor which is cutting across lines of force *at the rate* of N lines in t seconds.

Or, since one volt (§603) is equal to 10^8 C. G. S. units of potential,

$$E = \frac{N}{10^8 t} \qquad (E \text{ in volts})$$

To prove this relation suppose a circuit, such as is shown in figure 416, consisting of two straight parallel conducting rails connected together at one end and also connected by a cross conductor AB which can slide in the direction of the arrow; and let this circuit be in a magnetic field of strength H in which the lines of force are perpendicular to the plane of the circuit. Then if AB is slid along by hand at the rate of x cm. per second, an induced electromotive force will be produced which will cause a current I in the circuit.

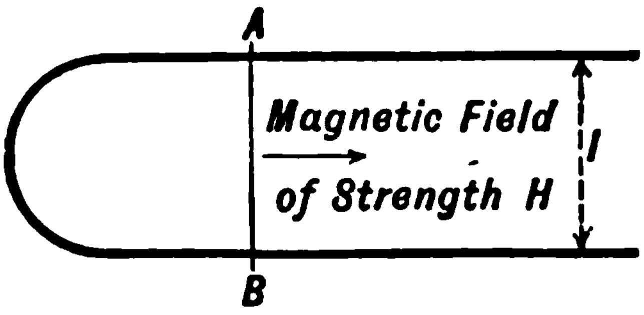


Fig. 416.

The energy expended per second by this current will be IE (§652), but this energy is supplied by the work expended in moving the conductor along and must be equal to it. But a conductor of length l which carries a current I across a magnetic field of strength H, is acted on by a force $F = HIl$ (§684) and if in one second the conductor is moved against that force through a distance x the work done in one second is $Fx = HIlx$. We have then,

$$IE = HIlx \quad \text{or,} \quad E = Hlx$$

but lx is the area moved over by the conductor AB in one second, and so Hlx equals the number of lines of force cut across per second.

Thus the electromotive force of induction in C. G. S. units is shown to be numerically equal to the number of lines of force cut across per second by the moving conductor.

714. Illustration.—For example, suppose a straight conductor AB, one meter long (Fig. 417), is moved in the direction of the large arrow at the rate of 3 meters per sec., and suppose it is in a magnetic field of strength 0.5 (about as strong as the earth's field) in which the lines of force are down perpendicular to the paper. Then the number of lines of force cut per second will be the area in centimeters swept across per second by the conductor, multiplied by the number of lines of force per square centimeter which in this case is 0.5, or $E = 100 \times 300 \times 0.5 = 15000$, which is the electromotive force in *C. G. S.* units; to change it to volts it must be divided by 10^8, hence

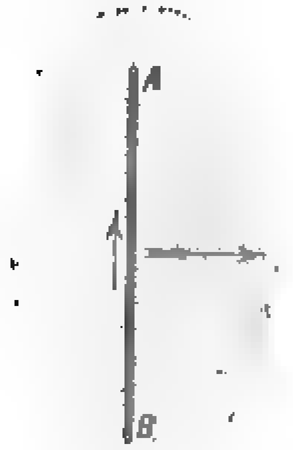


FIG. 417.—Wire moving across lines of force.

$$E = 0.00015 \text{ volt}$$

which is the difference of potential between the ends of the wire, since it is disconnected and no current can flow.

715. Why Induction Depends on Change in Number of Lines of Force through a Circuit.—We are now prepared to understand why it is that the resultant electromotive force induced in a circuit depends on the *change* in the number of magnetic lines of force passing through the circuit.

It has already been seen that when a coil of wire lying on a table is slid along, no induced current is produced although the wires of the coil cut across the lines of force of the earth's magnetic field (§711). The explanation of this is that electromotive forces are induced, but in such a way that they balance each other. For suppose the coil is moved from A to B as in figure 418 and that the lines of magnetic force are straight down perpendicular to the diagram, then the sides of the coil cut across lines of force in such a way as to cause electromotive forces in the direction of the arrows. The electromotive forces induced in the two sides therefore act against each other in the ring, but they are equal because each side of the ring cuts across the same number of lines of force in the same time, therefore the electromotive forces balance and *there is* no current.

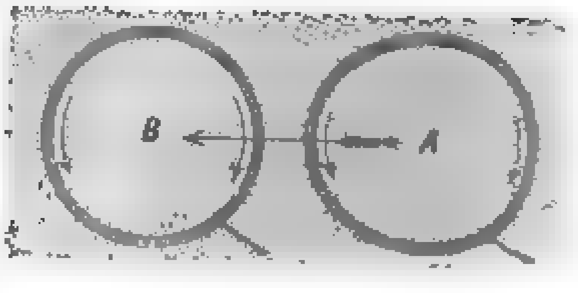


FIG. 418.—Coil moved sidewise in a magnetic field.

If the field is not uniform so that more lines of force pass through the coil in the second position B than in the first position, then more lines of force must have cut into the coil across its left-hand side than have cut out of it across its right-hand side. The electromotive force developed in the left-hand side of the coil will then be greater than the other and will cause a current to flow around the coil counter-clockwise. Therefore there must be a resultant electromotive force whenever the number of lines of force through a coil is increased or diminished.

716. Induced Electromotive Force.—Since the electromotive force developed in any part of a conductor by induction is equal to the number of lines of force which cut across it per second (§713), it follows that **in any circuit or coil the electromotive force of induction is equal to the change per second in the number of lines of force included by the circuit.**

This is expressed by the formula

$$E = \frac{N_1 - N_2}{t}$$

which gives the *average* electromotive force during the time interval t when N_1 is the number of lines of force through the circuit at the beginning of the interval and N_2 the number at the end.

By taking the time interval very short we approach the instantaneous value of the electromotive force as a limit.

If there are several turns of wire in the coil, to get the total electromotive force the above expression must be multiplied by the number of turns.

It is clear from the above that the more quickly the change in the number of lines of force takes place the greater the electromotive force.

717. Induced Current and Total Flow.—The induced current at any instant is by Ohm's Law

$$I = \frac{E}{R} \quad \text{and since} \quad E = \frac{N_1 - N_2}{t}$$

we have

$$I = \frac{N_1 - N_2}{Rt}$$

The *instantaneous* value of the induced current is therefore

greatest when the induced electromotive force is greatest; that is, when the change in the number of lines of force through the circuit is taking place most rapidly.

But It, the product of current by the time that it flows, is the whole quantity of charge or electricity that passes in time t; thus

$$It \text{ or } Q = \frac{N_1 - N_2}{R}$$

A simple integration shows that this expression holds true in every case, at whatever rate the lines of force through the circuit may be changing. **The total quantity of electricity passing a given point in the circuit in consequence of induction is equal to the change in the number of lines of force through the circuit divided by its resistance.** If C. G. S. units are used for N and R the quantity Q will also be in that system. To find it in coulombs it must then be multiplied by 10.

It is to be remarked that **the total quantity of the induced flow is independent of the time during which the induction takes place.** It is the same when a magnet is put into a coil as when it is pulled out and whether it is moved slowly or rapidly.

718. Energy in Induction.—Every current of electricity possesses energy, and therefore energy is required to produce induced currents. During the changes which produce an induced current energy is supplied to it, and it dies out immediately when the inductive action stops because its energy is expended in heat in the conductor if in no other way. When induced currents are set up by making or breaking the current in an adjoining primary circuit the energy comes from the primary battery. When the induced current is caused by the motion of a conductor in a magnetic field the energy is supplied by the agency which causes the motion.

For instance, more energy must be expended when a magnet is thrust into a coil in which the ends of the wire are connected forming a closed circuit than if the ends had not been joined, for there is an induced current in the first case and not in the other. But in order to expend energy resistance must be overcome, and so the induced current must cause a force which resists the magnet as it is pushed into the coil. For the same reason the current which is induced when the magnet is with-

drawn must exert a force to resist the withdrawal of the magnet so that more work is done than if the current could not flow.

719. Lenz' Law.—The general law suggested in the last paragraph was first stated by Lenz and is known by his name. It may be stated thus: **An induced current is always in such a direction as to resist by its electromagnetic action the motion by which it is produced.** This law is a direct consequence of the conservation of energy, as has been already indicated.

720. Illustrations of Lenz' Law.—Thus in case of Faraday's disc experiment (§712) the induced current tends to rotate the disc in the opposite direction (see Barlow's wheel, §686) so that it is harder to turn the disc while the induced current is flowing than if the circuit were disconnected.

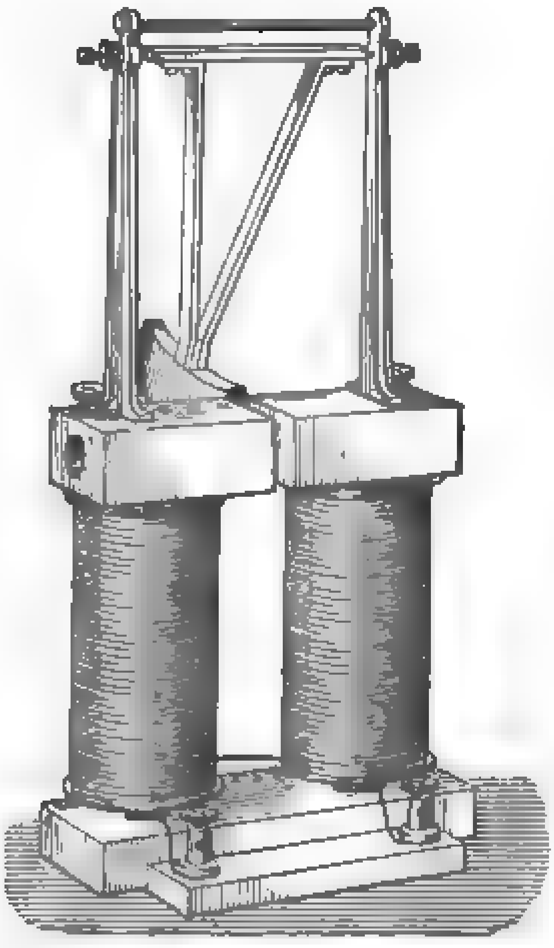


Fig. 419.—Copper pendulum and magnet.

If a thick strip of sheet copper is hung like a pendulum so that it can swing edgewise between the poles of a powerful electromagnet, it may swing down with a rush, but is instantly checked as it comes between the magnet poles, since there are induced in the copper, currents of electricity which resist the motion, transforming the energy of motion into current energy which finally results in heat in the copper. In some forms of galvanometer a bell magnet is employed, so called because it is shaped like a cylindrical bell of steel slit part way up, the poles being on the two sides. If such a magnet is suspended in a slightly larger cylindrical cavity in a copper block, it generates by its motion induced currents which quickly bring it to rest. This mode of stopping the vibrations of a magnetic needle is called **electrical damping.** The damping of the coil of a D'Arsonval galvanometer *(§692)* is also explained in the same way.

721. Arago's Disc.—A celebrated experiment of Arago's, which was first explained by Faraday, is illustrated in figure 420. A copper disc is rotated rapidly under a magnetic needle from which it is separated by a sheet of glass or parchment which prevents air currents from having any influence on the needle, and the needle is carried around with the disc. Induced currents are set up in the disc which resist the relative motion of the two, consequently the needle is dragged along after the disc. The lines of force due to the needle go down through the disc under the north pole and the induced currents are as indicated by the dotted curves. It is easily seen that the current flowing under the needle will tend to cause it to turn in the direction of the disc, as in Oersted's experiment (§671).

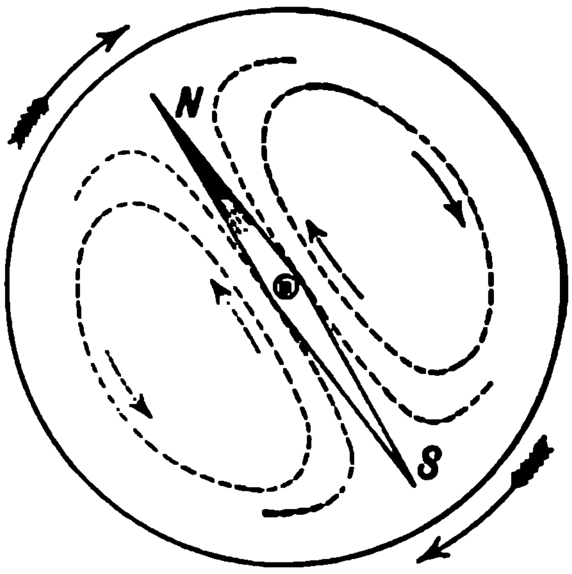


FIG. 420.—Currents in Arago's disc.

722. Rules for the Direction of Induced E.M.F. and Current.—Lenz' Law leads to the following rules for the direction of the induced electromotive force and resulting current:

Case of a Wire Moving Across Lines of Force

In this case the electromotive force induced in the wire is in such a direction as to cause a current which will strengthen the field immediately in front of the moving wire and weaken the field immediately behind it.

For it has been seen in §684 that such a current would urge the wire across the field in the opposite direction, thus resisting the motion.

Case of a Closed Circuit

When the number of lines of force through a circuit is increasing, the induced current is in such a direction as to set up lines of force through the circuit opposite to those already there, thus opposing the increase.

If the number of lines of force is decreasing, the induced current is in such a direction as to set up lines of force inside the coil in the same direction as those already there, thus opposing the decrease.

723. Self-induction.—When a current of electricity is set up in a coil of wire each turn in the coil experiences the inductive effect of the current starting in all the other turns. All act together to cause *an induced current in the coil opposite to the current which is starting.* The resultant current is therefore weaker than the steady current which will flow when the inductive action is over. When the circuit is broken *the self-induced current is in the same direction as the current which has been flowing;* it acts therefore *with* that current and prolongs its flow, causing a bright spark across the gap where the circuit is broken. The current induced on breaking connection is known as *the extra current.*

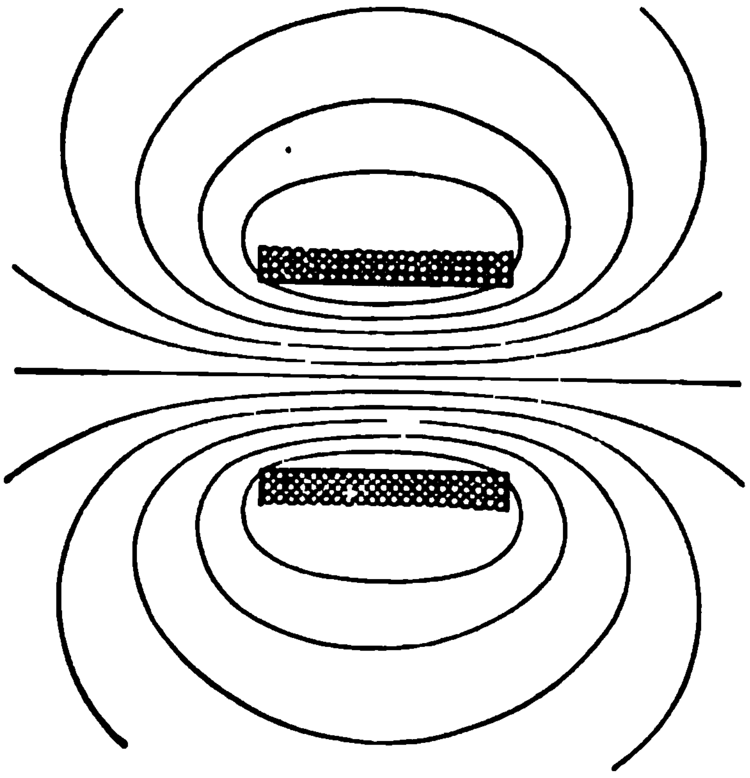


FIG. 421.

In this case, as in all other cases of induction, the action is due to that relative motion of conductors and magnetic field expressed by the phrase "cutting lines of force." The coil after the current is established has a magnetic field, and includes a large number of lines of force. These lines of force form closed curves surrounding the coil and may be considered as starting in the coil and spreading out in expanding curves as the current becomes stronger. Each turn of wire in the coil is cut by all the lines of force and hence the electromotive force of self-induction depends on the number of turns of wire in the coil and the total number of lines of force that are set up by its current. What is called the *coefficient of self-induction* of a coil is the product of the number of its turns of wire by the number of lines of force through the coil when unit current is flowing in it. Thus even a circuit consisting of a single turn of wire has some self-induction, but it is greatest in coils which have many turns of wire and include a great number of lines of force, as in electromagnets, where the iron core immensely increases the self-induction.

724. Experimental Illustration.—Take a large electromag-

net of low resistance having an armature across its poles and connect a small incandescent lamp across its terminals, as shown in figure 422. Then join the magnet to a storage battery which is strong enough to light up the lamp when connected to it alone, interposing a contact key. On pressing the key the lamp lights for an instant as the electromotive force of self-induction opposes the flow of current through the magnet and sends it through the lamp instead. The lamp dies out, however, as the current comes to its steady state and divides between lamp and magnet. On breaking the circuit the lamp again glows as the self-induced current rushes around through the lamp instead of leaping the gap at the key. *The phenomena of self-induction are observed only while the current is changing, hence in case of steady currents self-induction need not be considered,* but in dynamo machines and all alternating current apparatus it plays a most important part.

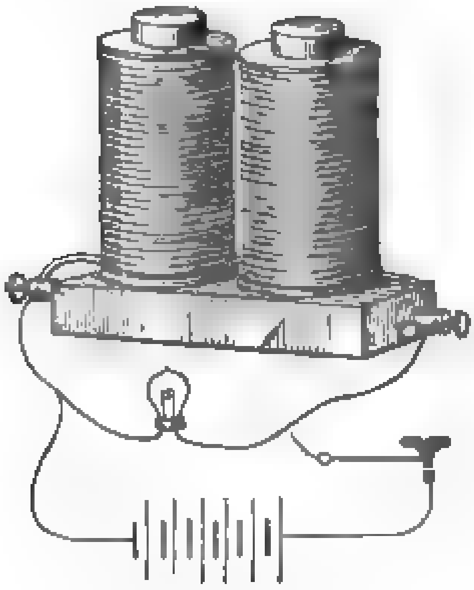


Fig. 422.—Self-induction.

In breaking connection in a circuit containing much self-induction, such as one including electromagnets or a dynamo machine, great care must be taken not to be touching the conductors on both sides of the gap when the contact is broken; otherwise a severe shock may be obtained from the *extra current* even when the ordinary voltage in the circuit is small.

725. **Energy of a Magnet.**—Every portion of the magnetic field whether within the iron core of the magnet or outside of it has a certain energy in consequence of its magnetization. It was shown by Maxwell that the energy per cubic centimeter in any part of a magnetic field is $\frac{B^2}{8\pi\mu}$, where B is the induction at that point or number of lines of force per square centimeter.

Therefore, when a current is starting in a coil or electromagnet it has to supply the energy of the magnetic field besides spending energy in heat owing to the resistance of the conductor. After the magnetic field is fully established, which may take several seconds in a large magnet, the current is steady and

pends energy only in heat in the conductor. **No energy is required to keep up a magnetic field when it is once established.**

The spending of energy by a current in making a magnetic field causes the current to delay in coming to its full strength and is the cause of the self-induced current on making connection. When the circuit is broken the field loses its magnetization and therefore gives up its energy again to the current. This causes the *extra current* or induced current on breaking the connection, and *the energy of this extra current is equal to the energy that was stored up in the magnet and surrounding magnetic field.*

726. Induction Coil. Ruhmkorff Coil.—The induction coil is a device for obtaining induced currents of very great electromotive force from an ordinary battery current. The construction is illustrated in figure 423. The *primary coil*, of a few layers of large copper wire so as to have small resistance, is wound about a central core which consists of a bundle of soft-iron wires. Outside of the primary coil and thoroughly insulated from it by a thick tube of hard-rubber is the *secondary coil*, made of an immense number of turns of fine wire the ends of which are brought to two insulated posts supporting the discharging rods *a b*. The diagram, for distinctness, shows only a few turns of wire in the secondary; but in the actual instrument there are thousands of turns, a coil to give a one inch spark must have something like a mile of wire in its secondary coil. The primary must be thoroughly insulated from the secondary by a thick tube of hard-rubber with hard-rubber flanges at the ends. The primary coil is connected to a battery of a few storage cells and when the current is interrupted the induced electromotive force in the secondary coil may be great enough to cause a discharge across between the discharging rods. *The primary current is automatically connected and broken.* A device commonly used is shown at *d*. A little block of iron on the end of a spring is mounted opposite the end of the iron core of the apparatus. The spring *rests* against the end of an adjusting screw,

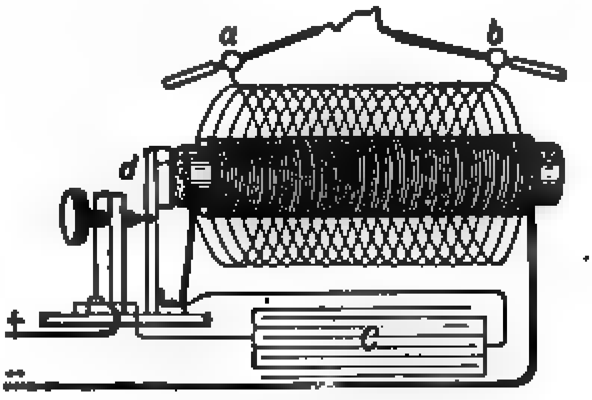


Fig. 423.—Induction coil.

the points of contact on each of them being made of platinum. The connections are made so that the primary current flows across the contact between spring and screw, and consequently as the core becomes magnetized and attracts the block of iron mounted on the spring, the connection is broken. But as the core then loses its magnetism the spring comes back and again makes the connection; and so the action is repeated, automatically making and breaking the current many times in a second.

The self-induction of the primary coil causes both the starting and stopping of the current to be prolonged, and consequently the E.M.F. of induction would be comparatively small if this were not obviated. It is found that if a condenser of suitable capacity is connected to the primary circuit, its two surfaces being connected one on each side of the point where the current is broken, the electromotive force produced on breaking is greatly increased. Such a condenser is represented at *C*; it is usually made of alternate sheets of tinfoil and paraffined paper, the odd sheets of tinfoil being connected together for one coating and the even sheets forming the other. By this construction a large capacity is obtained in very compact form. The condenser is often contained in the base of the instrument.

When the current is broken at *d* the extra current of self-induction rushes into the condenser and charges it instead of discharging in a spark across the gap at *d*. The flow of the extra current is thus very quickly stopped; but after the condenser is charged it immediately discharges itself back through the coil in a direction opposite to the original current, and so more perfectly demagnetizes the core or even magnetizes it oppositely.

By the use of the condenser, then, there is a greater change in the number of lines of force on breaking the current, and the change is more instantaneous, both effects serving to increase the electromotive force of induction; and at the same time the sparking at the gap *d*, which is very destructive to the platinum contacts, is greatly reduced.

727. Wehnelt Interrupter.—Instead of a mechanical interrupter for the circuit an electrolytic cell may be used, known as the Wehnelt interrupter from its discoverer. This cell consists of a vessel containing dilute sulphuric acid, having for the nega-

ɜlectrode a plate of sheet lead and for the positive electrode :inum wire or rod covered with a glass or porcelain sheathing at only the tip end projects into the acid. An adjusting ' is provided by which the amount projecting may be ated. If the voltage in the circuit is ient and the exposed tip of platinum is properly proportioned to the cur- the circuit will be rapidly interrupted, les of gas being given off at the plati- wire accompanied by flashes of light. frequency of interruption depends on elf-induction of the circuit and the omotive force of the battery as well on the adjustment of the platinum , and may be varied through wide s. With this form of interrupter a enser is of no advantage.

Fig. 424.—Electrolytic interrupter.

3. **Telephone.**—In the early telephone vised by Bell the receiver and trans- r were alike, the construction being n in figure 425. A hard-rubber handle contains a hard- cylindrical magnet, around one end of which is fixed a f many turns of fine wire the ends of which are brought to ng screws on the handle. A disc of thin sheet iron, supported at the edges so that it is free to vibrate in the middle, is mounted so that its center comes close to the end of the magnet and surrounding coil but does not touch them. A hard-rubber cap or ear-piece having e in the center fits over the disc and serves to clamp it / at the edges as well as to improve the quality of the by favoring the sound waves from the center of the disc.

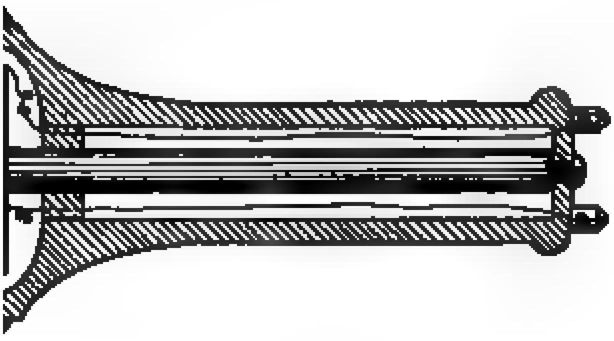


Fig. 425.—Telephone receiver.

ppose two such instruments with the coil in one connected sed circuit *with the* coil in the other. If a person speaks

into one the sound waves impinging on the center of the disc cause it to vibrate; but as it vibrates induced currents are set up, for when the disc approaches the magnet more lines of force pass through the coil into the disc, and as it springs away the lines of force spread out again cutting across the coil. These induced currents flow through the coil around the magnet of the receiving telephone and by alternately opposing and strengthening its magnetism cause the iron diaphragm of the receiver to vibrate in exact correspondence with that of the transmitter, so that the same motion is given to the air at one end as that which caused the disc to vibrate at the other, thus reproducing the sound.

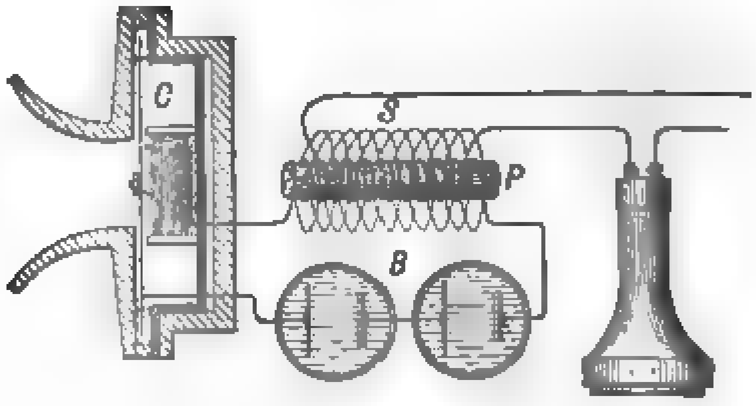


FIG. 426.—Transmitter.

There is a serious defect in this mode of transmission. All the energy of the induced currents must come from the sound waves which cause the disc of the transmitter to vibrate, and as a part of this energy is spent in heat in consequence of the resistance of the circuit, the sound heard at the receiver must be faint. To meet this difficulty another form of transmitter shown in figure 426 is ordinarily used. The cell C containing carbon granules between two plates of polished carbon is mounted between the thin metal diaphragm and the solid back of the instrument, and on each side of the cell is a metal plate connected in circuit with a battery B and the primary winding P of a small induction coil, of which the secondary S is connected to the line leading to the receiving station.

When sound waves fall upon the diaphragm of the transmitter the vibrations cause a variation in its pressure on the carbon cell and a consequent change in its resistance. The other resistances in the battery circuit are compared with

that of the granular carbon, hence variations in its resistance cause decided changes in the strength of the current. These set up induced currents in the secondary and produce corresponding vibrations in the diaphragm of the receiver, thus reproducing the sound.

By this arrangement the energy for transmission is supplied by the battery, and by taking a proper number of turns in the secondary coil the induced current can be adapted to the resistance of the line.

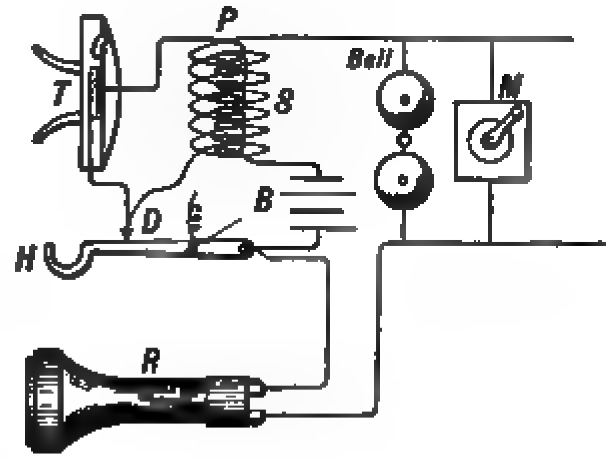


Fig. 427.

A telephone line is usually a complete circuit of two wires instead of using the earth, as in telegraphy, and the two wires are carried near together so that the inductive action of neighboring telegraph lines and lighting wires on one wire may be neutralized by their action on the other.

The arrangement adopted in the local battery system is shown in figure 427. When the receiver R is hung on the hook H the battery circuit is broken at D and also the secondary circuit so that no current flows from the

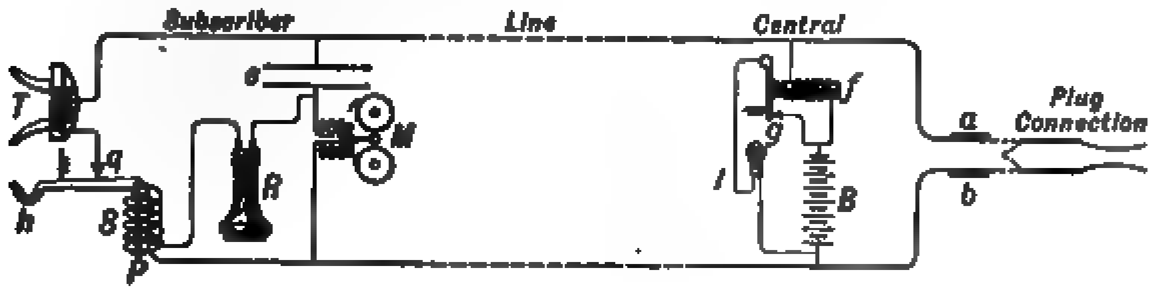


Fig. 428.—Telephone with central battery.

battery except when the line is in use. The call bell is of very high resistance so that only a very small part of the current is diverted through it, and the magneto M by which the bell is rung is so devised that it is connected to the line only while being used.

The local battery is often done away with and the current through the subscriber's transmitter supplied by a single battery at the central station. One method of connection is shown in figure 428.

A battery B of about 24 volts is connected to the line at the central station; but when the receiver R hangs on the hook h there is no current in the line, for the circuit is broken at g and no current can flow across through the call bell M because the condenser e is interposed. The subscriber may be called,

however, by connecting to the terminals *ab* any source of *alternating* current, which causes a surging of current back and forth in the line to the condenser, charging it so that first one of its coatings is positive and then the other, alternately. This current as it flows alternately into the condenser and out again rings the call bell *M*. On the other hand, if the subscriber wishes to call "central" he has only to lift his receiver from the hook. The current is then established through the contact points at *q* and flowing around the relay *f* closes at *g* the circuit through the signal lamp *l* which flashes out and shows that a connection is desired. The correspondent's line is then connected by means of a flexible cord having two conductors and terminating in a double contact plug which connects one conductor to *a* and the other to *b*.

The receiver *R* is so connected that only an extremely minute direct current from the battery can flow through it on account of the large resistance of the bell *M* (1000 ohms), but the *alternating* "talking" current induced in the secondary *s* is readily established through the condenser *e*. The "talking" current and the direct current from the battery through the transmitter are thus both transmitted over the same line without interfering with each other.

8407

ELECTROMAGNETIC UNITS

729. C. G. S. Electromagnetic Units.—The C. G. S. electromagnetic system is based on the unit magnet pole as defined in §485, unit current as in §602, and unit electromotive force as in §603. These units are determined from the above definitions by certain measurements of length in centimeters, of mass in grams, and of time in seconds. They have the advantage of being directly connected with the fundamental mechanical units of the C. G. S. system. Thus the product of current by electromotive force measured in these units gives the rate of spending energy in ergs per second.

The C. G. S. unit of resistance is the resistance of a circuit in which the above unit electromotive force will produce unit current of the same system.

730. Practical System of Units.—The C. G. S. units are not of a convenient size for use in commercial measurements, but it is desirable that the practical units should be related to the C. G. S. units by ratios which can be expressed by simple powers of 10.

Thus *the volt, the practical unit of electromotive force, is chosen equal to* 10^8 *C. G. S. units of electromotive force,* because that particular power of 10 gives a value nearer to the electromotive forces of ordinary battery cells than any other would have done.

The ohm is defined as 10^9 times the C. G. S. unit of resistance.

This power of 10 was adopted because it corresponds very nearly to the Siemens unit* of resistance which was already in use and had been found convenient.

The ampère is 10^{-1} times the C. G. S. unit of current. It is determined by Ohm's law as the current which results from an electromotive force of 1 volt in a circuit having a resistance of 1 ohm.

The coulomb is the unit of charge. It is the charge transmitted in 1 second by a current of 1 ampère. It is almost exactly equal to three thousand million electrostatic units of charge as defined in §525.

The farad is the unit of capacity. It is the capacity of a condenser which will hold a charge of one coulomb when the difference of potential between its coatings is 1 volt. This unit is so large that ordinary condensers are rated in microfarads, or millionths of a farad.

The henry is the unit of inductance.† It is the inductance of a circuit in which an increase in current strength at the rate of 1 ampere per second produces a back electromotive force of 1 volt.

Elaborate experiments have been made to determine how the units as above defined may be realized in practice, and the following experimental values have been obtained:

The ohm is the resistance of a column of pure mercury 106.3 cm. long and 1 sq. mm. in cross section at the temperature of melting ice.

The ampère is a current which will deposit 0.001118 grm. of silver per second in a silver voltameter.

The volt may be determined from a standard Clark cell, the electromotive force of which at 15°C. is found to be 1.4322 volts.

731. To Change from the Electrostatic to the Electromagnetic System.—The ratio of any electrostatic unit to the corresponding electromagnetic unit is in every case some power of the velocity of light (3×10^{10}) cm. per second.

Electrostatic quantity of charge $\div (3 \times 10^{10})$ = charge in C. G. S. electromagnetic units.

* The Siemens unit is the resistance of a column of pure mercury 1 meter long and 1 sq. mm. in cross section, at the temperature of melting ice. Named from Sir William Siemens, the distinguished German physicist and engineer who advocated it.

† Named in honor of *Joseph Henry*, a distinguished American physicist and first Secretary to the Smithsonian Institution, who discovered self-induced currents.

Electrostatic quantity of charge $\div (3 \times 10^9)$ = coulombs.

Electrostatic potential $\times (3 \times 10^{10})$ = potential in C. G. S. electromagnetic units.

Electrostatic potential $\times 300$ = volts.

Electrostatic capacity $\div (3 \times 10^{10})^2$ = capacity in C. G. S. electromagnetic units.

Electrostatic capacity $\div (9 \times 10^{11})$ = farads.

Electrostatic capacity $\div (9 \times 10^5)$ = microfarads.

Problems

1. A coil of wire of 10 turns, each turn enclosing an area of 900 sq. cm., is turned from position A to B (see Fig. 414) in $\frac{1}{2}$ second. Find the induced E.M.F. in volts when the strength of the magnetic field is 0.5.
2. A metal spoke in a wheel is 80 cm. long. If the wheel makes 300 revolutions per minute in a plane perpendicular to the lines of force of the earth where the field strength is 0.5, find the difference of potential between the center and rim of the wheel. Which part is at the higher potential when the wheel rotates clockwise as seen by one looking in the positive direction of the lines of force?
3. Show that the work expended in producing an induced current by turn- a coil over in a magnetic field becomes 2 times as great when the time of the operation is reduced $\frac{1}{2}$.
4. A railway train runs south on a straight track with a velocity of 25 meters per sec. If the vertical component of the earth's magnetic force is 0.50, find the electromotive force induced in a car axle 120 cm. long; also which end, east or west, is at the higher potential.
5. When the vertical component of the earth's magnetic force is 0.50, find the electromotive force induced in a coil of 10 turns of wire 3 meters in diameter which while lying on the ground is in $\frac{1}{2}$ second pulled out into a loop so long that the sides touch.
6. When a circular coil of 100 turns of wire 1 meter in diameter lying on the floor is turned over in 0.3 seconds, find the average electromotive force, earth field being as above.
7. If the resistance of the coil in the last problem is 2 ohms, find the total flow of electricity in coulombs, also the average current in ampères, also the energy spent in producing the current.
8. A magnet which includes 6000 lines of force is pulled out of a coil of 160 turns of wire which closely surrounds it, in $\frac{1}{10}$ second. Find the induced electromotive force in volts.
9. A disc of iron 60 cm. in diameter mounted in a uniform magnetic field so that 4000 lines of force per sq. cm. pass perpendicularly through it, rotates like Faraday's disc (§712), making 30 revolutions per sec. Find the difference in potential between the edge of the disc and its center.

10. A rectangular loop of wire 20 × 30 cm. is rotated about an axis parallel to the long sides and half-way between them, in a magnetic field of strength 2000. If the axis is perpendicular to the lines of force of the field, what will be the average electromotive force in a half rotation between reversals (§733) when the loop makes 20 revolutions per sec.?
11. What is the maximum electromotive force in the case specified in the preceding problem?

Dynamo Electric Machines and Motors

Part I.—Direct-current Dynamos

732. Introductory.—The first machine by which a continuous current of electricity was developed by electromagnetic induction was Faraday's rotating copper disc (§712).

A machine developing current by electromagnetic induction consists of a strong magnet between the poles of which an *armature* rotates which contains the conductors in which the currents are induced. Such *generators*, as they are called, are known as *magneto* machines when permanent steel magnets are used, and *dynamo* machines when electromagnets are employed.

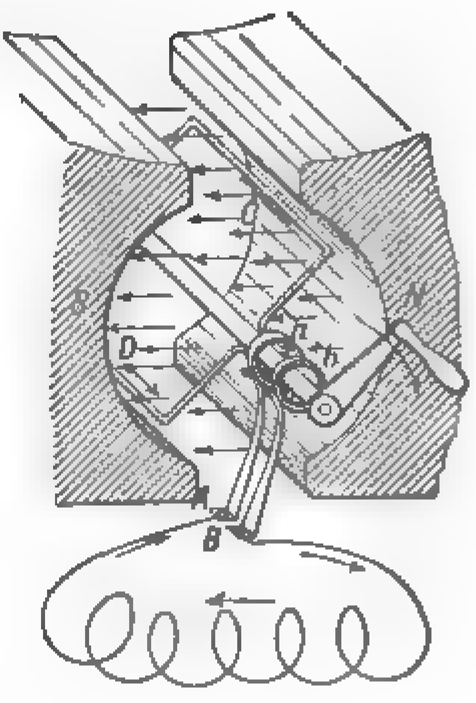


Fig. 429.—Induction in simple loop armature.

733. Rectangular Armature.—Suppose that a simple rectangular frame of wire is rotated between the poles of a powerful magnet as shown in figure 429, and that its ends are connected to two rings *a* and *b* which are mounted on the axle, and against which press two springs connected to the ends of the outer circuit. In the position shown the upper bar *C* is rapidly cutting across lines of force. By the rule of induction (§722) the induced electromotive force is in the direction of the arrow. So also electromotive force is developed in *D*. These two electromotive forces act together to cause a current in the outside circuit from *B* to *A*. This will be the direction of the current so long as *C* is moving down across the field of force and D is moving upward. When the coil is in the vertical position both C

and D will be moving parallel to the lines of force so that there is no electromotive force in the circuit at that instant. Then as C comes up and D descends the electromotive force is reversed, causing a current from A toward B in the outer circuit, which reaches a maximum when the coil is horizontal, for then both C and D are cutting perpendicularly across the lines of force. The electromotive force again becomes zero when C reaches the top and D is at the bottom and then reverses again into the original direction.

In the vertical position of the coil the electromotive force is zero, although it includes the maximum number of lines of force, *because in that position a small motion of the coil* **does not appreciably change the number of lines of force** *which it embraces.* While **in the horizontal position the electromotive force is a maximum,** although no lines of force pass through the coil, because **the change is most rapid in that position.**

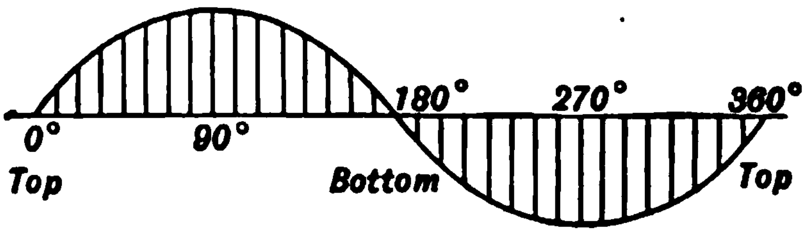


FIG. 430.—Diagram of alternating electromotive force.

The diagram (Fig. 430) exhibits what may be called the curve of electromotive force in such a case. The curve starts with C at the top, the abscissa at any point is the angle through which C has moved and the corresponding electromotive force is the ordinate, drawn above the horizontal when it is directed from B to A, and below when it is reversed.

The current produced is what is known as an *alternating* current and goes through a complete cycle in the time of one revolution of the armature. *An alternating current may be compared to the surging back and forth of water in a pipe in which a tightly fitting piston is moved to and fro.*

734. Commutator.—The terminals of the coil just discussed, instead of being joined to two rings, may be connected to the two halves of a divided ring or *commutator,* as shown in figure 431, on which rest springs or *brushes* which connect to the external circuit and are so placed that they slip from one segment

to the other at the instant when the electromotive force in the coil is reversing. In the above diagram, whichever side of the coil is descending, is connected with A, while the ascending side is connected with B, so that the current is always from B to A in the external circuit. The current curve in the external circuit will in such a case be as in figure 432, where ordinates represent the current and abscissas the corresponding instants of time. Each section of the curve represents half the period of a complete revolution of the armature. Such a current, though always in the same direction, is fluctuating.

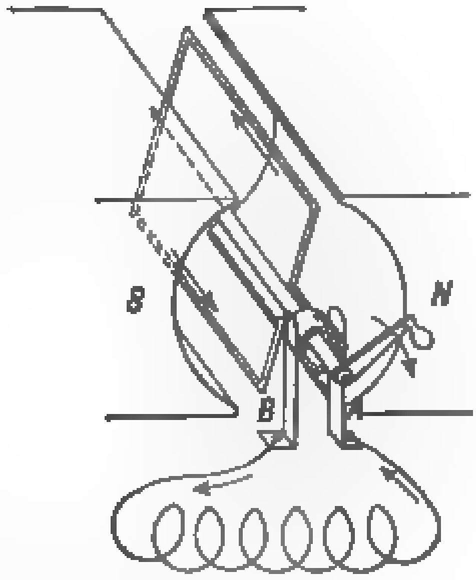


Fig. 431.—Loop armature with commutator.

735. The Ring Armature.—A valuable armature, devised by Pacinotti, is known as the Gramme ring from the French inventor who was the first to construct commercial machines using that type of armature. It consists of a soft-iron ring made of a coil of iron wire or a pile of ring-shaped plates of thin sheet iron, wrapped around with a coil of insulated copper wire, the ends of which are joined together forming an endless-ring solenoid with an iron core. For distinctness in the diagram (Fig. 433), the turns of copper wire are shown widely separated. Suppose the ring to be mounted on an axle and rotated between the poles of a powerful magnet as shown in the figure. The lines of force of the magnetic field pass from one pole to the other chiefly through the iron ring as shown by the dotted lines. This, of course, is in consequence of its great *permeability*. As the armature rotates, those parts of the copper winding which cross the outside of the ring cut across lines of force in the space between poles and armature. On the right-hand side the wires cut *down* across the field, and the electromotive forces in these turns will be from the front toward the back of the armature. This tends to cause a current in the windings in the direction shown by the small arrows. All

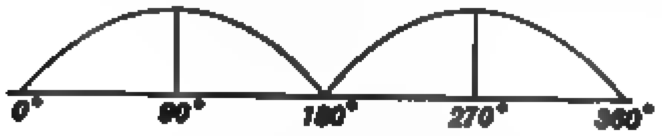


Fig. 432.

of the turns on one side act together like so many little battery cells in series, though those in the middle are most effective. The outer sides of the turns on the left-hand side of the ring, next the south pole of the field magnet, cut up across the field of force, and hence the electromotive force in them is from the back toward the front of the armature, and so they conspire to produce a current on that side in the direction of the small arrows. But it will be observed that in consequence of the winding of the wire,

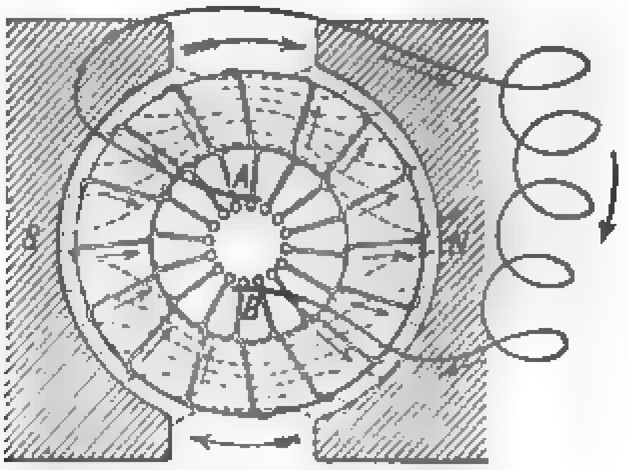


FIG. 433.—Gramme ring armature.

the induced electromotive force on each side acts to cause a flow around the coil working from the bottom toward the top of the ring, and hence the top of the coil will be a point of high potential and the bottom a point of low potential, when the poles and winding of the armature are as shown in the diagram, but there will be *no flow* around the coil for the electromotive force on one side balances that on the other.

To obtain a current, the top and bottom of the coil must be connected with an outside circuit. This is accomplished by the commutator which consists of a number of segments of copper insulated from each other and mounted in cylindrical form around the axis, each segment being connected with a corresponding point in the copper coil.

The sections of the armature coil included between the points where connection is made to the commutator, all have the same number of turns. In the diagram only one turn is shown for each section, but any number may be used.

If the ends of the external circuit are connected to the two *brushes A* and *B* there will be a current from A to B as indicated.

For the brush A rests upon the upper segment in the commutator which is connected with the top of the wire coil, and is in this case a point of high potential, while similarly the brush B is in connection with the bottom of the coil where the potential is low.

The flow of current within the armature coil is around on each side as shown by the arrows, the two currents coming together at the top and flowing out through the commutator at A, around through the external circuit, and in at the bottom of the armature coil where the current divides, half flowing around on one side and half on the other. The case resembles an external circuit connected to two batteries joined in parallel (Fig. 434), the electromotive force of each battery corresponding to that of one side of the armature.

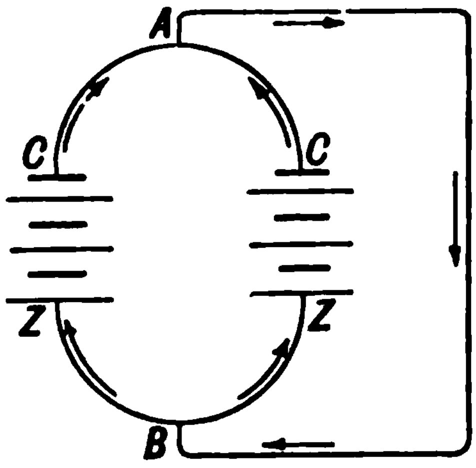


FIG. 434.—Two batteries in parallel.

736. Drum Armature.—Of every turn of wire on a ring armature part lies on the inside of the ring, and this does not contribute to the electromotive force. Whatever slight effect it may have, due to the weak magnetic field inside of the ring, is in opposition to the outside part. It is desirable to have as little inactive wire as possible in an armature since it adds to its resistance.

The drum armature is like a ring armature where the opening in the ring is filled up with iron and the turns of copper wire pass clear across the ring from one side to the other, so that the only inactive wire is that across the ends.

The core is a cylinder of iron made of a pile of thin sheet-iron plates bolted together, around which the coils of wire pass longitudinally lying in grooves made for them. In winding, the wire starts at one of the commutator segments, is passed around the core lengthwise in one of the grooves the desired number of times, suppose twice, and then is connected to the next commutator segment. It is then carried right on around the core in the next groove in the same direction as before, making two more turns, and then connected to the third segment of the commutator. This process is continued until the segment is reached where the winding began and there the end is made fast. In this way an

endless coil is constructed just as in the Gramme ring, and between each commutator segment and the one opposite there are two paths by which the current may flow within the armature, so that the current divides in the armature just as in the ring armature.

737. Foucault Currents.—In each of these armatures the inductive action which causes electromotive force in the copper coils also causes a similar electromotive force in the iron core tending to set up currents within the core itself. Such currents would spend energy in heat, and the double disadvantage would result that more work would have to be spent in turning the armature, and this useless expenditure of energy would go to unduly heat the machine.

In order to prevent these *Foucault currents*, or *eddy currents* as they are often called, the iron core is laminated or made up of thin plates insulated from each other by varnish, or paper, and lying across the direction in which the currents would flow. The thinner the sheets of iron the more perfectly is this waste of energy prevented.

738. Electromotive Force of Armature.—The electromotive force of a ring armature is easily reckoned. *The electromotive force of the ring is the same as that of one side,* since the two sides of the ring act *in parallel.* Let N be the number of lines of force passing through the armature, n the number of revolutions per second, and C the number of turns of wire on the ring, then since each turn cuts down on one side across all N lines of force once in every half revolution, that is in $\frac{1}{2n}$ second, the average electromotive force induced in each coil as it moves across the field must be

$$N \div \frac{1}{2n} = 2Nn.$$

But all the coils on one side of the ring act together or in series, hence if there are C coils of wire on the ring the total electromotive force must be

$$2Nn . \frac{C}{2}$$

thus

$$E = NnC \quad \text{in C. G. S. units,} \quad \text{or} \quad E = \frac{NnC}{10^8} \text{ volts.}$$

The electromotive force depends on three factors: the number of lines of force through the armature, the number of revolutions which it makes per second, and the number of coils of wire upon it.

The electromotive force of a drum armature is calculated from the same formula, C representing the whole number of wires on the armature which cut across lines of force.

739. Field Magnets.—In most dynamo machines and motors the armature rotates between the poles of an electromagnet which receives its exciting current from the armature. Three

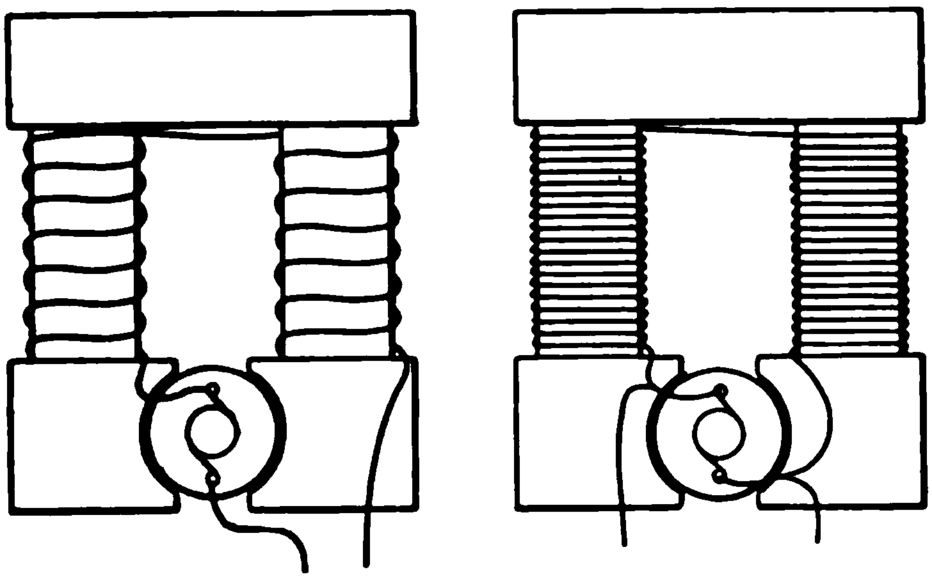


FIG. 435.—Series and shunt field magnets.

modes of winding are in use, *series*, *shunt*, and *compound*. In the first diagram in the figure is shown a *series*-wound dynamo. The whole armature current passes around the field magnets and through the external circuit. Any resistance introduced into the external circuit, causing the current to diminish, weakens the magnetic field and therefore makes the electromotive force of the machine less. When there is no current flowing its electromotive force is zero except for the residual magnetism.

In the *shunt* arrangement the current in the armature divides, part flowing around the magnet and part to the external circuit. In order that but a small current may be taken for the magnet, it is wound with many turns of rather fine wire.

The current through the shunt coil depends only on its resistance and on the difference of potential of the brushes; hence it is constant and the strength of the magnet is constant so long as the difference in potential of the brushes is unchanged. The electromotive force of such a dynamo is very nearly constant.

but is slightly greater when no external current is flowing, for with increasing current in the external circuit there is more current and a greater *fall of potential in the armature itself.*

Compound winding is a combination of the shunt and series arrangements, in which there is a shunt coil and also a few turns carrying the whole current around the magnets. In this way a dynamo may be made to maintain a nearly constant potential at the terminals, though the external current may vary greatly, or it may be *over*-compounded so that its terminal electromotive force may be greater with large currents than with small.

Part II.—Direct-current Motors

740. Motors.—An electric motor is an appliance in which an electric current gives motion to an armature, thus producing

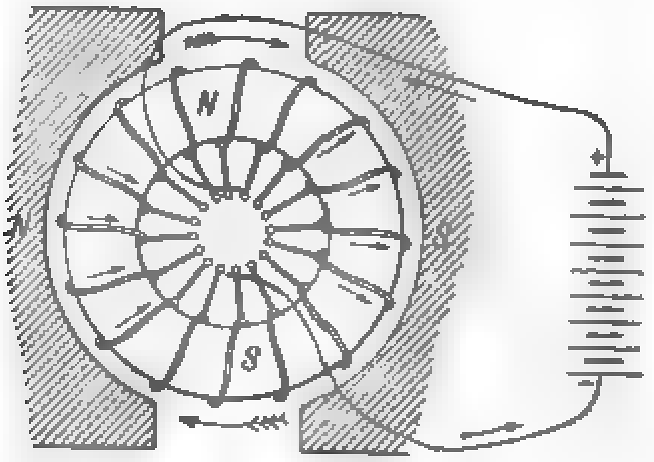


Fig. 436.—Motor with ring armature.

mechanical work. Small direct-current motors usually have ring or drum armatures and are in most respects like dynamos.

The action of the ring armature in a motor may be understood from the diagram (Fig. 436). The current from a battery or other source is shown as flowing in at the upper brush and out at the lower one. Within the armature the current divides, half flowing around and down through the coils on one side and half through those on the other side as shown by the arrows, and the effect of these currents in the armature is to make each half of the ring a magnet with its north pole at the top and south pole at the bottom. The attractions and repulsions between these poles and those of the field magnet cause the armature to rotate in the direction of the large arrows.

Another aspect of the action is worth considering. The gaps etween the pole pieces and the armature are regions of intense agnetic force, and the wires on the outside of the armature arry currents directly across these lines of force, *up* (perpendicu- ır to the paper) on the left and *down* on the right; there is, there- ore, a force (§684) urging these wires to move across the lines f force toward the top of the diagram on the left and toward he bottom on the right.

741. Energy Spent in Motor.—While the motor is running mechan- al work is being done in addition to the energy which is spent as heat in the rmature in consequence of its resistance. But the total energy spent per cond in the motor is equal to the product of the current strength by the fference of potential between the brushes. Therefore if the current is kept nstant the difference of potential between the brushes must be greater hen the motor is running and doing work than when the armature is at rest. This increase in the difference of potential between the brushes due to the otion of the armature is *the back electromotive force* of the motor. **There ust be such a back electromotive force in every kind of device in which otion results from the flow of an electric current.**

et $V_1 - V_2$ = difference of potential between brushes of motor.
IR = drop in potential due to the resistance of the armature.
$V_1 - V_2 = E + IR$ where E is the back electromotive force.

Total watts spent in motor = $\left\{\begin{array}{l}\text{watts spent in turning armature} +\\ \text{watts spent in heat}\end{array}\right.$

r in symbols

$$I(V_1 - V_2) = IE + I^2R.$$

742. Back Electromotive Force.—Connect an electric motor to battery by which it may be driven and introduce into the cir-

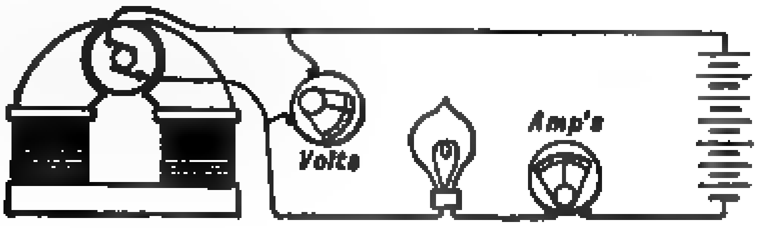


FIG. 437.

uit an incandescent lamp which will glow with full brilliancy hen the armature of the motor is held stationary. *On letting he armature run the lamp grows dim,* and an ammeter in circuit hows that *the current has diminished,* but *a voltmeter connected o the brushes of the motor will show a much greater difference of otential between them than when the armature was at rest.*

Since the electromotive force of the battery and the resistance of the whole circuit is unchanged by the running of the motor, it is clear that **the current can have been diminished only by the development of an electromotive force in the circuit back against the driving current.** The motor, in fact, while running acts like a dynamo and develops an electromotive force, called its **back electromotive force,** because it acts in opposition to the electromotive force of the driving battery.

743. Starting a Motor.—In starting a motor there is at first no back electromotive force to oppose the current, and in order to prevent the current being excessive and "burning out" the armature before the motor is well started some such device as shown in figure 438 is commonly used.

The current is led to the motor through the wires *AB*, one of which is connected directly to the motor while the other is joined to the switch *S*. When the switch is turned from 1 to 2 the current flows through coils of wire having considerable resistance and starts the armature. As the speed increases, developing more back electromotive force, the switch is moved on to 3 and 4, reducing with each step the extra resistance until, as the armature comes to full speed, the switch on 5 makes direct connection, and the back electromotive force keeps the current moderate even though the armature resistance may be extremely small.

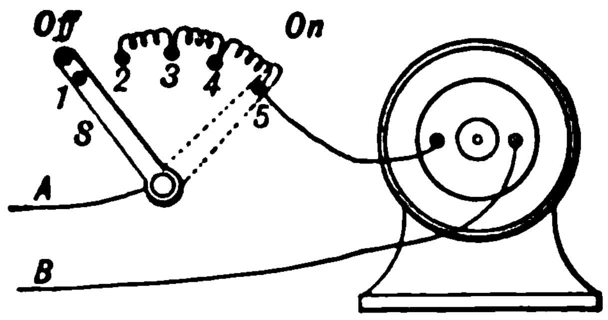


Fig. 438.—Starting connections of motor.

744. Dynamo and Motor.—Suppose a transmission system consisting of dynamo and motor and connecting circuit. Let the electromotive force of the dynamo be 200 volts, and suppose the resistance of the whole circuit including the armatures of both dynamo and motor to be 1 ohm, and let the back electromotive force of the motor be 180 volts at the working speed. Then the resultant or effective electromotive force in the circuit is $200 - 180 = 20$ volts, and the current is 20 ampères.

Power spent in the dynamo	$200 \times 20 = 4000$ watts.
Power used in motor	$180 \times 20 = 3600$ watts.
Loss in heat (I^2R) is the difference	400 watts.

If a motor is used which in running develops twice the back electromotive force of that just discussed, then with a current of 10 ampères as much power will be obtained as with the 20 ampères in the former case.

In this case the electromotive force of the dynamo must be 370 volts, and that of the motor being 360 volts the effective

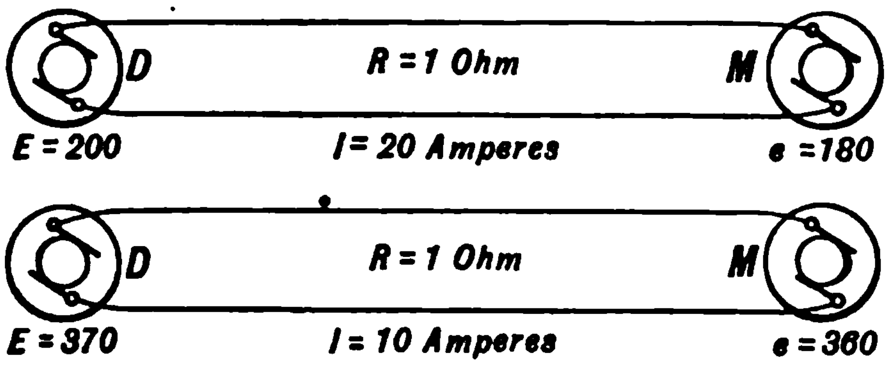


Fig. 439.

electromotive force is $370 - 360 = 10$ volts. The current will therefore be 10 ampères, and we have

Power spent by dynamo	$= 370 \times 10 = 3700$ watts.
Power used in motor	$= 360 \times 10 = 3600$ watts.
Power wasted in heat I^2R	$= 100 \times 1 = 100$ watts.

This is evidently a much more economical arrangement than the first and illustrates the general principle that electrical energy can be transmitted with least loss by means of small currents at high voltage.

Part III.—Alternating Currents

745. Alternating Currents.—Alternating currents have come extensively into use because of the ease with which a large alternating current at a low voltage can be changed to a small one at a high voltage. The small high-voltage current can be carried by comparatively small conductors to a distant point and then be transformed down again to a large current at a low enough voltage to be safely used for light or power.

746. Alternating-current Dynamos.—Almost any direct-current dynamo will give alternating currents if it is provided with two rings mounted on the axis and connected respectively to two diametrically opposite segments of the commutator. A circuit whose ends are connected to these rings by brushes will

have an alternating current. Such a case was illustrated in §733. To secure good insulation, high electromotive force, and sufficient frequency of alternation, alternating-current dynamos are usually multipolar, as illustrated in figure 440. In the type shown the field magnet poles, alternately north and south, project outward from the rim of a rotating wheel and are magnetised by the current supplied by a small separate direct-current dynamo called an exciter. This rotary field, or rotor, rotates within the fixed armature or stator, in which the poles project inward from the outside circular frame. These poles are of the same number as those on the rotor, and are laminated or built up of thin plates of sheet iron to prevent eddy currents. Around the poles

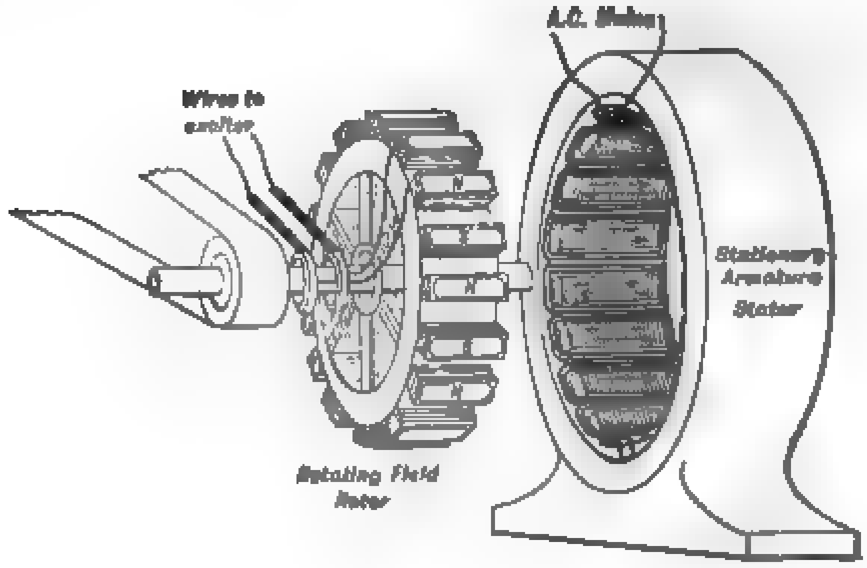


FIG. 440.

of the stator the armature coils are fitted, passing through slots between them and wound alternately clockwise and counterclockwise around successive poles. As the rotor turns and a north pole facing one pole of the armature moves over to the next, the lines of force from the pole of the rotor cut across the conducting wires lying between the poles of the stator and induce electromotive force in them which reverses as the succeeding south pole moves across, thus causing an alternation. But as the wires in one slot return in the opposite direction through the next slot, and as a north pole is moving across the one while a south pole is moving across the other, the electromotive forces induced in all act together at every instant, so that in the case figured where there are 16 poles, an alternating electromotive

force is produced 16 times as great as would be developed in a single coil.

If a low electromotive force is desired the several coils of the armature may be connected in parallel instead of in series as above described.

747. Virtual Amperes and Volts.—An alternating current is constantly varying in strength, as illustrated in the curve of figure 441, its average value is zero and it will not give a steady deflection of the needle in an ordinary galvanometer. A definition must therefore be given of what is meant by an alternating current of one ampère. Since the energy relations of a current are commercially the most important, an alternating current

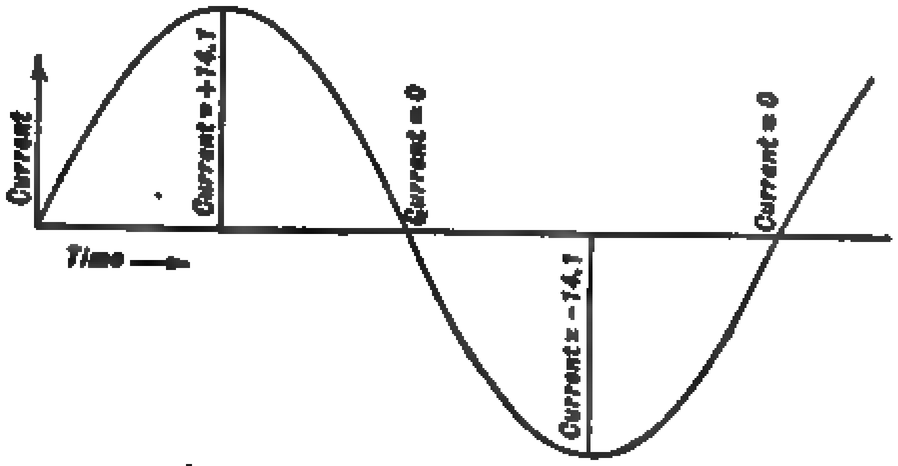


FIG. 441.—Alternating-current curve. Current = 10 Amperes.

is said to have the strength of 1 ampère, when it will develop the same amount of heat in a given resistance as would be produced by a direct current of 1 ampère. The heating effect of a current at any instant is proportional to the square of its strength at that instant, so also the deflection produced by a current in an electrodynamometer is proportional to the square of the current strength (§693); therefore an electrodynamometer measures directly the virtual ampères of an alternating current just as it does a direct current.

If the alternating-current curve is a *sine* curve, the virtual strength as defined above is to its maximum value in the ratio of 1 to 1.41; thus an alternating current of 10 ampères ranges from + 14.1 to − 14.1 ampères in its instantaneous values.

So also the virtual value of an alternating electromotive force is *said to be 1 volt* when it will develop an alternating

current of 1 ampère in a resistance of 1 ohm having no self-inductance.

748. Effect of Self-induction.—It has already been shown (§723) that the effect of self-induction in a circuit is to cause an electromotive force *contrary* to an increasing current and *with* a decreasing current. In case of alternating currents, the effect is twofold. **First, it causes an apparent increase in resistance.** It may be proved that the current produced by an alternating electromotive force E in a coil whose coefficient of self-induction is L and whose resistance is R, is

$$I = \frac{E}{\sqrt{R^2 + (2\pi nL)^2}}$$

where n is the number of complete cycles per second, the current and electromotive force being measured in virtual ampères and volts, the resistance in ohms and the inductance in henrys. The denominator is known as **the impedance of the conductor.**

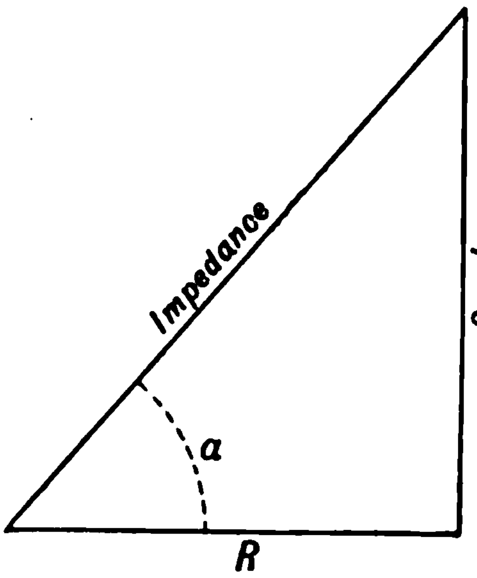


Fig. 442.

If the self-induction of the coil is large and if there are a large number of alternations per second, the impedance may be large although the resistance is small.

Second, self-induction causes the phase of the current to lag behind that of the electromotive force, so that the current does not reach its maximum value at the same instant that the electromotive force is a maximum, but a certain fractional part of a period later, which is called the lag.

The relations of these quantities are shown by the triangle in figure 442. If the base of the right-angled triangle represents the resistance of a coil, and the altitude, the quantity $2\pi nL$, then the hypothenuse represents the impedance, and the angle a at the base, the angle of lag. That is, the current maximum lags behind the maximum of electromotive force the same fractional part of a complete period that a is of the whole angle about a point.

749. Theater Dimmers.—If an electric glow lamp, connected

in series with a coil of wire having very low resistance, is lighted by means of an alternating current, the light may be dimmed by inserting a laminated core of soft iron inside the coil. The self-induction of the coil is greatly increased in this way and the current is decreased, but there is no waste of energy as there would have been if the current had been reduced by introducing resistance. This method is used for dimming theater lights.

750. Transformers.—Alternating currents are easily changed from low voltage to high, or vice versa, by means of transformers.

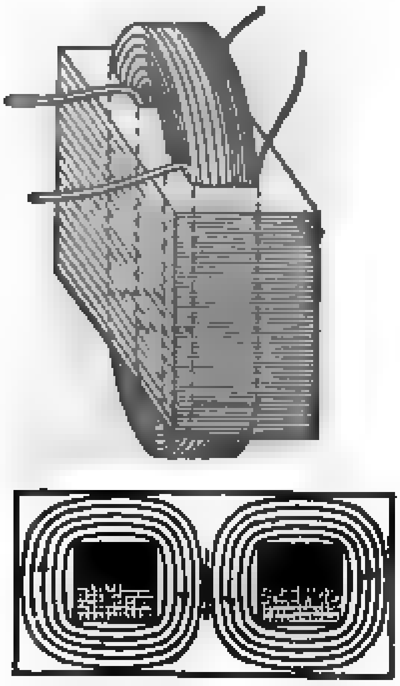


Fig. 443.—Transformer and section.

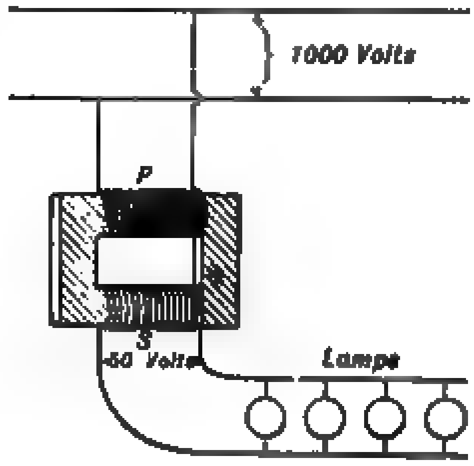


Fig. 444.—Transformer connections.

A transformer consists of two coils side by side, having a common core of soft iron. In the form shown in figure 443, the iron core is made up of a pile of thin sheet-iron plates of the shape shown in the section. The core thus formed is a block of soft iron having two rectangular holes through it in which the two coils lie side by side, one coil having many turns of fine wire and the other a few turns of coarse wire as shown in the diagram. When an alternating current is set up in one coil it magnetizes the iron core, setting up lines of force which at one instant are in the direction shown by the arrows in the diagram and a half period later are exactly opposite. But the lines of force pass through the second coil as well as the first and therefore an alternating induced current is set up in the secondary coil.

The *same lines of force* cut across one coil as the other and

consequently the electromotive force induced in one coil is to that in the other as the number of turns of wire in the coils.

Suppose it is required to transform from 1000 volts down to 50. The fine wire coil which is connected to the 1000-volt circuit must have 20 times as many turns of wire as the coarse wire coil which is connected with the lamps. If no lamps are turned on, there is no current in the secondary coil and the magnetic field through the primary coil causes such a strong back electromotive force that only a very small current flows through it and there is but a small loss of energy.

When lamps are turned on in the secondary, a current flows which, by the laws of induction, opposes the changes in the magnetic field by which it is produced, and therefore more current must flow in the primary coil to keep up the magnetism necessary to produce the back electromotive force in the primary which balances the electromotive force of the main line.

In this way the transformer is self-regulating, the primary current being very nearly in the same ratio to the secondary as the number of turns of wire in the secondary coil is to that in the primary. In the example considered, the current in the primary would be one-twentieth that in the secondary

The energy spent in the secondary circuit is equal to that which the transformer takes from the main line except for a small amount, perhaps 5 per cent., which is lost as heat in the transformer.

751. Advantage of Transformers.—Large currents cannot be transmitted long distances without great loss in heat unless large conductors of low resistance are used, in which case the cost and interest charges are high. By means of a transformer a large electrical power may be transmitted by a small current at high voltage. Thus in districts where scattered houses are to be lighted a small current at high voltage is used on the street line and transformed down, giving large currents at low potentials at the points where lights are used.

In many lines where power is to be transmitted a long distance transformers are used at both ends of the line. Thus at Niagara dynamos develop currents at 2200 volts, which are then transformed *up* to 22,000 volts, and so transmitted 20 miles to Buffalo, where they are transformed down again for power and lighting purposes.

752. Electric Welding.—An important application of large electric currents is in fusing bars of metal together. Two bars of iron as large even as a man's wrist may be placed end to end and fused together in a few seconds. For such a purpose a very large current is required just at the spot to be heated. Accordingly a transformer is used in which the secondary may consist of only a single turn or two built of heavy copper bars, the terminals of which are clamped to the bars to be welded, one on each side of the junction. The primary coil is made of many turns of wire and takes a comparatively small current at high voltage.

753. Alternating-current Motor.—There are two principal types of alternating-current motors, the *synchronous motor* in which the armature will not start of itself, but must be brought by some accessory motor to such a speed that its armature coils move from one field pole to the next in exact synchronism with the alternations of the driving current. When brought to speed, it will continue to work when driven by a single alternating current.

A second type is the *induction motor* in which a rotary magnetic field is produced by polyphase currents.

754. Rotary Magnetic Field.—Suppose that a laminated ring of soft iron, having four poles projecting inward as shown in figure 445, is wound with two independent circuits, one of which magnetizes the *A* and *C* poles and the other the *B* and *D* poles. And let an alternating current be established in each circuit, the phase of the current in the *E* circuit being a quarter of a period ahead of that in the *F* circuit, as shown in the curves *E* and *F* in figure 446, so that one reaches its maximum value, either positive or negative, at the instant that the other is passing through its zero value.

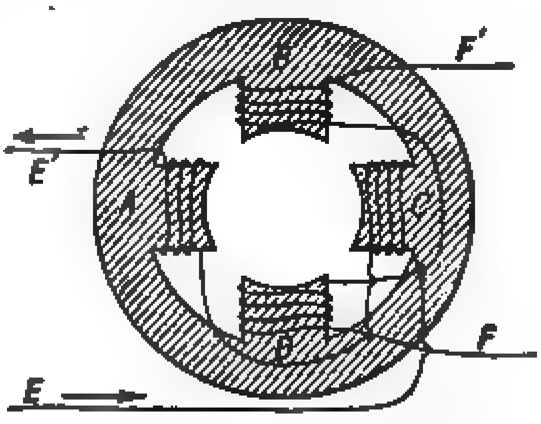


FIG. 445.—Rotary field magnet.

The corresponding changes in the direction of the lines of force in the field between the poles are shown in the lower diagrams of figure 446. Thus in the first diagram the current *E* is a maximum, *and is supposed* flowing from E to E' (Fig. 445) making

A a north pole and C a south pole, while at that instant the F current is zero. But as the F current increases that in E diminishes until F becomes a maximum and E zero. The north pole has now passed to B, while A and C have lost their polarity as shown in diagram 3. The sign of the E current is now reversed and it begins to flow from E' toward E, making C a north

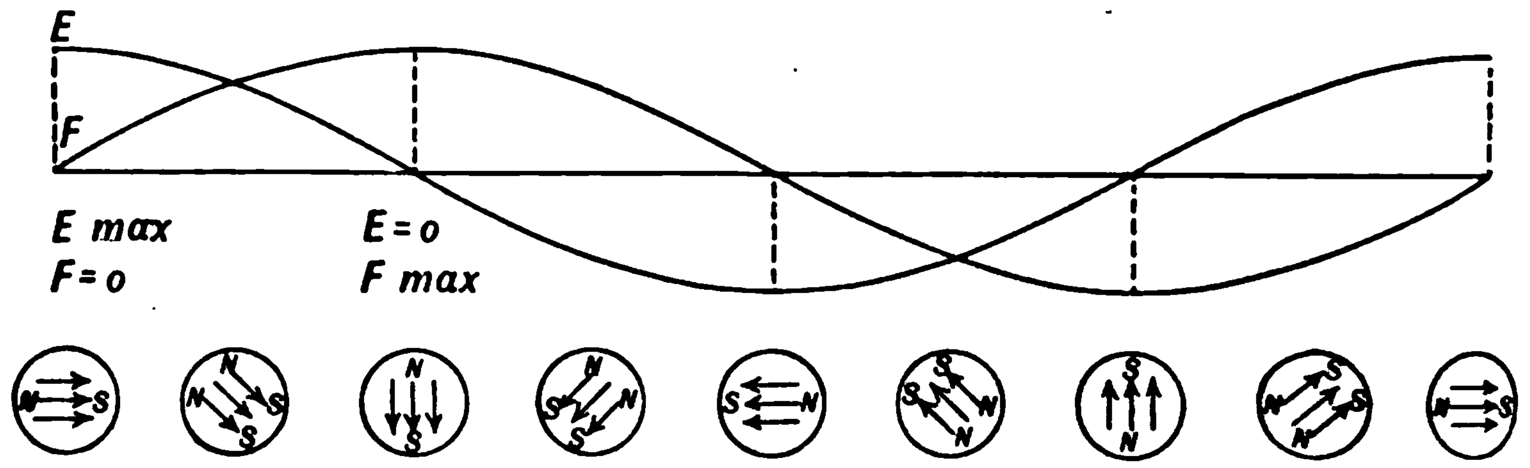


Fig. 446.—Diagram of two phase currents and rotary field.

pole as shown in 4; at the same time the F current is decreasing and becomes zero in 5, where E reaches its maximum negative value. In this way what is known as a *rotary magnetic field* is produced in which the north and south poles move around the ring making one revolution for every complete period of the current.

755. Induction Motors.—Between the poles of the rotary field just described there is mounted a cylindrical-shaped armature having a set of parallel rods of copper at equal intervals around the circumference, like the bars in the wheel of a squirrel cage, connected across the ends by copper plates. And to strengthen the lines of force through the armature, it is filled with a soft-iron core made of a pile of circular plates of thin sheet iron. As the lines of force of the field rotate they cut across the bars of the armature, inducing currents which by Lenz' law are in such a direction as to resist the relative motion of armature and field, and the armature is therefore carried around in the direction in which the field rotates. But clearly in such an induction motor, *the armature cannot rotate as fast as the magnetic field, because it is the difference between the motions of the two that causes the induction on which the rotation of the armature depends.*

756. How Currents in Different Phases are Obtained.—Imagine a Gramme ring armature as shown in figure 447 provided with four insulated brass rings mounted on its axis, each of which is connected permanently to one of the points $EFE'F'$, which are just one-quarter circumference apart on the ring. If one circuit is now connected to the brushes e and e' and another

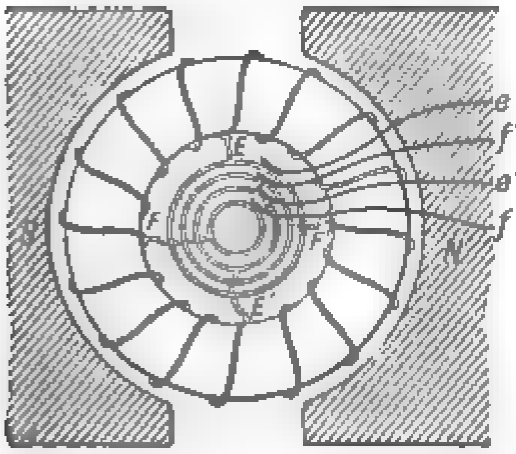


FIG. 447.—Connections for currents in quadrature.

to the brushes f and f' which rest on the rings, the currents in the two circuits will be one-quarter period different in phase as represented in figure 446.

757. Three-phase Motors.—The usual form of induction motor uses *three-phase currents*, or three currents which differ in phase by one-third of a period, and requires only three line wires instead of four. The generator has three rings connected, re-

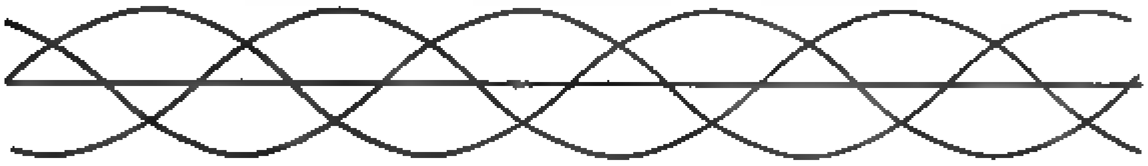


FIG. 448.—Three-phase currents.

spectively, to three equidistant points in the armature, so that the currents developed in the three line wires are related as shown in the curves of figure 448. It will be noted that the sum of the ordinates of any two of the three curves taken at any point along the base is equal and opposite to the ordinate of the third curve at that point; that is the sum of the currents in any two of the three line wires at any instant is equal and opposite to the current in the remaining line wire, the three are, therefore, connected

together at the farther end and *each serves as the return wire for the other two*, as shown in figure 449.

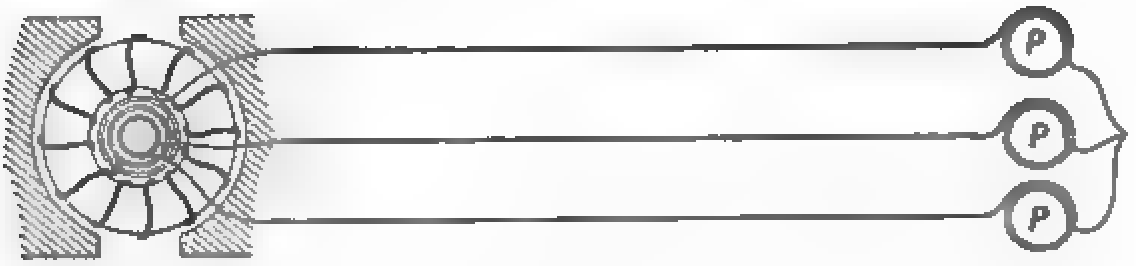


FIG. 449.—Connections of three-phase generator to the poles P of the field magnet of the motor.

Three-phase motors are usually multipolar, each principal pole being subdivided into three parts. The figure shows a field having twelve small poles which are so wound as to form a rotary field with two north poles and two south poles. How this is done may be understood from the diagram in which the field ring is supposed to be cut at one point and bent out flat so that we look directly at the faces of the twelve poles.

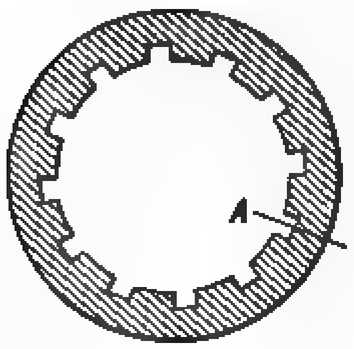


FIG. 450.

For simplicity the wire is represented as carried only once through each groove. It will be seen that when the current in 1 is a maximum in the direction of the arrow the poles will be situated as shown in the upper row of letters. A third of a period later the current in 2 will be a maximum in the same direction,

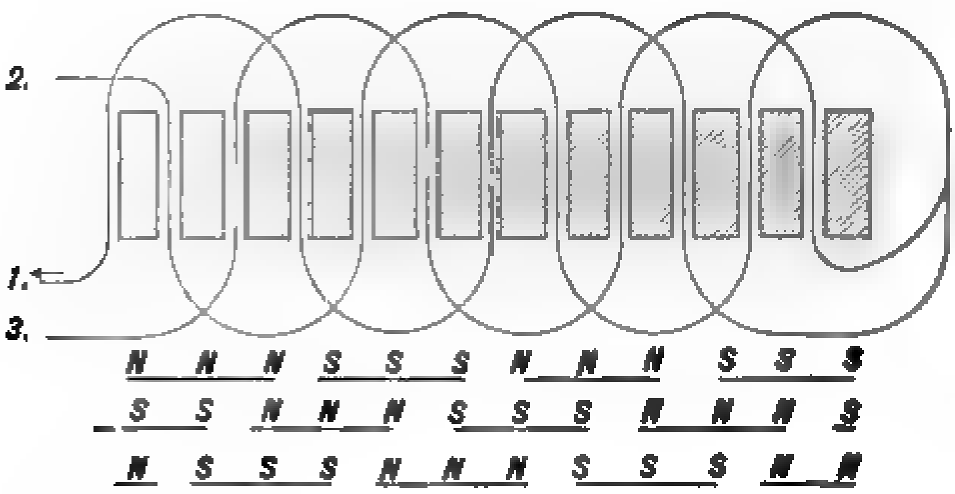


FIG. 451.—Windings of a three-phase field magnet.

and the poles will then be as indicated in the second row. Then after another one-third of a period current 3 will have reached

its maximum and the poles will have shifted to the positions indicated in the third row of letters. There is thus produced a steady movement of the poles around the ring, moving over the distance between two similar poles in the time of one complete period of alternation of the current. In the above case, if the current has a frequency of sixty periods per second, the field will make thirty revolutions per second.

Problems

1. The core of a Gramme ring armature has a cross section of 6×10 cm. How many turns of wire must it have that it may give an electromotive force of 20 volts when making 800 revolutions per minute in a magnetic field so strong that where the lines of force in the ring are most concentrated there are 6000 per square centimeter?
2. A certain dynamo armature when making 1000 revolutions per minute is supplying a current of 50 ampères at 100 volts. Find the horse-power required to drive it and thence the moment of force or torque in pound-feet required to turn the armature at the given speed.
3. When the armature of a certain motor is held fixed a current of 10 ampères through it causes a difference in potential between its brushes of 5 volts. When the armature is permitted to run at 600 revolutions per minute the current is 4 ampères and difference of potentials at the brushes is 30 volts. Determine the back electromotive force of the motor.
4. The core of a drum armature is a cylinder of iron 30 cm. long and 15 cm. in diameter, the induction through its middle longitudinal section is 6000 lines of force per square centimeter. If there are 50 complete turns of wire on the armature, or 100 longitudinal wires in grooves on its surface, what is its electromotive force when making 1200 revolutions per minute?
5. A transformer has a coil of 250 turns; what must be the size of the iron core in order that an average electromotive force of 100 volts may be developed in this coil while the number of lines of force in the core changes from $+6000$ to -6000 per sq. cm., the current alternating at the rate of 60 complete periods or cycles per second?
6. A certain transmission line has a resistance of 20 ohms. How much power will be lost in the line when 100 kilowatts are transmitted at 2000 volts? How much when the same power is transmitted at 20,000 volts?
7. A multipolar generator having 16 poles (Fig. 440) makes an alternating current of 60 cycles per sec. How fast does it rotate? If there are 30 turns in each armature coil, what E.M.F. is developed when each pole of the rotor gives rise to 100,000 lines of force?

Electric Oscillations and Waves

758. Oscillatory Discharge of a Leyden Jar.—It has been already stated (§585) that when the resistance of the discharge circuit is sufficiently small the discharge of a Leyden jar is oscillatory. This was discovered by the American physicist Joseph Henry, who, as early as 1842, found that when a Leyden jar was discharged through a wire wound around a needle the latter was magnetized, but sometimes one end was made the north pole and sometimes the other, although the jar was always charged the same way. He believed that this was caused by the oscillation of the discharge current which kept reversing the magnetism of the needle back and forth until the current became too small to have a further effect. This opinion was confirmed by eating off the surface layer of the needle with acid, when the interior was found magnetized opposite to the outer layer.

Lord Kelvin, in 1855, quite unaware of Henry's discovery, showed by the principle of energy that the discharge must oscillate back and forth until all the original energy of charge is expended in sound, heat, light, and radiation, and that when the resistance of the circuit is very small the period of oscillation is given by the formula

$$P = 2\pi\sqrt{LC}$$

where L is the coefficient of self-induction of the circuit and C is the capacity of the jar. In case of an ordinary gallon jar discharged by a short discharging rod, the period of oscillation may be as small as two ten-millionths of a second, while Lodge, by using a battery of large capacity and discharging it through a very long circuit having large self-induction, was able to make the alternations so slow as to give out a distinct musical note. Feddersen, in 1859, first analyzed the spark by a rotating mirror, as already related (§585).

759. Electric Resonance.—When a Leyden jar is discharged not only may there be oscillations in the discharge circuit itself, but in consequence of induction there are set up electric oscillations or surgings in neighboring conductors. In general these are but feeble, but if the free period of the surging happens to be the same as that of the oscillations in the discharge circuit, quite energetic surgings may result, just as a tuning-fork will excite

rong vibrations in a resonator which is in tune with it. **The ·cuits are then said to be in resonance.**

760. A Case of Electrical Resonance.—The influence of elec-cal resonance is well shown in the following experiment due Lodge. Two Leyden jars of nearly equal capacities are chosen. ıe which can be charged by an electrical machine or induc-ın coil is provided with a short circuit of thick wire which is ;ached to the outer coating and terminates in a knob separated a short spark gap from the knob of the jar. The second jar s a strip of tinfoil reaching from the inner coating over the ge and terminating in a point at e near the upper edge of the ter coating; its inner and outer coatings are connected by a re circuit, part of which, ırked AB in the figure, can be d along changing the length the path. When the two jars placed, say, a foot apart th the two circuits parallel, a sition for the slider AB may found by trial, such that ıenever the first jar discharges ross between the knobs, a ark leaps the gap between the foil strip and the outer coat-g of the second jar. If the der is moved a short distance away from this position in her direction, the sparks at e cease. Lodge calls the sparks e the "slopping over" of the powerful surgings due to the two ·cuits being in resonance.

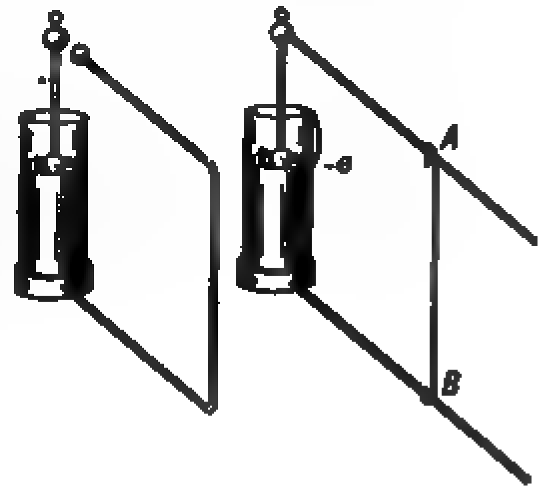


Fig. 452.—Sir Oliver Lodge's resonance experiment.

761. Electric Waves in Wires.—When one end of a long straight re is given a charge or touched to a battery pole, a wave of elec-.c pressure or potential runs along the wire with a velocity which pends on the insulating medium immediately surrounding the re. *In case of a straight bare wire in air the wave has the velocity light;* but when the wire is coiled, forming a closely wound lix, the wave travels much slower on account of the greater lf-induction of the coil.

On reaching the end of the wire the wave is reflected back, just a sound wave is reflected at the end of a stopped organ pipe.

If, instead of a single impulse, a series of alternate positive and negative charges are given to the end of the wire in exactly the right frequency, it may be set in strong electrical resonance just as a stopped pipe vibrates powerfully when a tuning-fork of the proper frequency is sounded at its mouth. Resonance will occur when the period of the electrical impulses is four times as long as it takes a wave to run the length of the wire, exactly as in case of a stopped organ pipe.

The resonance of waves in wires may be beautifully shown by the following experiment due to the German electrician Seibt:

A large Leyden jar has its coatings connected by a circuit having a spark gap at S with zinc knobs. By moving the slider

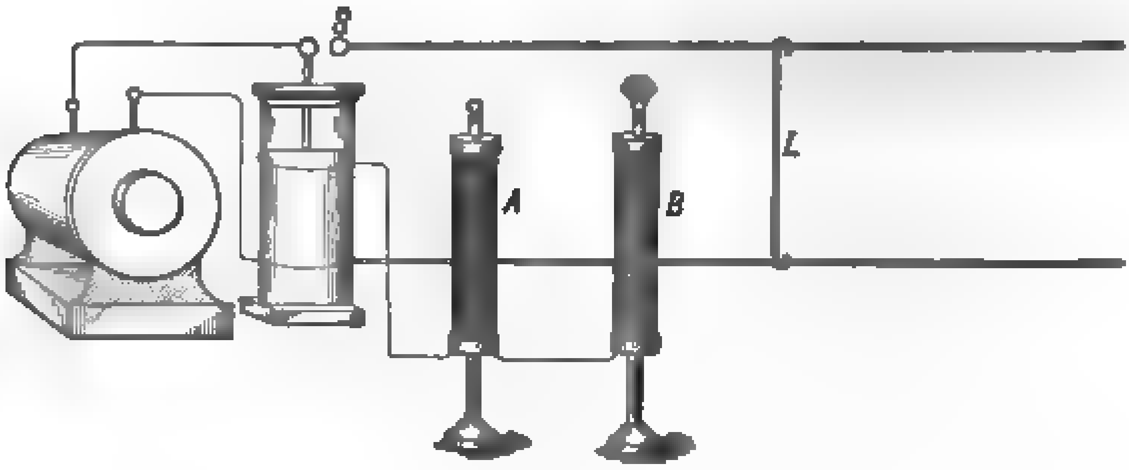


FIG. 453.—Resonance experiment.

L nearer to the jar or farther away, the length and self-induction of the discharge circuit may be varied and consequently the period of the oscillatory discharge can be adjusted.

Two long helical coils of wire A and B are mounted on insulating stands. They are both connected at the bottom to one of the coatings of the Leyden jar while each terminates above in a point. One helix is wound with a much greater length of wire than the other.

If by means of a powerful induction coil the Leyden jar is caused to discharge across the gap S, each discharge will be oscillatory and consequently a series of impulses is communicated to the lower ends of the helices A and B, and when the slider is in such a position that the period of oscillation of the discharge is the same as the period of oscillation in the wire on A, a strong brush discharge will be observed from the upper

point of that helix; while by moving the slider until the jar circuit is in resonance with B, the discharge will take place from the top of B instead of from A.

762. Electromagnetic Waves in Air.—As early as 1862 Clerk Maxwell, who followed Faraday in recognizing the important function of the dielectric in all electric phenomena, showed that it was probable that when a current is stopped or started in a conductor, *the inductive action on other conductors is not communicated instantly, but is propagated through the intervening dielectric with a velocity equal to*

$$\frac{1}{\sqrt{K\mu}}$$

where μ is the magnetic permeability of the medium and K is its specific inductive capacity.

The quantity $1/\sqrt{K\mu}$ can be determined by electrical experiments in a variety of ways and is found to have a value in air of very nearly 300,000,000 meters per second, which agrees with the velocity of light.

Of course, if induction is propagated with a definite velocity, an alternating current sending out first one kind of inductive disturbance and then the reverse must produce a series of electrical waves, just as a tuning-fork giving a series of impulses which travel successively forward through the air produces a train of sound waves.

763. Hertz' Experiments.—Maxwell's conclusions as to electric waves were not *directly* demonstrated until 1884, when the German physicist Hertz obtained such waves and measured their velocity.

The difficulty was twofold: to set up waves short enough to be studied—for if their velocity was 186,000 miles per second, an alternating current with a frequency of even 186,000 per second would produce waves a mile long—and, *second*, to devise some method of detecting and measuring them.

Hertz succeeded in obtaining waves sufficiently short to measure by using those sent out in the oscillatory discharge of the apparatus shown in figure 454.

Two rectangular metal plates were mounted as shown, with polished knobs close together. The plates were connected to the secondary of a powerful induction coil so that when charged by

the coil they discharged with oscillations across the spark gap between the knobs. Thus a group of short waves was sent out by the oscillatory discharge every time the induction coil acted, and this may have been 200 times a second, but each group died out absolutely before the next was formed.

To detect the waves, Hertz used an *electrical resonator*, a hoop

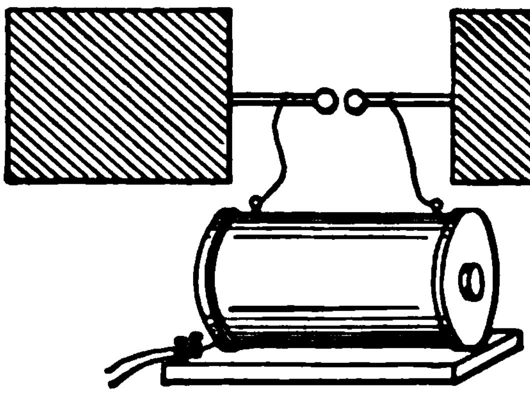


Fig. 454.—Hertz oscillator.

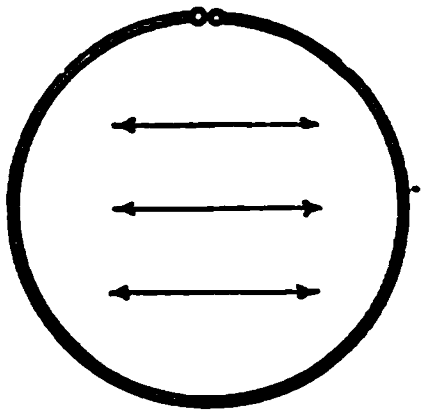


Fig. 455.—Hertz resonator.

of metal having at one point a minute spark gap between two knobs. The resonator was adjusted to be in resonance with the vibrator so that in a darkened room a small spark could be seen at the spark gap of the resonator at every discharge of the vibrator, even when it was 10 or 12 meters distant.

In order to test whether the disturbance was propagated as

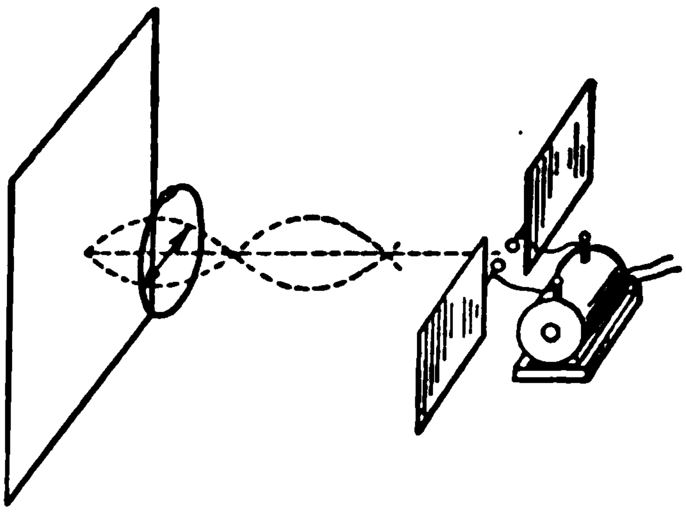


Fig. 456.—Hertz nodes and loops.

a wave motion, Hertz set up the vibrator in front of a great reflector of sheet metal, as shown in figure 456, so that the reflected waves meeting the advancing ones might cause nodes and loops just as in any other case of wave motion. *He then found by means of the resonator that there actually were points of maximum disturbance and points half-way between them where the effect was a minimum.* In this way the existence of electrical waves was *proved* and the wave length measured.

From the wave length and period of oscillation the velocity of the waves was calculated *and found to be, as nearly as could be determined, the same as the velocity of light*, thus confirming the anticipations of Maxwell.

764. Other Experiments.—Later experimenters have devised oscillators of other forms more suitable for obtaining short waves. One of the best arrangements is that of Righi shown in figure 457. Two brass balls are mounted near each other, the space between being filled with oil contained in a surrounding glass cylinder. Just outside of these are other balls connected with the poles of the induction coil. A large difference of potential between the two inner balls is required before a spark can burst through the oil, and consequently the vibrations are so much the more energetic.

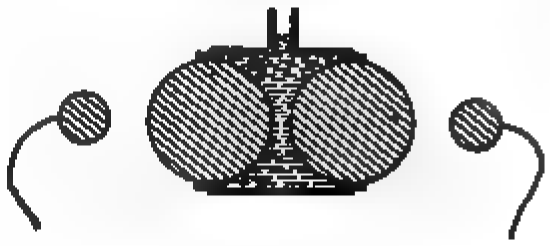


FIG. 457.—Righi oscillator.

Using oscillators of this form electric waves only a few millimeters long have been obtained and measured, and have been reflected, refracted, and polarized, like waves of light.

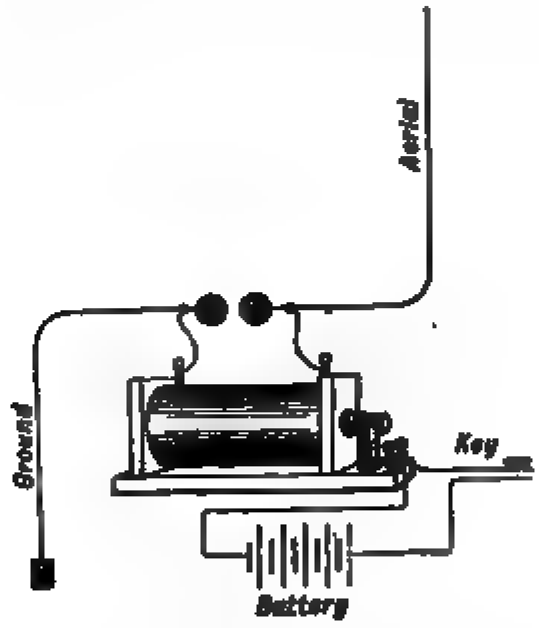


FIG. 458a.—Simple wireless sender.

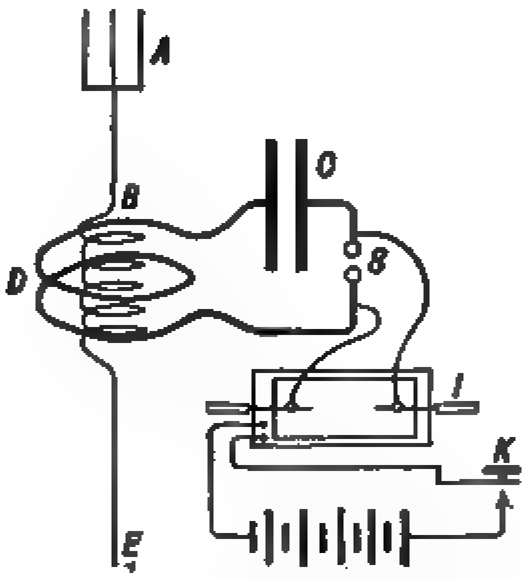


FIG. 458b.—Wireless sending circuit.

765. Wireless Telegraphy.—An important application of electromagnetic waves has been made by Marconi in wireless tel w. The waves are sent out from a tall *antenna* or w h is *connected* to one pole of an induction coil, *which is* connected to earth. Between the

two there is a spark gap across which the oscillation takes place. Such an arrangement sends out waves on all sides, the most energetic waves going out at right angles to the wire. No energy is sent directly upward.

The aerial wire or antenna is given a variety of forms; a common type consists of several copper wires stretched at a height of from 50 to 100 ft. above the ground and all connected to the earth through a single wire at one end.

In order to set up more powerful oscillations the arrangement shown in figure 458*b* is commonly used. The antenna *A* is shown directly connected to the ground *E* through a coil of perhaps a dozen turns of heavy copper wire. The condenser *C* has its coatings connected by a circuit which takes a few turns close around the coil *B* in the antenna circuit, and includes a spark gap *S*. When the condenser is charged by an induction coil or high-tension transformer, discharges take place across the spark gap, accompanied by

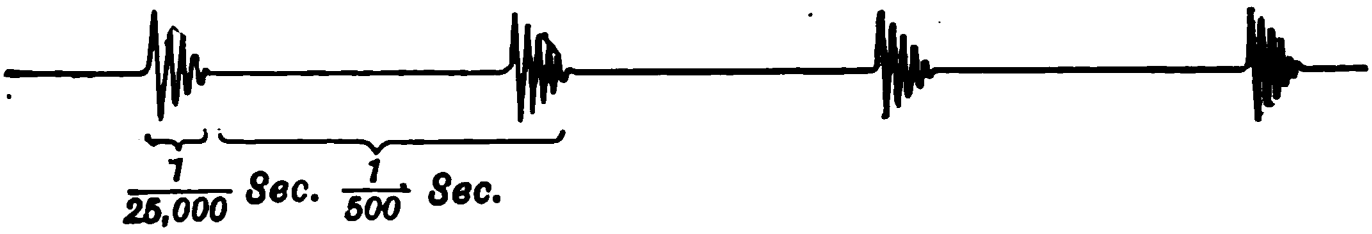


FIG. 459.—Current in Antenna. Fifty such groups may be in one "dot."

oscillations or surgings in the condenser circuit, and these, by induction between the coils of wire in *B* and *D*, set up corresponding oscillations in the antenna circuit, and if one circuit is in resonance with the other the oscillations will be strong. When the key *K* is pressed for a tenth of a second to send a "dot" of the telegraphic code, perhaps 50 sparks will pass at *S* each causing a group of surgings in the antenna which rapidly die out. It is these oscillations that determine the length of the waves that are sent out. If the wave length is 600 meters each oscillation in the antenna will have a period of one five-hundred-thousandth of a second, and each group will die out after perhaps 20 such oscillations or in $\frac{1}{25,000}$ second, so that the current in the antenna may be represented by the above diagram, the current having zero value between the groups of oscillations.

766. Receiving Apparatus.—An early form of receiving apparatus using a coherer as a detector is indicated in figure 460*a*.

The coherer consists of two small silver rods fitted closely into a short glass tube and having a narrow gap between their ends partly filled with sharply cut nickel and silver filings.

When electric waves meet the antenna of the receiving station, they excite oscillations which surge alternately down through the wire to the ground and back again. These surgings pass through the filings in the coherer and have the effect of causing the particles to cling together and so their electrical resistance is greatly diminished permitting current from the battery *C* to pass. This operates the relay and sounder giving the signal. There is also a *tapper* which slightly jars the coherer and restores it to its original high

resistance, thus interrupting the current from C so that all is ready for the next signal.

Electric waves may also be detected by a simple microphone consisting of a needle laid across two sharp edges of carbon connected in series with a telephone and battery cell.

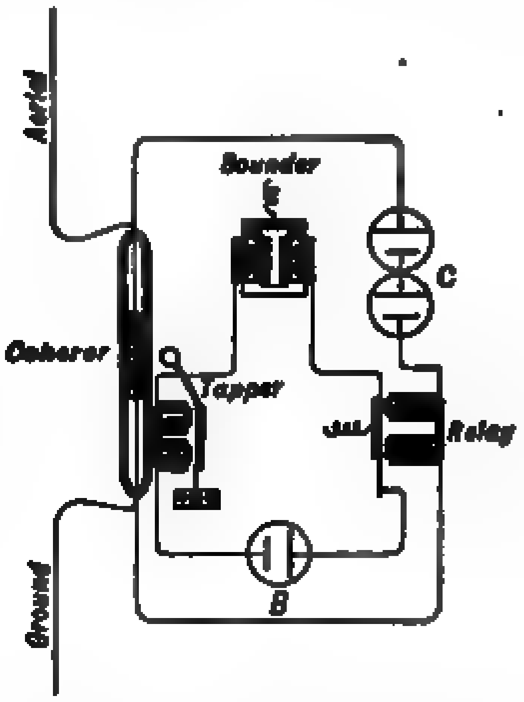


FIG. 460a.—Wireless receiver.

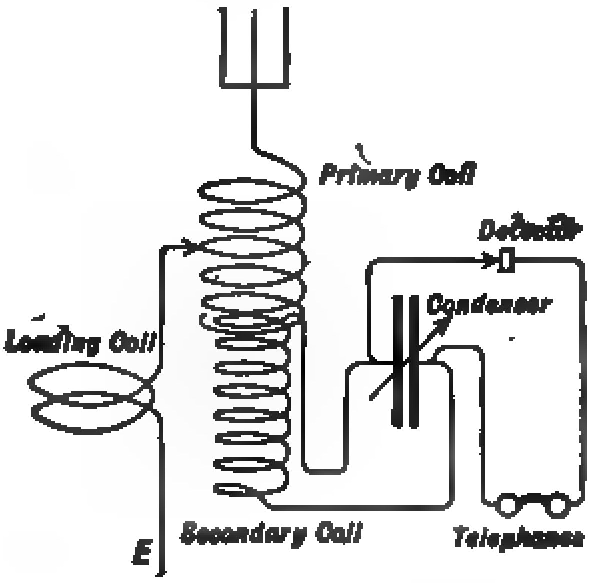


FIG. 460b.

A simple form of receiving circuit widely used in wireless telegraphy is shown in figure 460*b*. The antenna is connected directly to earth through a cylindrical coil known as the primary coil. Another coil, the secondary, of smaller diameter, is mounted so that it can be slid inside of the primary coil or moved away from it when it is desired to weaken the inductive action of one coil upon the other. The terminals of the secondary coil are joined to the coatings of a variable condenser of small capacity.

When electric waves of a certain period fall on the antenna, oscillations are set up which will be strongest when the natural period of electrical surgings in the antenna and primary coil are the same as the period of the oncoming waves. This adjustment may be effected by regulating the number of turns used of the primary coil by means of a sliding contact, or by connecting additional turns from a so-called "loading coil." The secondary also is brought into resonance with the primary, by the adjustment of the variable condenser and distance between the two coils.

767. Detectors.—The detector system is attached to the terminals of the variable condenser and may consist of a pair of receiving telephones in series with a crystal detector. The receiving telephones are specially wound with a great number of turns of very fine wire so that they may respond to the slightest current. The crystal detector may consist of a crystal of galena which is lightly touched by the sharp tip of a fine brass or steel wire; or it may be a fragment of silicon against which a point of steel piano-wire is gently pressed; or an *electrolytic* detector may be used in which the tip of a

very fine Wollaston wire of platinum just touches the surface of a little dilute sulphuric acid, a battery cell being in series with the arrangement in this case.

But the most sensitive type of detector in use is the so-called audion of DeForest, which makes use of the fact that a glowing incandescent filament in a vacuum gives off negative electricity more readily than positive.

These various detectors act as valves or rectifiers which permit current to flow easily in one direction but oppose it when it is reversed. The electrical oscillations which are received are too rapid to affect the telephone diaphragm, but the detector *rectifies* the oscillations in each group (Fig. 459) so that for each group of oscillations there is a little pulse of current in one direction through the telephone; and this being repeated with the frequency with which the groups come along, causes a distinct tone to be heard.

768. Wireless Telephony.—For wireless telephony it is necessary to send out a sustained series of oscillations which although too rapid to affect a telephone directly are, by means of a suitable transmitter, made to fluctuate

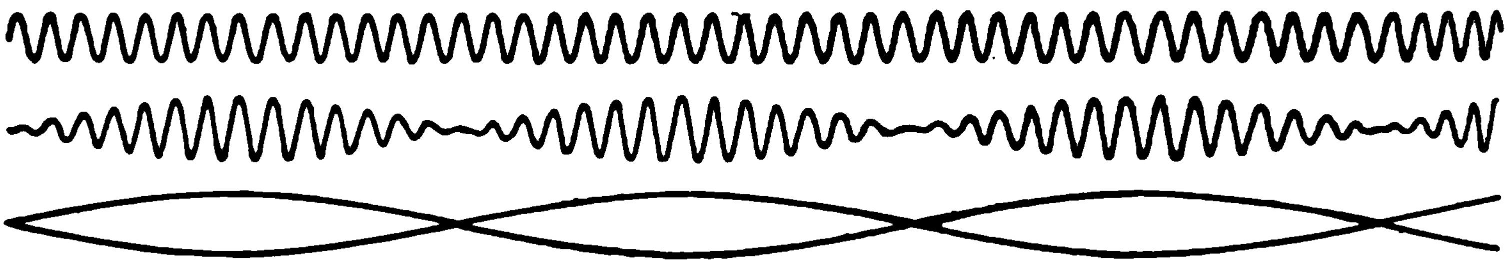


FIG. 461.—Current curves in wireless telephony.

in intensity just as an ordinary telephone current fluctuates in response to the voice, and it is these fluctuations in intensity that act upon the receiving telephone and reproduce the sounds to be transmitted. For instance, suppose the transmitting station sends out a continuous series of what are called *undamped waves* 10,000 meters long. These will come along at the rate of 30,000 to the second, and if the receiving station is *in resonance* with these waves a strong electrical oscillation is set up represented by the upper curve in figure 461, but these oscillations are far too rapid to set in vibration the telephone diaphragm.

Now if sound waves acting through the transmitter cause fluctuations or variations in intensity in the original waves the effect will be such as is indicated in the middle curve. This also would not directly affect a telephone, for the positive and negative currents exactly balance each other. But if a suitable rectifying device, such as an audion, is used, which only permits currents to flow in one direction in the detecting circuit, then the current in the telephone though intermittent will on the whole vary in intensity as indicated in the lower curve and the diaphragm will vibrate in response, thus reproducing the sound.

References

POINCARÉ AND VREELAND: *Maxwell's Theory and Wireless Telegraphy.*
RUPERT STANLEY: *Wireless Telegraphy*

Electric Dicharge through Gases

769. Discharge at Atmospheric Pressure.—The difference in potential between two knobs required to cause a spark to pass between them at atmospheric pressure appears to be nearly the same whatever metal is used for the knobs, but it depends on their curvature.

For knobs over 2 cm. in diameter and more than 2 mm. apart, the number of volts required for spark discharge is approximately given by the formula

$$V = 30{,}000d + 1500$$

where d is the distance between the knobs in centimeters.

Table of Spark Potentials between Slightly Curved Surfaces in Air

Spark length	Voltage	Voltage per centimeter
0.0015 cm.	426	284,000
0.01	948	94,800
0.1	4,419	44,190
0.5	16,326	32,652
0.8	25,458	31,822
1.0	31,650	31,650

Heydweiler

770. Effect of Diminished Pressure.—On diminishing the pressure, the potential necessary for discharge is lowered until a certain critical degree of exhaustion is reached. Beyond this point the higher the exhaustion, the greater the potential difference required to produce discharge, until at the highest exhaustions a spark can hardly be made to pass, but discharge will take place through several inches of air at atmospheric pressure, in preference to 1 mm. in the vacuum. The critical pressure is less than a millimeter of mercury when the electrodes are more than a few millimeters apart, but if they are very close it is somewhat greater.

The appearance of the discharge may be conveniently studied in a wide glass tube 3 or 4 ft. in length, closed at the ends with caps in which the electrodes are mounted, and connected with an air-pump. *If the* electrodes are connected with an in-

duction coil or electrical machine, the discharge will take place through the tube after a few strokes of the air-pump. At first there is a crackling, flashing discharge along narrow flickering lines, but as the exhaustion proceeds the lines of discharge widen out and fill the whole tube, which glows with a steady light.

The discharge at first is between certain points on the electrodes, but with higher exhaustion the luminous glow entirely covers the surface of the negative electrode.

In this stage the characteristic features of the discharge are as follows: A faint velvety glow covers the surface of the negative electrode or cathode; just outside of this is the Crookes dark

FIG. 462.—Discharge in gas at low pressure.

space which surrounds the cathode and has nearly a constant width everywhere. Then comes a luminous region, called the negative glow, and then a dark space, the so-called Faraday dark space, after which a luminous column, known as the positive column, reaches all the way to the anode.

The positive column is commonly not continuous, but shows alternate bright and dark layers across the path of discharge; and these bright layers or *striæ* become broader and farther apart as the exhaustion is increased.

If the distance between electrodes is increased the appearance at the negative electrode is not particularly changed, the positive column, however, is increased in length and reaches as before nearly to the negative glow. Professor J. J. Thomson examined the discharge in an exhausted tube 50 ft. long, and found that the positive column reached the entire length of the tube to within a short distance of the negative electrode and was stratified throughout.

771. Geissler Tubes.—Geissler tubes, so called from the name of a well-known maker who showed great skill and ingenuity in their construction, are tubes of glass especially designed for the purpose of exhibiting the phenomena of discharge. They are exhausted to a pressure of about 1 mm. of mercury, and are provided with aluminum electrodes attached to platinum

wires sealed into the glass. They are usually wide near the electrodes, but often a part of the tube is quite narrow, and here the concentration of the discharge makes the illumination particularly brilliant and the stratification very noticeable.

In these tubes a marked fluorescence of the glass is produced by the discharge, some kinds of glass glowing with a yellowish-green light, while other kinds appear bluish. When such a tube is surrounded by a solution of sulphate of quinia or fluorescein or other fluorescent liquid, the characteristic fluorescence is strikingly brought out.

The character of the light from such a tube depends on the gas which it contains, a tube containing nitrogen or atmospheric air appears of a reddish-violet color, a hydrogen tube is much bluer, while carbon dioxide gas shows a pale whitish illumination. The light of each when analyzed by a spectroscope is found to be made up of certain particular wave lengths characteristic of the gas.

772. Cathode Rays.—The Crookes dark space closely surrounding the negative electrode or cathode is the seat of a re-

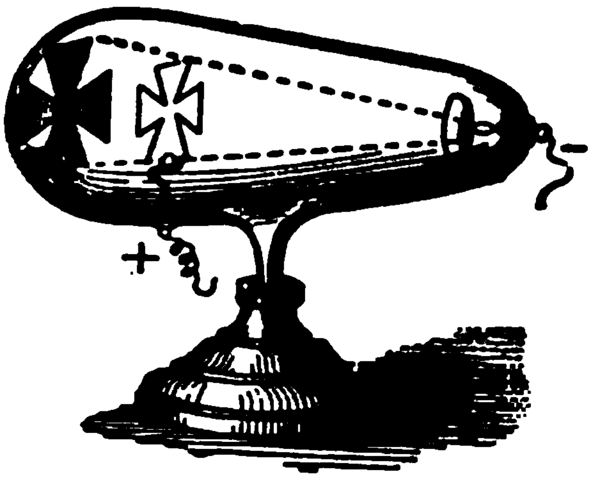


Fig. 463.—Crooke's tube with screen intercepting cathode rays.

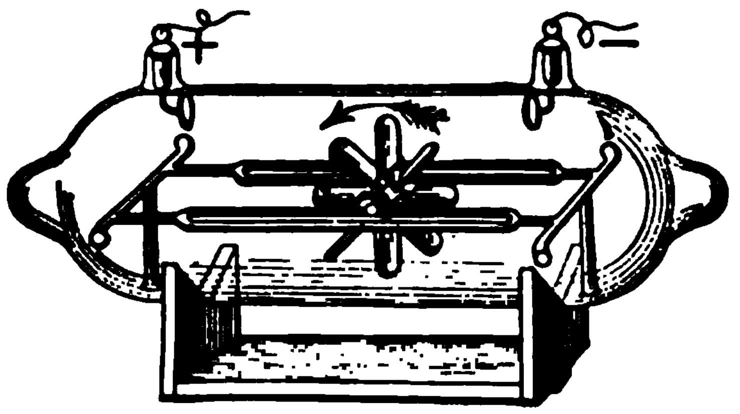


Fig. 464.—Wheel driven by cathode rays.

markable kind of discharge called the *cathode rays*, the properties of which are best studied in tubes that are exhausted far higher than ordinary Geissler tubes, or to a pressure of about one-thousandth of a millimeter of mercury. In such a tube the positive column is faint and inconspicuous, while the dark space around the cathode occupies a considerable space in the tube and is pervaded by a discharge which streams out nearly at right angles to the surface of the cathode without reference to the position of the positive electrode. In the tube shown in figure 463 a sharply defined shadow of the metal cross is cast on the

end of the tube opposite the cathode, for the rays excite brilliant yellowish fluorescence in the glass wherever they strike it directly. The position of the positive electrode is immaterial.

A crystal of Iceland spar or calcite on which the cathode rays fall glows with orange-red light which persists some seconds after the discharge has ceased.

If the cathode is concave, the discharge may be concentrated in a focus and produce intense heat.

That there is also a mechanical effect produced by the cathode rays was shown by the English chemist Crookes by means of the tube shown in figure 464, in which a carefully balanced little wheel with light vanes of mica or aluminum rests with its axle on horizontal rails of glass. The electrodes are mounted at each end of the track facing the upper part of the wheel, and when the discharge passes the wheel is driven away from the cathode.

J. J. Thomson has shown that the rotation of the wheel in this case is probably caused by the unequal heating of the two sides of the vanes as in the radiometer (§465) rather than by the direct mechanical effect of the cathode rays.

773. Deflection of Cathode Rays.—In the tube shown in figure 465 the rays from the cathode after passing through a narrow

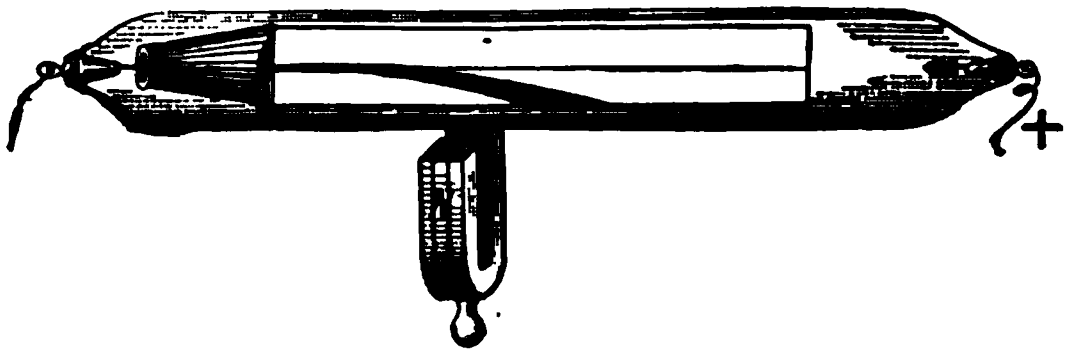


Fig. 465.—Cathode rays deflected by magnet.

opening in the screen S fall upon a sheet of mica running lengthwise the tube and covered with fluorescent material so that the path of the rays is distinctly seen. The boundaries of the luminous path are seen to curve outward slightly as though the discharge consisted of a stream of particles charged with electricity whose mutual repulsion causes them to separate while they are streaming forward.

If a horseshoe magnet is now held with its poles on opposite sides of the tube so that its lines of force are at right angles to *the path* of discharge, the rays are deflected to one side. If

es of force are down, perpendicular to the paper, the ill be bent into the position shown by the shaded curve, would be expected of particles of matter negatively l and projected forward.

as also been shown by J. J. Thomson that the stream iode rays is deflected when a positively or negatively l body is brought near it, being attracted by the former pelled by the latter.

Nature of Cathode Rays.—The experiments described last paragraph all point to the conclusion that *the cathode nsist of a stream of negatively charged particles projected ie cathode.* From the amount of deflection of the rays agnetic field of known strength, taken in connection with lection caused by a known electrostatic field, the English st J. J. Thomson has estimated that the particles forming hode rays have each a mass about $\frac{1}{1800}$ that of a hydrogen nd move with a velocity which depends on the fall of al at the cathode, but may be about $\frac{1}{10}$ of the velocity , or at the rate of about 18,000 miles per second, and each a negative charge of electricity equal to one elementary arge (4.77×10^{-10} electrostatic units) as found by Millikan. mass and charge of the cathode-ray particles have since udied by other physicists and measured in different ways *e found to be the same whatever may be the nature of the lectrodes or of the residual gas in the vacuum tube.*

e particles have received the name *electrons* (a contrac- *electrical ions*), and are the smallest known particles of

Röntgen Rays.—An entirely new kind of radiation ing from those parts of a Crookes tube which are acted the cathode rays was discovered in 1896 by the German st Röntgen. This radiation, called Röntgen rays or X- strongly sent out from an oblique plate of platinum on the cathode rays are converged, as in the tube shown in 66.

Röntgen rays are detected by photography, as they act lly on an ordinary dry plate, and also by their power to luorescence. They are not deflected by a magnet nor d by prisms or lenses, and are but feebly reflected. They

have a remarkable power of penetrating substances of small density, such as wood, pasteboard, or flesh, which are opaque to light. They also penetrate thin sheets of metal of small density, such as aluminum, while they are screened off by lead or platinum.

A fluorescent screen made of pasteboard covered with fine crystals of barium platino-cyanide or calcium tungstate will

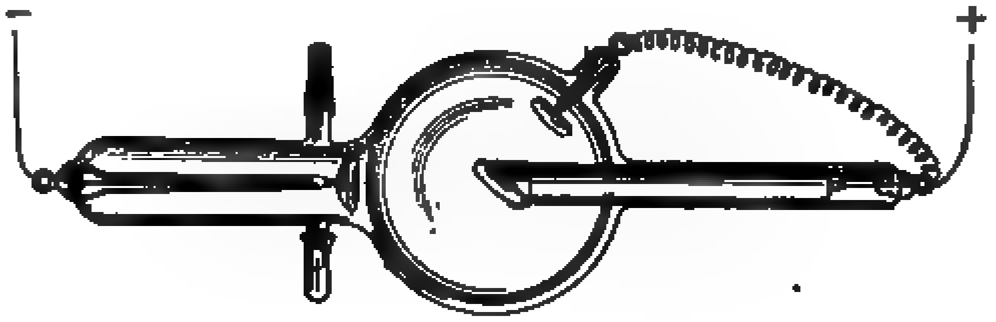


FIG. 466.—Röntgen-ray tube.

glow brightly if brought in front of a Röntgen-ray tube in a darkened room. If the hand is interposed between the tube and the screen, the flesh, being most easily penetrated by the rays, will show but faintly while the shadow of the bones is strongly marked. If a photographic plate enclosed in the usual plate holder with slides of hard rubber, wood, or pasteboard is substituted for the fluorescent screen, a photograph is obtained on development such as is shown in figure 467.

FIG. 467. Radiograph of hand.

776. The Nature of Röntgen Rays.—There has been considerable uncertainty as to the nature of the Röntgen radiation, though the fact that these rays are not deviated by a magnet shows conclusively that they cannot consist of a stream of charged particles like the cathode rays.

But the recent discovery by Laue that diffraction effects (§935) are

ained when a pencil of Röntgen rays is passed through a thin plate of stal, and the studies of the reflection of these rays from crystal surfaces W. H. and W. L. Bragg, which grew out of Laue's discovery, have made it m quite certain that this radiation is like light but having wave lengths ich in some cases are less than one-thousandth the shortest visible wave gths of ordinary light. It is found that the radiation from an X-ray tube generally quite complex, being made up of radiations of very different ve lengths, the shortest being the most penetrating.

777. **Characteristic X-rays.**—In an ordinary X-ray tube where the 's are produced by the convergence of cathode rays upon an oblique plati- m target there are certain wave lengths in the radiation which are ecially energetic. If the target is made of some other metal the wave gths given off are different from those given by platinum, and in general it s been discovered that each element, so far as it has been possible to test matter by experiment, when subjected to bombardment by cathode rays 'es off X-rays of certain special wave lengths, designated by Barkla, their coverer, *the characteristic X-rays* of that substance. The investigations Moseley have shown that the *frequency of vibration of the characteristic rays of any element is proportional to the square of its atomic number*, the omic number of an element being an interger which represents its posi- n in the series of elements arranged according to their atomic weights. us the position of any element in the series is at once defined when the ve lengths of its characteristic X-rays have been determined. An alloy h as brass gives off simply the characteristic X-rays of its two constit- nts copper and zinc.

778. **Canal Rays.**—Still another kind of rays, known as *canal ys*, was discovered by Goldstein in 1886. When the cathode a highly exhausted vacuum tube is pierced full of small holes, e canal rays may be observed as streams of light coming through e holes and out on the back of the cathode, and are thus directed ay from the positive electrode and are in the opposite direc- n to the cathode rays. They derive their name from the small enings or canals through which they pass. These rays are ınd to consist of streams of *positively* charged particles hav- g masses more than a thousand times as great as the masses the electrons in the cathode rays, but their velocity is very ıch less than that of the cathode-ray particles.

779. **Gaseous Conduction.**—At ordinary temperatures and at- ospheric pressure gases are almost complete non-conductors electricity. Careful experiments have shown, however, that ere is a slight loss of charge through the air, which is believed be due to the presence in the air of a few *ionized* molecules, molecules *which are* broken up each into a positive ion and

a negative ion or electron. Positive ions coming in contact with a negatively charged body receive negative charge and become neutral.

The ionization of a gas may be brought about by intense chemical action and high temperature, and so flame gases readily conduct electricity. Also it may result from the passage of an electric spark through the gas, or from a very intense electric force as in case of discharge from electrified points. A mass of gas may also be ionized and made conducting by exposing it to Röntgen rays. When a charged electroscope is exposed to Röntgen rays it quickly loses its charge.

It seems probable that in Geissler tubes the initial ionization is caused by electrons driven off with great velocity from the cathode and striking against gaseous molecules.

In the process of conduction a gas soon loses its ionization partly because the ions are neutralized in the act of conduction and partly because the recombination of positive and negative ions is constantly going on.

Radioactivity

780. Radioactivity.—In 1896 the French physicist Becquerel discovered that minerals containing uranium gave out a radiation which resembled Röntgen rays in acting on a photographic plate through an envelope of black paper. He also showed that the uranium or Becquerel rays, as they were called, had the power to discharge electrified bodies, so that by using a sensitive electroscope they could be easily and accurately detected and their intensity measured. The photographic action was very slow, an exposure of several days being required to produce a distinct impression.

Following out this discovery, Mme. Curie made a systematic search for other active substances and found that thorium possessed a similar power of *radioactivity*, as it now came to be called, a discovery which was also independently made by Schmidt.

It also appeared from these investigations that radioactivity is an *atomic* phenomenon. For the radioactivity of any given compound of uranium was found to be simply proportional to the amount of uranium in the substance, and in no way dependent on its physical or chemical condition.

But pitchblende, a mineral ore of uranium, was found to be ᵣeral times more active than could be accounted for by the ıount of uranium which it contained, and hence Mme. Curie ıcluded that it must contain some unknown and highly radio-.ive substance, and resolutely set out to isolate it. As a re-t of the laborious treatment of several tons of pitchblende l uranium residues, a few hundredths of a gram of a new l astonishingly active element were obtained (in 1898) to ich the name *radium* was given. This substance is obtained ıally as a chloride or bromide, and is estimated to have in the re state about two million times the activity of uranium.

Some radium compounds glow with a faint luminosity in dark, though pure radium bromide is only feebly luminous. ookes showed that if a minute particle of radium bromide is ported about a millimeter in front of a surface coated with osphorescent zinc sulphide, the latter lights up with flashes of ıt, probably due to its bombardment by alpha particles from radium.

'81. **Complex Character of the Radiation.**—The radiation m uranium, thorium, or radium, is found to contain three tinct kinds of rays known as alpha, beta, and gamma rays.

These different rays are distinguished from each other in two ys: by their penetrating power and by their deflection in a magnetic electric field.

The *alpha* rays are completely stopped by a few centimeters air or a layer of aluminum foil 0.05 mm. in thickness. They ısist of positively charged particles having a mass about ır times that of the hydrogen atom and are projected with a ocity of about 20,000 miles per second.

The *beta* rays have about 100 times the penetrating power the alpha rays. They consist of negatively charged particles ving only about $\frac{1}{1800}$ the mass of the hydrogen atom and m to be of exactly the same nature as the cathode rays in a uum tube except that the velocity of projection of the par-es in the *beta* rays is much greater than is usual in cathode 's, and in case of radium is found to be from 0.3 to 0.9 the ocity of light.

The *gamma* rays are the most penetrating of all; these rays m 30 mg. of radium bromide having been detected by their

effect on an electroscope after passing through 30 cm. of iron. They are not deviated in a magnetic or electric field and seem to be of the same nature as the Röntgen rays from a "hard" X-ray tube. They probably originate in the collisions of *beta* particles with the substance itself in its interior, just as Röntgen rays arise from the impacts of cathode-ray particles against some obstacle.

782. Ionizing Power of the Rays.—The power to discharge electrified bodies is possessed by all three kinds of rays, and is explained by their ionizing effect (§779) upon the gas through which they pass. The *alpha* particles with their comparatively great momentum have the greatest ionizing power and as they rush through a gas knock out electrons from an immense number of atoms along their paths. The power which the *gamma* rays have of making a gas conducting, and this is true also of Röntgen rays, appears for the most part to be indirect, and due to their effect in causing *beta* particles of small velocity to escape from atoms of gas along their path, the ionizing being mainly due to the direct action of these *beta* particles.

783. Paths of Alpha and Beta Particles.—The ionizing effect of alpha and beta particles upon a gas has been demonstrated in a very remarkable series of photographs obtained by C. T. R Wilson, two of which are reproduced in figures 468 and 469. The method which he employed makes use of the fact that when a gas saturated with moisture is suddenly expanded a cloud of condensed vapor forms. Wilson found that by regulating the exact degree of expansion the condensation might be caused to take place only on atoms of gas that were ionized, so that, since an alpha particle in shooting through a gas produces a great number of ions along its path, the sudden expansion of the gas just after the particle has passed causes the condensation of microscopic drops on these ions and if these drops are instantaneously illuminated by an electric spark, before they have time to scatter or dissipate, they may be photographed and the path of the particle thus outlined. Figure 468 shows the intense ionization caused by the motion of an *alpha* particle from radium, something like 30,000 ions per centimeter being produced along its path. As its velocity grows less its path is more affected by the atoms through which it strikes, as shown

by the sudden changes in direction toward the end. Finally, its velocity becomes too small to ionize the gas and its path becomes invisible.

Figure 469 shows the paths of *beta* particles which are set free from atoms of gas by a narrow pencil of X-rays. Each beta particle produces but few ions along its course compared with the number produced by an alpha particle, and their paths are

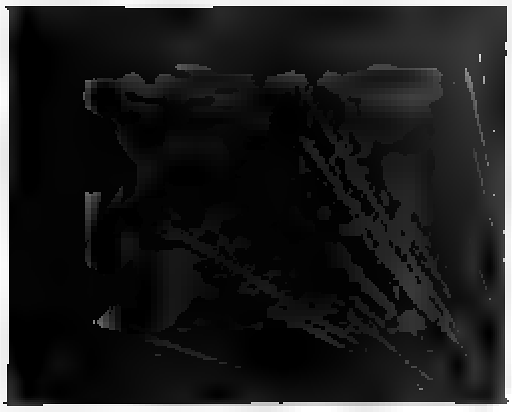


Fig. 468.

Fig. 469.

very crooked and irregular because the momentum of one of these particles is so small that its direction of motion is greatly changed by impacts against atoms of gas.

784. Energy and Heating Effect of Rays.—Although the velocity of the *alpha* particles is less than that of the *beta* particles, yet in consequence of their greater mass *the energy of motion of each* **alpha** *particle is something like* 120 *times that of a* **beta** *particle*, consequently the heating and ionizing effect of the rays is mainly due to the *alpha* particles.

It was found by Curie and Laborde, in 1903, that radium was a constant source of heat, one gram of radium bromide giving out about 100 gram-calories of heat per hour, or more than enough to melt its own weight of ice in an hour. This extraordinary development of energy is believed to be mainly due to the escaping alpha particles which are absorbed within the mass of radium itself and in the enclosing vessel, their energy of motion being transformed into heat. Originally the energy must have existed in the radium atoms before the alpha particles were given off.

785. Radioactive Transformations.—Evidently an atom of radium cannot give out alpha and beta particles with a corresponding expenditure of energy and remain the same as before.

It is therefore believed that the atoms of the radioactive substances have such slight stability that at every instant some of them are reaching a condition of instability and breaking up or exploding with the expulsion of alpha or beta particles and coming to a new state of equilibrium. In case of radium, about half the particles in a given mass will have broken up in this way in the course of 2000 years. When such a change has taken place, and an alpha particle has been expelled, the new state of equilibrium is apt to be even less stable than the original one, and consequently another alpha particle is soon expelled and another state of equilibrium reached which itself may also be short-lived, and so there takes place a series of changes in the course of which the various kinds of rays are given off, and finally a

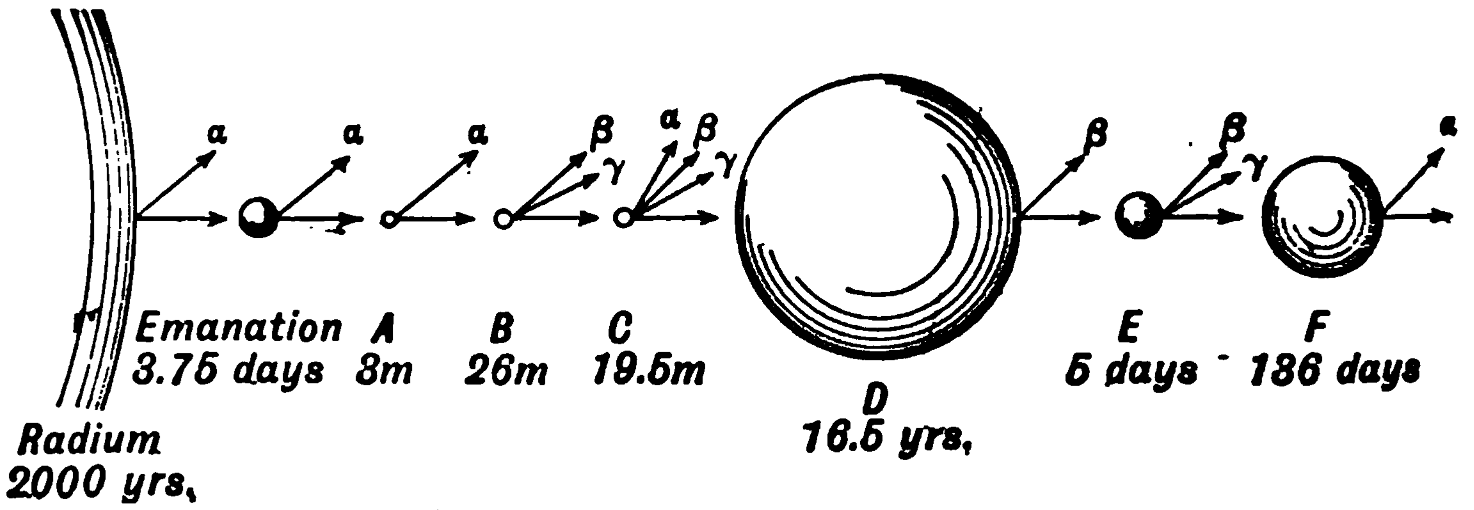


Fig. 470.—Diagram of radioactive transformations.

condition of such stability is reached that no further change is detected. This disintegration theory was proposed by Rutherford and Soddy in 1902.

The diagram indicates the series of changes of the radium atom as followed out by Rutherford, and under each step is given the time in which that product is half transformed. Thus, while radium itself is half transformed in 2000 years, the instability of the emanation is so great that any given portion of it will be half transformed in 3.75 days; while radium A is the most unstable of all, its period of half transformation being only 3 minutes.

The sizes of the spheres in the diagram represent the relative amounts of the various products that are present in a mass of radium which is undergoing change, and has come to such a state of equilibrium that each product is being produced from the one preceding it in the series, just as rapidly as it is being

ansformed into the next succeeding one, and is therefore neither creasing nor decreasing in amount.

Since radium is being constantly though slowly transformed, must disappear from the earth in the course of a few thousand ears unless it is being produced in some way. It is now believed at radium is a disintegration product of uranium, for uraum has the greater atomic weight, they are always found tother, and it has been shown that the amount of radium in any dioactive mineral always bears a constant ratio to the amount uranium which it contains, viz., about 1 to 2,630,000.

The transformation of uranium itself is so slow that it will quire, according to Rutherford, a period of at least ten thound million years for any large fraction of it to be transformed. Boltwood has been able to separate a radioactive substance, hich he has named *ionium*, which appears to be one of the termediate steps in the change from uranium to radium.

786. Helium.—In 1868 Lockyer proposed the name *helium* r an unknown substance existing in the sun and causing a line the solar spectrum which could not be obtained from any iown terrestrial substance. Nearly thirty years later Ramsay, n English chemist, identified it with a gas obtained from some dioactive minerals. Its occurrence in these minerals suggested Rutherford and Soddy that it might be a product of the disingration of radium. Later Ramsay and Soddy collected in a ibe some of the emanation from radium, and though at first iere was no indication of the presence of helium, after five days s complete spectrum was obtained. Helium is a gas having rice the density of hydrogen, and it seems probable that the pha particles given off in the transformations of radium form ie atoms of helium. This result is remarkable as the first se in which one stable element has been derived from another.

787. Final Product of Radium.—If the atomic weight of the pha particle is 4 (the same as helium), then, since the atomic eight of uranium is 238.5, the loss of three alpha particles ould bring it down to 226.5, which is almost the same as 225, e observed atomic weight of radium. Then, as five alpha rticles are given off from radium in the series of changes dicated above (§785), the atomic weight of the final product ould be expected to be 206.5, which is in close agreement with

206.9, the atomic weight of lead. The suggestion that lead may be the final product of the radium changes is due to Boltwood and is supported by the fact that lead is always found associated with radioactive minerals which are rich in uranium.

It has been shown recently that lead from radioactive minerals has an atomic weight slightly less than that of *ordinary* lead. The two kinds of lead differ slightly in density, but in many of their properties including their spectra they seem identical.

These discoveries that radium, helium, and possibly lead also may be products of the disintegration of uranium give new emphasis to a very old suggestion that the atoms of what we call the elements are a number of stable aggregates built up from some simple primordial atom. Whether the elements as we now know them are to be regarded as so many stages in the gradual disintegration of originally more complex atoms of high atomic weight or are various stable aggregates formed independently and perhaps simultaneously under physical conditions that we cannot now distinctly conceive, is a question that cannot now be answered, though the absence of any perceptible radioactivity in the case of most substances points toward the latter origin.

788. Internal Energy of Atoms.—That there exists an enormous amount of energy in the interior of atoms is evident from the heat which is given out by radium in its slow transformation. Thus it has been shown that one gram of radium gives out 100 gram-calories of heat per hour, but as it requires 2000 years for one-half of a gram to be transformed, it is calculated that the whole amount of heat emitted by one gram of radium in the process of transformation is about 10,000,000,000 gram-calories, or more than a million times as much heat as is given out in the combustion of a gram of coal. And it must not be forgotten that this enormous amount of energy can represent only a small fraction of the total internal energy of the atoms in a gram of radium, being merely *that part which is given out when the atoms pass from one state of equilibrium to another.* This energy is detected in radioactive elements only in consequence of their disintegration, but there is no reason to suppose that the internal energy of these elements is of a different order of magnitude from that of the other elements.

[redacted]

There is no known process by which the store of energy locked up in the atoms of matter may be made available for the service of man, for it does not seem possible to disturb the equilibrium of the electrons in the atoms by any outside force. The radioactivity of radium is not changed either by cooling to the temperature of liquid air or heating it to 200°C.

789. The Electron Theory of Matter.—It has been seen (§774) that the small negatively charged particles, or *electrons* as they are called, which make up the cathode discharge in a vacuum tube, have the same mass and charge and are apparently identical whatever may be the residual gas in the tube or whatever metals may be used for the electrodes. Similar electrons also are given out from a glowing carbon or metal filament, and from a zinc plate acted on by ultra-violet light, and are emitted at high velocities, as *beta* rays, from radioactive substances. These discoveries have led to the idea that electrons are fundamental units entering into the structure of all kinds of atoms.

The study of radioactive substances has shown also that in their disintegration there are given off not only the negatively charged *beta* particles or electrons, but also *alpha* particles, each having a positive charge twice as great as the charge of an electron and a mass equal to that of four hydrogen atoms. It is a remarkable fact that alpha particles given off in the disintegration of uranium or of radium appear to be identical with those given off in the radioactive changes of thorium and actinium, suggesting that the alpha particles, as such, may enter into the structure of different kinds of matter.

Various attempts have been made to form a theory of atomic structure that would serve to explain the facts of radioactivity and would lend itself to an interpretation of the chemical relationships of the atoms and of their power to radiate light and heat, assuming that the only forces concerned are electrical. One theory, suggested by Lord Kelvin and developed with great ingenuity by Sir J. J. Thomson, assumes that an atom of matter consists of a sphere of uniformly distributed positive electrification in which revolve in approximately circular orbits and with great velocities a number of electrons, the number being such as to neutralize the positive charge in case of an ordinary neutral atom. If an electron becomes detached the atom becomes positive, or if an extra electron is gained, the atom becomes negative.

But in order to account for the sudden changes in direction in the paths of alpha particles when they strike through gases or layers of gold foil, Sir E. Rutherford has advocated the theory that the positive charge in the atom is not diffused but concentrated in a neuclus about which the electrons revolve. Thus the neutral hydrogen atom has been conceived as made up of a positive nucleus having a charge $+e$ (when $-e$ is the charge of an electron) about which there revolves a single electron. The greater part of the *mass* of the atom is conceived as in the positive nucleus, as an electron has a mass only about 1/1800 of that of the hydrogen atom.

The *alpha particle may* be thought of as made up of four positive nuclei

such as that in the hydrogen atom, in equilibrium with two electrons. This would account for its mass and positive charge equal to $2e$. The neutral atom of helium may then be thought of as having an alpha particle as its nucleus with two additional electrons revolving about it.

The atoms of other elements may then be conceived as built up in some such way with various groupings of hydrogen nuclei, alpha particles, and electrons, forming a central positive nucleus around which a certain number of electrons move in circular orbits, the number of these external electrons being believed by Rutherford to be, for the more complicated atoms, about half the number expressing the atomic weight of the element.

The electron is estimated to have only one hundred-thousandth the diameter of the atom (by the diameter of an atom meaning the shortest distance between the centers of two when they collide in ordinary thermal agitation), so that *if an atom were magnified to be 100 ft. in diameter each electron would have a diameter of only about one hundredth of an inch.* It is easy, therefore, to conceive that in an atom there may be many electrons moving with great velocity and yet without interfering with one another.

According to this theory, when such an atom loses an electron it becomes electropositive and when it gains one it becomes negative. A negatively charged body has an excess of electrons, while in a positively charged body there is a deficiency of them, and an electric current is conceived as the streaming of free electrons through a conductor.

It has been shown by J. J. Thomson that such a conception of atomic structure affords a possible explanation of the periodic law of the elements as well as valency. It is supported also by the observed high velocities with which electrons escape from radioactive elements, for it can hardly be supposed that these enormous velocities can have been given to them wholly in the act of escape. And Lorentz* has shown that the assumption that light waves originate in moving electrons in an atom not only explains the Zeeman effect (§970), but leads to a value of the ratio of the mass of an electron to its electric charge which agrees with that found by J. J. Thomson for the electrons in cathode rays.

Whether such a theory shall stand or fall depends on whether it will enable the physicist to coördinate his knowledge and form a clear mental picture of the interrelation of the various isolated facts known about atoms. A satisfactory theory besides giving a basis for the explanation of radioactivity and the chemical relations of the elements must lend itself to an explanation of the peculiar series of lines in the spectra of the elements as well as of the other peculiarities of atomic radiation revealed by the spectroscope.

References

Righi: *The Modern Theory of Physical Phenomena.*
J. J. Thomson: *The Discharge of Electricity through Gases.*
J. J. Thomson: *Electricity and Matter.*
Rutherford: *Radioactive Transformations.*
Crowthers: *Molecular Physics.*
Soddy: *Interpretation of Radium.*

* Professor of Physics in the University of Leyden.

LIGHT

Shadows and Photometry

790. Light.—In a perfectly dark room we cannot see any objects—we have no sensation of sight—showing that vision requires something more than simply the eye and the object to be seen. That additional something is called **light**. We may define it as the agent which excites the sensation of sight.

When a candle is lighted in the room we see it and also the other objects near. The candle flame is said to be self-luminous and a source of light.

Conditions of Vision.—When the candle is so screened that its light falls only on the eye of the observer but not on other objects in the room, then only the candle itself is seen. It thus appears that illuminating the eye does not give it power to see other objects from which light is excluded. **Light must fall upon the objects themselves if we are to see them.**

And even when an object is illuminated, if a screen is interposed across the straight line from the object to the eye, the former is hidden and we see the screen but not the object behind it. **This leads to the inference that in order that a body may be seen light must pass from it to the eye, and usually this takes place along straight lines.**

791. Transparent and Opaque Bodies.—Bodies differ greatly in their capacity for transmitting light. Those that transmit it freely are said to be **transparent**, while those that intercept it are called **opaque**. Opaque bodies are of two kinds: those that turn back the light at the surface and those into which light penetrates and is absorbed and transformed into heat. The opacity of metals is largely of the first kind, while that of most other substances is due to absorption.

Substances like paper or milk-glass, or milky or muddy waters, which transmit light but through which we cannot see objects, are said to be **translucent**. They are not homogeneous bodies, but light in passing through them is scattered in all directions at the surfaces of innumerable little particles throughout the

mass. Even transparent bodies, such as glass or water, turn back or reflect *at the surface* a part of the light that falls upon them.

Bodies may be transparent for some kinds of light and opaque for others, and this is largely the cause of the colors of bodies.

792. Light Advances in Straight Lines.—Light travels out from the source in straight lines so long as it remains in a homogeneous medium.

Thus a carpenter sights along the edge of a board to see whether it is straight; and the boundary of a shadow is roughly

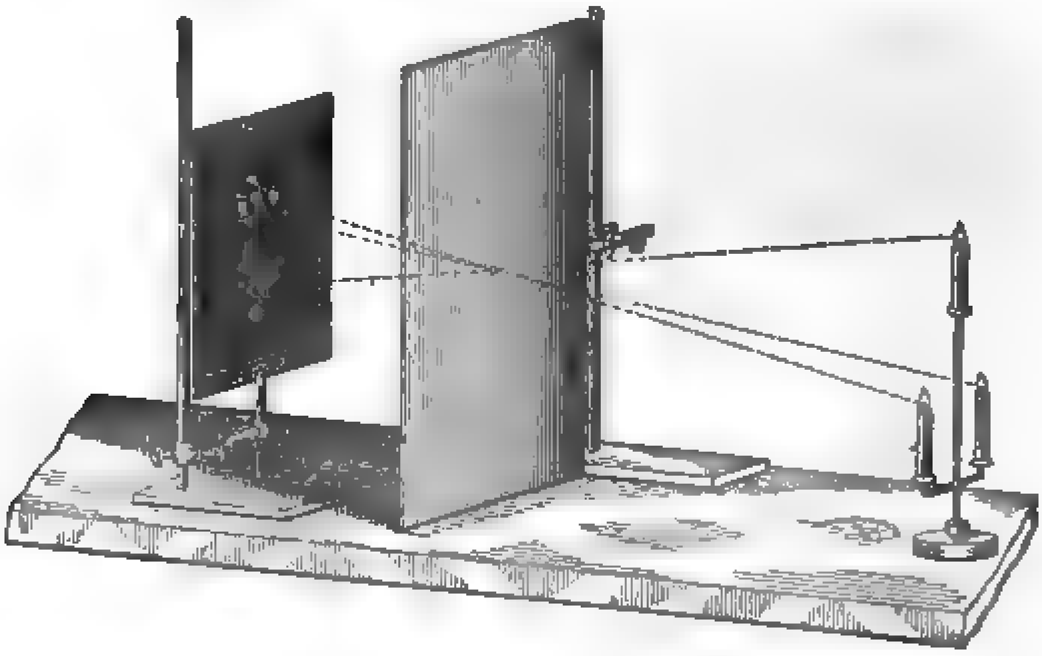


Fig. 471.—Light travels in straight lines.

defined by straight lines through the source of light and tangent to the obstacle.*

But the most convincing evidence of this fact is the exactness with which surveys are made. All measurements of angles made by surveying or astronomical instruments assume that light from the distant object comes to the observer's telescope in straight lines if the medium is homogeneous.

When light is admitted to a dark chamber through a small hole, an inverted image of the outer landscape is formed on a white screen opposite the opening. For as light goes through the opening in straight lines each point on the screen is illumi-

* On close examination of a shadow, even when the source is the merest point of light, it is seen that there is not a sharp transition from light to dark at its edge, but it is marked *by a series of alternate dark and light diffraction bands.*

ated by light from a single point in the landscape, and, there-ore, the relative brightness and colors of objects in the landscape re reproduced at the corresponding points on the screen. A hite screen is used because it reflects to the eye all colors equally rell.

793. Shadow, Umbra and Penumbra.—When the source of ight is a broad luminous surface, as in case of an ordinary gas lame, shadows are not harply defined, but shaded it the edges. For example, n case of the sun and earth, is shown in figure 472, the egion between B and C is in ull shadow, and is known as the **umbra**; while the outer region hades from full illumination at A to complete shadow at B, and s known as the **penumbra.**

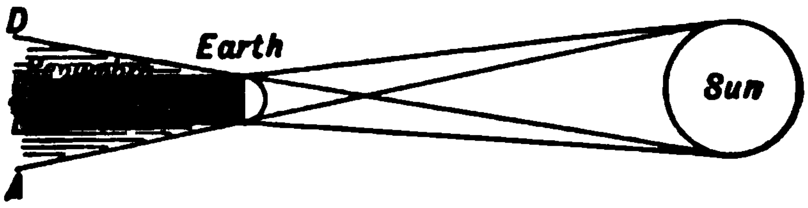


FIG. 472.—Umbra and penumbra.

794. Intensity.—If the source of light S, figure 473, *is a point*, t is clear that a surface A if moved to B, twice as far from the source, will intercept only one-fourth as much light as in its original position; if its distance from the source is increased three times it will intercept only one-ninth as much light, etc. Hence **the intensity of its illumination or the quantity of ight which it receives per unit surface varies inversely as the quare of its distance from the source.**

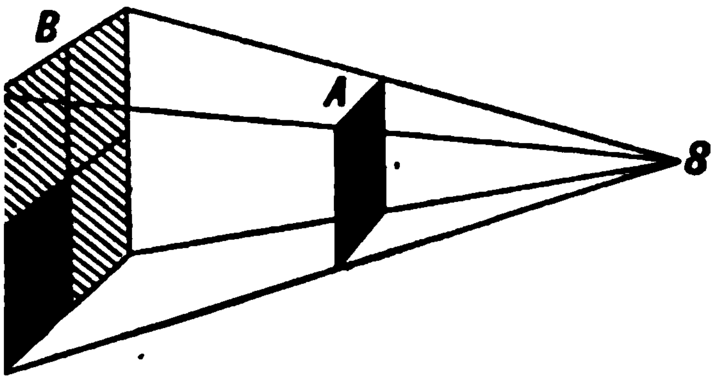


FIG. 473.

That is, if the intensity at unit distance from a point source s I_0 then the intensity I at a distance r from the source is

$$I = \frac{I_0}{r^2}.$$

795. Oblique Incidence.—Suppose a square surface is placed erpendicular to the rays of light, as shown at AB in figure 474, nd is illuminated with light of intensity I. If it is inclined hrough an angle x into the position AC, the beam of light falling pon it will be narrower than before in the ratio AD to AB. The intensity of illumination in the inclined position will there-

fore be to the intensity when perpendicular in the ratio of AD to AB, that is, as $\cos x$ is to 1.

796. Actual Sources.—In all practical cases the source is not a point, but a luminous surface or region from every point of which light is sent out; and it is clear that however close the illuminated surface may be placed, its intensity of illumination cannot be greater than the brightness of the source.

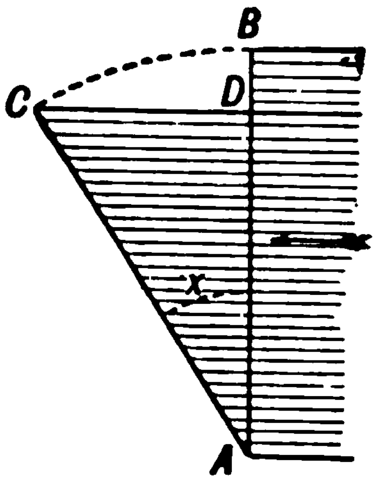


FIG. 474.

The law that the intensities at two points are inversely proportional to the squares of their distances from the source holds very closely when they are in the same direction from the source and are so far away from it that it subtends only a small angle. If the source subtends an angle of 10° the error is less than 1 per cent.

797. Photometry.—The measurement of the relative amounts of light given out by two sources is called *photometry*. When both lights are of the same color the comparison may be made by several methods with much accuracy. If the lights differ in color they may be analyzed into their component colors and the corresponding components of each be compared by means of a spectrophotometer to be described later (§914).

In all photometric measurements care must be taken that the only light falling on the photometer screen comes *directly* from the lights which are compared. The experiments must therefore be conducted in a room from which daylight is excluded, and by means of black screens all light reflected from white walls or other objects must be kept from the photometer screen.

798. Rumford Photometer.—The simplest form of photometer is that devised by Count Rumford and shown in figure 475. An opaque rod is mounted a short distance in front of a white screen. The lights to be compared are so placed that the two shadows of the rod are side by side and of equal intensity. The shadow cast by A is illuminated only by B and that cast by B is illuminated only by A; if therefore the shadows are equally intense the illumination of the screen must be the same by A as by B. Let b represent the brightness of A, meaning by *brightness* the intensity with which it illuminates a surface at

unit distance from it, and let b_1, represent that of B and let d and d_1 represent their respective distances from the screen. Then

$$\frac{b}{d^2} = \frac{b_1}{d_1^2} \quad \text{or} \quad \frac{b}{b_1} = \frac{d^2}{d_1^2}$$

or the brightnesses are proportional to the square of the distances measured from the lights to the screen.

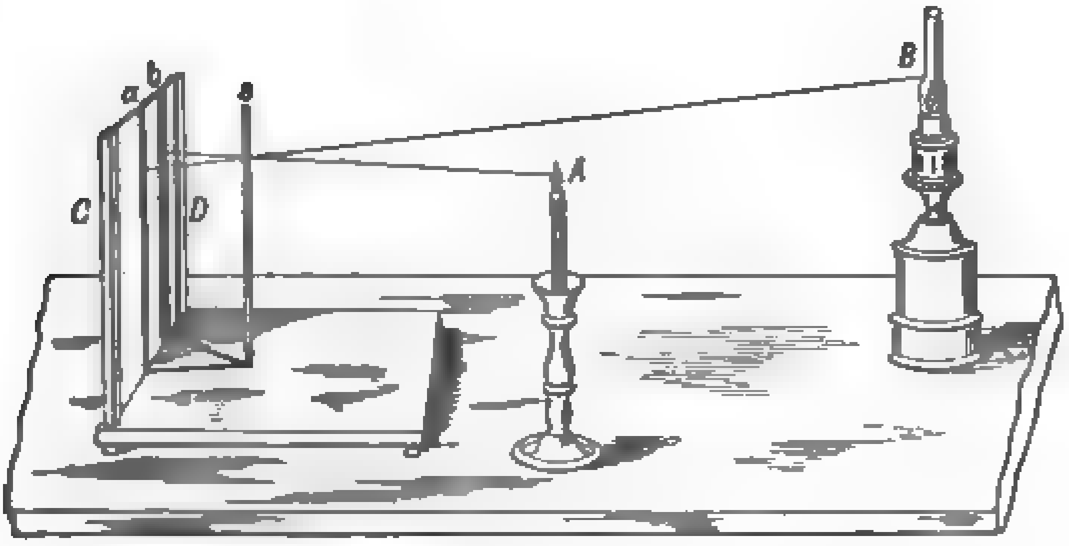


FIG. 475.—Shadow photometer.

799. Bunsen Photometer.—A better form of photometer, free from penumbral disturbances at the edges of shadows, is the grease-spot photometer devised by Bunsen, which consists of a screen of white paper having a spot at its center rendered

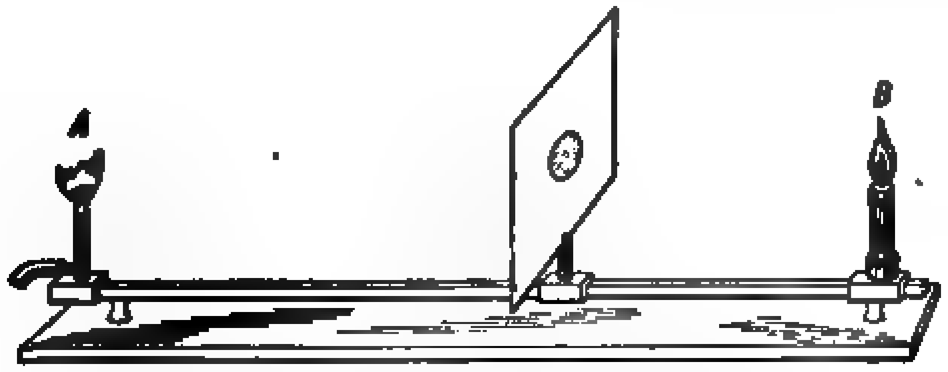


FIG. 476.—Grease-spot photometer.

translucent by means of grease or paraffin. The screen is placed between the lights to be tested so that one side is illuminated by one light and one by the other. The translucent spot transmits light quite freely, and therefore if the paper is lighted only on one side the illuminated side will appear bright with a dark spot at the center while the side away from the light

will be darker with a bright central spot. If the two sides of the paper are equally illuminated the spot disappears. The intensities of the lights are then proportional to the squares of their distances from the screen.

800. Lummer-Brodhun Photometer.—The Lummer-Brodhun photometer is a development of the idea embodied in the Bunsen photometer. Its construction is shown in figure 477. At S is an opaque screen with perfectly white plaster-of-Paris surfaces. M and M' are mirrors and P and P' are two polished prisms of glass, one of which, P', has the whole of the diagonal surface completely polished, the other, P, has only a central round spot on the diagonal face polished, the rest being ground away so that the two prisms touch only in this central polished spot. When freshly polished they are put together under great pressure so that they cohere firmly like a solid block of glass. If the lights to be compared are now placed in the direction of the arrows A and B, respectively, light from the side of the screen illuminated by A after reflection at M passes directly through the central spot in the block of prisms to the observer at O, while light from the side toward B after reflection at M' is reflected to the observer at O from that part of the diagonal face of P' which surrounds the central spot, while the light from M' which falls upon the central spot simply passes through and is not reflected to the eye. Thus to the observer at O the brightness of the central spot depends on the illumination of the left-hand side of the screen by A, while the brightness of the surface around the central spot depends on the illumination of the right-hand side of the screen by the light B. The distances of the lights from the screen are varied until the central spot and surrounding surface appear equally illuminated.

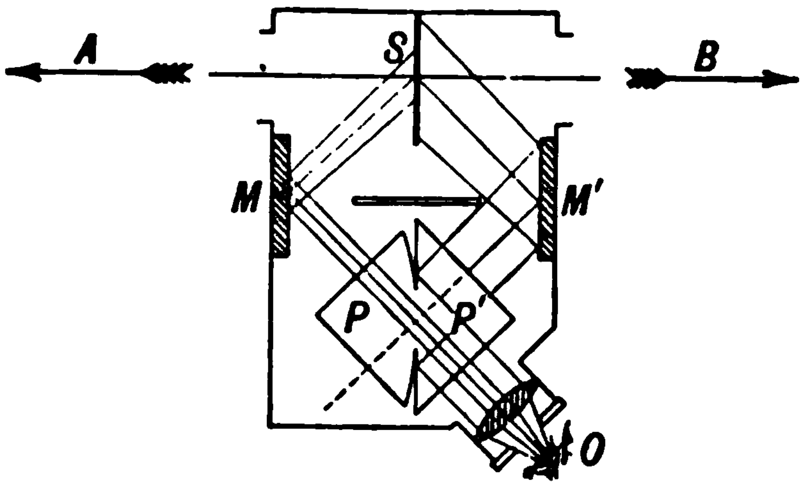


Fig. 477.

801. The Rood-Whitman Flicker Photometer.—When two lights differ in color it is difficult to compare their intensities by the above methods, for the two parts of the photometer screen cannot be made to look alike. In such cases the relative luminous intensities may be approximately found by the flicker photometer. In this instrument light from the source B falls on a white screen F, fixed at an angle of 45° in front of the eye tube through which it is observed. Light from the source A falls on a second white screen which also makes an angle of 45° with the eye tube and is rotated slowly about the axis shown in the figure.

This screen is made with projecting sectors which, as it rotates, come between the eye tube and the fixed screen F, so that the observer sees during one-quarter of a revolution only the rotating sector illuminated by the light A, while in the next quarter revolution the fixed screen illuminated by B is exposed.

Thus the two screens are alternately exposed to view for equal times, and by careful adjustment the speed of rotation is made such that a very disagreeable flickering effect is noticed unless the illuminations due to A and B are of equal intensity. The distances of A or B are varied until the flickering is a minimum, when the intensities of the two lights are proportional to the squares of their distances from the screens.

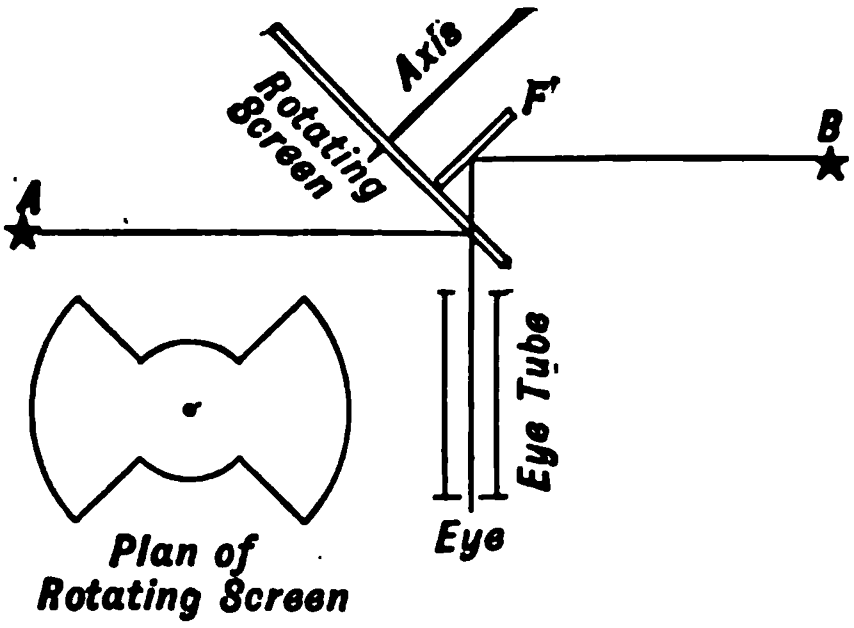


FIG. 478.

802. Standard of Light Intensity.—Many standards of light have been proposed for commercial and scientific purposes, but none are altogether satisfactory.

The English Electrical Standards Committee has defined one candle-power as one-tenth the candle-power under standard conditions of a particular pentane lamp kept in the National Physics Laboratory. The standards of light used by our Bureau of Standards are certain incandescent lamps which have been compared directly or indirectly with the English, French and German standards.

The English standard candle formerly used was a spermaceti candle made to burn 120 grains per minute with a flame height of 45 mm.

803. Illumination.—The illumination of a surface is measured in foot-candles, one foot-candle being the illumination produced by a 1-candle-power lamp at a distance of 1 ft. or by a 16-candle-power lamp at a distance of 4 ft.

Some Values of Illumination

Good illumination for reading............	4 foot-candles.
Poor illumination for reading.............	1–2 foot-candles.
Full moonlight.........................	.02 foot-candles.

Velocity of Light

804. Early Experiment.—It has been seen that light seems to pass from the source to the eye in straight lines. This at once suggests inquiry whether or not the eye *instantly* experiences the sensation of light when a candle is uncovered.

The earliest attempt to solve this question was made by the Florentine Academy after a method proposed by Galileo. A light on an eminence was uncovered and flashed to a station on a distant hill where a second observer also having a covered light was watching. As soon as the flash was seen by the second observer he uncovered his light, sending an answering flash back to the first station. The first observer was to note the exact time between the uncovering of his light and the sight of the return flash. The experiment showed that if any time at all was required for light to travel from one station to the other it was too short to be detected by that method.

805. Roemer's Discovery.—The first evidence that light required an appreciable time to pass from one point to another

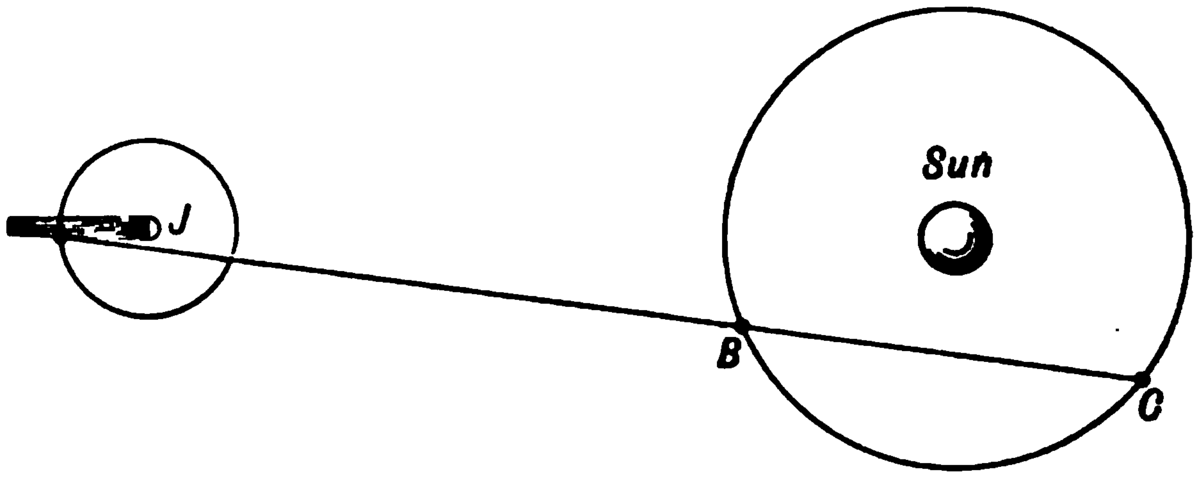


Fig. 479.

was obtained by the Danish astronomer Roemer, in 1676, by the following method:

The first satellite of Jupiter passes into the planet's shadow and disappears or is eclipsed every time it revolves around the planet. Some years before Roemer's discovery Cassini had carefully determined the periodic time of the satellite and had prepared tables showing when the eclipses might be expected to take place for several years ahead. On comparing these tables with the recorded times of observed eclipses Roemer found that they were observed sooner than predicted when the *earth* was on the side of its orbit nearest to Jupiter, and later

[redacted]

han predicted when it was on the opposite side. He concluded hat the discrepancy was due to the velocity of light; for evi- ently if it takes 10 minutes for light to cross the earth's orbit rom B to C, then an eclipse would be seen 10 minutes later if the arth were at C than if it were at B. The observations indicated hat light requires 16 minutes to cross the whole of the earth's rbit, or approximately 8 minutes to go from the sun to the earth r, more exactly, 498 seconds to traverse the 92,900,000 miles etween sun and earth, making the velocity of light in inter- planetary space 186,600 miles or 300,200 kilometers per second.

806. Bradley's Discovery.—No further evidence of the veloc- ity of light was obtained until 1727 when the English astronomer

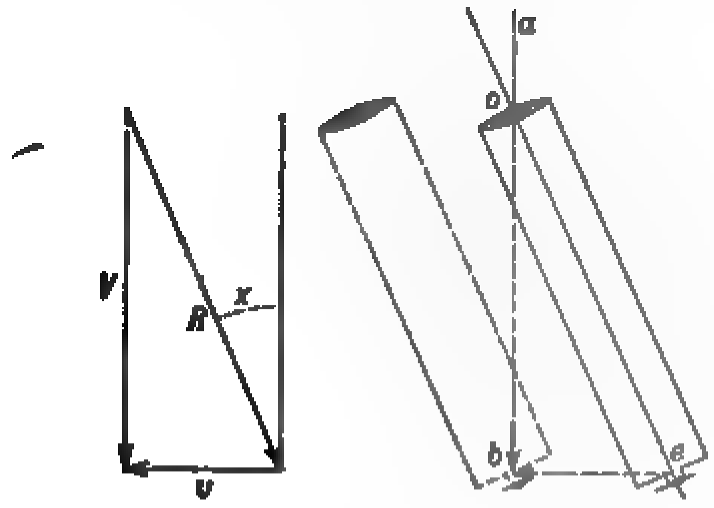


FIG. 480.

Bradley discovered that **the stars in any given part of the heavens were apparently displaced from their mean positions by an exceedingly small amount which depended on the position of the earth in its orbit.** The explanation of this phenomenon, which is known as *aberration*, was finally suggested to him by the observation that the position of a flag on a small boat de- pended on the velocity and direction of motion of the boat as well as on the wind. He said to himself that the apparent direction in which light comes to the earth from a star must be affected by the velocity of the earth, just as the apparent direc- tion of a breeze to a man in a boat depends on the motion of the boat.

For suppose light coming from a star in the direction *ao* (Fig. 480) enters at *o* a telescope which is being carried along side- wise in the direction *eb*, and that the light advancing in the

direction *ob* reaches *b* at the same instant that the eye-piece reaches there as it moves from *e* to *b*. The light will be received by the eye and the telescope will seem to be pointing at the star. To accomplish this the telescope evidently must incline forward so that $ob : eb :: V : v$ where V is the velocity of light and v is the component, perpendicular to the star's direction, of the velocity with which the telescope is carried along by the earth, and the apparent direction of the star differs from its true direction by the angle x, such that

$$\textit{tangent } x = \frac{v}{V}$$

When the earth is moving directly toward or away from a star there is no displacement or aberration, while stars in directions at right angles to that in which the earth is moving have maximum displacement. The apparent position of a star therefore changes slightly as the earth moves from one part of its orbit to another, so that by careful determinations of its apparent position made during an entire year the maximum displacement or *aberration constant* may be determined.

Recent observations give as the aberration constant 20.492″. Now, the mean velocity of the earth in its orbit is 18.51 miles per second, and we may calculate the velocity of light V from the relation

$$\tan\,(20.492'') = \frac{18.51}{V}$$

which gives 186,400 miles or 299,930 kilometers per second.

807. Fizeau's Method.—On account of the enormous velocity of light it was not until 1849 that a method of measuring it was devised which did not involve astronomical measurements. In that year the determination was made by Fizeau by the following method. A telescope and collimator were set up 8.633 kilometers (more than 5 miles) apart. A beam of sunlight L (Fig. 481) sent through an opening in the side of the telescope was reflected by a small oblique plate of glass G so that it passed directly out through the lens of the telescope to the distant collimator which was provided with a mirror M at its back. The collimator and mirror were so adjusted that the beam of light was reflected directly back into the telescope again, and, passing

through the plate of glass G was received by the eye at E. Thus light came to the eye at E after traveling to the distant mirror M and back again. At S in the telescope is a small opening which is alternately opened and closed by the teeth of a cogged wheel W which revolves immediately in front of it. If the wheel is slowly rotated, light from L passing through a gap between two teeth travels to the distant mirror and back again through the same opening to the eye at E. If the notches and teeth in the

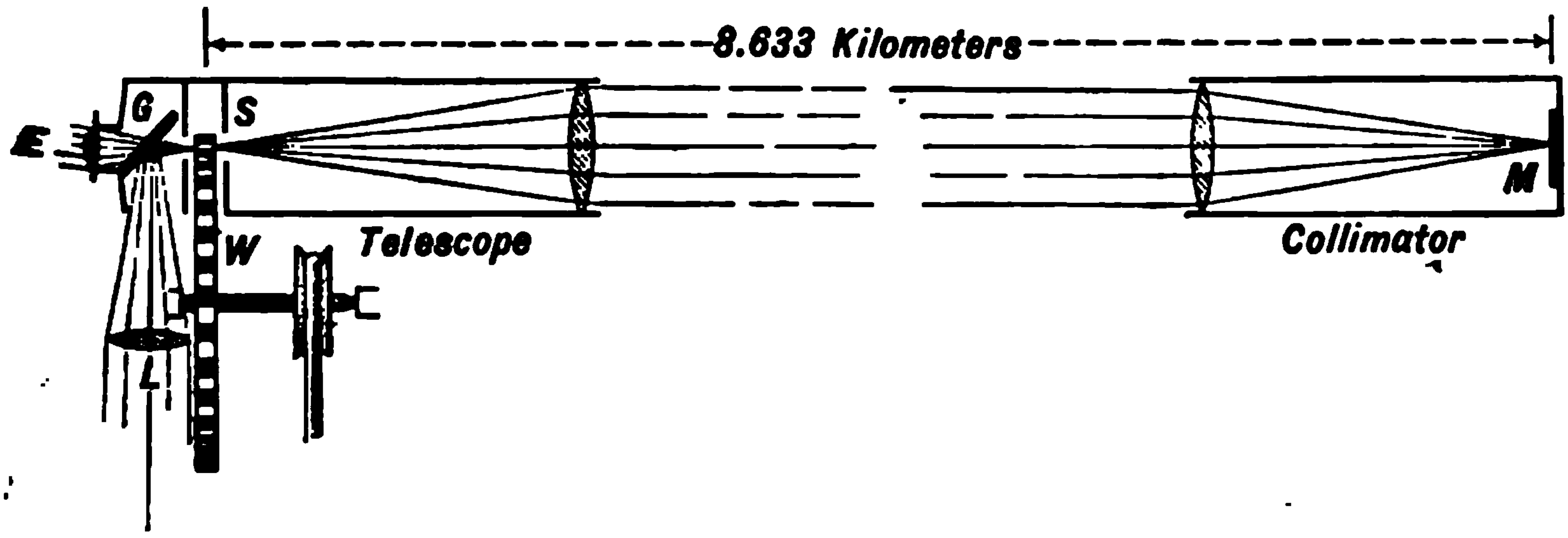


FIG. 481.—Fizeau's apparatus for measuring the velocity of light.

wheel are of equal width and if the speed of rotation is such that a tooth moves forward just its own width in the time that light requires to go to the distant station and back, light which must have passed out through an opening will on returning find the opening closed by a tooth and will therefore be cut off from the observer at E. If the speed is then doubled, light passing out through one opening will return through the next one; at a still higher speed it will be eclipsed again, etc. It is therefore only necessary to observe the speeds at which the light is completely eclipsed to be able to determine the velocity of light, when the distance between the two stations and the number of teeth in the wheel are known. In Fizeau's apparatus there were 720 teeth in the wheel and the first eclipse was noticed when the wheel made 12.6 revolutions per second. Therefore the time required for light to travel twice the distance between the two stations was only

$$\frac{1}{720 \times 12.6 \times 2} = \frac{1}{18143}\text{ sec.}$$

The stations were 8.633 kilometers apart, making the velocity of light 313,000 *kilometers per* second. The same method carried

out by Cornu in 1874 with improved apparatus gave $V = 304{,}000$ kilometers per second.

808. Foucault's Method.—Another method of measuring the velocity of light was devised and carried out by the French physicist Foucault in 1850. The essential features of the apparatus are shown in the diagram, figure 482. A beam of sunlight concentrated on the narrow slit S passes through it and through the inclined plate of glass G and the lens L to a small mirror m, from which it is reflected to a concave mirror M whose center of curvature is exactly at the center of the mirror m. The light is

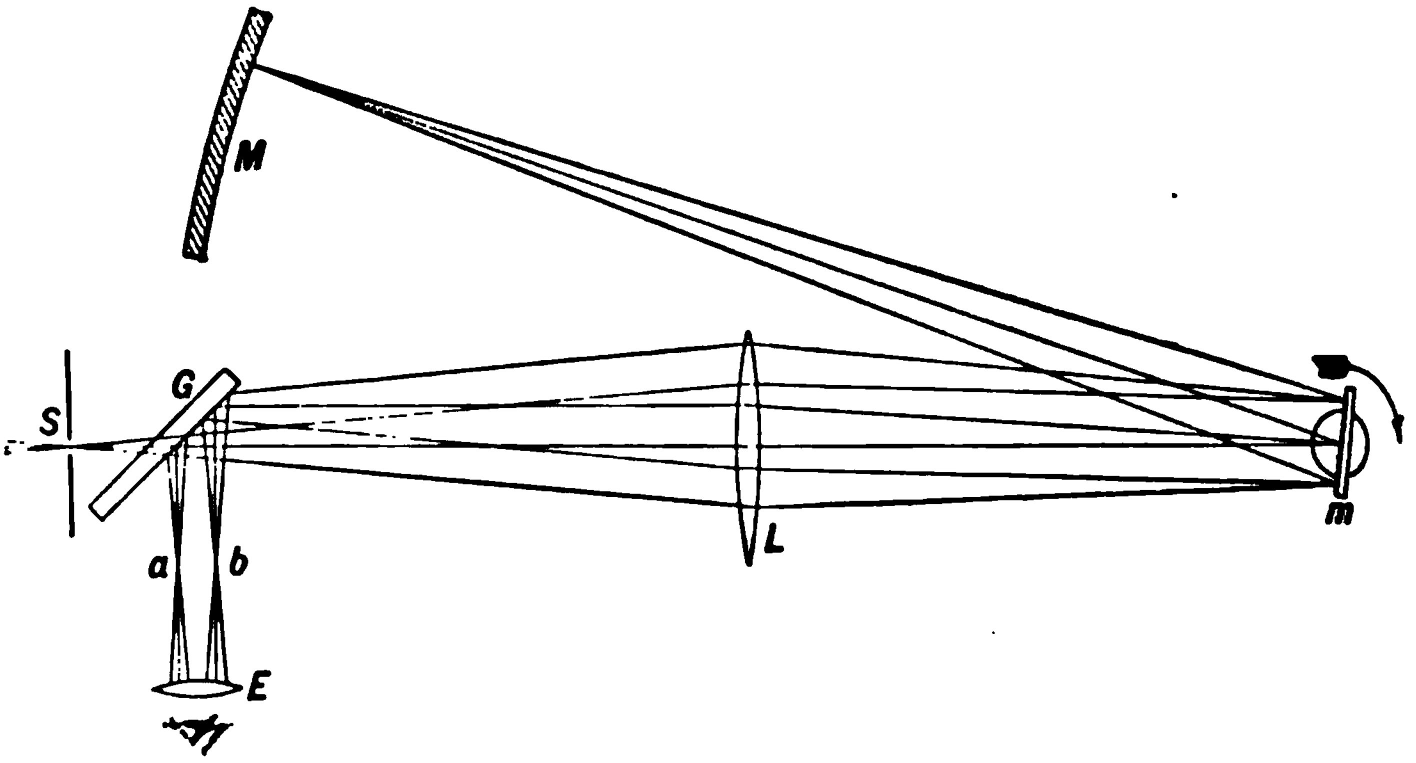


Fig. 482.—Velocity of light measured by rotating mirror.

reflected perpendicularly back from M to m and thence back to the glass plate G, which reflects it aside into the eye-piece E. A bright image of the slit S is formed at a by means of the lens L, and this is seen by the observer at E. If the mirror m is now slowly rotated in the direction of the arrow, the image of the slit will be formed at a only when m is in such a position, as it revolves, that the beam of light reflected from it meets the concave mirror M; consequently the image at a will disappear and reappear once in each revolution; but as the speed is increased to more than about 10 revolutions per second the eye no longer detects the intermittence, but sees a continuous image of the slit. As the speed of rotation increases, it is noticed that the image of the slit is no longer at a, but is displaced toward b, the amount of the displacement being proportional to the speed

rotation of the mirror. This displacement is due to the fact iat while light is traveling from m to M and back again, the iirror has turned forward through a small angle, and conseuently the returning light is reflected slightly upward, as shown ı the above figure, and not back along its original path, causing ıe image of the slit to be displaced to b. The displacement ab ıay be measured by a micrometer, and from it the angle through rhich the returning beam is turned upward may be determined. But this angle will be twice the angle through which the mirror urns while light is traveling from m to M and back (§820). All hat remains therefore is to determine the distance mM and the peed of revolution of the mirror; the velocity of light may then be asily calculated.

In Foucault's experiment the distance mM was only 4.12 neters, and the greatest displacement ab was about 0.3 mm. when the rotating mirror was making 800 turns per second, a dis)lacement too small to give a very accurate result. But he tried he very important experiment of introducing a long tube of water between m and M through which the light was sent, and was able to show that *the velocity of light in water was less than in air*, a result of the greatest significance in determining the nature of light (§842).

809. Michelson's Modification.—In 1879 Michelson, then at the United States Naval Academy, modified Foucault's method oy substituting for the concave mirror M a lens through which he light passed to a distant mirror where it was reflected ack. In his first experiments the distance from the revolving nirror to the fixed mirror was 605 meters. This great increase n the distance between the mirrors caused a correspondingly reater displacement which could be measured with far less ercentage of error. His experiments in 1879–82 and those onducted according to his method by Newcomb in 1882 are he most accurate determinations of this important constant hat have been made.

In some of Michelson's experiments the displacement to be ıeasured by the micrometer was 13.3 cm., or 400 times that btained by Foucault.

The results obtained by this improved method are as follows:

Observer	Kilometers per sec.	Miles per second
Michelson, 1879...........	299,910	186,380
Michelson, 1882...........	299,853	186,345
Newcomb, 1882...........	299,810	186,317

810. Velocity Same for All Colors.—In these various determinations of the velocity of light, the light employed was either sunlight, starlight, or light from the electric arc. But although these lights are complex, there was not found in any case a perceptible difference in velocity between light of different colors, when measured in air or in interplanetary space.

Problems

1. How far from a screen must an 8 candle-power lamp be placed to give the same illumination as a 16 candle-power lamp 10 ft. distant?
2. When a photometer screen is equally illuminated by a 32 candle-power lamp at a distance of 2 meters, and an arc lamp 12 meters away, what is the candle-power of the arc light?
3. A rifle bullet has a speed of 2000 ft. per sec. How many inches will it advance while light travels a mile, and how far while light travels 25,000 miles (the circumference of the earth)?

Wave Theory

811. Mode of Propagation.—Only three methods are known by which energy may be transmitted from one point of space to another. *First, by the movement as a whole of some medium reaching from one point to the other*, as in the case of ropes, belts, or shafting.

Second, by projectiles, as in the case of a shot from a gun or a ball thrown.

Third, by waves, as in case of sound or water waves.

We have found that light is communicated from one point to another with a velocity of about 300,000,000 meters or 186,400 miles per second. By which of the above processes is it propagated? We may evidently reject the first as inconceivable. The second was advocated by so great a philosopher as Sir Isaac Newton, while Huygens, the celebrated Dutch physicist, urged

the claims of the third. While much of the most convincing evidence will be found in phenomena that must be taken up later in our study, there are some considerations which even at this point may help us to reach a tentative conclusion.

The velocity with which a projectile travels depends on the initial impulse. If light is communicated by means of particles shot out from the luminous body we should expect to find the velocity depending on the source and that particles emanating from the sun would have a different velocity from those from an electric light.

On the other hand, **the velocity of a wave depends only on its wave length and the nature of the wave (whether compressional or transverse, etc.) and the properties of the medium of which it is a disturbance.** Sound waves from fiddle, pipe, or drum advance with the same speed through air. If light is a wave motion we may expect to find light waves, whatever their source, traveling with the same velocity through space, and this is precisely what experiment shows to be the case. This consideration therefore points to its being a wave motion.

812. The Ether.—On the other hand, if light is propagated by waves, they are waves of what? Light passes through interstellar space and through the most perfect artificial vacua that can be produced. If there are light waves, they must be in some medium which extends throughout space as far as the most distant star from which we receive light, it must fill all vacua and permeate all bodies through which light can pass. And yet no resistance to the motion of earth or planets through this medium has ever been detected. Yet in spite of these objections such a medium must be supposed to exist if light is communicated by waves, and it has been named **the luminiferous ether** or simply **the ether.**

813. Other Evidence for the Ether.—It is remarkable that there is independent evidence for the existence of such a medium obtained from the study of electricity and magnetism. Electric and magnetic forces act through vacua, and may be produced as Faraday supposed by tensions and pressures in a surrounding medium. When a magnet draws a piece of iron to itself we may in imagination see it pushed up to the magnet by the stresses in the ether.

But far more important is the direct evidence of Hertz' experiments; for **electric waves have been proved to exist and are found to have the same velocity as light.** *There is, therefore, a medium in which waves may exist and travel with the velocity of light.*

814. Electromagnetic Theory.—And since the velocity of a wave depends both on the properties of the medium and on the kind of wave motion, it is highly probable that the vibrations in light waves are exactly the same as in electric waves; or, in other words, that **light waves are electric waves.**

This theory of the nature of light waves is known as the *electromagnetic theory of light*, it was proposed and developed by Maxwell in 1865. Its conception and establishment, next to that of the conservation of energy, is the most remarkable achievement of physical science in the nineteenth century.

In our further study we shall endeavor to test the probability of the hypothesis that light is a wave motion by inquiring whether it affords simple and natural explanations of the various phenomena as they arise.

815. Form of Light Waves.—If light comes from a source as a series of waves, the form of a wave, as it advances in all directions with equal velocity, must be spherical, and the direction of advance being radial is at right angles to the wave front.

816. Beams and Rays.—When light shines through a small opening, the stream of light is called a *beam*, and a very narrow beam is called a *ray*. When the beam comes from a very distant source, the rays of which it may be conceived as made up are parallel, and it is called a parallel beam; in that case the wave fronts are planes.

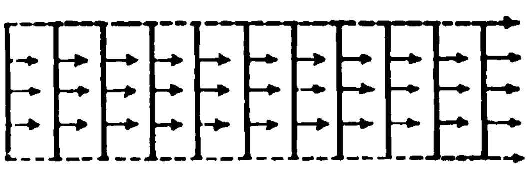


FIG. 483.—Parallel beam with plane waves.

When light comes from a point, the rays diverge radially from the source and the wave fronts are spherical segments having the source as their center. Such a beam is divergent, and its waves enlarge as they advance.

By means of a lens or curved mirror, a beam of light may be made to converge toward a point which is called the focus, in which case the wave fronts must be concave spherical surfaces which contract as they approach the focus.

817. Geometrical and Physical Optics.—The study of light is also called *optics*. That method of treating the subject which ignores the existence of waves and treats a beam of light as a bundle of rays is called *geometrical* optics, while the other method which investigates the dependence of the various phenomena of light on the properties of waves is known as *physical* optics.

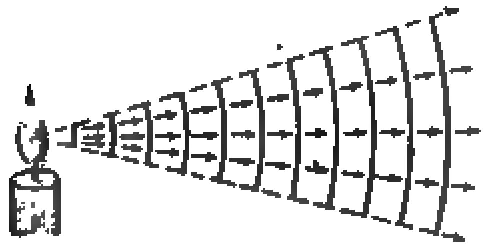


Fig. 484.—Divergent beam with convex expanding waves.

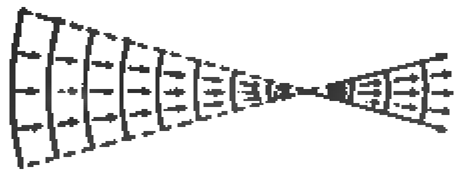


Fig. 485.—Convergent beam with concave contracting waves.

Reflection of Light and Mirrors

818. Reflection: Regular and Diffuse.—When light reaches a surface where there is a change of medium, some is reflected or turned back into the first medium while some penetrates into the second medium.

When the reflection takes place at a flat polished surface, light comes to the eye as though directly from the distant objects themselves, and if the polish is perfect none of the light seems to come from the reflecting surface, but we seem to be looking through an opening at objects beyond. This is known as **regular reflection.**

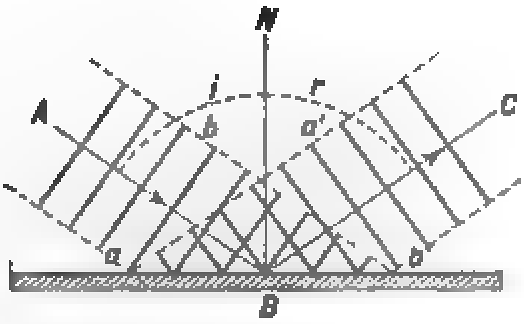


Fig. 486.—Regular reflection of waves.

If the surface is now ground with coarse emery, we no longer see reflected objects, but light goes out from the surface itself in all directions as though it were a source of light. This is known as **diffuse reflection.** It takes place at the surface of such bodies as wood, paper, cloth, etc., and seems to be due to the breaking up and scattering of light waves by the roughness or irregularity of the reflecting surface. To polish a surface so that it reflects like a mirror the very finest emery and polishing rouge must be used, a fact which indicates that the length of

light waves must be extremely small. In some conjurers' illusions advantage is taken of the invisibility of a well-polished mirror surface.

819. Regular Reflection.—The law of regular reflection is the same for light waves as for other forms of wave motion. If AB is the incident ray and BC the reflected one, the angles i and r which they make with the normal BN are called the *angles* of *incidence* and *reflection,* respectively. In case of regular reflection, *the angles of incidence and reflection are equal and lie in the same plane.* This plane is called the *plane of incidence.*

It will be observed that by reflection a wave front such as ab is turned into the position $a'b'$.

820. Mirror Turned Through an Angle.—If a beam of light SO (Fig. 487) meets the mirror perpendicularly, it is reflected directly back on its path. But if the mirror is turned through the angle x, the reflected beam will take the direction OP where the angle of reflection y is equal to the angle of incidence x. By the motion of the mirror through the angle x, the reflected beam has therefore been turned through an angle $x + y = 2x$.

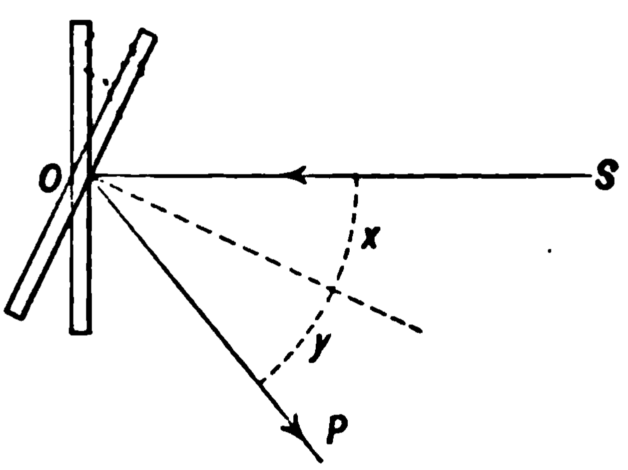


Fig. 487.

Suppose the mirror is attached to the needle of a galvanometer and reflects the light from an incandescent lamp upon a graduated scale. It is clear from the above that when the needle turns through a small angle, the reflected beam of light must move through twice that angle.

821. Plane Mirror and Images.—If a source of light is placed at O in front of a plane mirror MM', figure 488, the light after reflection will appear to come from a point O' as far behind the mirror as O is in front of it. (See also §292.) For trace the incident and reflected rays OB and BC, and produce the latter backward meeting the perpendicular OP at O'. Then the triangles OPB and $O'PB$ have right angles at P, and the side PB common, and the angles PBO and PBO' are equal because of the law of reflection, therefore $PO' = PO$. But B is *any* point *in the* reflecting surface, therefore all reflected rays from O if

produced backward will pass through the point O'. Light waves from O after reflection from the mirror come to the eye as if they had come from O', and consequently *O' is said to be the image of O. It is a virtual image as distinguished from a real one, because the light does not actually pass through the point O'.*

If an object represented by the arrow OE is placed in front of a plane mirror the image is virtual and in the position $O'E'$, each point in the image being as far behind the mirror as the corresponding point in the object is in front of it.

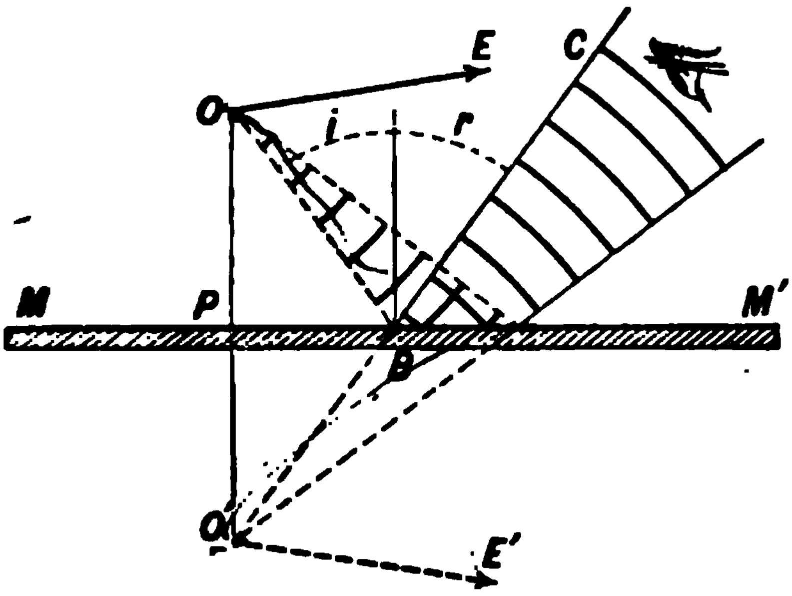


FIG. 488.—Image by plane mirror.

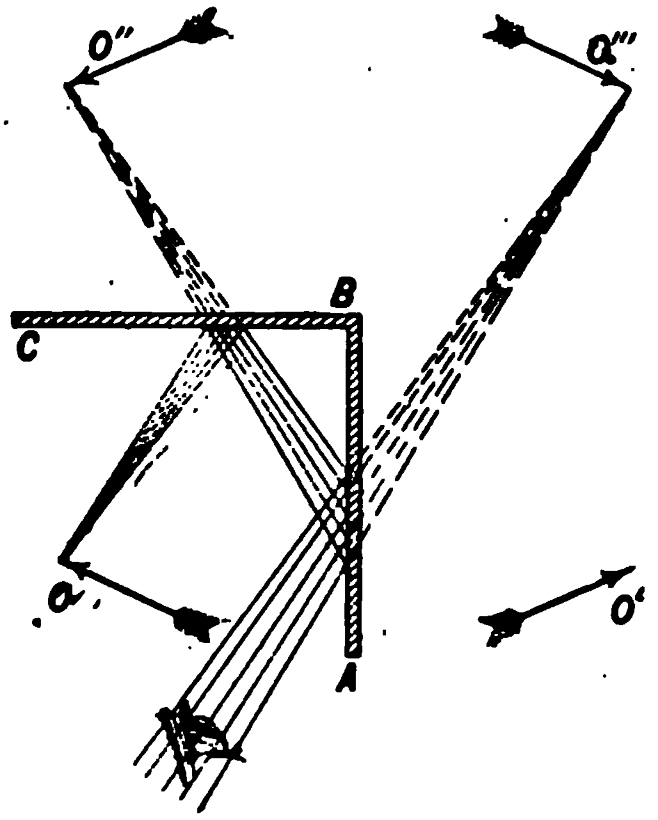


FIG. 489.

822. Multiple Reflection.—When two plane mirrors are placed at right angles to each other as in figure 489, three images are formed of an object O placed between them. O' is the image of O in AB, O'' is the image of O in BC, while O''' is the image of O' in BC or of O'' in AB. The rays which come to the eye as if from O''' are reflected twice, once by each mirror.

In the kaleidoscope three narrow strips of mirror glass are placed edge to edge, forming a triangular prism with the mirror faces turned inward. An observer looking in at one end of the contrivance sees a regular hexagonal pattern formed by the repetition of some figure formed by pieces of colored glass at the other end of the tube.

If two flat mirrors are placed parallel and facing each other, an observer standing between them with a lighted candle will see an infinite series of images of the candle stretching into the distance in each mirror. The first image is formed by a single reflection, the second by light reflected twice, once in each mirror, etc.

823. Concave Mirror.—The surface of a concave mirror is commonly a portion of a sphere, because spherical surfaces are ground and polished with comparative ease.

If such a mirror is held facing the sun, a bright spot of light, which is the image of the sun, is formed half-way between the mirror and its center of curvature. The angular size of the focal image as seen from the mirror is the same as that of the sun itself, so that the shorter the radius of curvature of the mirror, the smaller this image is. The point where the image is formed is called the *principal focus* of the mirror (Lat. *focus*, a hearth). With a large mirror of short focal length so great a concentration of the sun's rays may be obtained that lead may be melted and paper and wood ignited at the focus.

If a candle is held at A, between the center of curvature of the mirror and its principal focus, a real image of the flame will

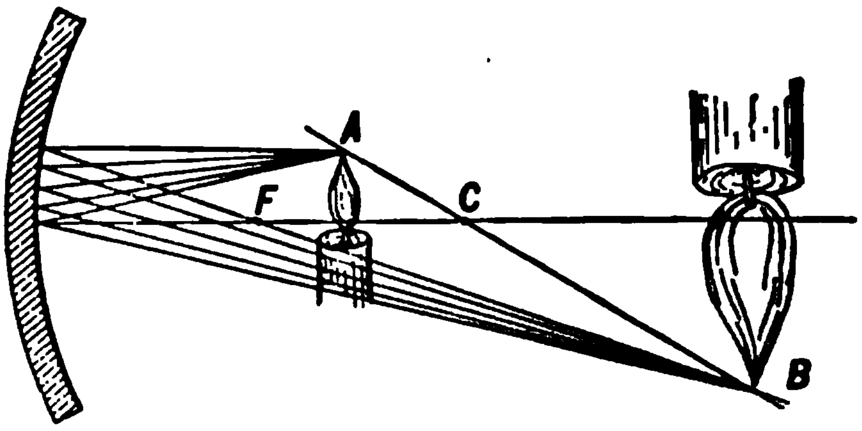


FIG. 490.—Image by concave mirror.

be formed at a certain point B beyond C. This image, being real, may be seen on a white screen placed there. It is inverted and as much larger than the object at A as it is farther from the mirror.

When the candle is moved away from the mirror, the image moves toward it and they meet at C where the image is of the same size as the object and still real and inverted. As the candle is moved still farther from the mirror, the image approaches F *as a limit* and when the distance of the candle is many times the radius of curvature of the mirror, the image is formed almost exactly at F, the size of the image being smaller the farther off the candle is placed.

824. Conjugate Foci.—The positions of candle and image, A and B, are interchangeable,—the candle may be placed either at A or B and the image will be formed at the other point. *Two points so related that one is the image of the other are known as conjugate foci.* The *principal focus* is conjugate to a point on the axis infinitely distant from the mirror.

825. Principal Focus.—The principal focus of a mirror may be defined as **that point where all rays parallel to the axis meet after reflection,** ***it is half-way between the mirror and its center of curvature.*** This may be proved as follows:

Let C be the center of curvature of the mirror, and MC its axis. A ray OP parallel to the axis MC and meeting the mirror at P will be reflected into the direction PF, such that the angle of reflection r is equal to the angle of incidence i. But since OP is parallel to the axis, the angles i and x are equal, and consequently $r = x$, and the triangle FPC is isosceles and $PF = FC$. If the point P is not too far from M, PF and MF are very nearly equal, so that $MF = FC$, and F is therefore half-way between M and C.

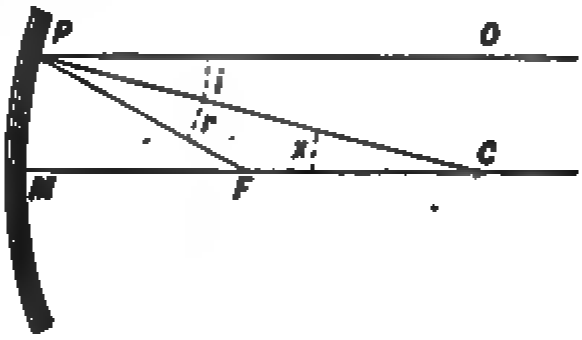


Fig. 491.

It is clear from the above that *all* rays parallel to the axis of a concave spherical mirror do not meet exactly at the same point after reflection. This imperfection is known as aberration. When a concave mirror is only a very small portion of a sphere this aberration is slight.

826. Construction of Image.—The size and position of the image which a concave mirror forms of an object in front of it may be determined by the following construction:

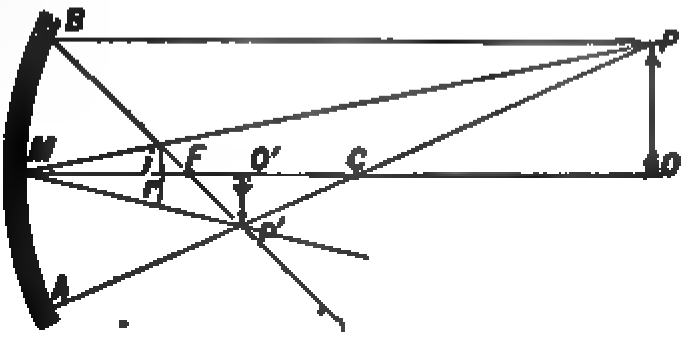


Fig. 492.

Suppose it is required to find the image of the arrow PO (Fig. 492). **Trace two rays from P, and the point where they intersect after reflection is the image of P.** One ray easily traced is PA through the center of curvature C. This ray meets the mirror perpendicularly and is reflected back along the same line ACP. Another ray to be taken is PB, which is parallel to the

axis and is, therefore, reflected back through the principal focus F, and intersects the first ray at P'. This point is therefore the image of P. The line PCP' through the center of curvature is known as the *secondary axis* through P.

The image of the point O on the axis will be at O', also on the axis, so that $P'O'$ will be the image of the arrow PO. This image is evidently *inverted*, it is also *real*, for rays of light from various points in the object PO *actually pass through* the corresponding points in the image $P'O'$.

827. Size of Image.—The size of the image is to the size of the object as their distances from the mirror. For if we draw the rays PM and MP' reflected at M, the angles i and r are equal by the law of reflection, hence the triangles POM and $P'O'M$ are similar and $PO{:}P'O'{::}OM{:}O'M$.

It is also evident from the construction that the size of object and image are proportional to their distances from the center of curvature C.

828. Virtual Image.—When the object is moved nearer to the mirror than the principal focus F, an *erect virtual* image is

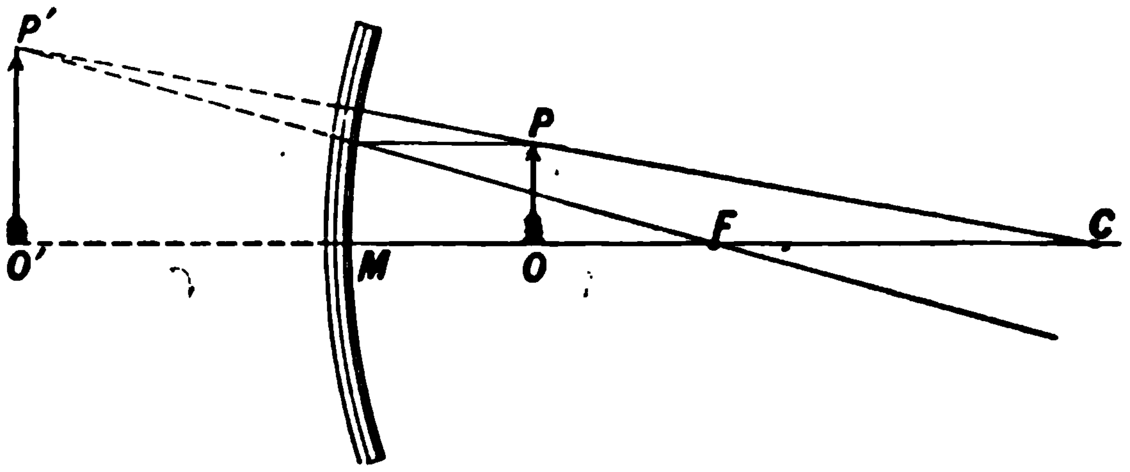


FIG. 493.—Virtual image by concave mirror.

formed back of the mirror, as will be clear from the following construction. Trace as before two rays from P, one parallel to the axis and reflected through F, the other perpendicular to the mirror and reflected through C; they will *diverge* after reflection and must be produced backward to find the point of intersection P'. This is the image of P, and is *virtual* because the light from P does not actually pass through P'. The sizes of object and image are proportional to their distances from C, hence the virtual image is larger than the object. As the object is moved toward the mirror the image also approaches it and they meet at M.

839. Formula for Concave Mirror.—A simple formula which expresses the relation between the radius of curvature of a mirror and the distances from it of two conjugate foci, may be obtained as follows: By similar triangles

$$OP : O'P' :: OC : O'C$$

also

$$OP : O'P' :: OM : O'M$$

therefore

$$OC : O'C :: OM : O'M \tag{1}$$

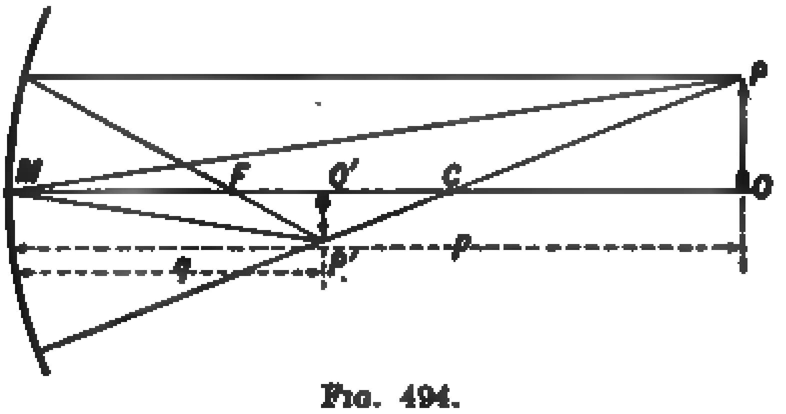


Fig. 494.

Let p and q be the distances from the mirror of O and O' respectively, and let r be the radius of curvature of the mirror, then $OC = p - r$, $O'C = r - q$, $OM = p$, $O'M = q$, and we have by substituting in (1)

$$p - r : r - q :: p : q$$

and multiplying means and extremes

$$pr - pq = pq - qr$$

dividing through by pqr, we obtain *the mirror formula,*

$$\frac{1}{p} + \frac{1}{q} = \frac{2}{r} \tag{2}$$

When O is at a great distance from the mirror, or p is infinitely great, we have

$$\frac{1}{p} = 0, \qquad \text{and therefore} \qquad q = \frac{r}{2}.$$

The point O' is in that case at F, half-way between M and C, a result which we have already obtained in §825.

When p is less than $\frac{r}{2}$, a negative value of q is obtained, showing that in that case the image is formed *back of the mirror* or is *virtual*.

830. Illustrations.—If a vase mounted on an open box in which a bouquet of flowers brightly illuminated is hung upside down, is set in front of a concave mirror at the distance of its center of curvature, and if the mirror is properly inclined, a real

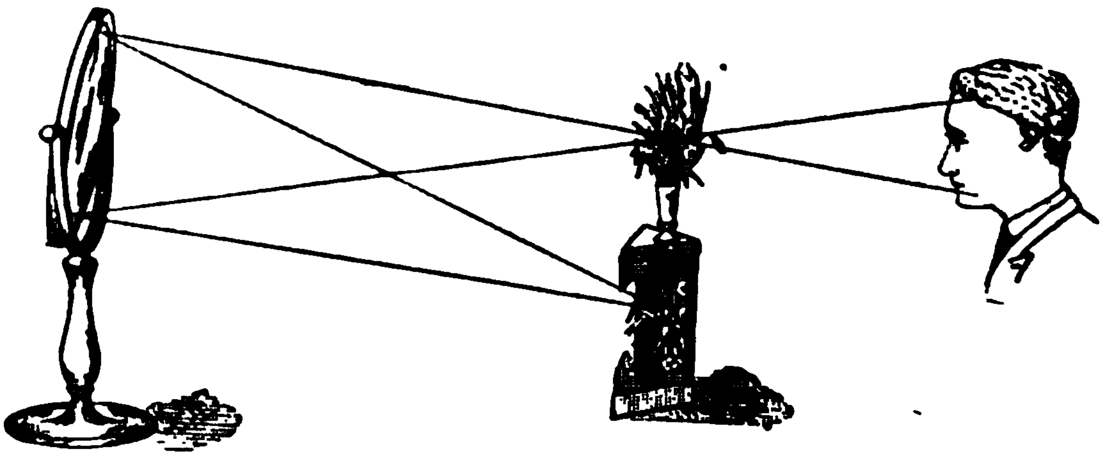


FIG. 495.

image of the bouquet will be formed exactly over the vase, so that to an observer looking over the vase into the mirror the vase appears to hold the flowers. Here object and image are of the same size since equally distant from the mirror.

Standing back of the center of curvature of a concave mirror and looking into it, an inverted and diminished reflection of the face is seen; if, however, the face is held within less than the focal distance MF, the image is virtual, erect, and enlarged, and we have a magnifying mirror.

831. Convex Mirror.—In case of a spherical *convex* mirror, the formula obtained in §829 applies if the radius of curvature is taken negative. Thus for convex mirrors

$$\frac{1}{p}+\frac{1}{q}=-\frac{2}{r}$$

expresses the relation between the distances of image and object from the mirror, and its radius of curvature r.

It will be observed that whenever p is positive, it will give *a negative* value of q, indicating that wherever *the object may be*

placed in front of the mirror, the image will be formed behind it; that is, it will be virtual.

The image may be constructed in size and position as before, by tracing from a point P in the object two rays, one which is parallel to the axis and therefore after reflection is directed away from the principal focus F, and another which is directed toward C and meets the mirror perpendicularly so that it is reflected back along the same line. These two rays diverge after

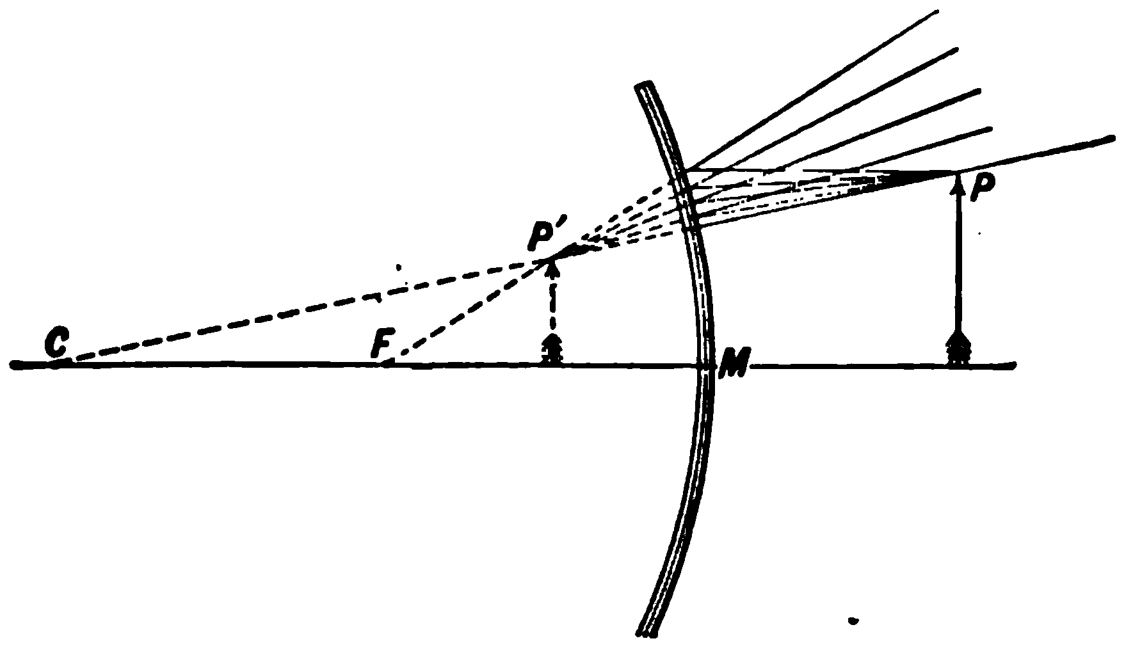


FIG. 496.

reflection and if produced intersect at P' where the virtual image is formed.

The relative sizes of image and object are proportional to their distances from C. **The image is therefore erect, virtual, and smaller than the object and nearer to the mirror than the object is.**

832. Illustrations.—When a polished ball is placed in direct sunlight, the brilliant spot of light seen in the ball is the virtual image of the sun, formed at the principal focus, half-way between the center of the ball and the surface. It is small, for it subtends an angle at the center of the ball equal only to the apparent angular diameter of the sun.

The reflected image of the face seen in a convex mirror is always virtual, erect, and diminished in size.

833. Perfect Mirror.—It is useful to consider from the point of view of the wave theory what form a mirror must have to reflect perfectly to a focus all the light that falls on it from a given point. If light waves going out from P as spherical waves

are to be converged to P', they must after reflection be spherical waves converging toward P'. That is, **all parts of the spherical wave which left P at a given instant must reach P' simultaneously,** and hence it must take light just as long to travel by the path PMP' as by any other path $PM'P'$ where M and M' are points

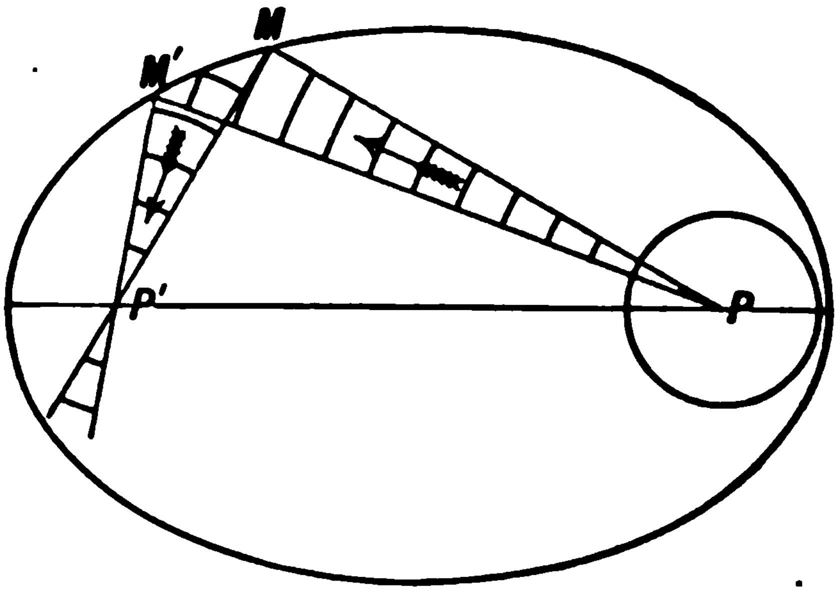


FIG. 497.

on the mirror, and therefore $PM + MP'$ must be equal to $PM' + M'P'$.

It is known that an ellipsoid of revolution having its foci at P and P' satisfies this condition, hence a perfect mirror should be a portion of the surface of such an ellipsoid. But even then it would be perfect only for light coming from one of its foci. Light from any other point would not be perfectly converged to a single point focus.

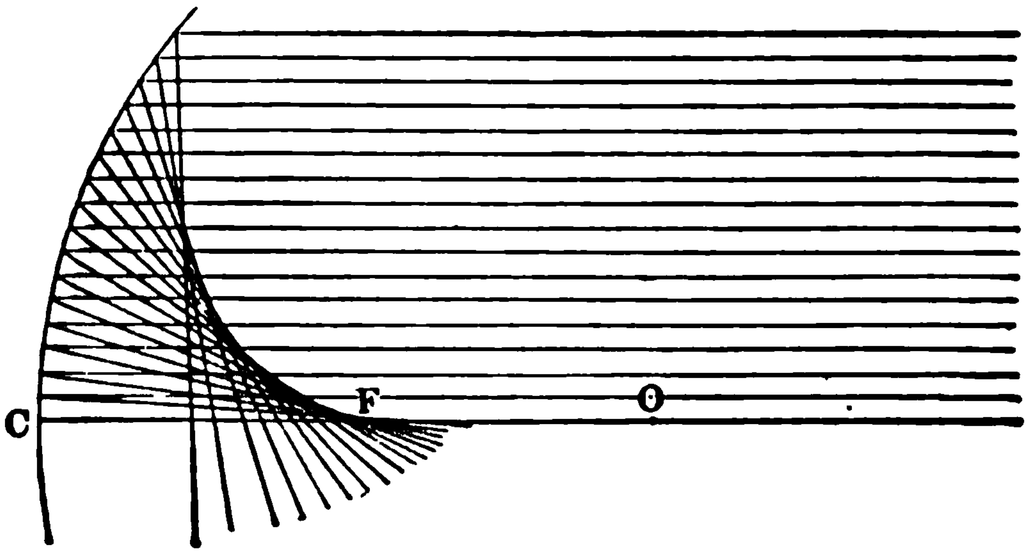


FIG. 498.—Caustic curve.

834. Aberration.—When light from a point in the object does not converge to a point in the image, there is said to be *aberration*. The nature of the aberration in case of a spherical mirror is well shown by reflecting a beam of parallel rays, or light from a distant object such as the sun, in a concave *cylindrical* mirror mounted over a sheet of white paper, as shown in figure

3. It will be observed that only the central rays are reflected through the :us F, those striking the mirror near the edge cross the axis decidedly to the t of F. The curve to which all the reflected rays are tangent is called a *ıstic*, and its cusp at F is the ordinary focus of the light reflected from the ıtral part of the mirror.

To avoid excessive aberration the diameter of spherical mirrors is ordirily small compared with their radius of curvature.

835. Parabolic Mirrors.—When it is desired to take a beam of light large angle and reflect it all in one direction, as in a searchlight, a parabolic rror is used. For it is a property of the parabola that a line joining any int P with its focus F and a line through P parallel to the axis make equal gles with a tangent at P, and hence a ray of ,ht parallel to the axis will be reflected to F ıd conversely all rays from F that meet the rface will be reflected parallel to the axis.

In searchlights and in locomotive headlights .e source of light is not a point but a lumi-ıus surface, and an image of this source is ·rmed by the mirror. The size of the image ırmed is to the size of the source as its dis-ınce from the mirror is to the focal length M. Therefore to illuminate a large region in ·ont of the reflector the source of light should e large and the focal length of the mirror small. Vhile to obtain a very intense beam illumi-ating only a small patch at a great distance ıe source should be small and intense and the ıcal length of the mirror large.

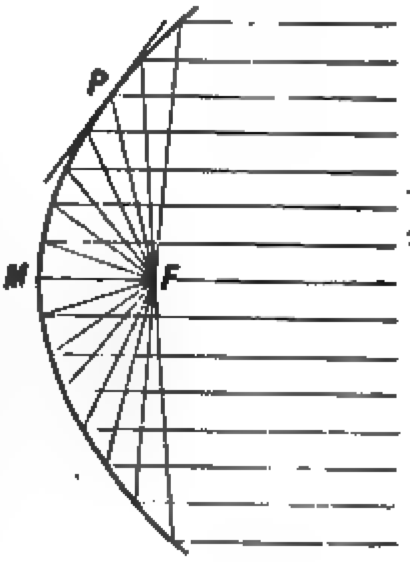


Fig. 499.
Parabolic mirror.

Problems

1. How high must a plane vertical mirror be in order that an observer 6 ft. in height standing in front of it may just see his whole figure?
2. What sort of mirror must be used and how placed that a ruler in front of the mirror and its image may form two sides of an equilateral triangle?
3. Make a construction showing the size and position of the image formed by a concave mirror having a radius of curvature of 3 in., of an object ½ in. long placed 4 in. in front of the mirror. Make full sized drawing.
4. Make a construction showing the size and position of the image formed by a convex mirror, the object being 4 in. in front of the mirror. Use the same radius of curvature and size of object as in the last problem.
5. A candle is placed 3 ft. in front of a concave mirror having a focal length of 1½ ft.; where is the image, and how large?
6. Where must an arc light be placed in front of a mirror having a radius of curvature of 6 ft. in order that its image may be focused on a screen 20 ft. from the mirror?
7. If a light is placed 2 ft. in front of a concave mirror having a radius of curvature of 6 ft., where will its image be, and how large?

8. How far must a man stand from a concave mirror having a focal length of 2 ft. in order that he may see an erect image of his face just twice its natural size?

1 9. Which would make the hottest image of the sun, a mirror with a focal length of 6 in. or one with 2 ft. focal length, supposing both to be of the same diameter? Why?

10. How big is the bright image formed when sunlight is reflected by a polished sphere 10 cm. in diameter and where is the image situated? Take the distance of the sun as approximately 110 times its diameter.

11. What sort of mirror must be used and what must be its focal length, in order that it may form an erect image 3/5 as large as an object placed 2 ft. in front of it? What kind would give an inverted image all other conditions being the same?

Refraction

836. Refraction.—When a beam of light passes obliquely from one medium into another, it is usually bent at the surface separating the two. This is known as *refraction*. It may be conveniently studied by the aid of the apparatus shown in figure 500. This consists of a circular glass vessel with flat sides and half-full of water into which a narrow beam of sunlight is directed in a darkened room. If smoke is blown into the space above the water and if the water is very slightly soapy or colored with fluorescein, the path of the beam may be distinctly traced both in the air and water. It is then observed that when the beam is sent vertically downward it is not bent, but when it is inclined it is sharply bent downward at the surface, and the bending is greater the more obliquely the beam meets the surface.

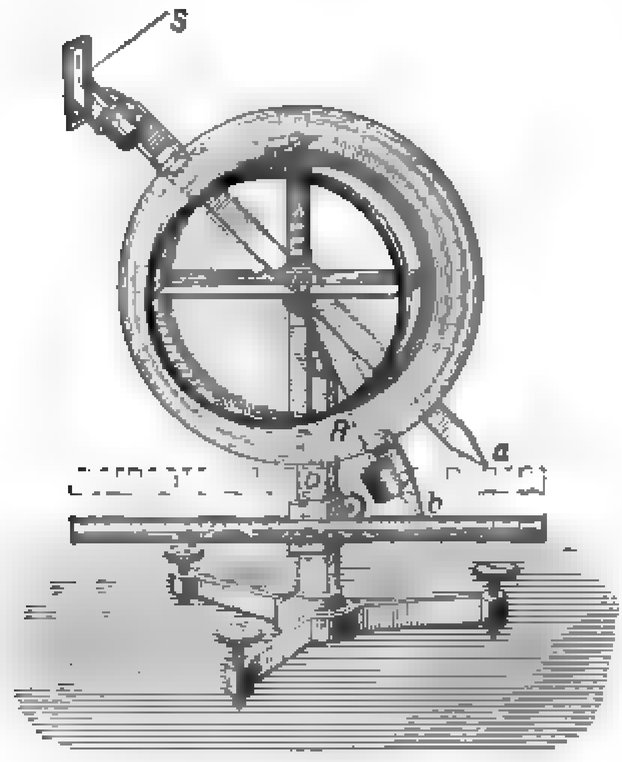


Fig. 500.—Refraction of light.

The bending also takes place when light passes from water to air, as in case of the coin in the dish shown in figure 501. The

coin C is out of sight of the observer's eye so long as the dish is empty, but on filling the dish with water, light coming from the coin is bent into the direction OE and comes to the eye as if from C', and the coin seems lifted into view.

In the same way, because of refraction, an oar appears bent upward where it enters the water; and a tank of water, to one looking down into it, looks shallower than it really is, and the more obliquely the bottom is seen the shallower the tank appears.

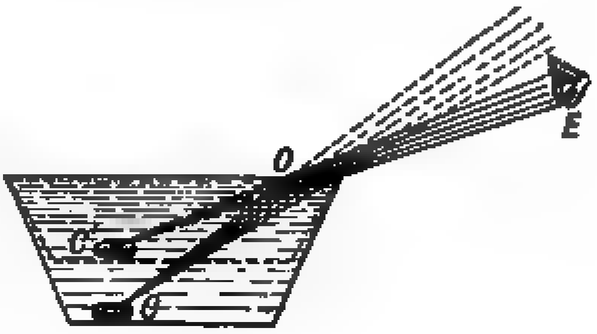


FIG. 501.—Coin in dish.

837. Law of Refraction.—The exact law of refraction was discovered by the Dutch physicist Snell about 1620, and may be thus stated:

When light passes from one isotropic medium into another, the ratio of the sine of the angle of incidence to the sine of the angle of refraction is constant for light of any given wave length, whatever may be the inclination of the incident beam, ***and the incident, reflected, and refracted rays are all in the same plane,*** called the plane of incidence, which is normal to the surface.

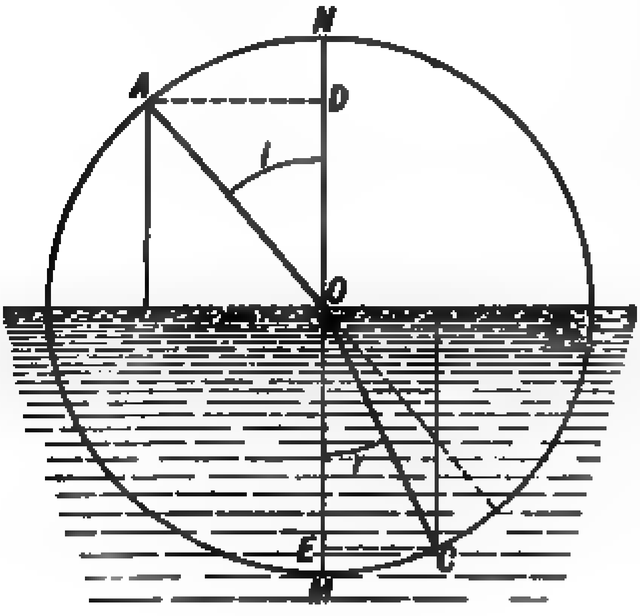


FIG. 502.

Thus in figure 502, AD and CE are proportional to the sines of the angles i and r, respectively, and the law states that whatever may be the direction of the incident ray AO, the refracted ray OC will be so inclined that AD will be to CE in a constant ratio which depends on the nature of the two media and on the kind of light. If the upper medium is air and the lower water, AD is very nearly $\frac{4}{3}$ of CE for yellow light, while in case of air and crown glass the ratio of AD to CE is more nearly $\frac{3}{2}$ for the same kind of light.

838. Index of Refraction.—This constant ratio of the sine of the angle of incidence to the sine of the angle of refraction is called the *relative index of refraction* of the two media concerned, and the more it differs from unity, the greater the bending of the ray in passing from one medium to the other.

The relative index of refraction when light passes from *air* into a substance is commonly called simply *the index of refraction* of the substance.

The *absolute index of refaction* of a substance is that which holds when light passes from vacuum into the substance; it differs from the ordinary index by only about one part in 3500. It may be determined by multiplying the index from air into the substance by the absolute index of refraction of air, which is 1.000292 at standard conditions.

Indices of Refraction of Some Common Substances for Sodium Light

Glass, very dense flint	1.71
Glass, light crown	1.51
Rock salt	1.54
Diamond	2.47
Water	1.33
Alcohol	1.36
Carbon bisulphide	1.64
Air	1.000292

839. Cause of Refraction.—The velocity of light in water was measured by Foucault and found to be about ¾ that in air; and later Michelson found the velocity of light in bisulphide of carbon to be still less than in water. In each case it was found that **the ratio of the velocity of light in air to that in the substance was equal to the index of refraction of the substance.** Let us now inquire whether the assumption that a beam of light consists of a train of waves which experience a change of velocity in passing from one medium into another will account for the above result, and also whether it affords a satisfactory explanation of the law of refraction as established by experiment. In the following paragraphs we shall trace the consequences of this assumption.

840. Perpendicular Incidence: No Change in Direction.—When a beam of light in air is perpendicular to the surface of another substance, as water, in which its velocity is less, the

[redacted]

wave fronts are parallel to the surface AB, and consequently all parts of a given wave front meet the surface AB at the same instant, and advancing into the lower medium with the same velocity everywhere, the wave front in the lower medium must remain parallel to the surface AB, and the ray direction remains unchanged.

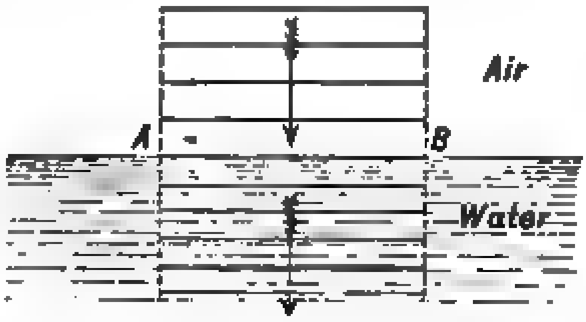


FIG. 503.—Perpendicular incidence of waves.

The wave length, however, in the lower medium must be less than in air in the same ratio as the velocity of light in the substance is less than its velocity in air. For it must advance one wave length in the substance in the same time that it advances one wave length in air, since just as many waves per second enter the lower medium as leave the air.

841. Oblique Incidence: Change in Velocity and Direction.—If the incident beam falls obliquely on the refracting surface then the change in velocity in passing from one medium to the other causes a bending of the ray or change in direction, as shown in figure 504.

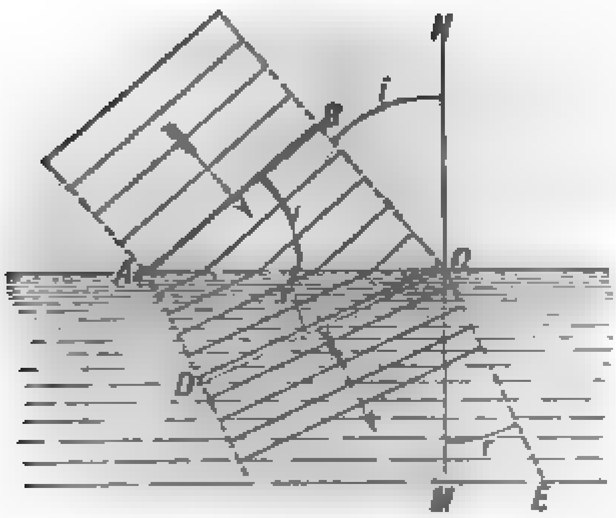


FIG. 504.—Refraction of oblique waves.

For, let the heavy lines represent wave fronts one wave length apart, advancing in the direction of the arrows, and let the second medium be one, such as glass, in which the velocity is less than in air. As soon as the edge of the wave enters the glass at A it is retarded, while that part which is still in air continues to advance with the same velocity as before. Consequently the direction of the wave front is changed into the position DC.

Now, BC is the distance that a wave travels in the upper medium in the same time that it travels a distance AD in the lower medium; therefore

$$BC : AD :: V : v$$

where V is the velocity of light in the upper medium and v its velocity in the lower one.

Let i be the angle of incidence BCN or BAC and let r be the angle of refraction ECM or ACD, then

$$BC = AC . \sin i$$
$$AD = AC . \sin r$$

hence dividing, we find

$$\frac{BC}{AD} = \frac{\sin i}{\sin r}$$

but

$$\frac{BC}{AD} = \frac{V}{v}$$

therefore

$$\frac{\sin i}{\sin r} = \frac{V}{v}$$

From this it appears that **the ratio of the sine of the angle of incidence to the sine of the angle of refraction is the same as the ratio of the velocities of light in the two media, and must, therefore, be constant for all angles of incidence.**

842. Adequacy of the Wave Theory.—The above interesting result is in exact agreement with the law of refraction as discovered by Snell, and it also leads to the conclusion that **the relative index of refraction of two media is simply the ratio of the velocities of light in those media**, a conclusion substantiated by the measurements of the velocity of light in water and in bisulphide of carbon by Foucault and Michelson.

We thus find that the wave theory leads to a simple and natural explanation of the facts known about refraction, a result which must strengthen our conviction of the essential soundness of the theory.

Also the index of refraction of a substance takes on a new interest when we think of its physical significance as the ratio of the velocity of light in air or vacuum to that in the substance.

843. Total Reflection.—When light passes from one medium into another in which the velocity of light is *greater*, as when it passes from water or glass into air, the refracted ray is bent *away* from the normal. Thus a ray of light coming up from below and meeting the surface of water on the under side, as shown by AO in the first diagram of figure 505, is in general partly refracted,

ıd bent away from the normal in the direction OB and partly flected along OD. But as the direction of AO is changed and ade more oblique, OB is bent away more strongly until when O takes the direction shown in the middle diagram of the ;ure, the refracted ray OB emerges at an angle of 90° and grazes ong the surface. The angle AON in this case is called *the 'tical angle.* When the angle of incidence is greater than the

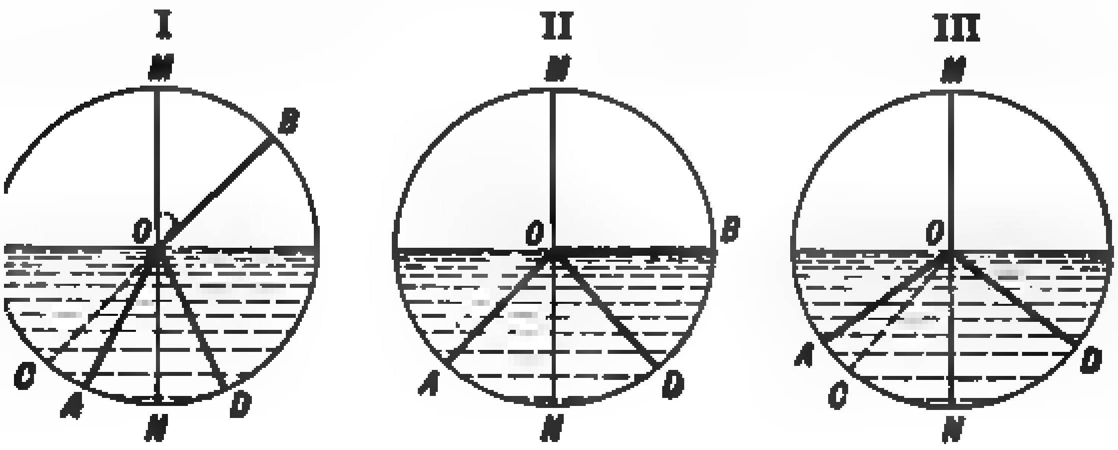


FIG. 505.

tical angle, as shown in the third diagram, none of the light is racted but the beam is totally reflected along OD, as if the face of the liquid were a polished metal mirror, for there is no responding direction in which it can emerge into the upper dium.

344. **Critical Angle.**—The angle AON in the middle diagram ove, beyond which refraction cannot take place, is called the *'tical angle.* From the law of refraction,

$$\frac{\sin MOB}{\sin AON} = n,$$

t $\sin MOB = 1$ since MOB is a right-angle, therefore

$$\sin AON = \frac{1}{n}$$

the sine of the critical angle is equal to the reciprocal of the lex of refraction.

345. **Illustration of Total Reflection.**—If a tumbler full of ter and having smooth sides is held in the hand, on looking wn obliquely into it the sides are seen as polished, mirror-like faces reflecting objects under the glass but the fingers holding glass *cannot be seen* through the surface if it is dry, as in

that case light coming up from below is totally reflected at the side. If the fingers are moist they will be seen only at the spots where they press against the glass.

A right-angled glass prism having all its sides polished may be used as a mirror to turn a beam of light through 90° if the light falls upon it as shown in the figure, for in that case it meets the oblique surface *inside* the glass at 45°, which is greater than the critical angle for glass and air. The intensity of the beam reflected in this way is far greater than if reflected from the *outside* of the same surface, for in that case a large amount of light is lost by refraction through the prism.

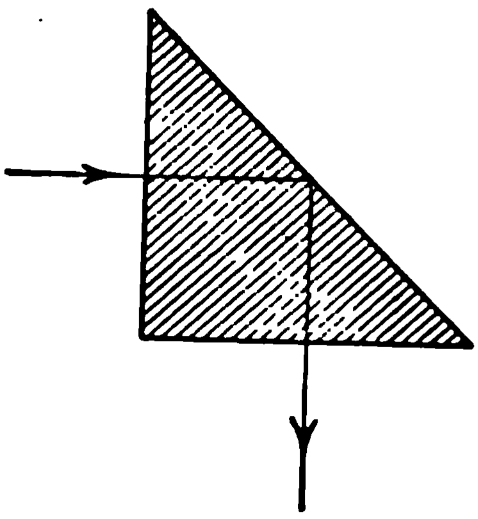


FIG. 506.—Total reflecting prism.

In figure 507 is shown a right-angled prism used as a reversing prism with a projecting lantern. The beam AA' which on entering the prism is directed downward, on leaving it is sloping upward, so also BB' is changed from an upward inclination on entering the prism to an equal downward slope on emergence.

846. Refraction of Gases.—The refracting power of gases is small compared with that of solids or liquids, the change in velocity when light passes from vacuum into air under ordinary conditions being only about one nine-hundredth part of the

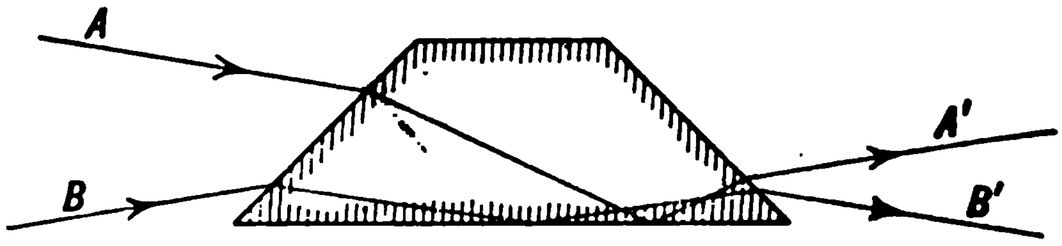


FIG. 507.—Reversing prism.

change in velocity when it enters water. Yet it is the variations of this small refractive power caused by the fluctuating density in the hot-air currents over a stove that cause the unsteadiness in the appearance of bodies seen through the stream of hot air.

847. Atmospheric Refraction.—In consequence of the refraction of the air the apparent angular distances of stars from the zenith is less than their true zenith distances, the rays being refracted just as much as if the atmosphere terminated abruptly in a level surface just above the observing telescope and all

above was vacuum, instead of gradually diminishing in density as it does. The sun or moon when seen near the horizon appears flattened in consequence of the lower edge being more raised by refraction than the upper edge, and when apparently just above the horizon it is really entirely below it.

848. Mirage.—When a layer of air next the surface of the earth becomes heated it may become less dense and less refracting than the cooler layers above it, so that the lower edges of light

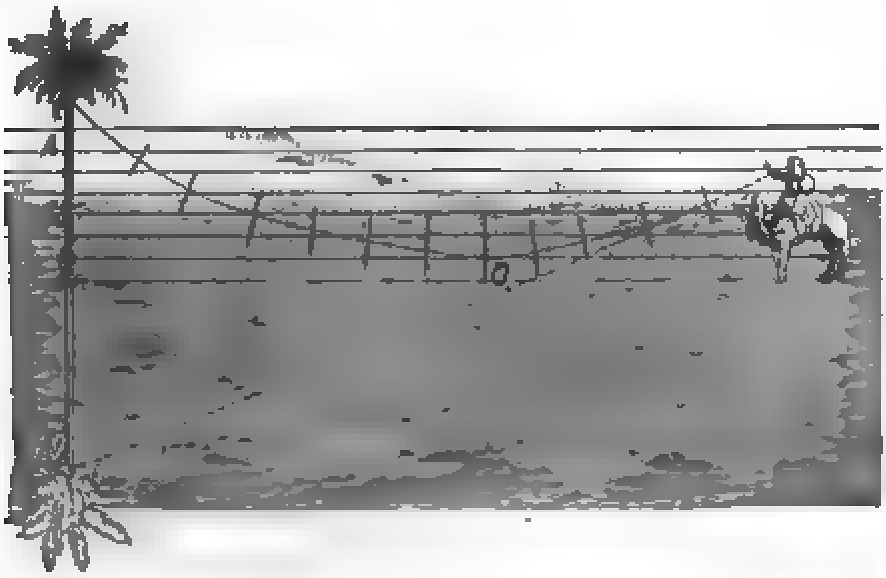


FIG. 508.

waves coming from a distant object are less retarded than the upper parts of the waves, and consequently the wave fronts swing around and come upward to the eye, as shown in figure 508. The distant object is thus seen inverted as if reflected in a horizontal mirror. In this way the familiar mirage of the desert may give the impression that objects seen are reflected in a sheet of water.

PRISMS AND LENSES

849. Refraction of Plane Waves by Plate with Parallel Sides.—In passing into the plate the beam is bent toward the normal, but since the two sides are parallel the waves within the plate make the same angle with one side as with the other and will therefore be bent as much on emerging from the plate as they were bent on entering, and the emergent beam will therefore be parallel to the entering one, but displaced sidewise by an amount which *depends* on the thickness of the plate. Light

waves from a *distant* point will therefore enter the eye of an observer in the same direction as if the plate were not there.

If the apparent position of a star shifts on interposing a piece of thick plate glass, even if held obliquely, it is because the sides of the plate are not perfectly parallel.

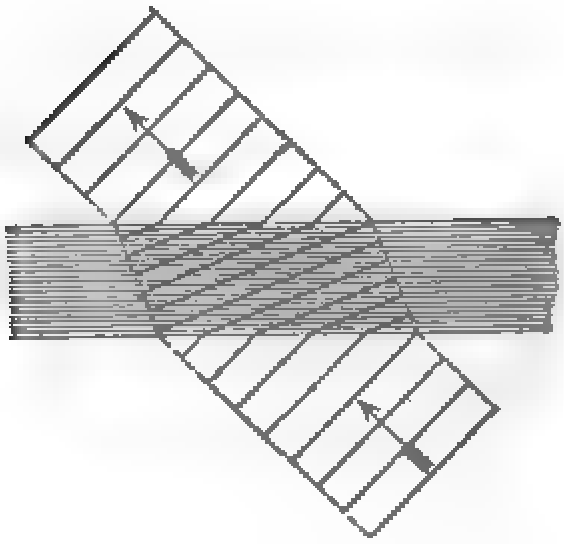


FIG. 509.—Refraction through plate with parallel sides.

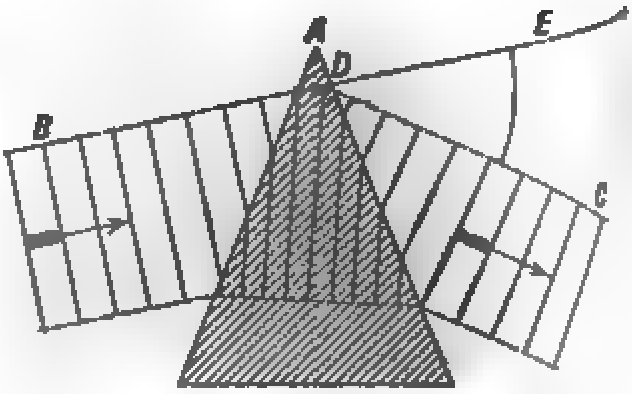


FIG. 510.—Refraction through a prism.

850. Refraction by a Prism.—Plane waves when refracted at a plane surface remain plane, and therefore will continue plane after any number of successive refractions at plane surfaces.

When a substance has two plane refracting surfaces which are inclined to each other it is called a prism, and the angle between the two refracting surfaces is called the angle of the prism.

In figure 510 the edge of the prism at A is supposed to be perpendicular to the plane of the paper, which is the plane of incidence. The beam of light at B enters the prism, is bent aside, and on emergence is again bent, and passes out in the direction shown at C. The total change in direction is represented by the angle CDE, which is called the deviation of the beam.

The beam is bent **toward the thicker part of the prism,** as shown in the figure, when the substance of the prism is more refracting than air, because that part of each wave is most retarded which is farthest from the edge of the prism and has to pass through the greatest thickness of retarding substance.

851. Minimum Deviation.—In such a position of the prism as that shown in figure 511, in which the incident beam makes

the same angle with the first face of the prism as the emergent beam does with the second, it is found that the deviation angle *CDE* is a *minimum;* turning the prism away from this position in either direction causes the angle *CDE* to increase.

If n represents the index of refraction of the substance of the

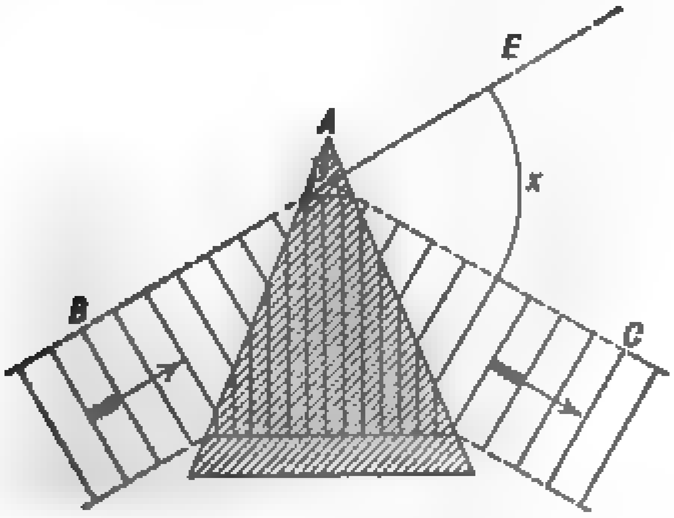


Fig. 511.—Minimum deviation.

prism, and if A is its angle and D the angle of minimum deviation, it may be easily proved that

$$n = \frac{\sin \frac{A+D}{2}}{\sin \frac{A}{2}}.$$

352. Lenses.—Lenses are pieces of glass or other transparent substance usually bounded by spherical surfaces, and are used in forming optical images. The line joining the centers of

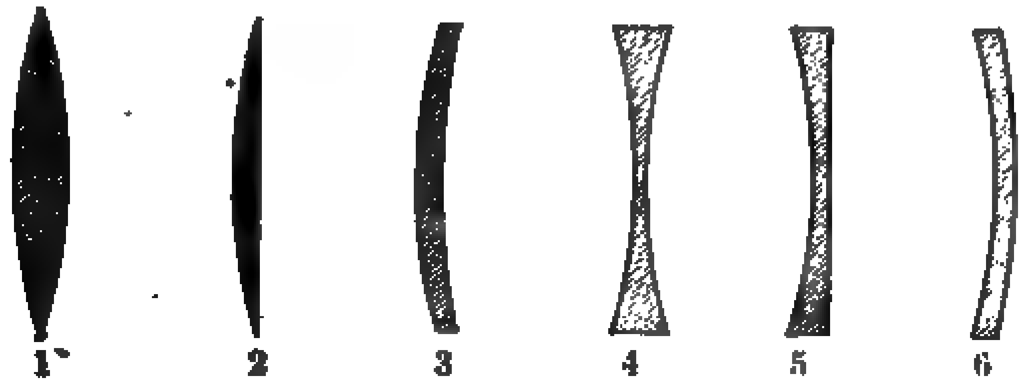


Fig. 512.

curvature of the surfaces of a lens is called its *axis.* Different types of lenses are shown in figure 512. These are distinguished as double convex (1), plano-convex (2), meniscus (3), double concave (4), plano-concave (5), and convexo-concave (6).

In the first three cases light rays parallel to the axis are converged to a point F, called the principal focus. The distance of this point from the lens is called its focal length. Such lenses are called **convergent**; they are thicker in the center than at the edges, and consequently plane-waves passing through them are more retarded at the middle than at the edges, and become of a concave spherical form converging on F.

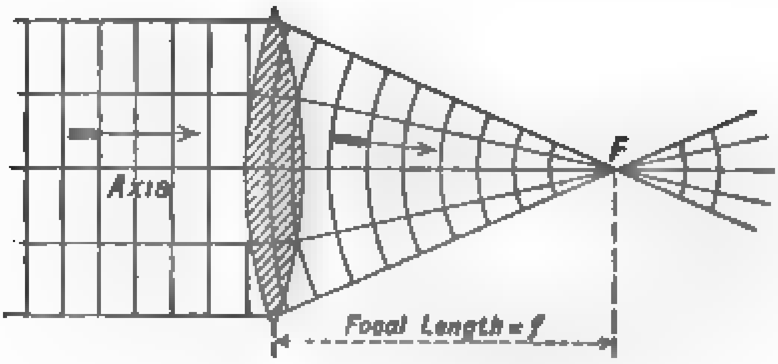


Fig. 513.—Convex lens. Focal length = f.

The last three forms of lens are thinner at the center than at the edges and are known as **divergent** lenses, for plane waves advancing along the axis of such a lens are more retarded at the edges than at the center and emerge from the lens as spherical waves expanding from a center F. This point from which rays parallel to the axis on one side of the lens appear to diverge on the other side is called the principal focus. In this case it is a *virtual* focus.

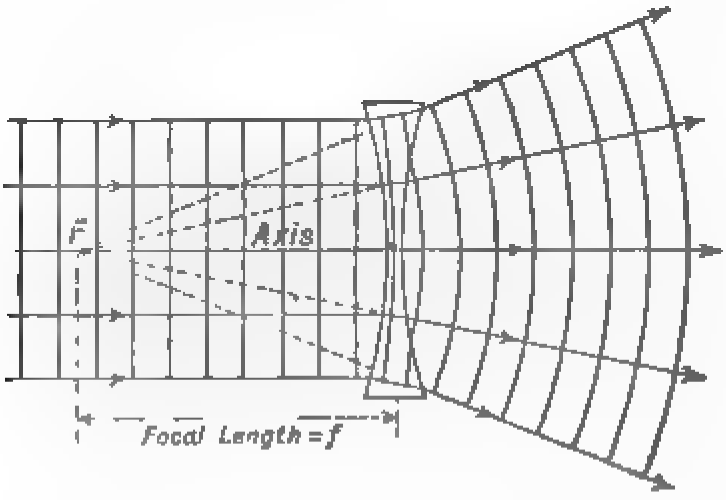


Fig. 514.—Concave lens. Focal length = f.

In general, when light from any point P passes through a lens, on emerging it is directed either toward or away from some

[redacted]

other point Q, and these two corresponding points are known as *conjugate foci*. The line joining them passes through the center of the lens if it is thin, and is called a *secondary axis* when it does not coincide with the principal axis of the lens.

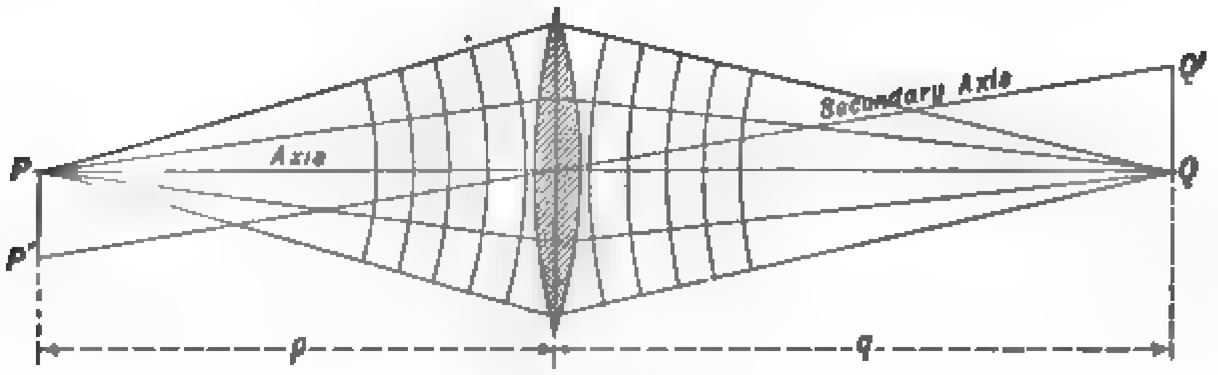


FIG. 515.

If the distances of the points P and Q from the lens are represented by p and q, respectively, then for thin lenses we have

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$

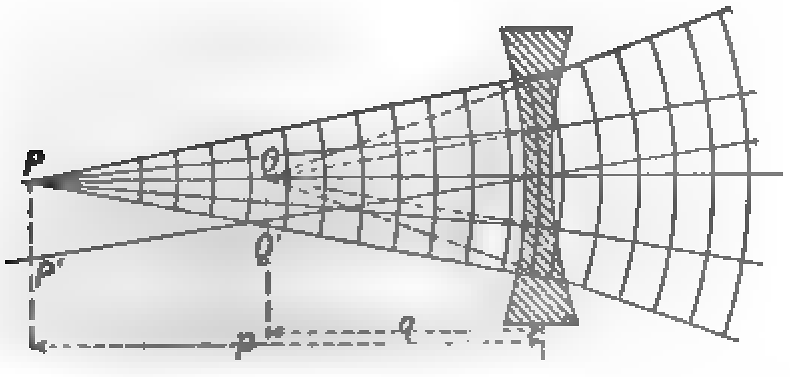


FIG. 516.

where f is the principal focal length of the lens and is taken *positive for convergent* and *negative for divergent* lenses. The proof of this formula will be given in the next paragraph.

853. Lens Formula by Method of Rays.—Let rays from A be converged to a focus at C by the action of the lens and let $AEFC$ be the path of a certain ray. Let D be the center of curvature of the first surface and B that of the second surface. Then since ND is normal to the first surface at E, the ray AE is refracted into the direction EF, so that by the law of refraction, if n is the index of refraction of the glass of the lens,

$$\frac{\sin NEA}{\sin FED}=n \quad \text{or} \quad \frac{\sin i}{\sin r}=n$$

so also at F

$$\frac{\sin MFC}{\sin EFB} = n \quad \text{or} \quad \frac{\sin i'}{\sin r'} = n$$

but if the angles $i\, r\, i'\, r'$ are *small* the ratio of the angles themselves is practically equal to the ratio of the sines. Hence we have

$$\frac{i}{r} = n \quad \text{and} \quad \frac{i'}{r'} = n$$

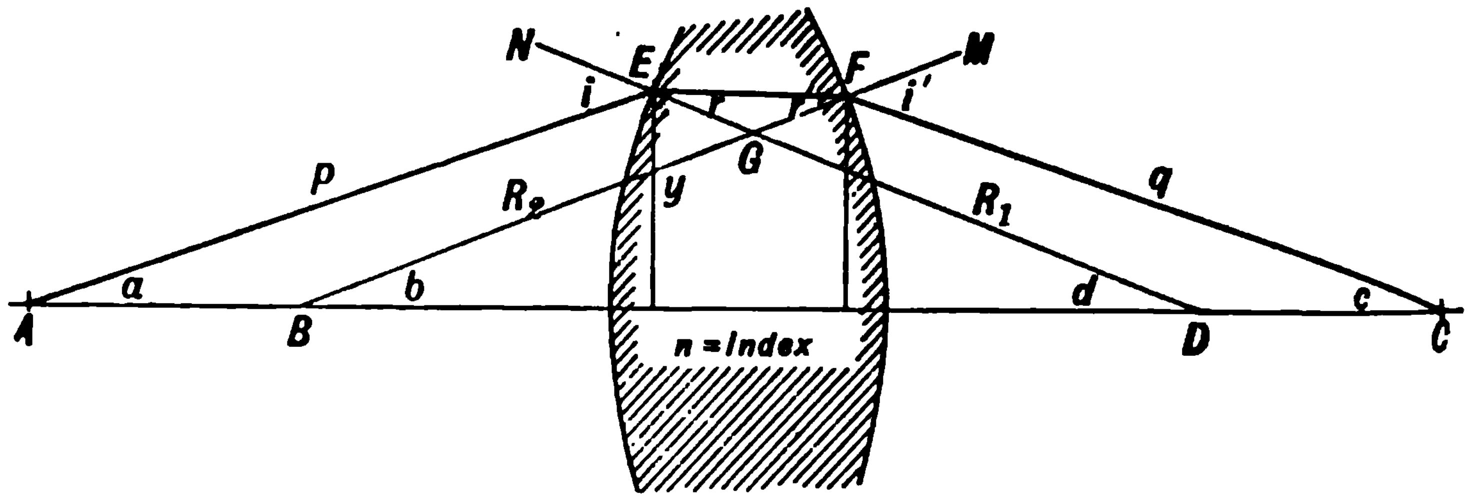


Fig. 517.

and therefore

$$i + i' = n(r + r') \tag{1}$$

Also the angle i which is external to the triangle EDA is equal to the sum of the opposite internal angles, therefore

$$i = a + d$$

and likewise

$$i' = b + c$$

adding, we have

$$i + i' = a + c + b + d \tag{2}$$

But the triangles EFG and BDG have the angles at G opposit and equal, therefore

$$r + r' = b + d.$$

Substituting this value of $r + r'$ in (1), we have $i + i' = n(b + d$ and now substituting this in (2) we find

$$n(b + d) = a + c + b + d$$

or

$$a + c = (n - 1)(b + d) \tag{3}$$

Now if the lens is thin E and F are practically at the same distance from the axis; call this distance y, and let p be the distance AE, q the distance CF, while R_1 and R_2 represent DE and BF, the radii of curvature of the lens surfaces. Then if the angles $a\,b\,d\,c$ are small, each will be equal to the arc that subtends it divided by the corresponding radius, but all the arcs may be considered equal to y, so that

$$a = \frac{y}{p} \quad b = \frac{y}{R_2} \quad c = \frac{y}{q} \quad d = \frac{y}{R_1}.$$

Substituting in (3) we have

$$\frac{y}{p} + \frac{y}{q} = (n-1)\left(\frac{y}{R_1} + \frac{y}{R_2}\right)$$

or finally

$$\frac{1}{p} + \frac{1}{q} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

which may be written

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

where

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

and f is a constant for the lens, depending on its index of refraction and the radii of curvature of its surfaces.

854. Discussion of Formula.—Consider first the case of a convergent lens with the source of light an infinitely distant point, in this case f is positive and $p = \infty$; therefore, $\frac{1}{p} = 0$ and $q = f$.

Therefore the light converges to a point at a distance f from the lens. This point is the *principal focus* and the distance f is the *focal length* of the lens.

As the point source P is moved along the axis nearer to the lens, the corresponding focus Q or *conjugate focus* moves away from the lens, so that when $p = q$, each of the points P and Q is at a distance from the lens equal to $2f$.

As P is now moved uniformly toward the lens, the rays on the farther side become more nearly parallel and Q moves off with

increasing speed till the distance p is equal to f when q becomes infinite and the rays go out from the lens parallel.

If P is now moved still nearer to the lens, the rays on the

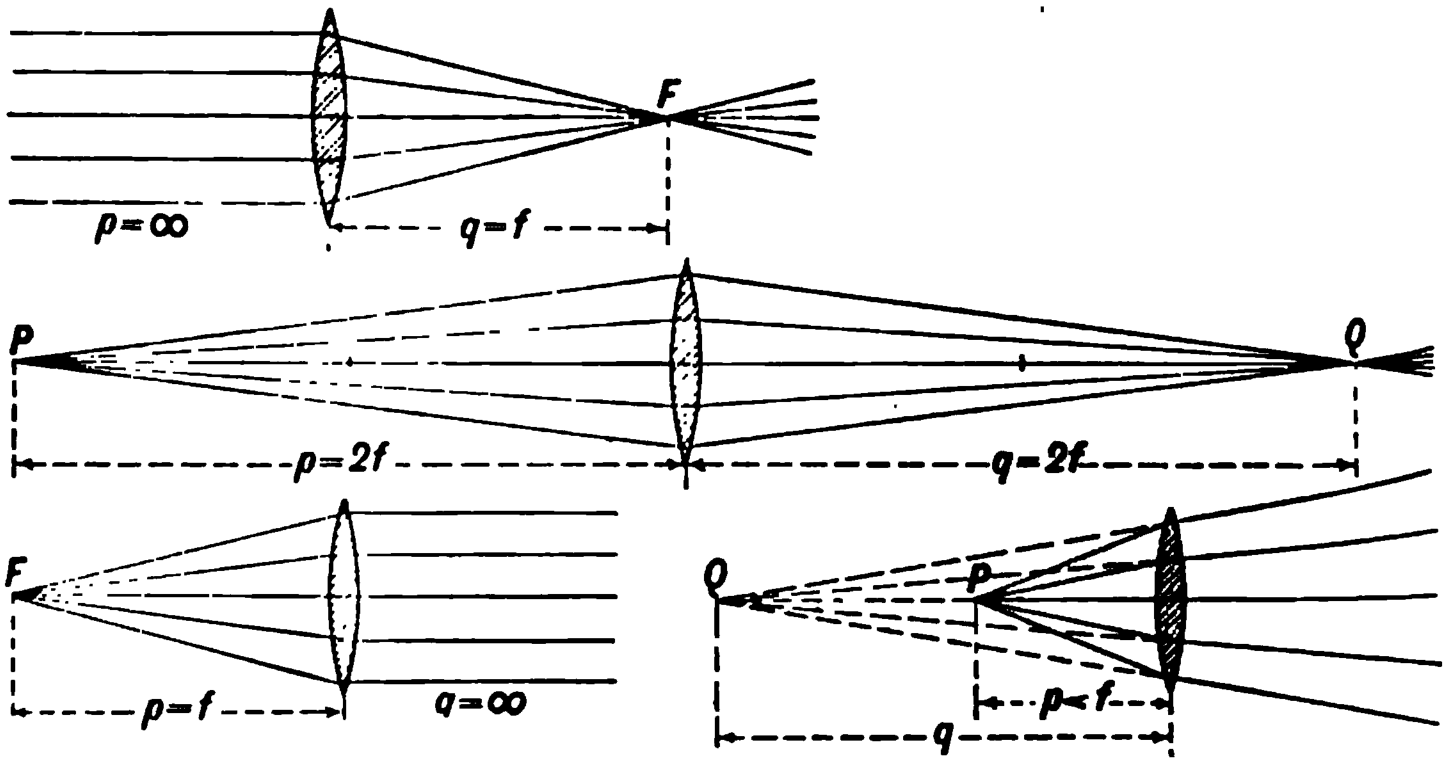


FIG. 518.

farther side diverge as if they came from a focus Q on the left of the lens. In this case the formula shows that q will be *negative*, and the focus Q is *virtual.*

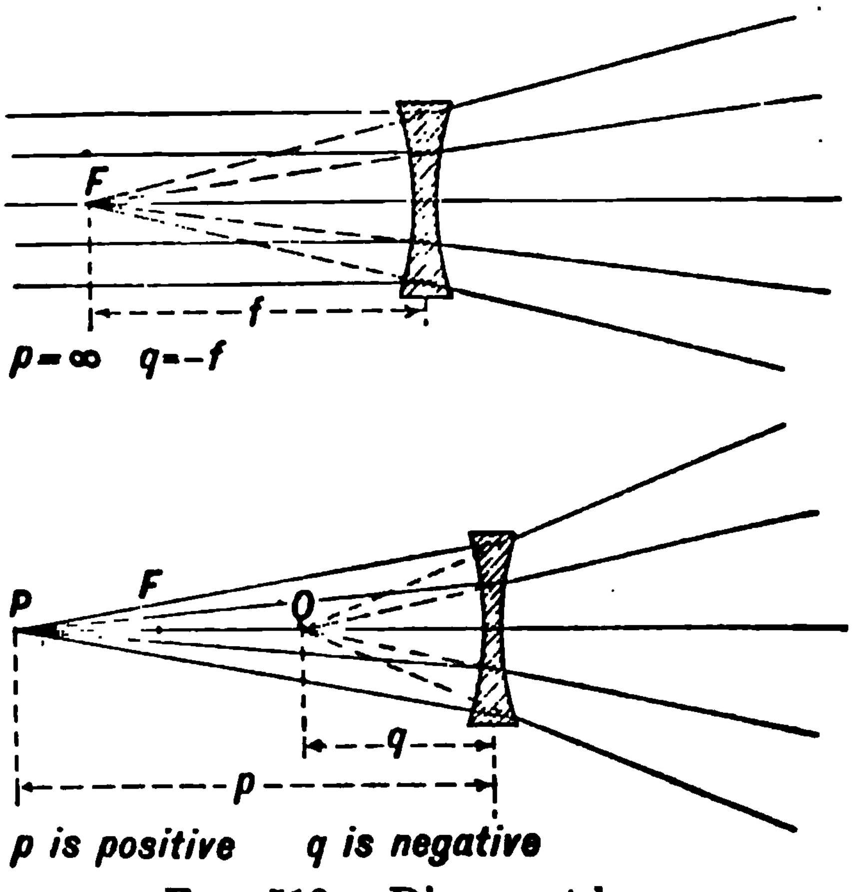


FIG. 519.—Divergent lens.

In case of a *divergent* lens, the focal length is ***negative,*** so that we have

$$\frac{1}{p}+\frac{1}{q}=-\frac{1}{f}.$$

Here if the luminous point P is at an infinite distance, we find $q = -f$, indicating that the rays diverge as if from a point F. This the *principal focus* and it is virtual.

As P moves from an infinite distance in toward the lens, the *conjugate focus* Q remains virtual and moves from F toward the lens, so that when $p = f$, $q = -\frac{f}{2}$, and when $p = 0$, $q = 0$.

855. Focus for Distant Objects.—It is important to observe that *when the point P is at a great distance from the lens compared with its focal length, the conjugate focus Q is very nearly at the principal focus*, and a great change in the position of P will cause only a slight change in Q. Thus when p = ten times f, $q = {}^{10}\!/_{9}f$, while if p is 100 times f, $q = {}^{100}\!/_{99}f$.

It is for this reason that in a photographic camera the focus for all *distant* objects is practically the same.

856. Rule for Use of Formula.—In using the formula

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

in the solution of lens problems care must be taken as to the *signs* of the various terms.

The focal length f is always *positive in case of a convergent lens and negative in case of a divergent lens.*

When the rays from the point P diverge toward the lens the sign of p is positive; if, however, the rays meet the lens as they are converging toward P then p must be taken *negative.*

So also *if rays leaving the lens converge toward a real focus the distance q of that focus from the lens is positive,* while if the rays after passing the lens diverge from a *virtual focus* the distance q is *negative* and is measured back of the lens.

Notice that we always consider a narrow pencil of rays *originating in a single point* in the object; and that p is determined by the rays of such a pencil as they *approach* the lens, while q relates to the rays *leaving* the lens.

The rule of signs may be illustrated by the case shown in figure 520. A convergent lens of 4 in. focus is placed 4 in. from a divergent lens of the same focal length; if an object P is placed 6 in. from the convergent lens, it is required to find the position of the *image formed* by the combination.

Rays from P diverge toward the lens L, therefore the distance 6 is positive, and as the focal length of that lens is also positive we have

$$\frac{1}{6}+\frac{1}{q}=\frac{1}{4}$$

which gives $q = +12$.

Hence the rays after passing the first lens converge toward a real focus at Q, 12 in. to the right of L.

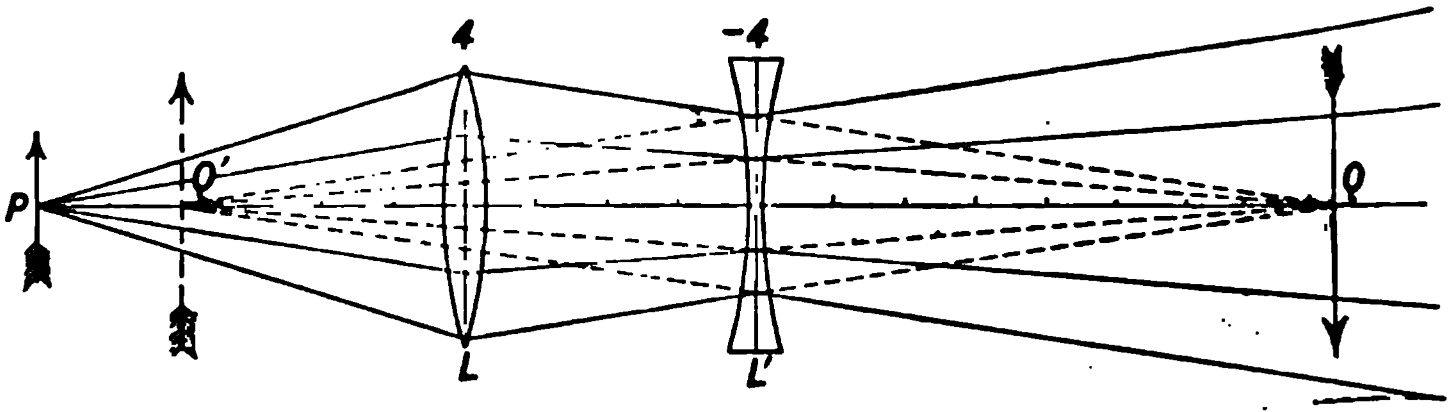


FIG. 520.

But in case of the second lens L' the rays approaching it are converging toward a point Q which is 8 in. beyond the lens; hence in this case $p = -8$, and as the lens is divergent $f = -4$; hence we have

$$-\frac{1}{8}+\frac{1}{q}=-\frac{1}{4}$$

which gives $q = -8$. The rays will therefore emerge as if coming from a point Q', 8 in. back of the lens L', and the final image is *virtual*.

857. Images by Lenses.—The most important use of lenses is in the formation of optical images. Let P (Fig. 521) be a point on

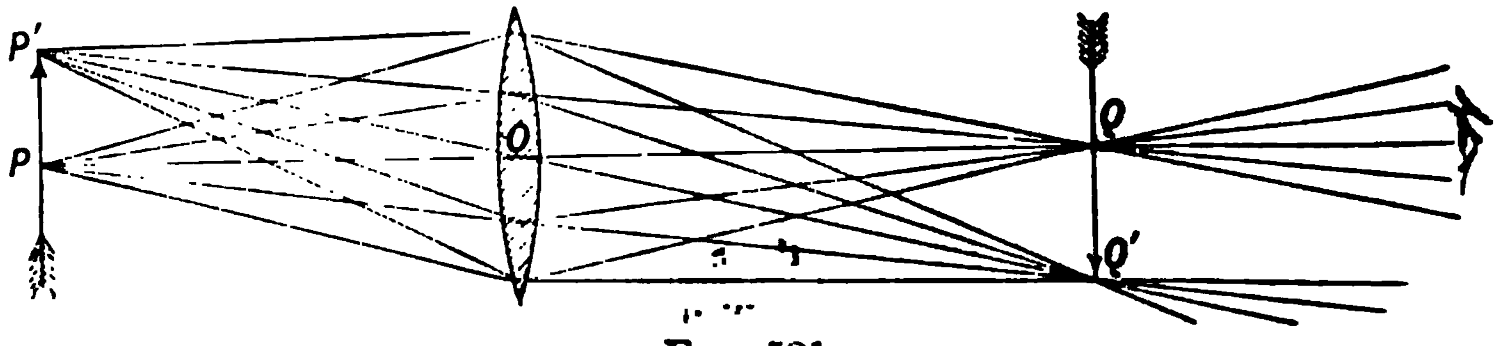


FIG. 521.

the arrow that lies on the axis of the lens; light from P will be converged at Q, its conjugate focus. So also for any other point P' in the object there is a conjugate focus Q' which must lie on the *secondary axis* or straight line $P'OQ'$ through the *center of the* lens, for at the center the opposite faces of the lens are

.lel and hence a ray passing through the center is not changed rection. If P and P' are in the same plane at right angles e axis of the lens, Q' will not be in the parallel plane through ut will be somewhat nearer the lens, making the image ed.

a white screen is placed at Q the light falling upon each t of it comes from the corresponding *conjugate* point on the r side of the lens and a picture or real image is therefore ed on the screen just as though the light had come through hole at O, but more brilliant. This is the principle of the ographic camera.

ie image formed by a lens may be seen directly by the eye ad of being received on a screen; for the eye may be placed ie right of Q at a distance from it of about 10 in., or the nce of normal distinct vision, and looking toward Q light enter the eye from Q just as it would have come from an t placed at that point, and accordingly the inverted image e arrow will be seen.

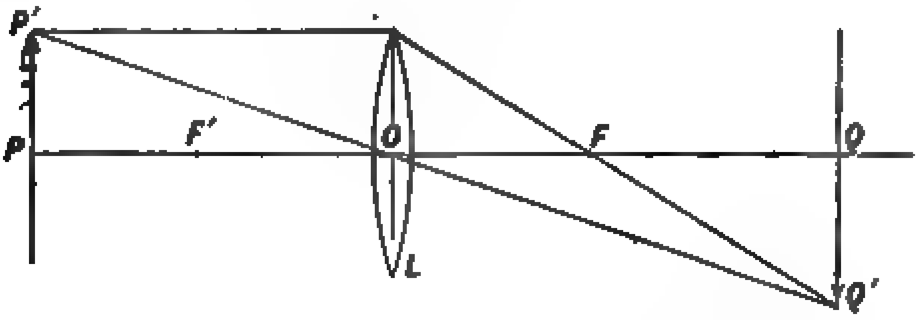


FIG. 522.

8. **Construction of Images.**—A simple geometrical contion will give the size and position of the image in any case, *t is only necessary to trace two rays from any point in the t to find by their intersection the position of the corresponding t of the image.* Suppose it is required to find the size and ion of the image of the arrow at P formed by the convergent L whose principal foci are at F and F' (Fig. 522). Since the s well as the position of the image is desired, we will choose a t P' not on the axis of the lens and trace two rays. One llel to the axis must after refraction by the lens pass through principal focus F. Another ray through the center of the O is undeviated, and where these two meet at Q' is the image

of the point P'. Since the image of P must be formed at Q, the *length of the object PP' is to the length of the image QQ' in the same proportion as their distances from the lens.*

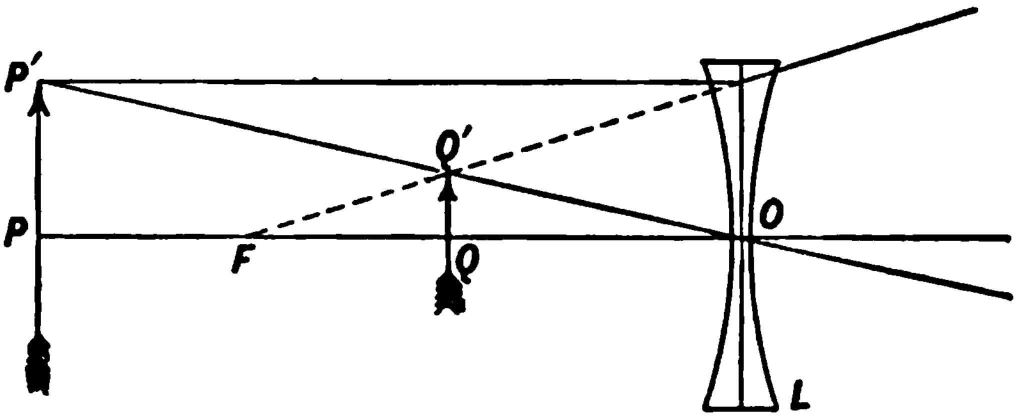


FIG. 523.

By a similar construction the size and position of the image formed by a divergent lens may be found, as in figure 523, where F is the principal focus of the lens L. A ray from P' approaching the lens parallel to the axis is refracted up as if it came from the principal focus F, while the second ray through the center is undeviated. The two rays after emerging from the lens diverge as if from the point Q', which is therefore the *virtual* image of the point P'.

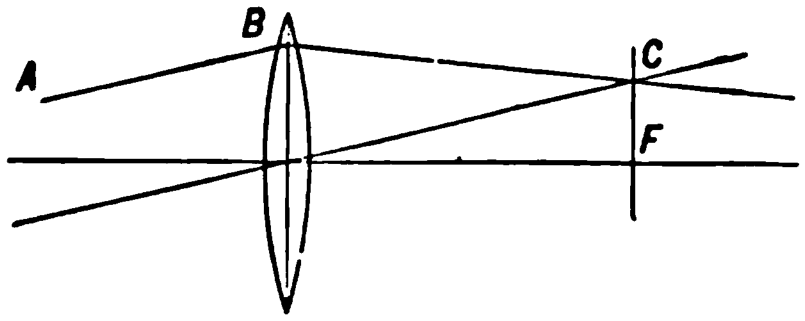


FIG. 524.

The relative size of object and image is, as before, the ratio of their distances from the lens.

859. Problem.—*To construct the direction of the refracted ray when the direction of the ray meeting the lens is given.* Let AB (Fig. 524) be the given ray, draw a parallel ray through the center of the lens meeting the *principal focal plane* through F at C, then BC will be the direction of the refracted ray if the lens is convergent. If the lens is divergent the parallel ray through the center meets the focal plane at C' (Fig. 525) on the first side of the lens, and the refracted ray BC diverges as if from C'.

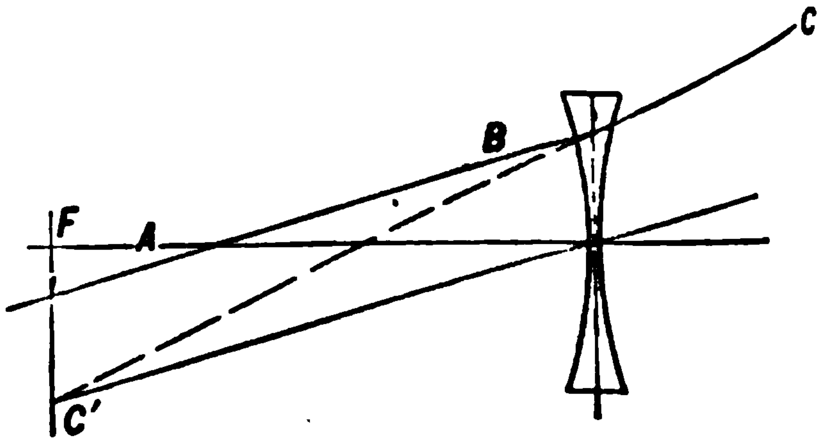


FIG. 525.

860. Thick Lenses.—It was shown by Gauss that for thick lenses or for a combination of lenses there are two *principal planes perpendicular to the axis* such that rays on one side of the lens which are directed toward any

point such as A (Fig. 526) in one plane will on emergence be directed as if from the opposite point A' on the other plane, and the points HH' (known as principal points) where these planes meet the axis have the additional property that rays directed toward H emerge in a *parallel* direction from H'.

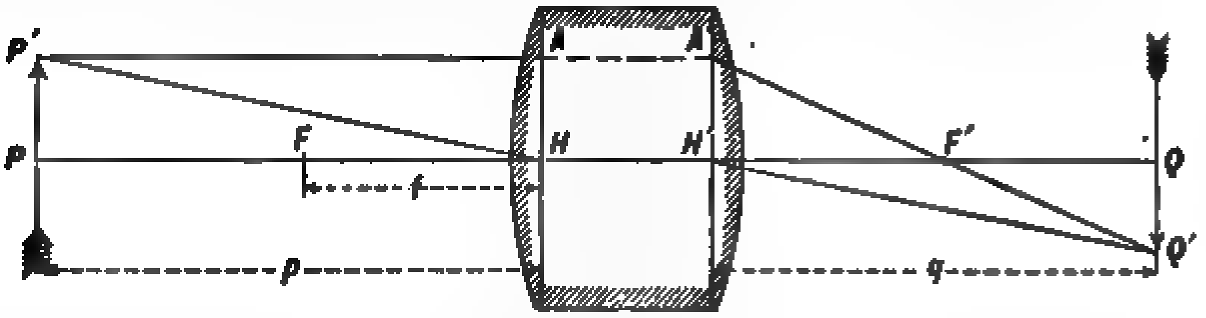


Fig. 526.

The principal focus F on one side is just as far from H as the other focus F' is from H'. This distance from either principal focus to the corresponding principal plane is called the *focal length* of the lens. If p and q are the distances of object and image, also measured from H and H' respectively, the simple formula

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$

holds just as in case of thin lenses *for all pencils of light that are but slightly oblique to the axis.*

The graphic construction of images, using the principal planes, is precisely similar to that explained above in §858 for thin lenses, except that $H'Q'$ is to be drawn *parallel* to $P'H$ instead of simply prolonging $P'H$. If the planes AH and $A'H'$ are made to coincide, the construction is that for a thin lens.

861. Defects of Images Formed by Lenses.—Besides the curvature of the image which is noticed when the object has large angular dimen-

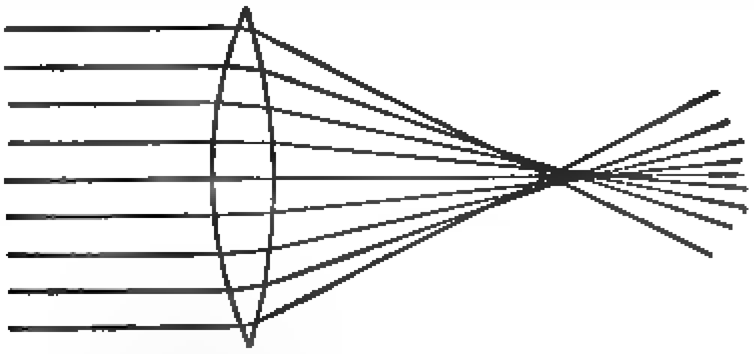


Fig. 527.—Spherical aberration.

sions as seen from the lens, there are other defects in the images formed by lenses which tax the skill of the optician to overcome.

Almost all lenses have spherical surfaces and are subject to a defect called *spherical aberration.* One kind of spherical aberration is that rays from a given point which are refracted on different portions of the lens do not meet accurately at a single focus, the rays refracted by the outer portions of the lens coming to a focus nearer the lens than those passing through its central region as indicated in figure 527. Another kind of spherical aberration is

that a pencil of rays from a point, passing obliquely through the lens, becomes astigmatic and converges through two focal lines instead of coming to a single point. These defects cause a lack of clearness and sharpness in the images formed. They are most serious when the diameter of the lens is a large fraction of its focal length.

The colors observed at the edges of images formed by lenses are due to the fact that ordinary lenses refract blue light more strongly than red light. This defect, known as chromatic aberration, will be discussed later (§868).

Problems

1. How deep is a tank of water which appears to be 4 ft. deep to a person looking vertically down into it?
2. An incandescent lamp is placed 6 ft. below the surface of a pond. Show why only a fractional part of the light can escape directly from the water.
3. If a beam of light has 50,000 light waves to the inch in air, how many to the inch will there be after it has entered water?
4. Find the velocity of light in water if the critical angle at the surface between water and air is 48° 30′.
5. When the index of refraction of water is 1.33 and that of carbon bisulphide is 1.67, what is the critical angle between water and carbon bisulphide?
6. The object-glass of the Yerkes telescope is a convergent lens 40 in. in diameter and having a focal length of 62 ft. What is the size of the sun's image formed by it? What effect has the size of the lens on the size of the image?
7. An incandescent lamp is 30 cm. from a convergent lens of 10 cm. focal length. Find the position and relative size of the image; is it real or virtual?
8. A candle is placed 1 meter from a divergent lens having a focal length of 1 meter. Where is the image formed and what is its size? Make a construction illustrating the case.
9. A lamp and a screen are 10 ft. apart. Where must a convergent lens of 2 ft. focal length be placed so as to form an image of the lamp on the screen? Show that there are two solutions and find the relative size of the image in each case.
10. A beam of sunlight falls on a divergent lens of focal length 10 in.; 20 in. beyond this lens is placed a convergent lens of 15 in. focal length. Find where a screen should be placed to receive the final image of the sun.
11. A convergent lens, focal length 10, is placed 12 in. from a gas flame; then 36 in. beyond the first lens is placed a divergent lens of focal length 16 in. Find the position and size of the final image; is it real or virtual?
12. A certain lens when placed 10 cm. from an object, forms a virtual image 5 times as large as the object. What kind of lens is used and what is its focal length?
13. What must be the focal length of spectacle lenses so that a man who can see distinctly objects 2 meters distant without the glasses can read print at 40 cm. distance with them, and what kind of lenses must be used?

Note.—The strength of spectacle lenses is expressed in diopters and is the reciprocal of the focal length expressed in meters.

14. A person, who without glasses cannot see distinctly objects more than 12 cm. from the eye, wishes glasses to enable him to see clearly distant objects. What must be the kind used and their focal length and strength in diopters?

Dispersion

862. Dispersion of Light by a Prism.—When a narrow beam of sunlight passes through a prism, the light is not only bent aside or *deviated,* it is also *dispersed* or spread out into a colored band called **the spectrum.**

Sir Isaac Newton placed a second prism (Fig. 528) in the spectrum so that light of only one color might fall on it. This

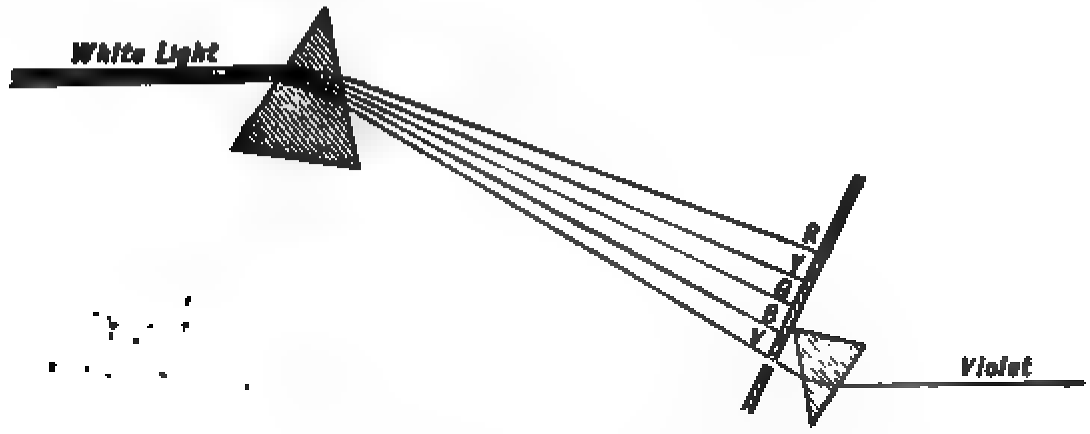


Fig. 528.—Newton's experiment.

light was refracted on passing through the second prism, but there was no further change in color, showing that the prism itself did not produce the different colors, but simply separated the various kinds of light already present in the beam of sunlight. The separation is effected because the various colored lights are differently refracted by the prism, the red being refracted least and the violet most.

863. Cause of Dispersion.—Since the bending of the rays by a prism depends only on the angle of the prism and the index of refraction of the substance of which it is made, it follows that **the index of refraction of the prism must be different for each kind of light in the spectrum,** being least for the red which is least refracted and greatest for the violet which is most strongly refracted.

Of course *the interpretation of this fact* is that red light must pass

through the substance of the prism with greater velocity than violet light. It will be shown later that the physical difference between one kind of light and another lies in their wave lengths. These vary from one end of the spectrum to the other, the longest waves being at the red end of the spectrum while the shortest are at the violet end.

It appears therefore that shorter waves of light are more retarded in passing through glass than longer ones.

864. Dispersive Power.—When two prisms of different substances have such angles that each produces the same deviation

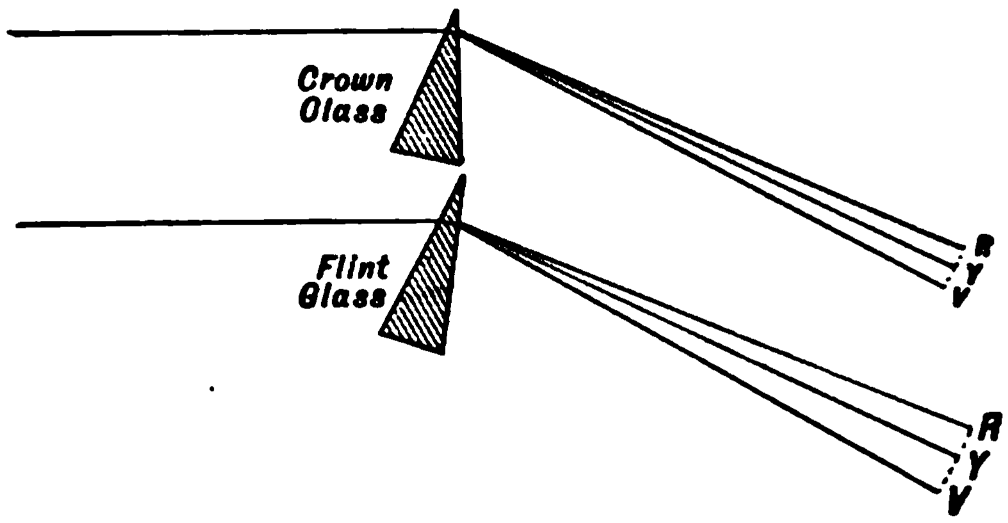


FIG. 529.—Prisms with different dispersive powers.

for yellow light, or light in the middle of the spectrum, the angular widths of the two spectra produced will usually not be the same, but are proportional to what are called the *dispersive powers* of the substances. The *dispersive powers* of some substances are as follows:

Water	0.042
Carbon bisulphide	0.145
Crown glass	0.043
Flint glass	0.061

Thus for an equal bending of the mean rays, carbon bisulphide will produce a spectrum $3\frac{1}{3}$ times as long as that produced by crown glass and $2\frac{1}{3}$ times as long as one formed by flint glass.

865. Calculation of Dispersive Power.—We have seen that the index of refraction of a substance depends on the *kind* of light. The following table gives three indices of refraction for each of four substances. The indices given in the first column are for light near the extreme red end of the spectrum, those in the second are for the yellow sodium light, while those in the third are for light near the violet end of the spectrum. These points in the spectrum correspond to three dark lines in the sun spectrum designated *A*, *D*, and *H* by Fraunhofer (§898).

Indices of Refraction

Substance	n_A	n_D	n_H	$n_D - 1$	$n_H - n_A$
Water at 16°C.	1.330	1.334	1.344	0 334	0.014
Carbon bisulphide at 10°C.. . .	1 616	1.635	1.708	0.635	0 092
A kind of crown glass	1 528	1.534	1.551	0.534	0.023
A kind of flint glass............	1.578	1.587	1.614	0.587	0.036

The next to the last column in the above table shows *the relative deviations of yellow sodium light caused by prisms of the different substances all having the same small angle.* Thus it appears that a prism of carbon bisulphide will cause nearly twice as great a deviation as a prism of water of the same angle, if the angles of the prism are small.

In the last column are given the *differences* between the indices of refraction for red and violet lights, which represent *the relative angular widths of the spectra produced by prisms of the various substances having the same small angle.* The spectrum formed by a thin prism of carbon bisulphide is therefore about 6½ times as long as that formed by a similar prism of water.

To obtain the relative *dispersive powers* given in the previous paragraph the figures in the last column must be divided by those in the next to the last.

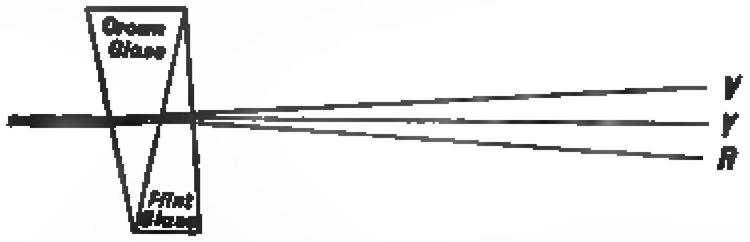


FIG. 530.—Dispersion without bending the mean ray.

866. Direct-vision Prism.—In consequence of the fact that the dispersive powers of substances differ it is possible to so combine two prisms of different substances as to produce dispersion without deviation of the mean ray, or to produce deviation without dispersion.

For example, if a prism of crown glass and one of flint glass are taken whose angles are small, and in the ratio 0.587 to 0.534, respectively, they will each deviate the D line of the spectrum by the same amount (see table §865), but the spectrum formed by the flint prism will be longer than that formed by the crown in the ratio of 0.061 to 0.043 (§864). If, therefore, the two are placed with their edges oppositely directed, as shown in figure
...he deviation of one will be balanced by that of the other
...ne or yellow light of the spectrum, but as the dis-

persive power of the flint is greater than that of the crown there will still be a spectrum formed with the violet toward the base of the flint and the red toward its edge.

Such a combination is known as a direct-vision prism. By using two prisms of flint combined with three of crown of suitable angles, as shown in figure 531, a very large dispersion may be produced with no deviation of the middle part of the spectrum.

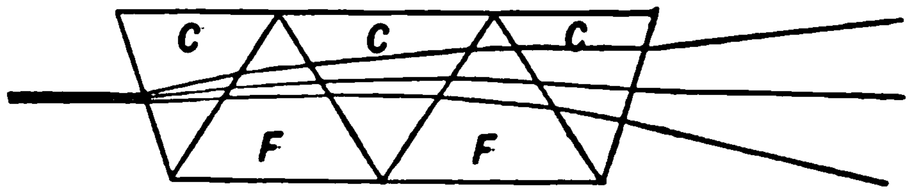


Fig. 531.—Amici prism.

867. Achromatism.—It is, however, of much more practical importance to produce *deviation without dispersion.* To obtain this result two prisms of crown and flint glass may be combined whose angles are in the ratio 0.036 to 0.023 or inversely as the ratio of the angular width of their spectra given in the last column of the table in §865. Two such prisms will give spectra of the same angular width, but the deviation by the crown-glass prism will be greater than that by the flint. If the two are now placed together so as to act oppositely, as shown in figure 532, the beam of light will be deviated toward

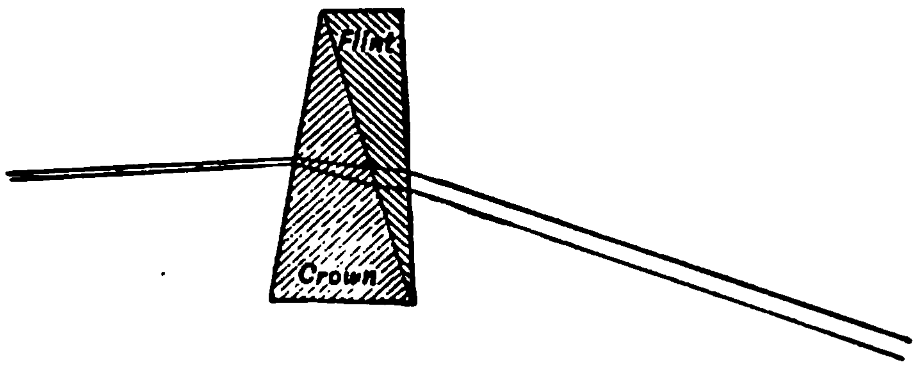


Fig. 532.—Ray bent without dispersion.

the base of the crown-glass prism, but there will be no dispersion, since in this respect the two balance each other. Such a prism is called *achromatic.*

868. Achromatic Lens.—If sunlight passes through an ordinary convergent lens made of a single piece of glass, it may easily be shown. by interposing successively a red glass and a blue glass, that the focus for red light is at a greater distance from the lens than that for blue light. For every little portion of the lens acts as a prism bending light toward the axis and at the same time dispersing it. With such a lens there is no

[redacted]

point at which a sharp image of an object will be formed by ordinary white light. All points in the image will be blurred and all lines of separation between light and dark portions of the image will be colored.

This serious defect may be remedied by combining a convergent lens of crown glass with a divergent lens of flint, as shown

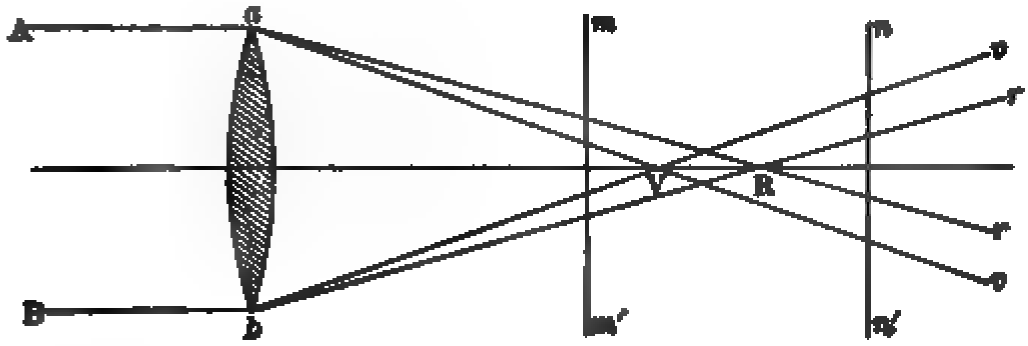


FIG. 533.—Different foci for violet and red rays.

in figure 534, to form a *convergent achromatic lens;* or if the crown-glass lens is divergent and the flint convergent, a *divergent* achromatic lens may be formed. In either case the curvatures must be so chosen that each little portion of the combination through which a ray passes will act like an achromatic prism as explained in the previous paragraph.

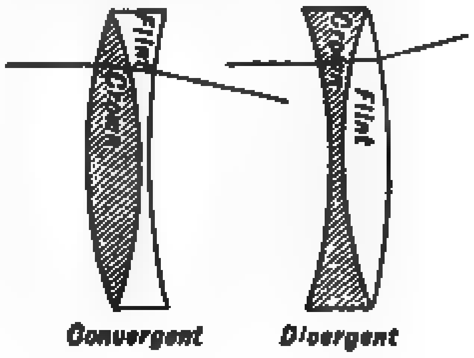


FIG. 534.—Achromatic lenses.

In case the two component lenses are in contact* and their curvatures are not great, *achromatism will be produced if the focal length of the crown-glass lens is to that of the flint in the same proportion as their dispersive powers* (§864).

If the ratio of the dispersive powers of two kinds of glass, such as flint and crown, was the same in all parts of the spectrum, a perfectly achromatic lens could be formed in this way, but unfortunately this is not the case with ordinary glasses, so that the image formed even by an achromatic lens shows some residual color.† Lenses intended for visual observation are made so as to bring as nearly as possible to one focus the orange, yellow, and green rays which form the brightest part of the spectrum;

* It is common in small lenses to cement together the two lenses of an achromatic combination with Canada balsam, in order to prevent loss of light by reflection from the inner surfaces.

† Certain kinds of glass are now made at Jena from which lenses are made which give images almost free from residual color.

while for photographic purposes the lens must be achromatic for the violet and ultraviolet rays which are most strongly *actinic* or active on a photographic film.

869. Ordinary and Anomalous Dispersion.—In most substances the shorter the wave length the more strongly the light is retarded or refracted. This is called ordinary or *normal dispersion*. But many substances exhibit what is called *anomalous dispersion*, especially such as absorb very strongly light of some particular wave lengths while transmitting compara-

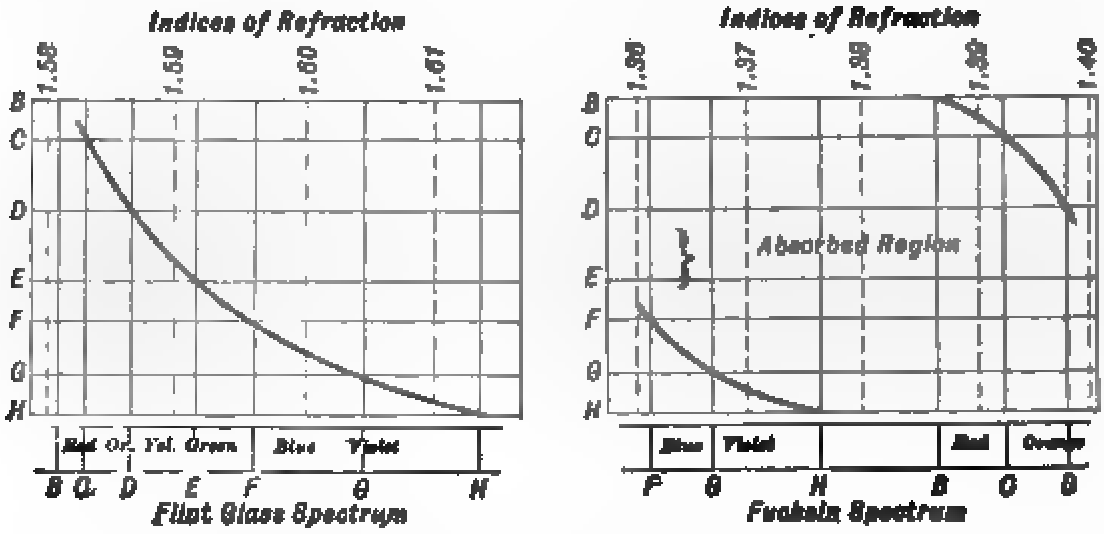


FIG. 535.—Dispersion curves.

tively freely those waves which are slightly longer or shorter than the ones absorbed. Figure 535 shows the normal dispersion curve of flint glass in which the index of refraction increases as wave lengths decrease. But the curve for fuchsin shows that when sun light is dispersed by a prism of this substance the middle part of the spectrum including green is wanting, being completely absorbed, while red and orange which in normal dispersion would be less refracted than blue, are so much more strongly refracted that they are beyond even the extreme violet. This substance thus illustrates the general law first stated by *Kundt*, that the effect of selective absorption is to increase the velocity of those light waves which are somewhat shorter than the ones most strongly absorbed and to retard light of greater wave length.

For an interesting discussion of anomalous dispersion, see EDSER, *Light for Students*.

870. Rainbow.—When parallel rays of light fall on a spherical drop of water, some of the light is refracted into the drop, suffers reflection at the opposite surface, and is then refracted out again. The direction in which it finally comes from the drop depends on the point where it entered. A ray *a* falling on the center at *A* is reflected back on its path, while *b* just above the center will be refracted as shown at *b'*. Rays meeting the drop farther from *A* are still further inclined downward on emergence, until we come to a group of rays, *efg*, meeting the surface at B, which have the maximum downward

tion as shown at $e'f'g'$; rays beyond B are again turned upward as
n at h' and k'. The pencil of rays refracted at B do not scatter on
ng the drop, but emerge as a nearly parallel beam in the direction CD.
e eye is so placed as to receive this beam the drop will appear very bright,
if the eye is above the line CD the drop will appear but faintly illum-
d in consequence of the scattering of the emergent rays, but if the point
ght is *below* the line CD, the drop will
ar dark, for no light at all is sent in
a direction.

d angle AOD between the bright *beam*
and the original direction AO depends
e index of refraction of the drop, and
s with the wave length, being about
or red light and 40° for violet.

consequence of this, drops of dew seen
ight sunlight at the proper angle may
ar as brilliant jewels colored red, yel-
green, or blue. In such a case it will be found that a slight change
sition of the eye may cause a red drop to change to green or blue.

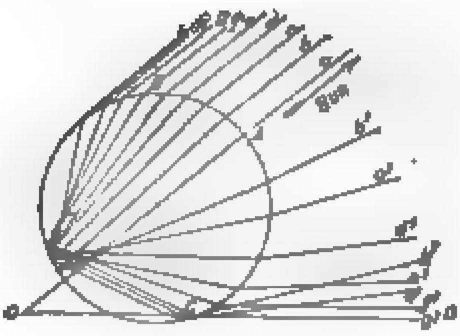


FIG. 536.—Refraction in rain drop.

e formation of the primary rainbow may now be readily understood
figure 537. Let $A\,B\,C\,D$ represent drops in the air all illuminated by
un's rays from S. From each drop there will emerge a bright parallel
l of red light in the direction R making an angle of 42° with S, and a

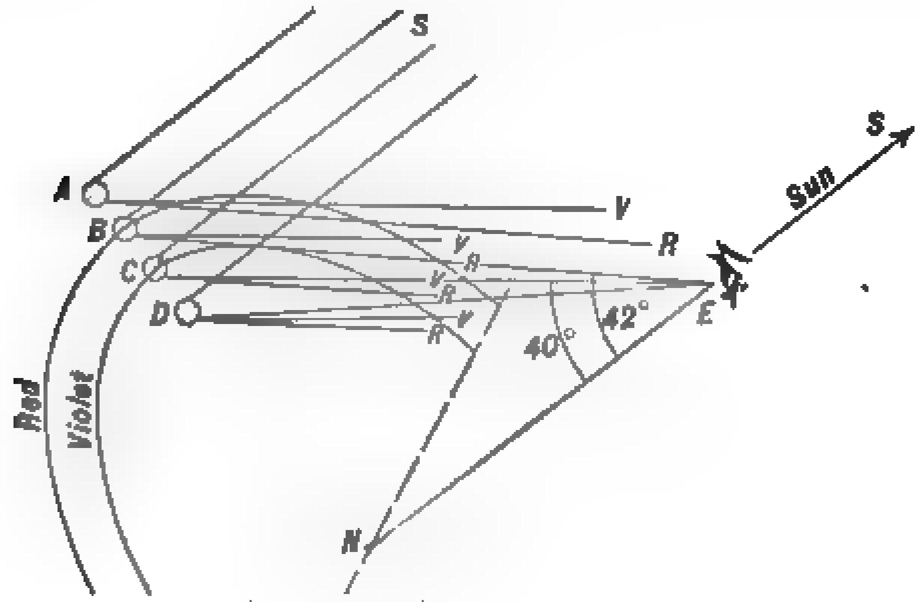


FIG. 537.—Primary rainbow.

t violet pencil V making an angle of 40° with S, the intermediate colors
e spectrum being between these extremes. An observer, therefore,
e eye is at E, will receive red light from the drop B and violet from the
C and intermediate colors from drops between them. If the diagram
w conceived to be rotated about the line NES, joining the eye and the
it is clear that every drop on the circle whose radius is BN, which is
ibed by the motion of the drop B, will send red light to the eye, while all

drops on the circle described by the motion of C will send violet light to the eye. In this way a colored circular band will be seen whose angular radius is between 40° and 42°.

It will be observed also that the drop A does not send any light at all to the eye at E, while scattered rays of all colors come from the drop D. The region, therefore, above the primary rainbow appears dark, while that within it is bright, and the red of the bow is nearly pure, while the violet is mixed with scattered rays of other colors and fades out into white.

871. Secondary Rainbow.—For those rays that suffer *two* reflections inside a rain drop there is also a certain direction in which the emergent rays are parallel, and therefore the light in that direction is particularly intense.

A colored bow will, therefore, be produced as shown in figure 538, the angular radius of the red being 51° while that of the violet is 54°. The sky

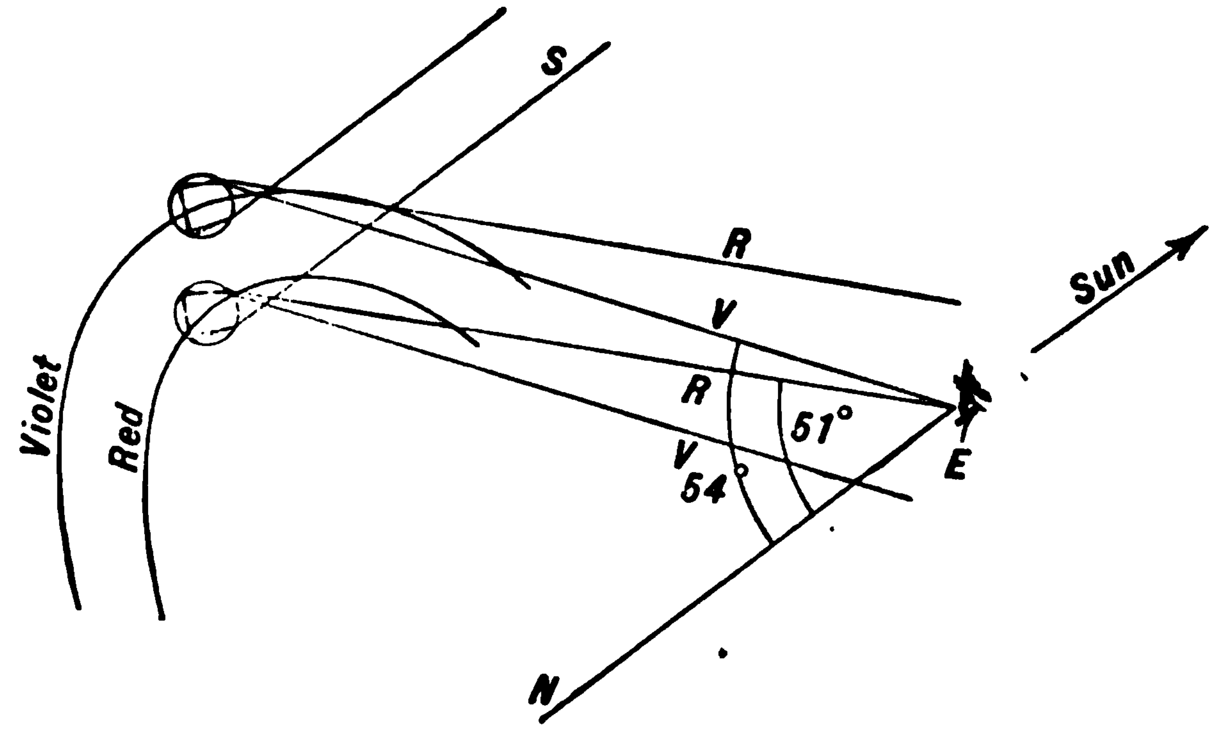


FIG. 538.—Secondary rainbow.

within this bow and between it and the primary bow will be dark while outside of it beyond the violet the sky will be bright with scattered light.

872. Supernumerary Bows.—The bows caused by more than two internal reflections cannot be seen. A second and even a third band of red may, however, be occasionally seen in the violet region of the primary bow. These are called *supernumerary* bows and are *diffraction* phenomena (§931). Their explanation is given in more advanced treatises, such as *Preston's Theory of Light.*

Optical Instruments

873. Optical Instruments.—There are two general classes of optical instruments, those which form a real image on a screen, as in case of the photographic camera and projection lantern (magic lantern), and those intended for direct eye observation in which the image formed is virtual. To the latter class belong the magnifying glass, microscope, and telescope.

To obtain a clear conception of the action of an optical apparatus it is desirable to study the effect of the instrument upon two pencils of light, starting from different points in the object and traced through to the corresponding points in the image. One pencil should be as oblique as can pass through the instrument.

874. Photographic Camera.—In the simplest form of photographic camera a single convergent lens forms a real image of a distant object on the sensitive plate. A diaphragm placed close to the lens limits the size of the pencil of light. The *quickness* of a photographic lens or the brightness of the image, will be proportional to A the area of the diaphragm opening, and

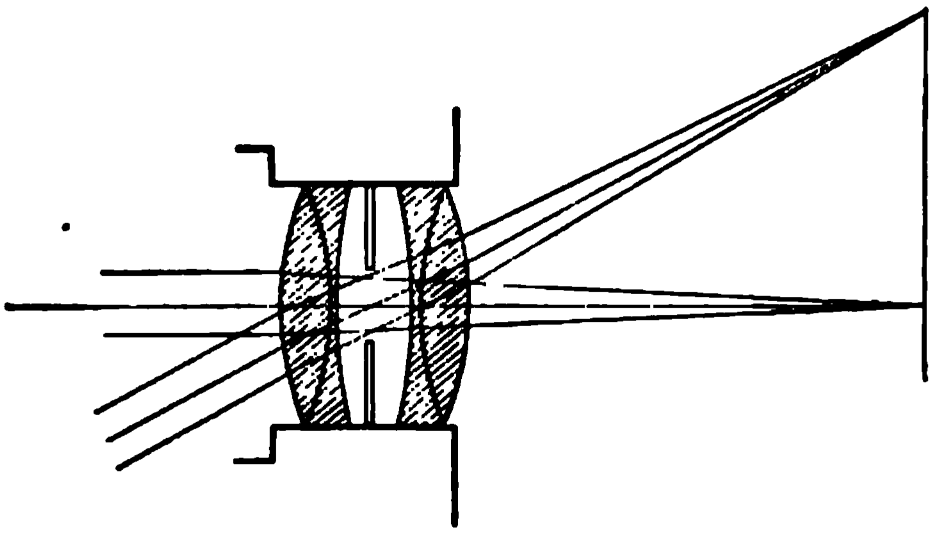


FIG. 539.

inversely proportional to the area of the image over which the light is spread. But the linear dimensions of the image are proportional to f the focal length of the lens, and so the *area* of the image is proportional to f^2. Hence, other things being equal, it is the ratio $\frac{A}{f^2}$ which determines the time of exposure.

Figure 539 represents a symmetrical or "rectilinear" lens consisting of two similar achromatic lenses, symmetrically placed, and having the diaphragm half-way between them. It will be observed that in this case the oblique pencil passes as much below the center of the front lens as above the center of the back lens, so that the beam is as much bent by one as by the other and emerges parallel to the incident pencil. This tends to cause straight lines in the object to be reproduced as straight lines in the image.

875. Distortion of Images.—The image of a grating with equal square openings may be distorted in either of the two modes shown in figure 540. The first or barrel-shaped distortion is seen when the center of the image is magnified relatively more than the outer portions, while the other form of distortion is caused by the greater relative magnification of the parts away from the center.

Either of these modes of distortion may be produced in projecting the image of the grating with the same lens, the form of distortion being determined by the mode of illumination.

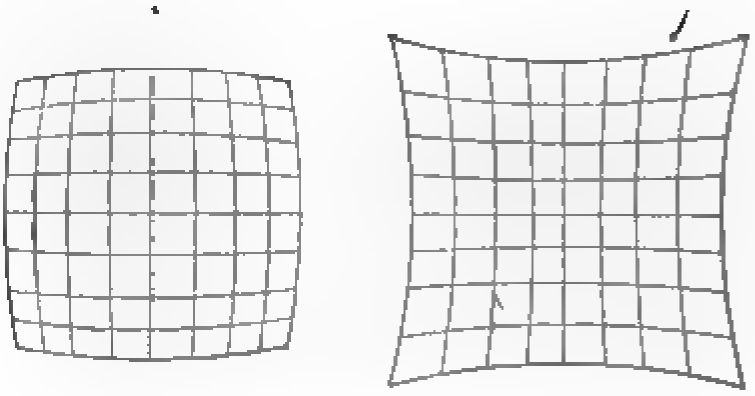


Fig. 540.

For let G (Fig. 541) be the grating and L the lens, then if the grating is illuminated by a beam of nearly parallel light, as direct sunlight, the light from the upper part of the grating will pass through the upper edge of the lens L, and being bent down too strongly in consequence of the spherical aberration of the lens (§861) will come to focus at P farther from the center than P' and will thus cause the distortion

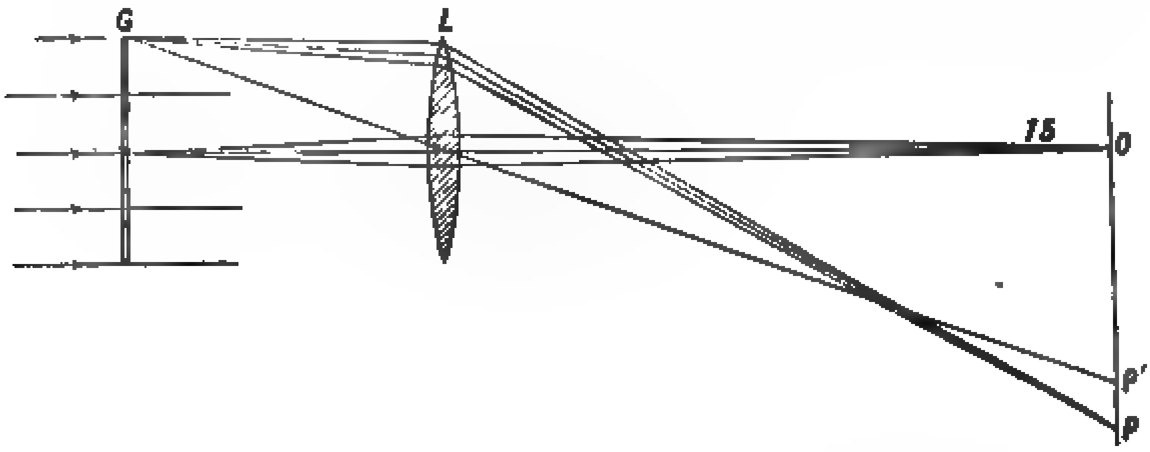


Fig. 541.—Parallel illumination. Pin-cushion-shaped distortion.

shown in the second diagram of figure 540. If, on the other hand, by means of a convergent lens the illuminating beam of light is converged so strongly that rays from the top of G are refracted by the bottom of the lens L, as shown in figure 542, the focus P will be too near the center and the distortion will be barrel-shaped.

It is clear that *the least spherical aberration and distortion will be secured when the illuminating lens converges the light toward the center of the lens L*, as shown in the diagram of the magic lantern, figure 543.

It is this same kind of spherical aberration which causes barrel-shaped distortion in the photographic image when the diaphragm is placed (outside) in front of the lens; while if the diaphragm is behind the lens the opposite form of distortion results.

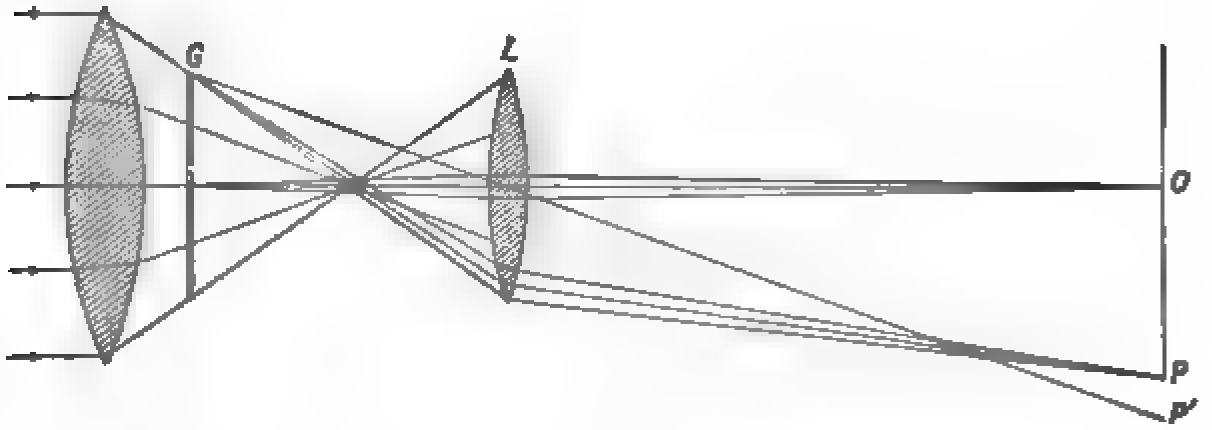


FIG. 542.—Barrel-shaped distortion.

876. Projecting Lantern.—The optical system of the *magic lantern, stereopticon,* or *projecting lantern* is shown in figure 543. It consists simply of a front lens or objective L which forms a real image of the slide S on the screen at S'; and an illuminating system which consists of the source of light at E and the condensing lens C which converges the light through the slide S toward the center of the lens L.

Since the screen S' is usually at a considerable distance, the distance from the slide S to the lens L is nearly the focal length

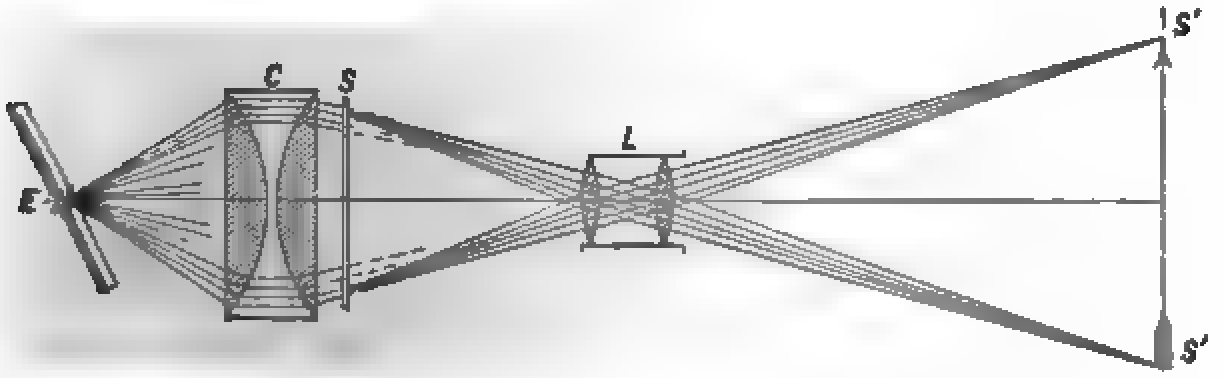


FIG. 543.—Projecting lantern.

of the lens or lens combination. So that the width of the image on the screen is to the width of the slide as the distance of the screen is to the focal length of the front lens L. Hence when the lantern is to be at a great distance from the screen a long-focus front lens should be used to prevent the image from being too large and dim.

The front lens is usually a combination of two lenses to secure flatness of field and freedom from color and distortion. The condensing lens consists of two plano-convex lenses with their convex surfaces almost touching. If the upper portions of the two condensing lenses are thought of as prisms, it will be noticed that with this construction each is nearly in the position of *minimum deviation* for the pencil of light passing through it; for the incident and emergent pencils make somewhat nearly equal angles with the two surfaces of each of the two lenses. Such an arrangement makes the spherical and chromatic aberration very much less than if the lenses had been placed with their flat faces together, which would make practically a single double-convex lens.

877. Projecting Microscope.—For the projecting of microscopic objects the lens L must have a very short focal length to

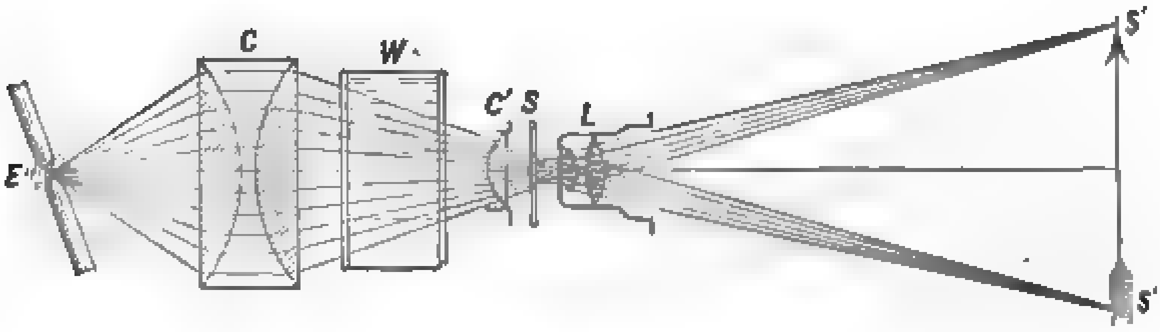


Fig. 544.

secure the requisite magnification. The object must also be intensely illuminated and so a second short focus condensing lens C' is introduced which converges the light from E to a bright focus at the slide S which is to be illuminated. To diminish the heat of the focus at S, which might ruin the slide, a tank of water 3 or 4 in. thick is introduced between C and C'. Water absorbs strongly the very energetic radiations whose wave lengths are too long to affect the eye, though the visible radiation or light is not sensibly weakened. An ordinary microscope may be used in this way for projection either with or without the eye-piece by turning the instrument into the horizontal position and converging a beam of sunlight on the slide S by a convergent lens of 8 or 10 in. focal length.

878. The Eye.—The human herical in shape, having an outer wall of firm of which the

transparent front is known as the *cornea*. Immediately back of the cornea is the *iris*, a variously colored membrane, having a round opening in its center called *the pupil*. The pupil contracts in bright light and dilates in the dark, the iris acting as a diaphragm to regulate the light admitted to the eye. Back of the pupil is the crystalline lens, of rather dense transparent tissue formed in layers and densest at the center. The space between the crystalline lens and the cornea is filled with a clear watery substance, the *aqueous humor*, while the main interior cavity back of the crystalline lens is filled with a transparent jelly-like substance, the *vitreous humor*. At the back of the eye, forming the inner coating of the outer wall, is the *retina*, a highly organized black membrane the surface of which is covered with minute structures called *rods and cones* in which the fibers of the optic nerve terminate.

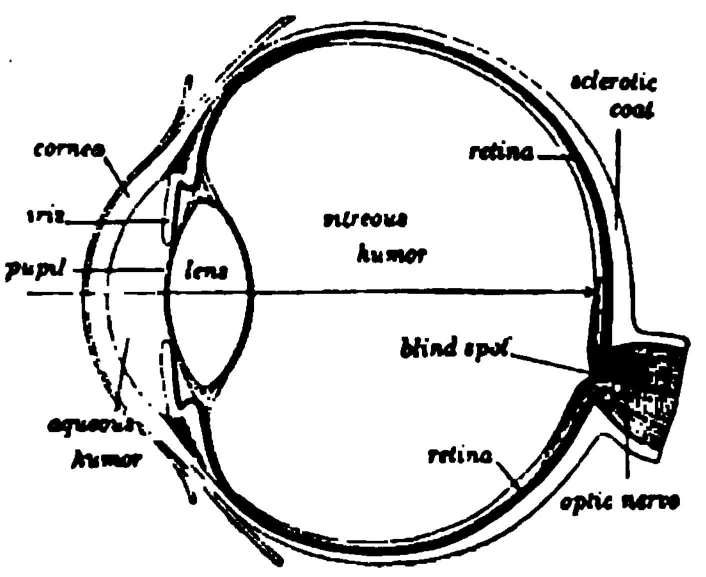


Fig. 545.

Rays from external objects are focused on the retina by the action of the crystalline lens and other refracting portions of the eye. An image is formed on the retina just as the image in a photographic camera is formed on the plate, and each portion of the retina thus receives a stimulus exactly corresponding to the illumination of the particular part of an external object which has its image at that point, and this stimulus of the optic nerve causes the corresponding sensation of brightness and color.

Of course the image formed on the retina is inverted, but we do not see that image; there is simply a correspondence between the retina and external directions, such that when light falls on a spot on the retina it excites a sensation which we describe by saying that it is bright in the corresponding direction.

At the center of the retina and just opposite the pupil and crystalline lens is a spot where the retina is much more highly developed than elsewhere, and to see objects distinctly their images must be formed on that spot. If the eye is directed at a particular point on a printed page only the words close to that point [illegible] s *distinctly*.

Where the optic nerve enters there is a *blind spot* in the retina. To verify this, make a small black spot on a sheet of white paper and covering the left eye look with the right eye at a point about one-fourth as far to the left of the spot as the latter is distant from the eye and the spot will disappear.

879. Accommodation.—A normal eye can change its focus or *accommodate* so as to form on the retina a distinct image either of distant objects or of those as near as about 8 in. This is brought about by muscles attached to the crystalline lens by which it is made flatter for distant objects and more convex for nearer ones.

880. Short and Far Sight.—When the lens of the eye is too convex, objects at ordinary distances are focused *in front of the retina*, so that the image on the retina itself is out of focus and blurred. In such a case objects can be seen distinctly only if held very near the eye, and the person is said to be *short*-sighted, or *myopic*.

If, on the other hand, the lens of the eye is too flat the image of a near object will be formed *back of the retina*, so that indistinct vision results. In such a case it may be that only distant objects can be seen distinctly or it may not be possible to see distinctly at any distance, and the person is said to be *presbyopic*, or far-sighted.

881. Spectacles.—If the lens of the eye is too convex as in case of short sight, a divergent lens may be used to correct the defect, while if the lens of the eye is too flat, as in far sight, a convergent lens must be used.

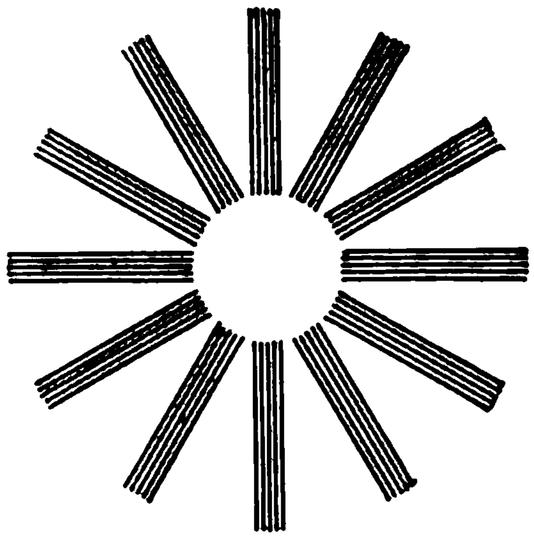


Fig. 546.

882. Astigmatism.—An eye is said to be *astigmatic* when a point of light, as a star, is seen as a short bright line, the direction of which is called the axis of astigmatism. In case of astigmatism all the lines in such a diagram as figure 546 will not appear equally distinct, but those in the direction of the axis of astigmatism will be sharply defined while those at right angles to them will appear broadened and blurred. This defect is caused by the lens of the eye having ellipsoidal instead of spherical surfaces and is corrected by the of cylindrical lenses.

883. Distance of Distinct Vision.—The nearer an object is brought to the eye the larger will be the dimensions of its image on the retina and the more detail will be brought out, provided it is not brought so near that the eye cannot properly focus the image.

A distance of about 10 in. or 25 cm. is the *normal distance of most distinct vision.*

884. Magnifying Glass.—A convergent lens produces a virtual and enlarged image of any object placed slightly nearer to the lens than its principal focus. In the diagram it will be seen that light from each point of the object O after passing through the

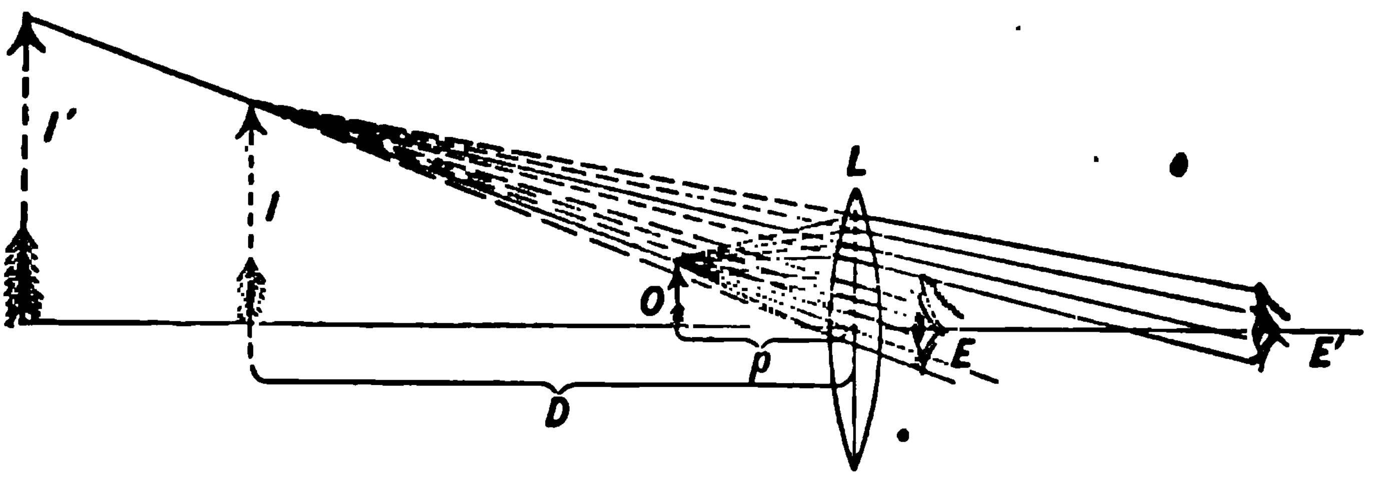


FIG. 547.

lens L comes to the eye as if it had come from the image I. The object is therefore seen enlarged or magnified.

In using a magnifying glass *the eye should be placed close to the lens as at E and the object then be brought up until its image is seen distinctly,* for in that case the rays from all parts of the object come to the eye through the *central part of the lens,* and are subject to the least spherical and chromatic aberration; whereas if the eye were placed at E' rays coming to the eye from the point of the arrow would be refracted near the edge of the lens and consequently there would be distortion of the image as well as other aberrations.

The angular dimensions of the image as seen by the eye placed at E close to the lens is practically the same whether the image is formed at I or I' or at a still greater distance. Therefore the *apparent size* of the image and consequently the magnification is substantially unchanged.

The magnifying power of the lens is the ratio of the length of the *image I, formed* at the distance of distinct vision D, to

the length of the object O; for in order to see the object distinctly without the lens it must be placed at the distance D from the eye.

To determine the magnifying power, let f represent the focal length of the lens and let p and D represent the distances from the lens of O and I, respectively, then by definition the magnifying power of the lens $= \frac{I}{O}$

but

$$\frac{O}{I} = \frac{D}{p}$$

and from the general lens formula (§853) we have

$$\frac{1}{p} - \frac{1}{D} = \frac{1}{f}$$

multiplying through by D this becomes

$$\frac{D}{p} = 1 + \frac{D}{f}$$

therefore the magnifying power of a lens $= 1 + \frac{D}{f}$

where D is the distance of most distinct vision and f is the focal length of the lens.

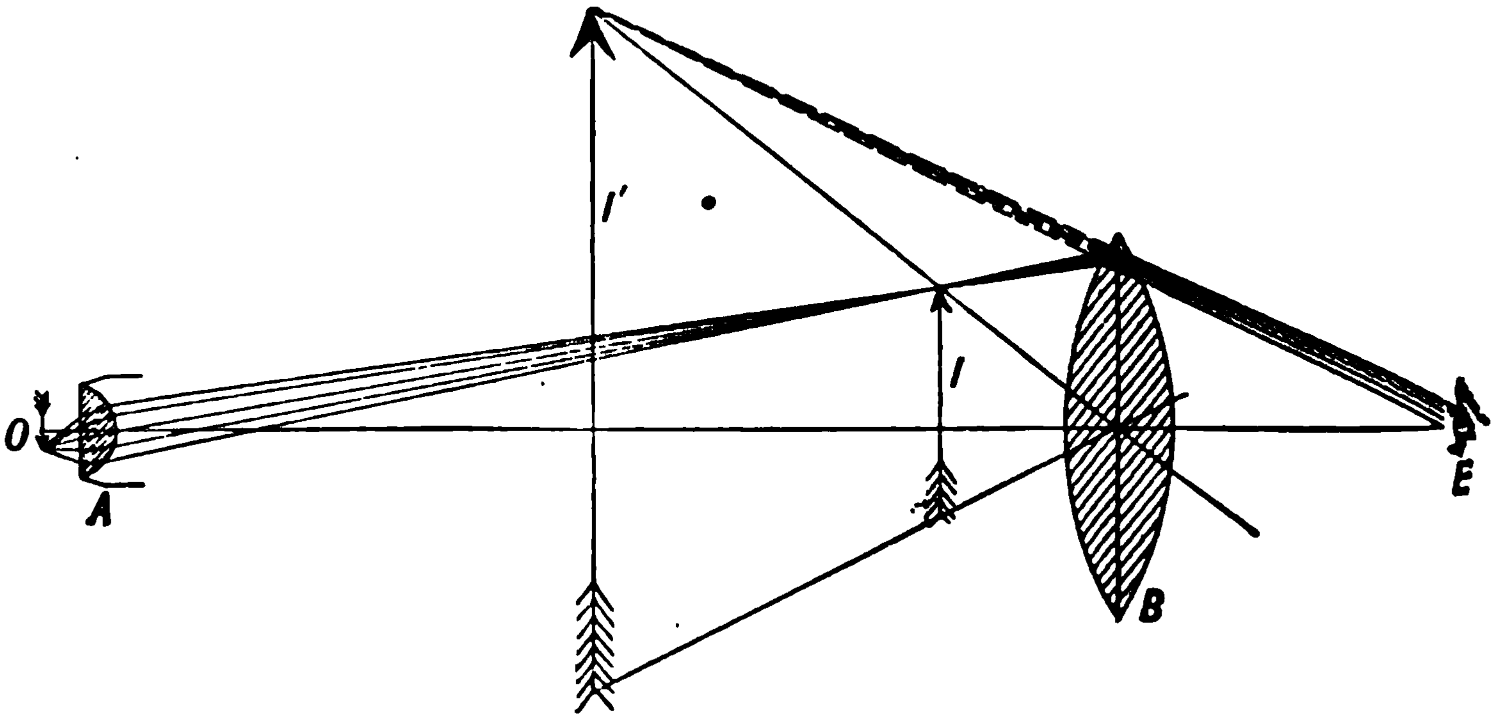


Fig. 548.—Compound microscope.

885. Compound Microscope.—For the highest magnification the compound microscope is used, the optical system of which is shown in figure 548. The object O to be magnified is placed just outside the focus of the *short-focus lens* A, called *the objective*, which forms a real image at I. Back of this image is placed a magnifying glass at a distance slightly less than its focal length

so that it forms a virtual image of I at I' which may be seen by the eye at E.

Since O is nearly at the principal focus of the lens A, the image I will be as many times greater than the object O as the distance AI is greater than the focal length of A. The distance AI in an ordinary microscope is about 150 mm., so that if the focal length of the objective is 5 mm. the image I will be 30 times as large as the object, and if the eye-piece or lens B has a magnifying power 10, the power of the combination is $30 \times 10 = 300$ diameters.

Many microscopes have a "draw tube" by which the distance between the objective and eye-piece may be increased. The effect of this is to increase the magnifying power of the instrument in the same ratio.

886. Eye-pieces and Micrometer. The *eye-piece* or *ocular* of a microscope usually consists of a combination of two lenses instead of the simple lens B shown in the previous diagram. Two forms are in use. One

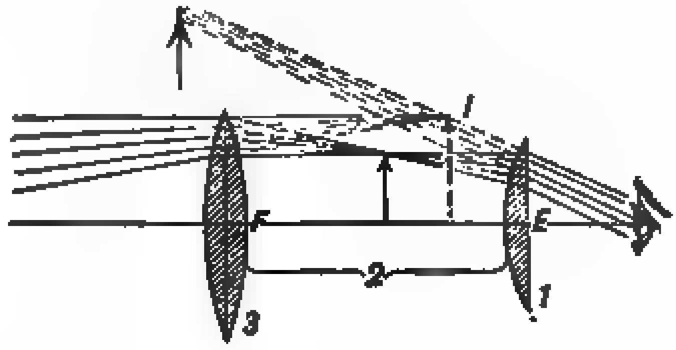


FIG. 549.—Huygens' or negative eye-piece. Focal length = $1\frac{1}{2}$.

of these, the *Huygens* or *negative eye-piece*, consists of two convergent lenses, a *field lens* F and an *eye lens* E, whose focal lengths are in the ratio 3 to 1, and the distance between them is twice the shorter focal length. It is equivalent to a single lens whose focal length is $1\frac{1}{2}$ times that of the eye lens, but it has the advantage of being effectively acromatic if both lenses are made of the same kind of glass, and gives less distortion and aberration than a single lens.

In this eye-piece the rays from the objective must fall on the field lens *before* coming to a focus at I, as shown in the figure.

For rough measurements this eye-piece may be provided with a scale ruled on glass, known as an *eye-piece micrometer*, which is fixed half-way between the two lenses so that it coincides with the image formed by F.

But for more exact measurements the microscope is provided with a *micrometer* (in which a cross-hair is moved by a screw having a graduated head) placed at I where the image is formed by the objective, and the eye-piece must be placed *back* of this point so as to magnify image and cross-hairs together.

For this purpose the *Ramsden* or *positive* eye-piece must be used. This form of ocular consists of two convergent lenses, a field lens and an eye lens of equal focal length the distance between them being two-thirds of the focal length of either lens. This eye-piece is placed back of the image *I* just as though a single lens were used. It is equivalent to a single lens whose focal length is $\frac{3}{4}$ that of its component lenses, and is called *positive* because it

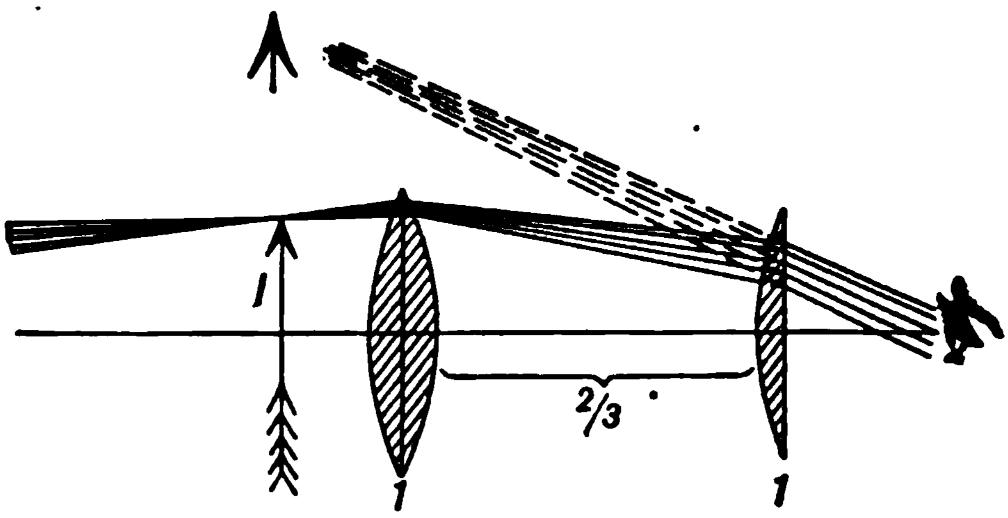


FIG. 550.—Ramsden's or positive eye-piece. Focal length = $\frac{3}{4}$.

can be used, like an ordinary simple magnifying glass, to magnify any small object. It is nearly achromatic and may give a flat field with little aberration.

887. Microscope Objectives.—The object-glass of a microscope usually consists of a nearly hemispherical front lens of crown glass with its flat face outward, having one or more achromatic combinations mounted back of it as shown in the figure. The curvatures of the lenses and their distances apart are calculated so as to give an image as free from aberration as possible. High powers are corrected for a particular tube length and thickness of cover-glass, and to obtain the best results these conditions must be satisfied. If between the front lens of the objective and the cover-glass a drop of oil is introduced having the same index of refraction as the glass, it is as though the object were imbedded in the glass of the front lens. In this way loss of light by reflection from the glass surfaces is avoided, no correction for the thickness of the cover-glass is required, and the brightness and definiteness of the final image are increased. But to secure these results the objective must be especially designed for this use. Such objectives are known as *oil* or *homogeneous immersion* lenses, and may have as short a focal length as $\frac{1}{12}$ or $\frac{1}{16}$ in.

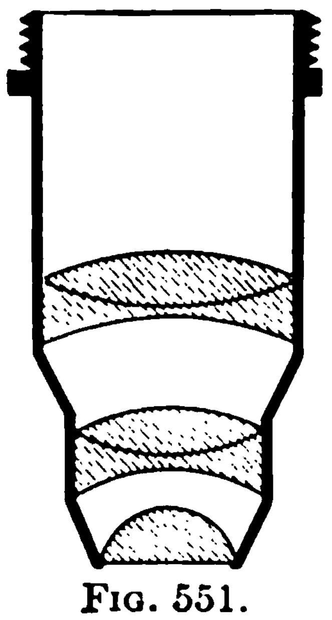


FIG. 551.

888. Numerical Aperture.—The size of the pencil of light transmitted through the microscope from a single point of the object has an important influence on the defining power (§937). *The ratio of the radius of the largest cross section of such a pencil to the focal length of the objective is known as its num*[illegible]*re*, and, other things being equal, the resolving *power* of an obj[illegible]onal to its numerical aperture.

889. Telescopes.—In telescopes a convergent lens, known as the *object-glass*, or a concave mirror, is used to form a real image of a distant object, and this image is then magnified by a suitable eye-piece. **Since the object is distant the image is formed at the principal focus of the object-glass or mirror, and consequently to have a large image and great magnifying power the object-glass must have long focal length.** There are three kinds of refracting telescopes, Galileo's, the astronomical, and the terrestrial forms.

890. Galileo's Telescope.—In Galileo's telescope a concave or divergent lens is used as the eye-piece. This lens is placed so that rays from the object-glass meet it *before* forming the image *I* as shown in the figure. If the distance from *L* to *I* is slightly

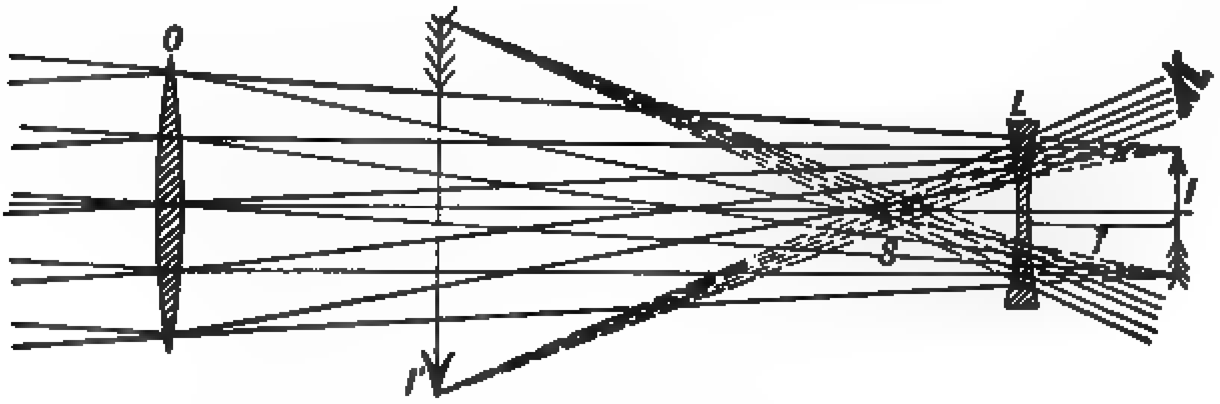


Fig. 552.—Galileo's telescope.

greater than the focal length *f* of the eye lens, rays approaching the point of the image at *I* will be bent upward and made to diverge as if from *I'*. An enlarged virtual image is thus formed which may be seen if the eye is placed so as to receive the emergent pencil. It will be observed, however, that the pencils of light from all points in the object come to the eye as if intersecting at *S*, so that it is as though the virtual image were seen through a small opening at *S* which restricts the field of view, only so much being seen in any one position of the eye as is in line with the eye and *S*. **The smallness of the field of view is the great defect in this form of telescope and causes its use to be restricted to low-power instruments, such as the ordinary opera-glass.** Its advantages are that it is shorter than the other forms of telescope and gives an erect image of the distant object.

891. The Astronomical Telescope.—In this instrument the eye-piece is a *convergent lens*, or an equivalent Huygens or Rams-

den eye-piece as in the microscope, §886. The eye lens, as shown in the diagram, is placed nearly its own focal length back of the image at I formed by the object-glass, so that a virtual enlarged image of I is formed at I'. It will be noted that the pencil of rays from the lower part of the distant object comes to the eye as if from the upper part of the image at I'. In this instrument, then, the image is inverted, and it is therefore used chiefly for astronomical observation. The various pencils of rays coming to the eye from different points in the virtual image all intersect at S, forming a bright spot known as the *eye spot*. If the pupil of the eye is held at this point all parts of the virtual image can be seen simultaneously, and the field of view is large, being limited only by the size of the lens L.

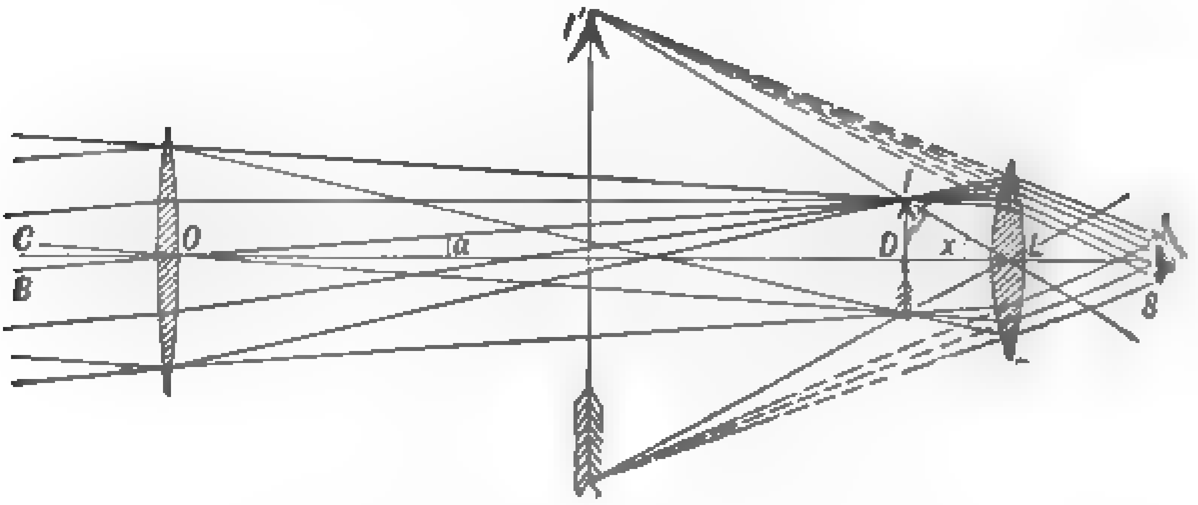


FIG. 553.—Astronomical telescope.

892. Magnifying Power of a Telescope.—The semi-diameter of the distant object as it would be seen without the telescope subtends the angle COB or IOD (Fig. 553) where CO is a ray coming from the middle of the object and BO is a ray from its edge; while the angle subtended by the corresponding part of the image seen in the telescope is $I'SD$ or ILD. The magnifying power of the instrument is therefore the ratio

$$\frac{\tan ILD}{\tan IOD}.$$

Since OD is equal to F, the focal length of the object-glass, and DL is nearly equal to f, the focal length of the eye lens, we have

$$\tan IOD = \frac{ID}{F}, \qquad \text{and} \qquad \tan ILD = \frac{ID}{f},$$

$$\text{therefore} \qquad \frac{\tan ILD}{\tan IOD} = \frac{F}{f}.$$

or the magnifying power of an astronomical telescope is equal to the number of times that the focal length of the eye-piece is contained in the focal length of the object-glass.

It may also be shown that the magnifying power is the ratio of the diameter of the object-glass to the diameter of the eye-spot S, for the latter is the image of the object-glass formed by the lens L.

It will be shown later (§937) that the *defining power* of a telescope is proportional to the diameter of the object-glass, supposing it to be perfect.

Hence for detecting close double stars or for investigating minute details on the surface of sun or planet where high powers must be used, it is necessary to employ a telescope of large aperture. Large lenses are used also on account of their light-gathering power in observing faint objects, such as nebulæ.

893. Terrestrial Telescope.—To obtain a large field of view and at the same time an erect image of distant objects the terrestrial telescope is used.

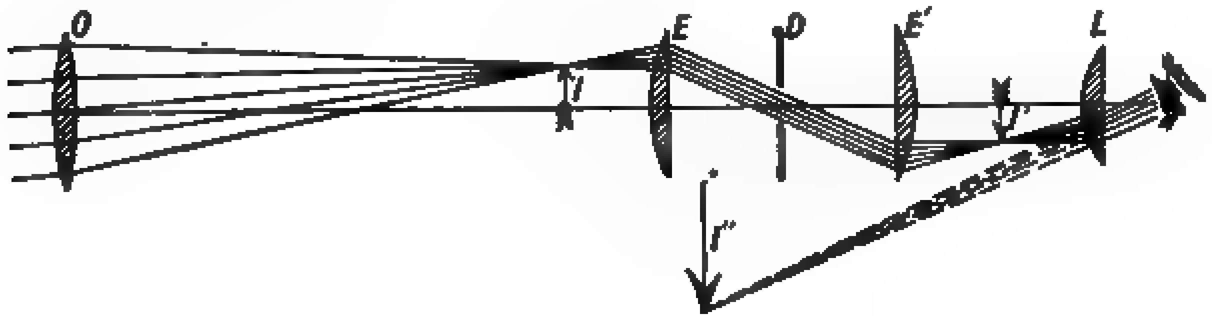


Fig. 554.—Terrestrial telescope.

This instrument is like the astronomical telescope except that there are two additional convergent lenses, E and E', introduced between the object-glass O and the eye lens L as shown in the diagram. These lenses invert the image I, forming another real image at I' with the point of the arrow downward as in the distant object. This image is magnified by the eye-piece at L, which forms the enlarged erect virtual image I''. In the ordinary spy-glass the lens L is replaced by a Huygens eye-piece, making four lenses in all besides the object-glass.

At D a diaphragm is introduced having a hole in the center just large enough to transmit the pencils of light which intersect at that point. This serves to stop any stray light which may be reflected from the sides of the tube.

894. Prism Binoculars.—Field glasses which combine high magnifying power and large field of view are now made according to the plan shown in figure 555.

The beam of light from the object-glass enters a right-angled glass prism and after two internal reflections, at *A* and *B*, is completely reversed in direction and travels back to a second prism, placed at right angles to the first, where it is again totally reflected, at *C* and *D*, and turned back again toward the eye lens *E*.

The reflections in the two prisms secure an erect image without using the reversing lenses of the ordinary terrestrial telescope; for

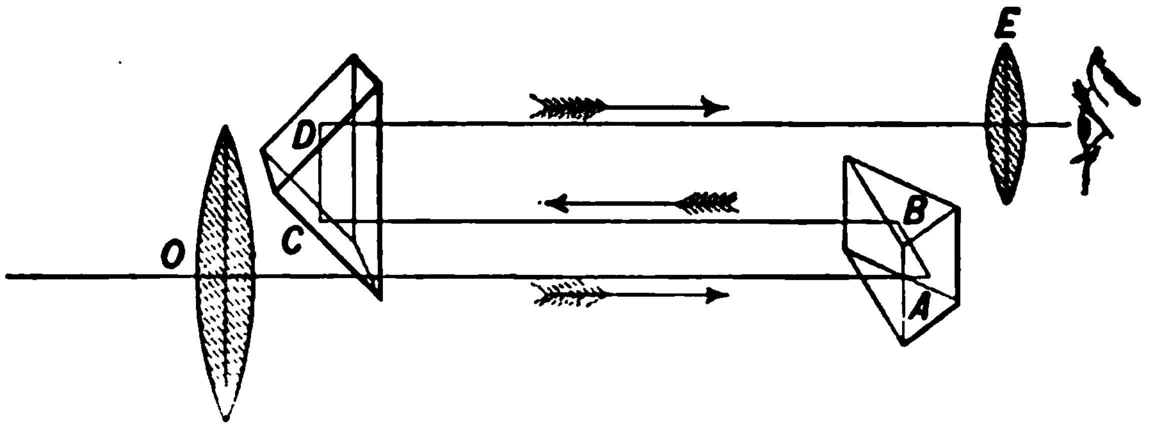


Fig. 555.—Path of rays in prism binocular.

one prism interchanges the two sides of the image, while the other makes it upright, thus restoring it completely to its natural position.

Also, on account of the length of the path of the rays from the object glass to the eye lens, the focal length of the object glass may be three times as great as in the ordinary field glass, and the magnifying power correspondingly increased.

895. Reflecting Telescopes.—Instead of the object-glass of a telescope, a long-focus concave mirror may be used. The arrangement shown in figure 556 was used in Sir William Hershel's great telescope, the mirror *M* being slightly inclined so that the eye-piece and the observer's head were not in the line of the rays falling in the mirror. In small reflecting telescopes the rays converging toward the image *I* may be reflected out sideways to the eye-piece by a small mirror placed directly in front of the large mirror, as was done by Newton. To obtain a perfect image free from spherical aberration the mirror must be parabolic instead of spherical. A mirror has the advantage of forming an image *free from chromatic aberration* since all wave lengths of *light* are reflected alike.

A telescope mirror is called a *speculum*, and may be made of an alloy called speculum metal which takes a fine polish and does not readily tarnish. Specula are now, however, usually made of glass, as this is harder and less dense than speculum metal. The surface is ground and polished to the required shape and then silvered.

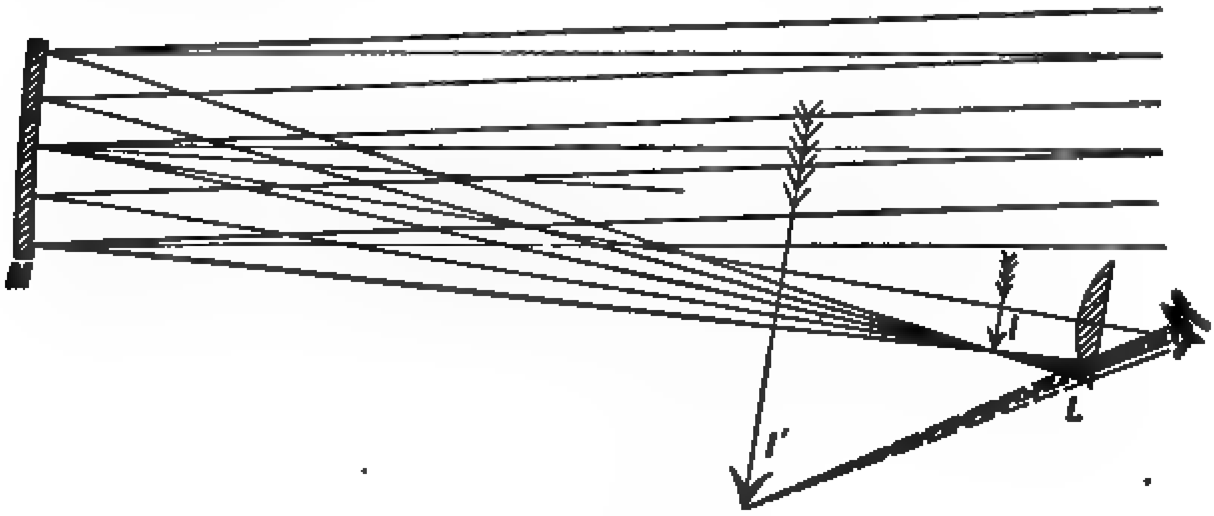


Fig. 556.—Herschel's form of reflecting telescope.

Problems

1. What is the magnifying power of a simple convergent lens of focal length 5 cm.? Take 25 cm. as distance of most distinct vision.
2. What is the magnifying power of an astronomical telescope in which the focal length of the object glass is 12 ft., while that of the eye-piece is 1.5 in.?
3. A compound microscope has an objective of 1 in. focus, the first image is formed 5 in. back of the objective and is magnified by an ocular of 2 in. equivalent focus. Find the magnifying power of the combination.
4. When a telescope is pointed at the sun, how should the eye-piece be placed to give a real image of the sun on a screen fixed back of the telescope? In this case is the image formed by an astronomical telescope erect or inverted?
5. In a projection apparatus it is desired that the pictures shall be 10 ft. wide on a screen 40 ft. distant, when the slides are 3 in. wide. What must be the focal length of the projecting lens used?
6. By means of a microscope objective, a scale having 50 lines to the millimeter is projected upon a screen 9 meters distant, and the distance between the lines in the image on the screen is 4 cm. What is the focal length of the objective used?
7. A certain binocular field glass has a power 8 and diameter of objective 25 mm. while another of power 6 has an objective 21 mm. in diameter. Which will give the brighter image and in what ratio?

Reference

J. T. Taylor: *Optics of Photography.*

Analysis of Light

The Spectrum

896. The Spectrum.—It has been seen that when light passes through a prism it is spread out in a colored band shading from red to violet and called the spectrum, showing that a beam of white light is complex and made up of different kinds of light which are separated by the prism in consequence of their different refrangibilities. These lights also differ in the color sensations which they excite, the least refrangible being the red rays while the most refrangible are the violet.

It will be shown later that the physical property which determines the refrangibility of a ray of light is *wave length;* so that *in forming a spectrum we are really spreading out the light in the order of wave lengths, the longest waves being at the red end of the spectrum and the shortest at the violet end.*

897. Pure Spectrum.—To obtain a complete analysis of light there must be no overlapping of different kinds, but each must

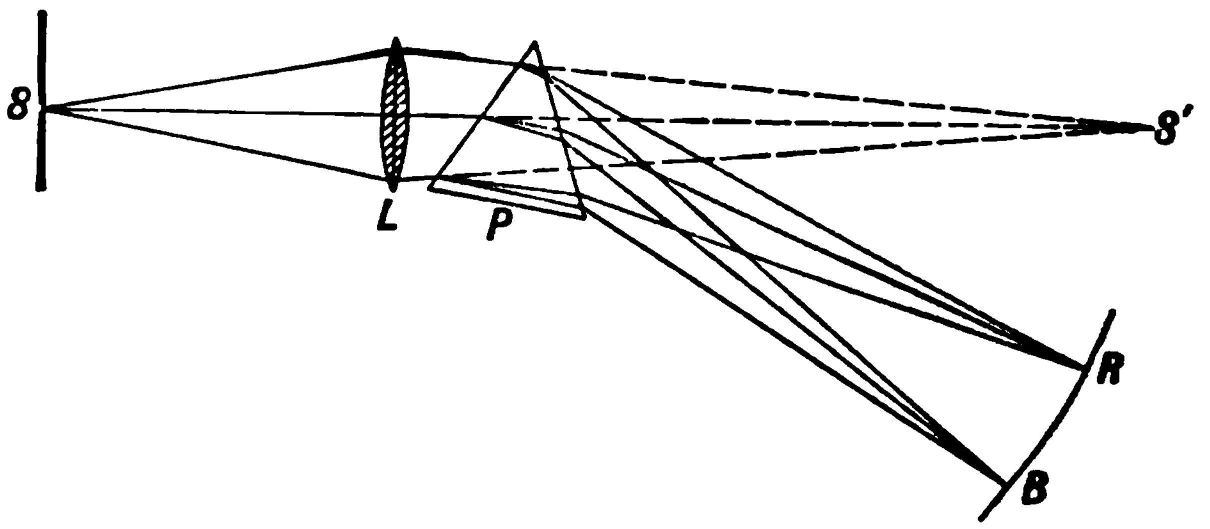
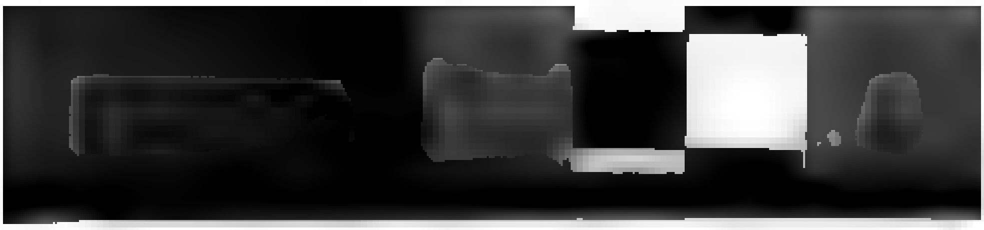


Fig. 557.

be separated by the prism from every other. To accomplish this the following arrangement may be employed.

The light to be examined enters through a narrow slit S, which is parallel to the edge of the prism and therefore perpendicular to the plane of the paper in the diagram. A converging lens L is so placed that it would bring the light to focus and form at S' an image of the slit if the prism were not interposed. By the action of the prism, however, the light is refracted downward and if there were only one kind of light present the whole beam would be equally refracted and the bright image of the



slit would be formed at R, say, instead of at S', but without any change in color. If, however, there were in the original beam two kinds of light which were differently refracted, there would then be formed two images of the slit, one at R and one at B. And since light waves that are refracted differently also act differently upon the eye, the images at R and B will be of different colors. But if the original beam contained waves of every

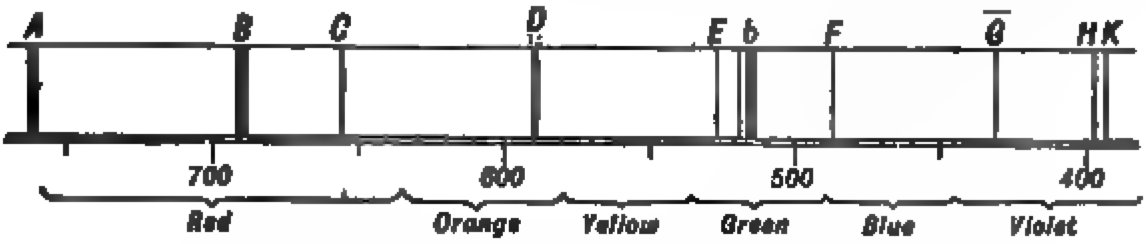


FIG. 558.—Fraunhofer lines. The wave lengths are given in millionths of a millimeter.

conceivable degree of refrangibility within certain limits, there would be an infinite number of images of the slit with no separation between them and even overlapping, producing a continuous band of color, shading from one extreme to the other. When the slit is narrow so that the amount of overlapping is small the spectrum is said to be *pure*.

898. Fraunhofer Lines.—When the *sun spectrum* is formed as above described, using a narrow slit so as to produce a pure spectrum, a large number of dark lines are observed which cross the spectrum parallel to the slit, showing that sunlight does not contain all kinds of light, but that certain wave lengths are lacking, and consequently no bright images of the slit are formed at the corresponding points of the spectrum. These dark lines characteristic of sunlight were first carefully studied by Fraunhofer and are known as the *Fraunhofer lines*. Some of the most prominent of them are designated by letters of the alphabet, and furnish convenient points of reference in the spectrum, the A line being almost at the limit of visibility in the red while the H line is near the extreme violet.

899. Analysis of Light by Spectroscope.—For the more exact analysis of the light from any source a *spectroscope* is used. The main features of this instrument are indicated in figure 559. Light from the source to be studied enters the narrow slit S at the focus of the *collimating* lens, diverges through that lens

and then passes as a parallel beam to the prism *P* where it is refracted and dispersed. After passing the prism the beam enters the telescope and forms a sharply defined spectrum at the principal focus of the object-glass of the telescope, a separate image of the slit being formed by each wave length of light present. This spectrum is magnified by the eye-piece of the telescope.

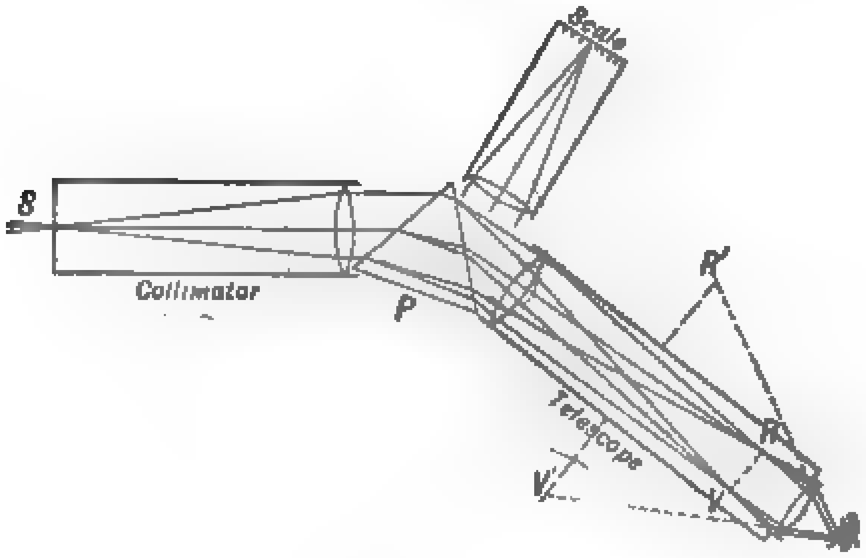


FIG. 559.—Spectroscope.

An illuminated scale is also sometimes mounted so that light from the scale, reflected at the second face of the prism, enters the telescope and forms an image of the scale along with the spectrum at *RV*. The observer can by this means locate a definite line in the spectrum by its position on the scale.

To compare the spectra from two separate sources a *comparison prism* is sometimes used. This is a small total reflecting prism covering one-half of the length of the slit. Light from one

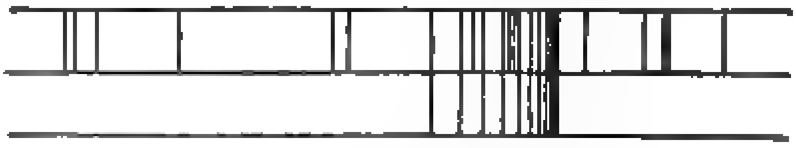
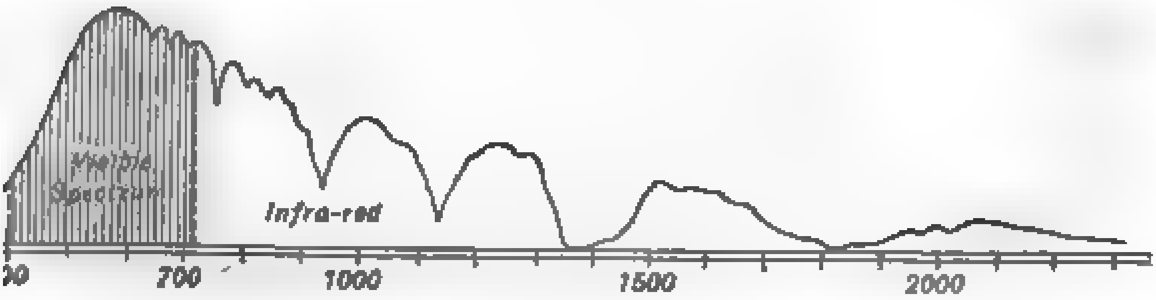


FIG. 560.—Diagram of a carbon band below; the upper spectrum has also lines due to other substances.

source enters the spectroscope directly through the uncovered half of the slit, while the other source is so placed that its light is reflected by the prism through the other half of the slit. The two spectra are formed side by side as shown in figure 560, the wave lengths of lines that match in the two spectra being the same.

o secure greater dispersion than can be obtained with a le prism a train of several prisms may be used, so placed that light passes through them in succession before entering the scope.

)0. The Complete Spectrum.—Instead of observing the :trum by the eye, it may be received on a photographic e and photographed. When this is done it is found that there rays beyond the violet light of the visible spectrum and still ·ter in wave length, which act on the photographic plate but invisible to the eye. These rays are called *ultra-violet* light. nother mode of examining the spectrum is by means of a cate thermopile or bolometer by which the heating effect he rays at each point in the spectrum may be determined. fessor Langley perfected an apparatus of this sort in which



:o. 561.—Energy spectrum, after Langley. Curve showing distribution ergy in the sun spectrum, plotted by wave lengths in millionths of a milli- er.

bolometer filament was carried along by clockwork from end of the spectrum to the other, while at the same time a m of light reflected from the mirror of the galvanometer fell a moving strip of sensitized paper recording the deflection :he galvanometer for every point in the spectrum. A curve ained in this way showing the heating effect of different parts the sun spectrum is shown in figure 561.

luch a study shows that beyond the red end of the sun spec- m there is an invisible region where the radiation has great rgy or heating power. This region is known as the *infra-red*, l its waves are longer than those of the visible red. The ded area in the diagram represents the distribution of energy :he *visible* spectrum.

f a plate of clear glass is held between the face and an open the *warmth of the radiation* is greatly cut off, though the

visible radiation is almost completely transmitted. The change is due to the power of glass to absorb the infra-red radiation. When the spectrum of fire light is taken by a bolometer first directly and then through a pane of glass a comparison of the two curves shows just what wave lengths are absorbed by the glass.

There is no reason to suppose that the infra-red or ultra-violet light is different except in wave length from ordinary visible light. The luminous body is to be thought of as giving out waves of different lengths from the extreme infra-red to the extreme ultra-violet; all of these waves have *energy* and consequently have heating effect, but only certain wave lengths can affect the eye. The luminous effect of the part called the *visible spectrum* is due to a peculiar response which waves of certain frequencies excite in the eye and which we call the sensation of light. The photographic effect of certain waves also depends on the responsiveness of the substances acted on, to waves of a special frequency of vibration.

The earlier writers speak of heat rays, luminous rays and chemical or actinic rays as though there were three different kinds of radiation. In the use of these terms care should be taken to guard against such a misconception.

901. Absorption in Spectroscopes.—A glass prism absorbs strongly long radiation beyond the visible spectrum and also the shorter waves in the ultra-violet. Hence *to study the energy spectrum in the infra-red* a prism of rock salt, fluor spar, or sylvite must be used.

Of these sylvite transmits waves up to 0.025 mm. in length, while the range of rock salt and fluorite is somewhat less, 0.020 mm. being the limit with the former and 0.011 mm. with the latter. For very long waves, more than 0.050 mm. in length, quartz is transparent, though it absorbs strongly the shorter wave lengths transmitted by rock salt and sylvite.

In the study of *the short waves in the ultra-violet* part of the spectrum quartz prisms and lenses are used. But the shortest waves are so strongly absorbed by air that they can be detected and studied only when the apparatus is in a vacuum. In this way Lyman and Schroeder have photographed light waves shorter than 0.0001 mm.

902. Kinds of Spectra.—There are three different kinds of ectra: *continuous, bright-line, and absorption spectra.*

A continuous spectrum contains all wave lengths between rtain limits, and if visible appears to the eye as a continuous ınd of color shading from one end to the other. Hot solids and ıuids give rise to continuous spectra.

Bright-line spectra are obtained when the source of light gives ıt only certain definite wave lengths. Each wave length gives bright line in the spectrum, the intensity of which depends on e energy of the corresponding mode of vibration. Gases and ıpors when rendered incandescent by heat or by the electric scharge give bright-line spectra.

These spectra are highly characteristic, every known substance ıving a different spectrum which depends to some extent on e method used to make it luminous. The investigation of bstances having lines in their spectra that could not be tributed to any known element has led to the discovery of a ımber of new elements. The identification of substances by eir spectra is known as *spectrum analysis.*

Absorption spectra are obtained when the light from some source ıich would give a continuous spectrum is made to pass through me absorbing medium and then analyzed by the spectroscope. Spectra due to absorption by solids or liquids usually show oad absorption bands shading off at both edges, while absorpon by gases and vapors gives rise to sharply defined black lines the spectrum, showing that only certain special wave lengths e absorbed. The Fraunhofer lines in the sun spectrum are lieved to be produced in this way.

903. Production of Bright-line Spectra.—If a loop of platinum re is dipped into a solution of some salt of sodium, potassium, rium, lithium, or strontium, and is then held in a very hot n-luminous flame, like that of a Bunsen burner, the flame comes colored, bright yellow in case of sodium, red by lithium d strontium, and violet by potassium; and when the colored ht is examined by the spectroscope, the spectrum is found consist of certain characteristic bright lines. Usually certain these lines are particularly bright and prominent and deterine the characteristic color, but a higher temperature will ten bring out others of less intensity.

To obtain the spectra of substances such as iron, copper, zinc, etc., which volatilize only at very high temperatures, the electric arc may be used, the substance to be studied being introduced into a cavity in the end of one of the carbons. In this case the spectrum of the carbon arc is also present and its lines must be distinguished from those due to the substance which is introduced. A better method is to obtain the arc between terminals of the pure metal which is to be studied, though there are practical difficulties in carrying this out.

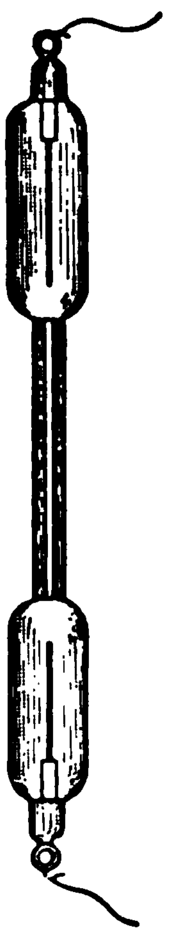


Fig. 562.

Still another method of obtaining spectra of these substances is by the *spark discharge* between terminals made of the substance whose spectrum is to be determined. The sparks are produced by an induction coil and intensified by the use of a Leyden jar having its inner coating connected to one terminal and its outer coating to the other. If the discharge is sufficiently intense, lines are observed which are characteristic of the substance of which the terminals are made.

The spectrum obtained by volatilizing a substance in the *electric arc* is generally somewhat different from its spark spectrum.

To study the spectrum of a gas it is usually enclosed in a tube such as shown in figure 562, known as a Plücker tube, and made to glow by sending the electric discharge from an induction coil between its two aluminum electrodes. The two bulbs at the ends containing the electrodes are connected by a capillary tube in which the discharge is concentrated and intensely luminous. It is this capillary portion which is placed in front of the slit of the spectroscope.

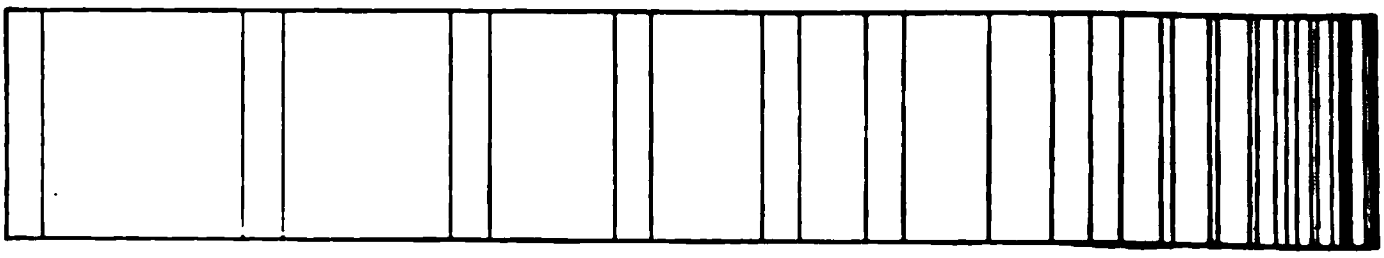
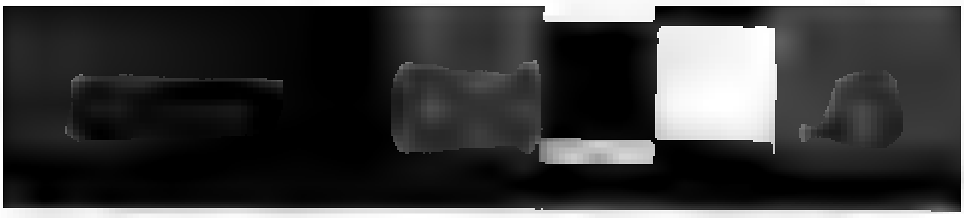


Fig. 563.—*B* group. Sun spectrum.

904. Fluted Spectra.—In some cases the spectrum presents the appearance of a series of shaded bands or *flutings*, well shown in the spectrum of nitrogen. But when examined with



a spectroscope of high dispersive power these flutings are each seen to be made up of a regular series of bright lines crowded closely together near the bright edge of the fluting and at distances apart which increase regularly from the bright to the faint edge of the fluting, somewhat as shown in figure 563 which represents a remarkable group of lines in the red end of the sun's spectrum, the *B* group.

905. Source of Light Waves.—Since each line in the spectrum of a substance is produced by waves of a certain wave length and period, it follows that there must be just as many different periods of vibration originating in the radiating atoms, as there are lines in their spectrum. This points to a considerable degree of complexity in the structure of the atom. It cannot be considered a little hard indivisible and structureless unit.

Evidence has already been cited (§814) that light waves are electromagnetic, and must therefore originate in electric charges which vibrate or oscillate in some manner in the atom. *According to the electron theory* (§789) *the source of light and of all radiation from the atom is found in the vibrations of its electrons.*

But it is not necessary to suppose that *each* atom of the substance gives out *all* the different wave lengths in its spectrum, for the spectrum of a gas may be supposed to be made up of some lines due to atoms in one state, other lines due to atoms in another state, etc., so that the observed spectrum is the combined effect of all the states possible for that sort of an atom. For example the electron theory supposes the hydrogen atom to be a positive nucleus around which a single electron rotates in an orbit, but according to a theory advanced by Bohr, there may be many possible distances from the nucleus at which the electron may move and be in equilibrium. When the equilibrium of the electron is disturbed, perhaps by the impact of some outside electron, it may be supposed to shift from one distance from the nucleus to another giving out radiation and energy until it settles down to its new state of equilibrium, the wave length given out in such a transition depending on the distance of the electron from the nucleus. The complete spectrum of hydrogen would include wave lengths corresponding to each possible transitional state, since some atoms would be in one state, and some in another, all possible states being represented.

An important cause of radiation is *ionization* (§779), where electrons are driven out of some atoms and recombine with others. Thus ionization accompanied by energetic radiation is especially marked in case of electric discharge through gases, in the electric arc, and in flames where there is high temperature and chemical

action. The bright stratified layers in a Geissler tube are believed to be regions where ionization is most intense.

In gases the average distance that the atoms move between impacts, or the mean free path as it is called, is so large compared with the period of oscillation of the electrons in radiation, that an immense number of undisturbed vibrations may take place between impacts, and these give the characteristic lines in the spectrum of the gas or vapor.

In liquids and solids, on the other hand, the atoms are so crowded together and are so incessantly clashing against each other at high temperatures, that the electrons in the atoms may be supposed to have their natural free periods of vibration constantly disturbed by the irregular impacts of other electrons, so that every possible wave length is given off and the spectrum is continuous.

906. Explanation of Fraunhofer Lines.—When the sun's spectrum and that of some elementary substance are photographed

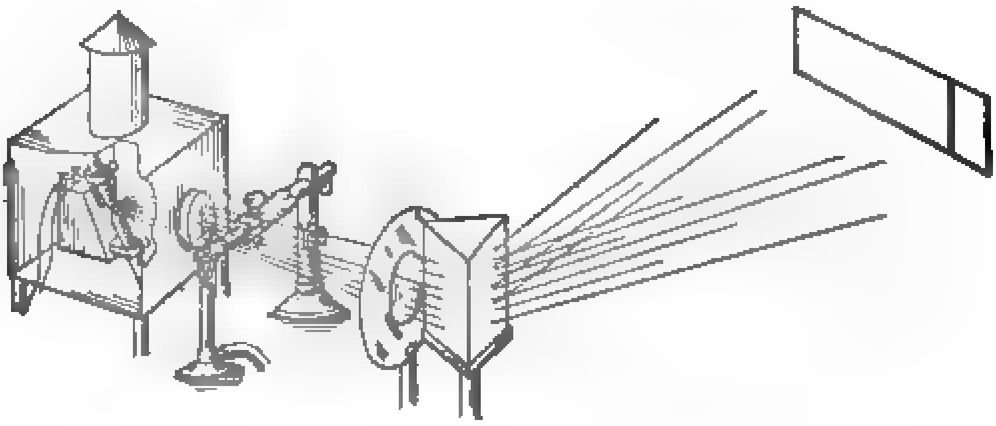


Fig. 564.—Absorption by sodium vapor.

beside each other on the same plate it is found in many cases that the bright lines in the spectrum of the substance exactly match, line for line, certain of the dark Fraunhofer lines in the sun's spectrum. For example, the two yellow sodium lines exactly coincide with the two *D* lines in the solar spectrum.

The explanation of these dark lines in the solar spectrum was given by the German physicist Kirchhoff, who showed that they might be caused by the absorption of light coming from the deeper layers of the sun in passing through the cooler vapors in the sun's outer atmosphere, and announced the principle that a vapor or gas will absorb most powerfully light of the same

wave lengths as the light which the same gas or vapor gives out when it is itself the source of radiation. This principle is illustrated by the following experiment. Form a pure continuous spectrum as described in §897, using as the source of light the glowing positive carbon of the electric arc, and volatilize a fragment of metallic sodium just below and close in front of the slit by means of an alcohol or Bunsen flame, or, better, put a fragment of metallic sodium in a little cavity in the lower carbon of the arc itself. The light from the arc passes through the dense cloud of sodium vapor and a dark line appears in the orange-yellow of the spectrum, just where bright lines are found in the spectrum of sodium, showing that the waves absorbed are of the same wave length as those given out by glowing sodium vapor.

The black line in this case is not strictly black as it is illuminated by the radiation from the sodium vapor, but this is so much less intense than the direct radiation from the arc that it appears black by contrast. It is clear, therefore, that to produce black absorption lines **the absorbing vapor must be colder than the luminous source, or at least its direct radiation must be less intense than that which it absorbs.**

Of course, **in a steady state of things a mass of vapor in the atmosphere of the sun must be radiating just as much energy as it absorbs,** otherwise it would be growing hotter or colder; but the radiation which it absorbs comes to it mainly in one direction, while it radiates equally in every direction; therefore the radiation which it sends to the earth must be much less intense than that which it intercepts.

907. Absorption Due to Resonance.—It has been seen in the study of vibrating bodies (§307) that when sound waves fall upon a tuning-fork having exactly the same frequency as the waves, the fork is set in sympathetic vibration. In such a case the waves cannot set the fork in vibration without spending energy and consequently suffering a partial absorption.

A precisely similar reaction must take place between light waves and molecules having the same natural time of vibration as the waves. **Thus the sympathetic resonance between molecules in the vapor and the light waves which they absorb affords a simple mechanical explanation of Kirchhoff's law of absorption.**

908. A cal Spectroscopy.—To examine the spectrum

of a star or of a particular portion of the sun it is only necessary to form on the slit of a spectroscope an image of the object to be examined. For this purpose the eye-piece of the telescope is removed and a spectroscope is mounted so that its collimator is in line with the axis of the telescope and its slit at the principal focus of the object-glass.

In stellar spectroscopy no slit is required since the image of a star is a mere point of light, and the spectra of neighboring stars may be simultaneously photographed on the same plate. But it is also a consequence of the smallness of the stellar image that the spectrum of a star is a mere line of light too narrow to show well the spectrum lines. The breadth of the spectra may be increased by using a cylindrical lens; or the motion of the telescope may be so regulated that the spectra shift slowly on the photographic plate, at right angles to their lengths, so that each leaves a broad trace on the plate.

909. Doppler's Principle in Spectroscopy.—Doppler's principle, by which the apparent pitch of a sounding body is raised when it is moving toward the ear and lowered when it is receding (§305), has also an important application in case of light waves. While a luminous body is moving *toward* the observer more waves of light are received in one second than are actually given out in that time, and consequently the wave lengths of the light received are shorter than if there were no motion. So also the motion of a luminous body *away from the earth* has the effect of increasing the length of the waves which are received from it.

Since the position of each line in the spectrum depends only on its wave length, it is evident that if a body giving a bright-line spectrum were moving toward the earth, every line in its spectrum would be shifted a little toward the blue end of the spectrum, while if it were moving away from the earth its lines would be slightly displaced toward the red. From the amount of the displacement the velocity of the source relative to the earth may be readily determined.

It has sometimes happened in examining the spectra of sun spots that when the image of a sun spot was formed on the slit of the spectroscope a part of the spot where there was a strong uprush of glowing gas has fallen on one part of the slit while a region of less disturbance has fallen on another part of it. In

such a case a line in the spectrum due to this gas shows a curious twist or distortion, being displaced more toward the blue at one point than at another. Displacements pointing to a velocity of uprush of hydrogen gas as great as 400 kms. per second were observed in one instance by Young in the spectrum of solar prominences.

This method has also been used to investigate the velocity of the sun's rotation on its axis, for it is clear that in consequence of rotation one edge of the sun in its equatorial region is moving away from us while the opposite edge is moving toward us with an equal velocity.

A very interesting application of Doppler's principle is illustrated in figure 565, which shows part of the spectrum of the

FIG. 565.—Spectrum of the star Beta Aurigæ. The lines in the lower spectrum are double by Doppler's principle. Photo by Prof. E. C. Pickering.

star Beta Aurigæ. This spectrum at times shows single lines as in the upper figure, but *once in every two days* these widen out and separate into two, "as is well shown by the calcium line near the middle of the lower diagram. The two great hydrogen lines on either side of it are too wide and too fuzzy to be separated; but the one on the left in the lower figure (which must have been printed very much more lightly than the other by some screening process) shows that at the core it is really double, too."*

This is precisely the kind of spectrum which would be given by the light from two stars of nearly equal brightness, revolving around their common center of gravity and having their common orbit turned somewhat edgewise toward the earth. At cer-

* PROF. HENRY NORRIS RUSSELL: *Scientific American*, Sept. 3, 1910.

tain times one star is coming toward us while the other is moving away from us and lines in the spectrum of the one are displaced toward the violet end of the spectrum while those in the spectrum of the other are displaced toward the red, so that at such times each line is double. When the two stars are moving sidewise, say one toward the right and the other toward the left, there is no displacement of the spectra and the lines coincide. Evidently this change takes place *twice* in each complete period of revolution. It is therefore concluded that the two stars make 1 revolution in their orbit in four days.

From the amount of the displacement of the lines the maximum relative velocity of the two in the line of sight is found to be 140 miles per second. This indicates an orbital velocity of not less than 70 miles per second; and this combined with the period of revolution shows that the orbit is at least 7½ million miles in diameter.

And now if we assume that the law of gravitation is the same for the stars as we know it to be in our solar system, and if we further assume that the two stars have equal masses, since they are equally bright, we may apply the formulas of §156 and find that each star has twice the mass of our sun.

All of these facts have thus been obtained by the spectroscope, although the distance between the two stars is "probably at most scarcely one-fiftieth as much as that of the closest pairs which can be seen double in our greatest telescope."*

910. Motion of Stars in Line of Sight.—If the spectrum of a star shows lines which agree exactly with the spectrum lines of some known substance, such as hydrogen or iron, except that all are displaced slightly toward the red or blue, it is inferred that the lines are due to that substance and that the displacement is caused by the motion of the star either away from or toward the earth.

When it is found that most of the stars in a certain region of the heavens are approaching the earth and those in the opposite part of the sky are receding from the earth, it may be inferred that these apparent motions are probably due to the motion among the stars of our sun with its attendant earth and planets. In this way it has been concluded that our solar system is moving

* Prof. Henry Norris Russell: Scientific American, Sept. 3, 1910.

toward a point in the constellation Bootes about 25° north of Arcturus with a velocity that probably lies between 12 and 18 kilometers per second.

Problems

1. A spectrum line having a wave length 656.30 is displaced in consequence of the motion of the star, the apparent wave length being 656.37. Find whether the earth and star are approaching or receding from each other and with what velocity.
2. If a star is moving toward the earth with a velocity of 18 miles per sec., find the per cent. of change in the wave lengths of its spectrum lines due to the motion.

Colors of Bodies

911. Colors of Bodies.—The colors of natural objects are due either to light waves which they themselves emit, or to their power of reflecting or absorbing the light that falls upon them from some external source. The first class includes all bodies that are self-luminous in consequence of:

(*a*) *high temperature,* as in red-hot or white-hot bodies,

(*b*) *chemical action,* as in flames,

(*c*) *electric discharge,*

(*d*) *stimulus of light from other sources,* as in fluorescence and phosphorescence.

The second class includes bodies whose colors are due either to:

(*a*) *selective absorption,* as in colored glass, pigments, and most colored bodies; or

(*b*) *selective reflection,* as in metals and bodies showing special luster.

912. Luminous Bodies.—*The color of the light from any source is the average effect of its radiation upon the eye; but the particular kind of radiation which causes the effect can be determined only by analyzing the light with a spectroscope.*

For example, a yellow gas flame is found to have in its spectrum all kinds of light, but the blue and violet rays are relatively less intense than in sunlight. It is this weakness in the blue and violet which gives it a yellow color. On the other hand, the spectroscope shows that the sodium light or the yellow light obtained when a bit of common salt, previously fused, is held by a loop of platinum wire in the pale blue flame of a Bunsen

burner, is yellow for an entirely different reason; for the spectrum of this light consists principally of two yellow lines so close together that they appear like one line in a spectroscope of low power. The light from this flame is therefore very nearly homogeneous and appears yellow because the only kind of light present is one that excites that color sensation and no other.

913. Non-luminous Bodies.—Non-luminous bodies show no color in the dark. They derive their color from the light by which they are illuminated. Let sunlight fall on a piece of colored glass or a vessel containing some strongly colored dye. The light *reflected* from the surface of the glass or solution shows no trace of color, indicating that such substances *reflect* all kinds of light equally. But light passing *through* the glass or colored solution is deeply colored and when examined by the spectroscope broad dark bands are seen in its spectrum, showing that certain constituents of sunlight have been strongly absorbed by the substance.

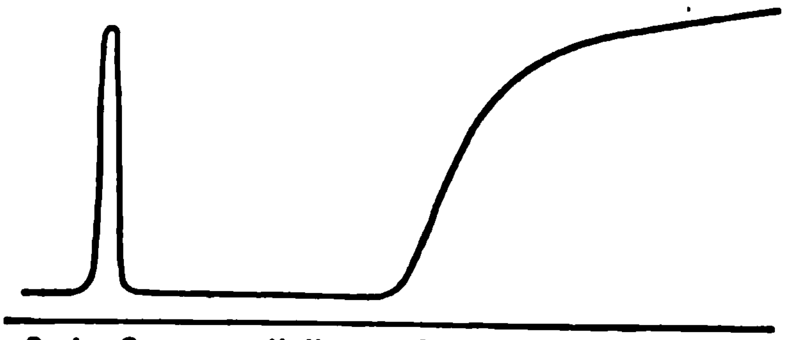


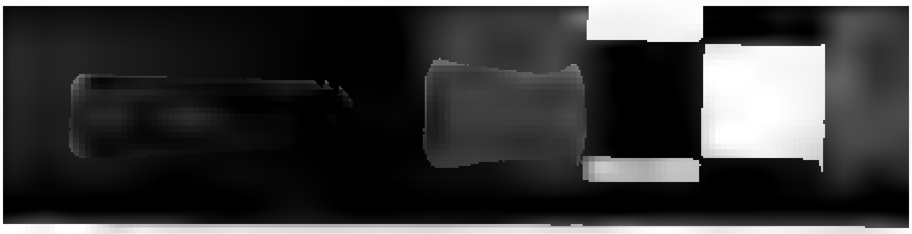
Fig. 566.—Spectrum of light transmitted by blue cobalt glass.

Thus a dark blue cobalt glass transmits green, blue, and violet, but absorbs strongly yellow and orange and most of the red. The curve in figure 566 represents by its height the intensity, in different parts of the spectrum, of light which has passed through a certain thickness of this kind of glass. Such absorption is called *selective.*

914. Spectrophotometer.—An instrument in which the spectra from two sources are formed side by side and with appliances so that the relative intensities of the two spectra can be determined for each point in the spectrum is known as a *spectrophotometer.*

Such a curve as that shown in figure 566 is obtained by means of an instrument of this kind, the spectrum of direct sunlight being compared with that of sunlight which has passed through the colored substance.

915. Absorption and Color of Powders.—The most common cause of the color of bodies is absorption. A crystal of copper



ılphate when seen by ordinary daylight appears blue because ght coming to the eye through the crystal has lost the red and ellow rays by absorption. The light received by the eye is, owever, not a pure blue, but is diluted with white light reflected rom its surface. If the crystal is broken into smaller fragments he thickness of crystal through which the transmitted light asses before meeting a reflecting surface is smaller and there is ccordingly less absorption and the blue color is not so marked. f the crystal is finely pulverized, the dry powder appears a pale hitish-blue, for light can penetrate only to an extremely small lepth before being reflected and scattered by the surfaces of the tiny fragments.

From the above considerations it is evident that *all clear colorless substances, such as ice, glass, Iceland spar, etc., must make white powders*, since they reflect and scatter the light from innumerable minute surfaces but absorb scarcely any of the visible rays. The light reflected to the eye by such a powder is, therefore, of the same quality as that which falls upon it, and when illuminated by white light it appears white.

916. Effect of Illumination.—Except in case of self-luminous bodies, *the color of a substance depends on the light by which it is illuminated*. When a piece of red paper is held in the red of a bright spectrum it appears bright red, but when held in the yellow, green, or blue parts of the spectrum it appears black, for it can reflect red rays, but it absorbs the yellow, green and blue. So a blue paper may reflect the violet, blue and green, but will appear black in the red, orange, or yellow parts of the spectrum, while a white paper reflects whatever color falls upon it.

Two kinds of light that appear very much alike in color may yet have very different effects on the colors of bodies. For example, an ordinary gas flame gives out a yellowish light not very unlike the yellow *sodium flame* in appearance, and yet bright-colored objects or pieces of paper are seen in their various colors when illuminated by the gas flame, but all appear of one color, either brighter or darker yellow or black, when illuminated with the sodium flame. This is because the ordinary flame gives out all kinds of light waves from red to violet, while the sodium light is nearly *homogeneous*. The peculiar, ghastly appearance of persons illuminated by a salted alcohol flame is due to this cause.

The difference between colors seen by daylight and gaslight is because light from the blue end of the spectrum is relatively far more intense in the former than in the latter.

917. Matched Colors.—Two colors that appear alike by daylight may yet be due to very different kinds of light. When an object appears yellow it does not follow that it reflects only rays from the yellow part of the spectrum. It means simply that the stimulus given to the retina by the various kinds of rays coming from the object excites the same sensation as the yellow light of the spectrum. What particular waves cause the color can be determined only by dispersing the light and examining its spectrum.

Consequently two colors which match perfectly when seen by daylight may differ very much when illuminated by some artificial light.

918. Mixed Pigments.—**When paints are mixed the resulting color is not a mixture of the colors that each would give separately, but is due to the double absorption which light suffers in the mixture.** For instance, a solution of gamboge yellow absorbs all rays but yellow and green, while a solution of Prussian blue absorbs all but blue and green, a mixture of the two will therefore transmit only the green.

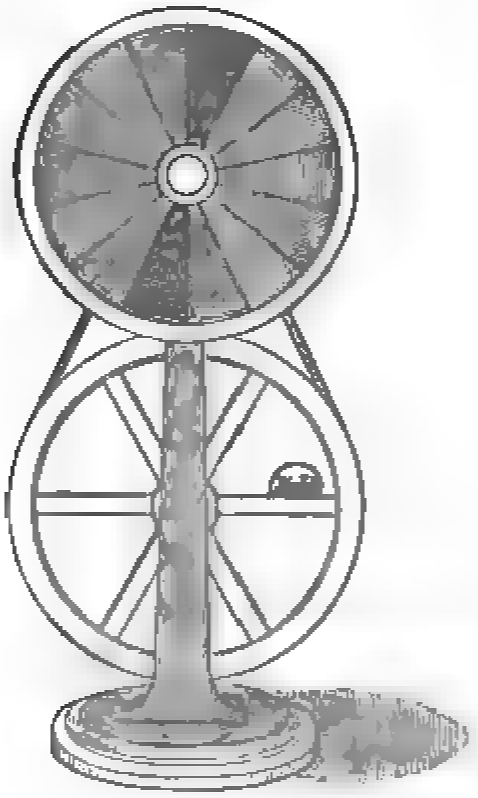


Fig. 567.—Newton's color disc.

919. Mixing Colors.—To find the color that will be produced by a mixture or blending of colored lights, the lights to be mixed may be made to illuminate simultaneously a white screen, or the color wheel may be employed. In this apparatus discs of colored paper, each slit from center to edge, and fitted together so as to expose a sector of each color, are mounted on a spindle and rapidly rotated. If the speed of rotation is sufficient the disc appears of a uniform color which is the mixture of the different colors used. The proportional amounts of the several colors depends on the widths of the

[redacted]

exposed sectors and may be changed by slipping the discs on each other. A larger and smaller disc may be mounted on the same spindle for comparison.

The effect in this case depends on the persistence of the sensation for a very short time after the stimulus to the retina has ceased. The various colors give their stimuli in such rapid succession that the effect is a blended sensation.

Newton found that a color disc painted in sectors to imitate the colors of the spectrum appeared grayish-white when rapidly rotated and could be matched with a black disc having a white sector, black being used to diminish the intensity of the white.

Complementary Colors.—Two colors which when combined produce white, are said to be *complementary.* By means of the color disc it is found that blue and yellow of the proper tints and intensities will make white, also green and red may be complementary.

920. Metallic Luster.—Metals owe their peculiar luster to their intense reflecting power. Polished silver reflects 90 per cent. of the light that falls upon it, while glass at perpendicular incidence reflects less than 5 per cent.

Sunlight reflected from red or blue glass remains white, but when reflected from gold-leaf it is yellow. This shows that the reflection of light in case of some metals is *selective,* some kinds of light being more strongly reflected than others. It is to this property that the colors of metals are due.

The light transmitted through a thin film of gold-leaf is not yellow, but green. The yellow light which is reflected is that also for which the absorbing power of the metal is greatest.

Some non-metallic substances also have the power of reflecting light like metals as is seen in the bronzy luster of aniline ink and in crystals of permanganate of potash. Such substances show strong selective absorption and anomalous dispersion.

921. Fluorescence.—When a strong beam of sunlight or light from the electric arc is sent through a block of glass colored with oxide of uranium the transmitted light is yellowish, showing that there has been absorption of the shorter wave lengths; but besides this, the whole block of glass is seen to glow with a greenish light which seems to come from each point in the glass itself, making the whole block seem turbid and milky. This is called *fluorescence,* for the phenomenon is strongly marked in fluorspar.

The subject was first carefully investigated by Sir George Stokes, who showed that fluorescence is really a kind of radiation from the molecules of the substance under the stimulus of the absorbed light. Light is absorbed by the block of glass, and the energy of the absorbed waves instead of appearing simply as heat, produces special molecular vibrations which give off waves of light, just as waves entering a harbor may set ships rocking and in consequence these become centers from which waves go out in all directions. Stokes announced the law that **waves of fluorescent light cannot be shorter than the absorbed waves to which they are due.**

It will be noticed that the block of glass fluoresces most strongly near the side where the incident beam enters, for as the beam penetrates into the block it loses by absorption the very rays which are effective in causing fluorescence.

The interposition of a piece of red glass in the path of the light cuts off all fluorescence, while a blue cobalt glass scarcely weakens it at all, showing that the effect is due to the shorter wave lengths which are transmitted by the blue glass but suppressed by the red.

Many substances show fluorescence, among others almost all mineral oils, especially the thick heavy oils, and crude petroleum, and even refined kerosene oil shows a delicate blue fluorescence in strong light. Some of the anilin substances are extremely fluorescent, notably fluorescein and eosin. Sulphate of quinine fluoresces a delicate blue as does also æsculin obtained from crushed horse-chestnut bark.

A white card covered with a thick paste of sulphate of quinine moistened with dilute sulphuric acid will fluoresce strongly in the invisible rays of the spectrum beyond the violet, the so-called ultra-violet region.

922. Phosphorescence.—When fluorescence persists after the illumination ceases the substance is said to be *phosphorescent.*

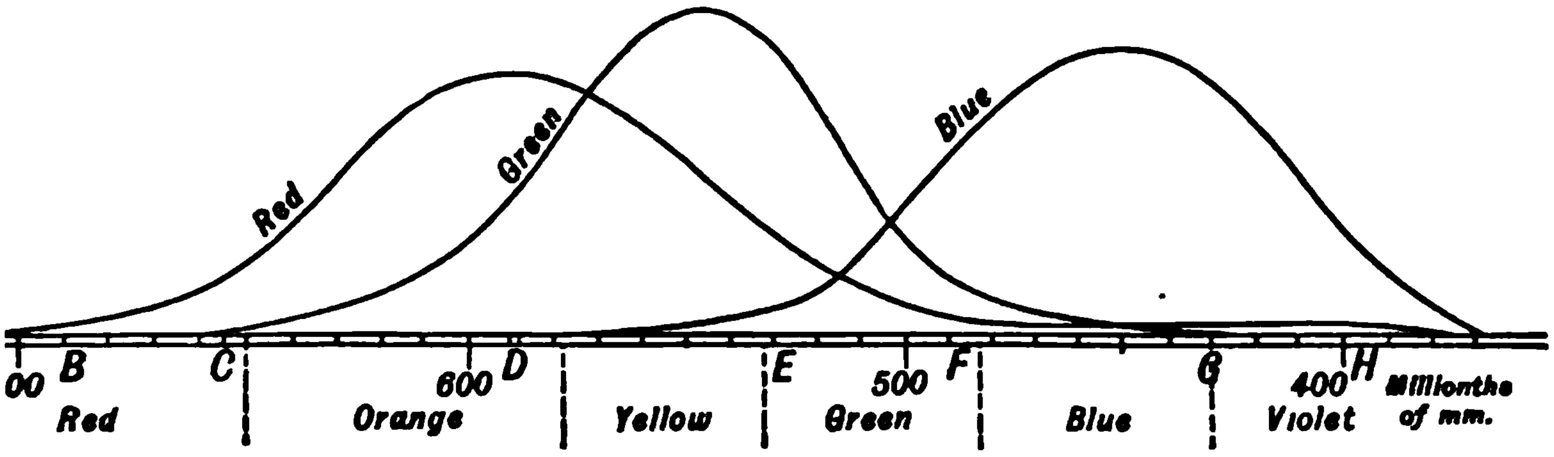
By a special contrivance, called a phosphoroscope, Becquerel found that many substances, including paper, bone, and ivory, not usually known as phosphorescent, glow for a fraction of a second after the incident beam is cut off.

The sulphides of calcium, barium, and strontium are strongly phosphorescent and the color of the phosphorescent light is greatly influenced by the presence of slight impurities.

This kind of phosphorescence may be called *physical* to dis-
nguish it from the glow of decaying vegetables, of fire-fly and
ow-worm, and of phosphorus itself, in which the light seems
› be due to *chemical* changes.

923. Theory of Color Sensation.—The Young-Helmholtz
heory of color sensation proposed by Thomas Young and
nodified by Helmholtz assumes that light falling on any point
n the central region of the retina where it is sensitive to colors,
xcites in general three primary color sensations, red, green, and
lue, the resulting color sensation depending on the relative
ntensities of these three primary sensations.

The sensation of red is found to be excited more or less by all
vave lengths in the visible spectrum, but most strongly by the
ong waves, as shown in the left-hand curve of figure 568. So



IG. 568.—Curves from Abney, showing variation of color sensation with wave length, according to the Young-Helmholtz theory.

hat if a person possessing only the red color sensibility and lack-
ng those of green and blue were to look at a bright spectrum it
ould appear to him red from one end to the other, but brightest
here the wave lengths are long as shown in the curve marked
ed. So, too, the curve marked *green* may be taken as exhibiting
he relative intensity of the green sensation excited by different
ave lengths of light, while the third curve shows how the
ensation of *blue* varies with the wave length.

In the normal eye, possessing all three sensibilities, a given
ave length of light excites all three sensations, the red pre-
ominating in case of long waves, green when the waves are
horter, and blue when they are shorter still, the intensity of
ach sensation being proportional to the height of its curve at
he point corresponding to the given wave length.

The sensation *of white results* when all three of the primary
ensations are equally excited.

[redacted]

What the three primary color sensations are, can be determined only by the study of color-blind individuals. By such a study Koenig finds that the primary sensations are the red, green, and blue found in the spectrum at wave lengths, 671, 505, and 470$\mu\mu$, respectively. By combining these three colors in proper relative intensities any color of the spectrum may be produced.

Helmholtz assumed that there were three kinds of nerve termini in the retina corresponding to the three primary sensations of color, while Hering supposes certain substances in the retina whose transformations under the influence of light give rise to the various primary sensations. For a further discussion of theories of color vision the reader may consult "A First Book in Psychology" by Calkins, or the article "Vision" in Baldwin's "Dictionary of Philosophy and Psychology."

Interference of Light

924. Introduction.—Up to this point in our study the theory that light is a wave motion has been supported by the fact that the velocity of light is the same as that of electric waves and by the simple explanation which that theory affords of the phenomena of reflection and refraction. But we have not yet found any direct evidence of the existence in a beam of light of a regular periodic oscillatory motion such as is characteristic of all kinds of waves. We now come to some phenomena which point unmistakably to just such a periodicity.

925. Interference of Waves.—Perhaps the most distinctive evidence of wave motion is afforded by the phenomena of *interference.*

When two trains of waves come together having the same wave length and amplitude and traveling in nearly the same direction, there will be found *points of rest or of very slight motion where the two systems of waves are in opposite phases and neutralize each other,* and other points where the waves coming together in *the same phase cause an amplitude of motion equal to the sum of the amplitudes of the component waves.*

The interference of water waves and sound waves has already been discussed (§319).

926. Young's Experiment.—The interference of light waves was first shown by Thomas Young in 1801 by the method illus-

ated in the diagram. In the path of a beam of sunlight shining rough a minute pinhole at S, is placed a screen of tinfoil having ro very small holes a and b close together. If light is now lowed to pass through only one of the openings, a round bright pot surrounded by faint dark and bright rings is formed on a creen at C. But if light passes through *both* openings, there

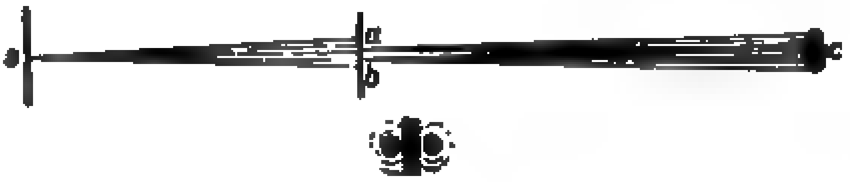


FIG. 569.—Young's experiment showing interference.

s seen between the two bright spots and at right angles to their ine of centers, a series of bright and dark bands, as shown in the ower part of figure 569.

The explanation of these bands will be understood by the aid of figure 570. Waves from S set up waves at a and b which start out simultaneously in the same phase, the two sets of waves spreading out in the medium beyond, one set from a and

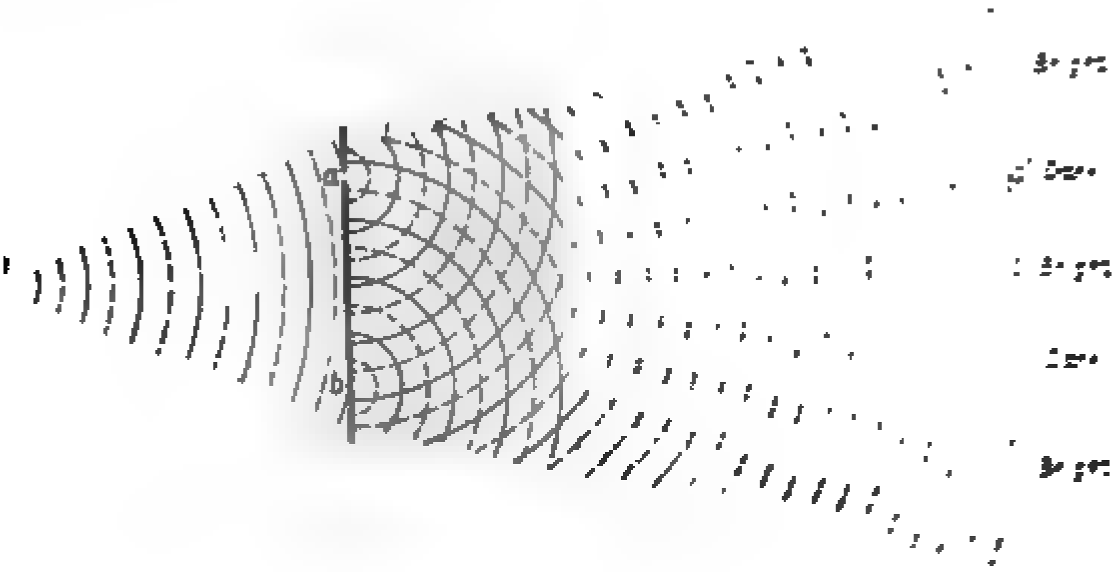


FIG. 570.—Diagram of interference of waves. Young's experiment.

ne from b, as shown in the diagram. The central point c is quidistant from a and b, so that waves leaving a and b at the ame instant meet at c in the same phase, reenforcing each other nd making c a bright spot. But d is *a half wave length farther rom b than from a*, and consequently waves from a and b reach here in *opposite phases* and neutralize each other, making a

a dark spot. In this way those points on the screen which are equidistant from a and b, or which are one, two, or more whole wave lengths farther from one opening than from the other will be bright, while points which are farther from one opening than the other by ½ or 1½ or 2½, etc., wave lengths, will be dark.

927. Fresnel's Interference Experiment.—In order to show that the above explanation of the dark bands obtained by Young was correct and that they were really due to interference of waves, Fresnel devised a most ingenious modification of the experiment, by which he avoided any disturbance of the light that might be imagined to result from its passing through the small openings a and b.

Waves of light from a narrow slit shown in section at S (figure 571) fell on two mirrors $M'M''$ inclined to each other at a small

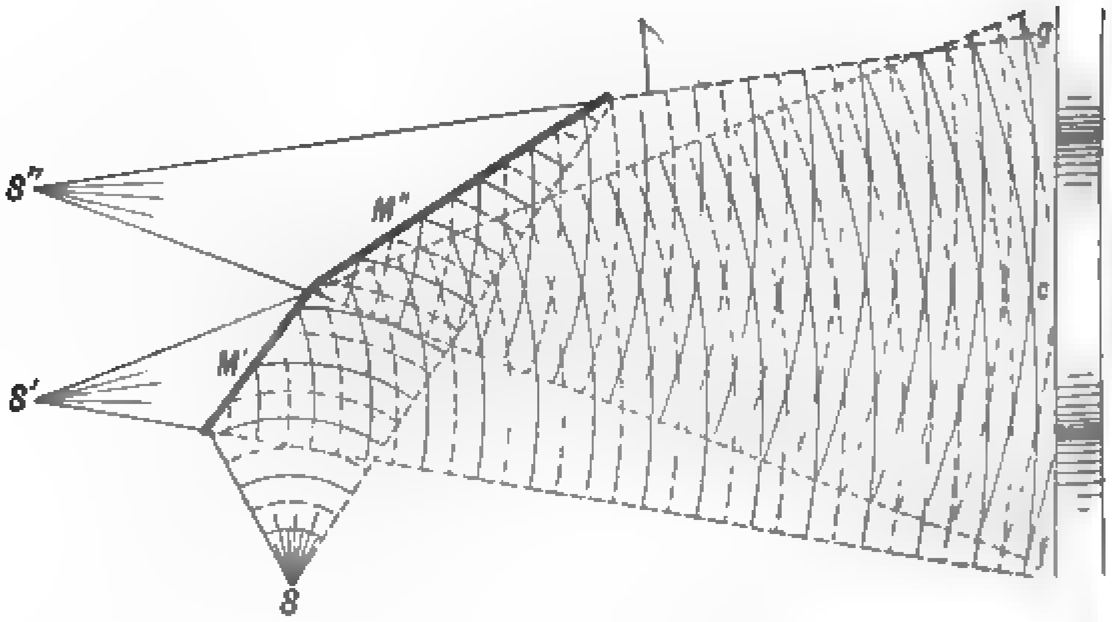
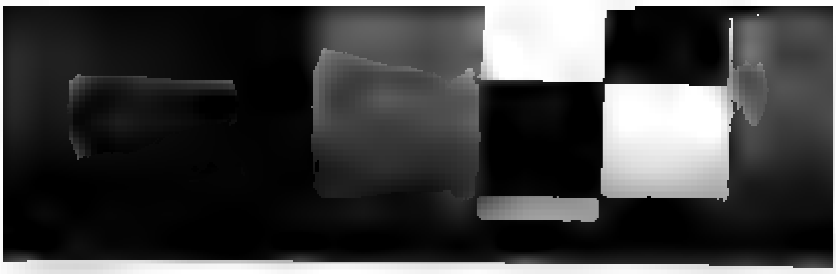


Fig. 571.—Fresnel's interference experiment with mirrors.

angle so that light after reflection from M' diverged as if from S', while that reflected from M'' came as if from S''. In this way two trains of waves were produced which gave bright bands at c, f, and g where the waves of the two sets were in the same phase, and intermediate dark bands, just as in Young's experiment.

928. Newton's Rings and Colors of Thin Films.—When a lens having a convex surface of *very* slight curvature is placed in contact with a flat glass plate a thin film of air is enclosed between the two plates which increases in thickness from the central



of contact outward. If this film is examined in white holding the eye so as to receive light reflected from its ·, a number of colored rings are seen surrounding the central of contact of the lens and each colored ring correspond- a definite thickness of the air

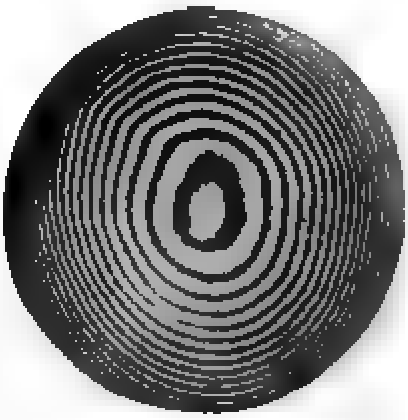


FIG. 572.—Newton's rings in sodium light.

lluminated with light of one length, as sodium light, the surface of the film is seen to be d with alternate dark and rings. When red light is each ring is larger than the onding ring in case of blue

colored rings observed in white light are due to the super- n of the sets of rings of different sizes due to different engths of light.

se bands are known as Newton's rings, for the experiment vised by him to determine the thickness of film correspond- a given color; for if the curvature of the lens surface is , it is easy to calculate the thickness of the air film for a any given radius. The following table shows some results:

Thickness of Film in Newton's Rings

Red light		*Blue light*	
ing,	0.00017 mm.	1st ring,	0.00012 mm.
ng,	0.00051 mm.	2d ring,	0.00036 mm.
ng,	0.00085 mm.	3d ring,	0.00060 mm.
rence,	0.00034 mm.	Difference,	0.00024 mm.

similar way, if two perfectly flat pieces of glass are laid on the other, touching at one edge and separated at the by a thin strip of tinfoil, the wedge-shaped film of air between them shows alternate dark and bright bands ogeneous light. These bands are straight and parallel edge of the wedge if the plates are flat, and the straightness bands affords a sensitive test of the flatness of the plates.

The English physicist, Thomas Young, first showed that Newton's rings could be explained easily by the interference of light waves as follows:

Let a train of waves advance upon a transparent film, and for simplicity suppose the incidence to be nearly perpendicular, as shown by the arrow (figure 573). Each wave on meeting the surface is partly reflected and partly refracted. An advancing wave meeting the surface at *a* is in part reflected along *ac*. Another part of the same wave passes into the film at *d*, meets the second surface at *b*, and is in part reflected to *a*, where it emerges along the direction *ac*. This portion of the wave will have had to *cross the film twice* and will therefore have fallen behind the part which was reflected directly at *a*, so that if the thickness of the film is one-quarter of the wave length of the light waves in the film, waves from *d* will reach *a* one-half wave length behind the corresponding waves reflected at *a*, and may therefore be expected to interfere, causing the film to appear dark at *a* to an observer looking in the direction *ca*. At points where the thickness of the film is one-half a wave length each wave from *d* reflected at *b* will be a whole wave length behind the corresponding wave reflected at *a* and may therefore be expected to reenforce the next succeeding wave, and so the film should appear bright at such points. But experiment shows that *exactly the reverse* is true, for the central point of contact in Newton's rings is dark by reflected light instead of being bright.

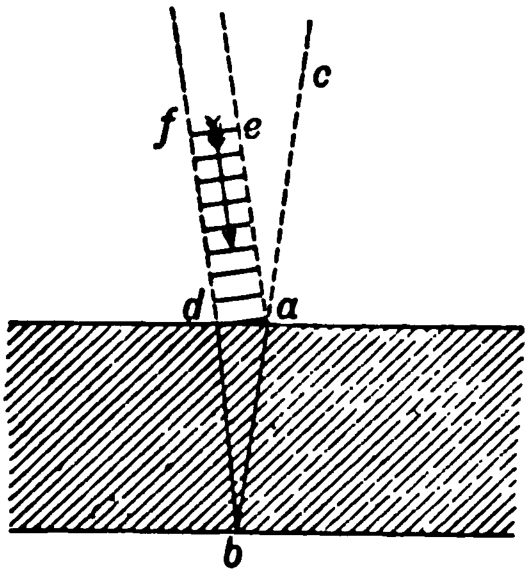


FIG. 573.—Interference in reflection from a film.

Thomas Young showed that this discrepancy is due to a *change of phase which takes place in the very act of reflection;* for at one surface of the film, waves in a less refracting medium are reflected where they meet the more refracting one, while at the other surface, waves in the more refracting medium are reflected on meeting the less refracting one. These two reflections are opposite in kind, just as reflection in a stopped organ pipe is opposite to that in an open pipe (§336), and one changes the phase of the reflected light while the other does not.

This opposition of phase brought about in reflection exactly reverses the conclusions reached above where merely the effect of the thickness of the film was considered, and consequently *those parts of the film where the thickness is an odd number of quarter wave lengths appear bright by reflected light.*

To test whether this explanation was correct, Dr. Young reflected light from a thin film of oil of sassafras between a lens of crown glass on one side and a flint glass plate on the other. The index of refraction of the oil of sassafras is more than that of crown glass, but less than that of flint glass, so that the change in phase due to reflection was the same at each surface. It was found in this case that *the central spot where the film was thinnest was bright* by reflected light, while dark bands were observed where the thickness of the film was one-quarter of a wave length, three-quarters of a wave length, etc., thus confirming Young's idea as to the cause of the reversal.

The thickness of the film for red and blue rings given on page 635 therefore leads to the conclusion that the average wave length of the red light used was about 0.00068 mm., while that of the blue light was 0.00048 mm.

The colors of soap bubbles and of thin films of oil or turpentine on water are also explained in the same way. For instance, where the thickness of the film is equal to a half wave length of red light it will be nearly equal to three-quarters of a wave length of blue light. The former will therefore be destroyed by interference, while the latter will be reflected and the film will appear blue.

A soap bubble does not show color unless it is very thin; for when the thickness of the film is, say, 0.0028 mm., it will be equal to 4 wave lengths of extreme red light and 6 wave lengths of extreme blue; these waves will be absent from the reflected beam because of interference, and also the waves whose lengths are such that in the thickness of the film there are included just 4½, 5, and 5½ wave lengths, respectively; while light having wave lengths intermediate to these will be reflected from the film. The film in such a case appears white because the reflected light contains so many different wave lengths that the average effect is white.

The intensely black spots seen in thin soap-bubble films by reflected light have been found by Reinold and Rücker to have a thickness of only about one-fiftieth of the wave length of sodium light, and appear black for the same reason that the central spot is black in Newton's rings.

929. Interference with Great Difference of Path.—In case of thin films the interfering waves differ in path by only a few wave lengths. There are some cases, however, in which interference has been obtained when the difference in path is very great.

A useful form of *interferometer*, as it is called, is that of Michelson, a diagram of which is shown in the figure.

A plate of glass A, having plane parallel surfaces, is mounted in front of the mirror M in an oblique position so that light from the source L on meeting the second surface at S is partly reflected to the mirror M and in part transmitted to a second mirror N which is at right angles to the first. The eye at E will therefore receive light from S which has been reflected at M, and also light from the same point which has been reflected at N, and when the adjustment of the instrument is correct, these two rays will interfere when they come together if the light in going from S to M and back, has to pass over a distance which differs by an odd number of half wave lengths from the distance from S to N and back again.

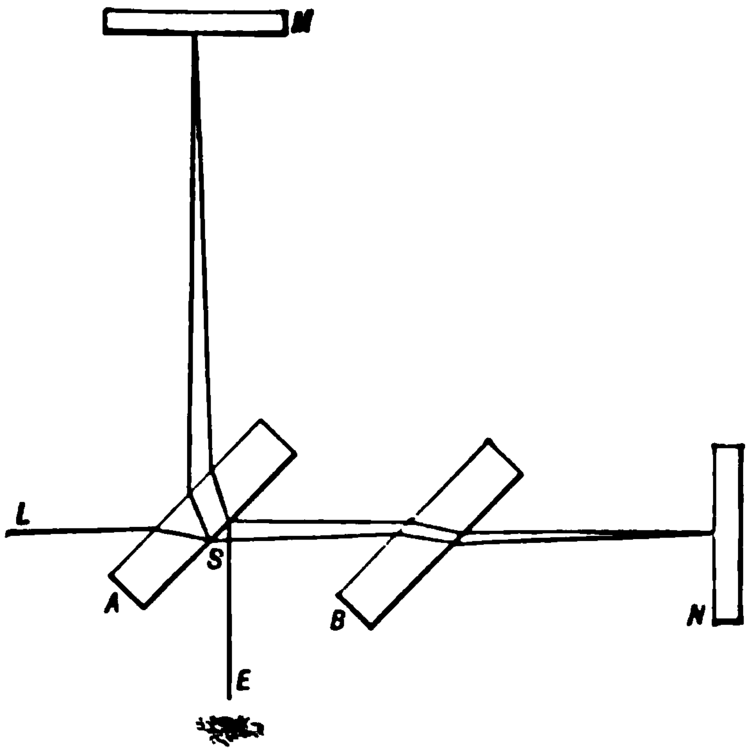


Fig. 574.—Michelson's interferometer.

In order that the interference may be complete, the reflecting surface at S has a very thin coating of silver, just sufficient to make the reflected and transmitted beams of light equally intense.

The mirror M is usually mounted so that its distance from S can be varied by means of a micrometer screw. The plate of glass B of the same thickness as A is mounted parallel to it, so that waves reflected at N have to pass through the same thickness of glass as those reflected at M. The interference bands in this case are circles, which expand and are succeeded by others, as the mirror M is moved away from S. A motion of M through one-half of a wave length will cause a shift in the position of the interference bands equal to the distance between two successive bands.

Using this method and employing light of one wave length from a Plücker tube (§903) containing mercury vapor, Michelson obtained interference bands when one path was longer than the other by 540,000 wave lengths; that is, the luminous atoms made 540,000 vibrations after giving out a wave of the first set before the interfering wave was sent out, and yet the waves had changed so slightly that interference could still be observed; a result which indicates a remarkable steadiness of vibration in the source.

By counting the bands that pass when the mirror M is moved backward a certain distance by the screw, the number of wave lengths of light contained in that distance may be exactly determined. By a very ingenious extension of this method, which the student will find described in detail by Professor

lichelson in *Light Waves and Their Uses*,* the length of the standard meter 'as determined by him in terms of wave lengths of light. The results ob- ined were

$$\text{meter} = \begin{cases} 1{,}553{,}163.5 \text{ waves of the red radiation from cadmium.} \\ 1{,}966{,}249.7 \text{ waves of the green radiation from cadmium.} \\ 2{,}083{,}372.1 \text{ waves of the blue radiation from cadmium.} \end{cases}$$

ll in air at 15°C. and normal pressure.

Of these results Michelson says: "It is worth noting that the fractions of a 'ave are important, because, while the absolute accuracy of this measure- ient may be roughly stated as about one part in two millions, the relative ccuracy is much greater, and is probably about one part in twenty millions."

930. Nodes and Loops with Light Waves.—When light waves re reflected directly back, we have in front of the mirror two sets of waves, he returning waves and the advancing ones, moving in opposite directions. Jnder these conditions, as we have already seen (§324), *standing waves* with odes and loops are formed.

The existence of these nodes and loops in case of light waves was first emonstrated by Wiener in 1889, by placing a photographic plate having a ery thin transparent film in a slightly oblique position in front of a mirror luminated with homogeneous light. On development it was found that 'here the plate crossed the loops it had been acted on by the light, while it 'as unaffected in the nodes.

The stratification of a photographic plate by these nodes and loops in front f a mirror is the basis of the color photographs of Lippmann, as the striae re closer together with short waves than with long.

References

A. A. Michelson: *Light Waves and Their Uses.*
Edwin Edser: *Light for Students.*

Diffraction

931. Diffraction Bands Around Shadows.—The observation f shadows suggests that light is propagated in straight lines. The form which the shadow of an obstacle would have if this 'ere the case is called the *geometrical shadow;* it is the projection f the obstacle upon the screen by straight lines radiating from he luminous source as a center.

Ordinary shadows are blurred at the edges because the angular agnitude of the source causes a penumbra. Hence to make an ccurate comparison of a real shadow with the geometrical hadow the source of light should be a mere point. But when the

* Univ. of Chicago Press.

experiment is tried, as, for example, when we examine closely the shadows cast by a moderately distant arc lamp, instead of finding a clear-cut boundary, *the entire edge of the shadow is observed to be surrounded by a series of alternate dark and bright bands, parallel to the edge,* very distinct next the shadow and gradually fading out into the fully illuminated region.

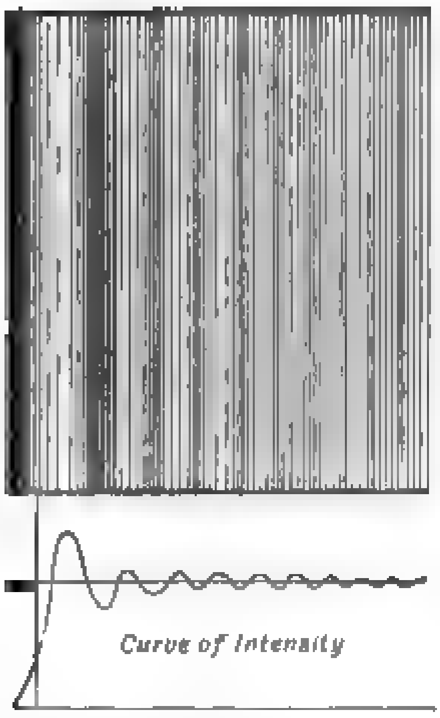


FIG. 575. – Diffraction bands at the edge of a shadow. The vertical line is edge of the *geometrical* shadow.

These bands were known at the time of Newton and were called *diffraction bands* or *fringes*, because to explain them on the emission theory it was supposed that the luminous corpuscles were bent aside from their straight course as they shot by the edge of the obstacle.

In the year 1816 a young French artillery officer, Joseph Fresnel, then less than thirty years of age, presented to the French Academy a memoir which marked an epoch in the science of optics, for in it he showed that the varied phenomena of diffraction are readily explained in every detail by the interference of light waves taken in connection with Huygens' principle, and without recourse to any additional hypothesis.

932. Huygens' Principle.—Let there be a train of waves advancing in the direction OP whose crests are represented by the parallel lines on the left of figure 576, and let AB be a row of particles parallel to the wave front; then as the waves sweep by AB the particles are all set vibrating simultaneously and in the same phase. Now, each of these vibrating particles may be considered as a center of disturbance from which spherical waves spread out into the region beyond. Thus, if we choose, **we may consider the vibration that is produced at the point P as due to the combined effect of all these little elementary waves or wavelets whose centers lie in the line AB, just as though the line of particles AB was the actual source from which waves spread out.** This is known as Huygens' principle.

iffraction by a Narrow Slit.—If sunlight shining through slit falls on a sècond narrow slit parallel with the first, be seen on a white screen held back of it, a central nd and on each side alternate bright and dark bands, len out when the second slit is made narrower. The ory affords a simple on; for let S be the ed down upon end- ich is *very* narrow with its distance from n at b_1 (Fig. 577), AB represent the agnified cross-section t in the plane of the Then the ether par- ng in ACB are kept ion by the successive ssing through the slit, uygens' principle these may be considered as

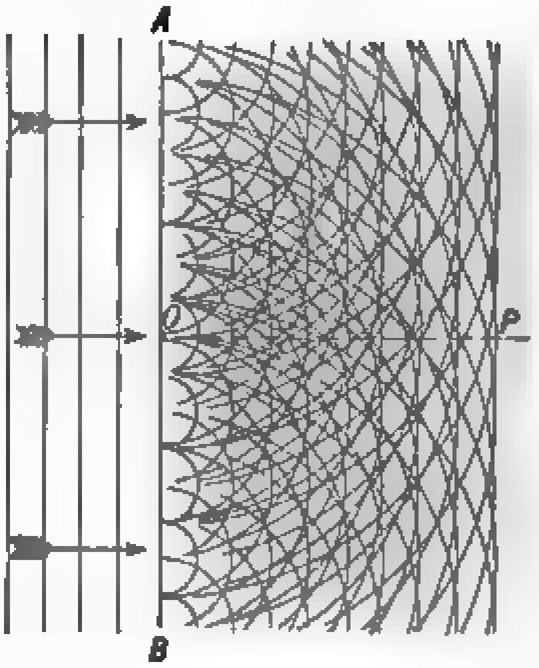
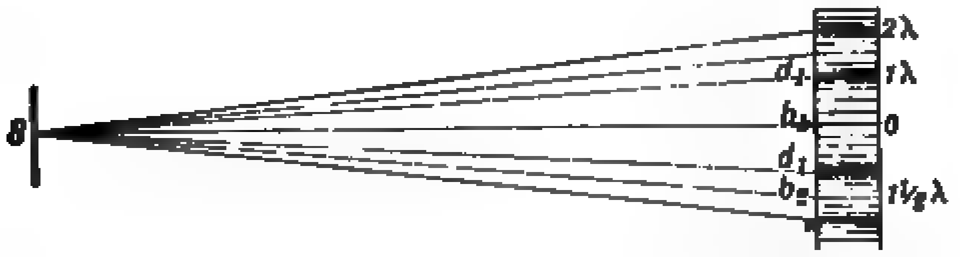


Fig. 576.—Huygens' principle.

s of wavelets which spread out in all directions and he effects which are observed. Now, on account of the arrowness of the slit, b_1 is practically equally distant



Diffraction through narrow slit perpendicular to the plane of the paper.

points along the line AB, and therefore the wavelets simultaneously at all points along AB reach b_1 in the se and so reënforce each other and make it a bright

low b_1 there will be a point d_1 which is a *whole wave* ther from A than from B. Then d_1 is a *half* wave ther *from* C than from B, and for every point between

B and C there is another between C and A which is just a half wave length farther from d_1. Therefore the wavelets going to d_1 from one-half of the slit will be exactly neutralized by wavelets from the other half, and d_1 will therefore be a dark spot in consequence of this interference. In the same way the dark spot d_1 above b_1 is explained.

But a little beyond d_1 there will be a point b_2 which is $1\frac{1}{2}$ wave lengths farther from A than B. In that case the wave front in the slit may be conceived as divided into three equal parts AD, DE, and EB, such that wavelets coming to b_2 from AD have a half wave length farther to travel than from the corresponding point in DE. Therefore waves from these segments interfere at b_2 while waves from the third segment will be effective and make b_2 a bright spot, though *much less bright than* b_1, since only one-third of the width of the slit is effective. Of course DE might be regarded as opposing EB, and in that case AD is effective.

The same reasoning shows that there will be a dark spot where the difference in path from A and B amounts to 2 wave lengths and again a bright spot where the difference amounts to $2\frac{1}{2}$ wave lengths. There will therefore be a series of alternate dark and bright spots on each side of b_1 as experiment shows.

If the slit is made narrower the line AB is shorter, and consequently the point d_1, which is one wave length farther from A than from B is farther away from b_1 than before. Therefore the bands spread out as the slit is made narrower, and if it had a *width* of *only one wave length* or less, light would go out from it in every direction, though the intensity would be less in oblique directions in consequence of partial interference.

934. Shadow of a Circular Obstacle.—When Fresnel's memoir was presented to the French Academy it was objected by Poisson that if his views were correct there should be a bright spot in the center of the shadow cast by a circular disc. Fresnel at once acknowledged the justness of the criticism and, making the experiment, *found the bright spot,* thus obtaining a triumph for the new theory.

The experiment may be made by fastening to a piece of plate glass a bicycle ball about $\frac{1}{4}$ inch in diameter and observing its shadow as cast by a *distant* arc light at a distance of 8 or 10 ft. back of the obstacle; the central

bright spot may be readily seen either by receiving the shadow on a card or by looking toward the object and viewing the shadow directly with a small pocket magnifier. Or the shadow may be received upon a sensitive film and photographed.

The central spot is bright because it is equally distant from every part of the edge of the obstacle, and therefore wavelets

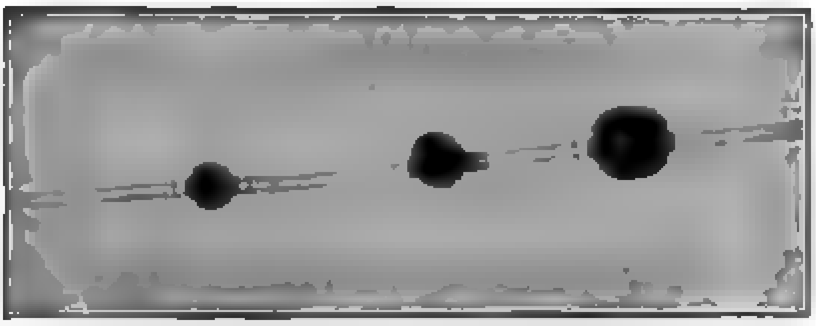


FIG. 578.—Shadow cast by small balls fastened to a fine wire.

coming from points just outside the edge around its whole circumference come together *in the same phase* at that point.

Similarly a bright line is found in the center of the shadow of a wire, since the central line is equidistant from the two edges and waves coming around the wire on both sides reach the central line in the same phase and therefore reënforce each other. (See figure 578.)

The student should observe through a pocket magnifier the diffraction bands formed by the wires of a mosquito netting or screen of thin silk, standing a few feet from the screen and looking through it toward a distant arc light.

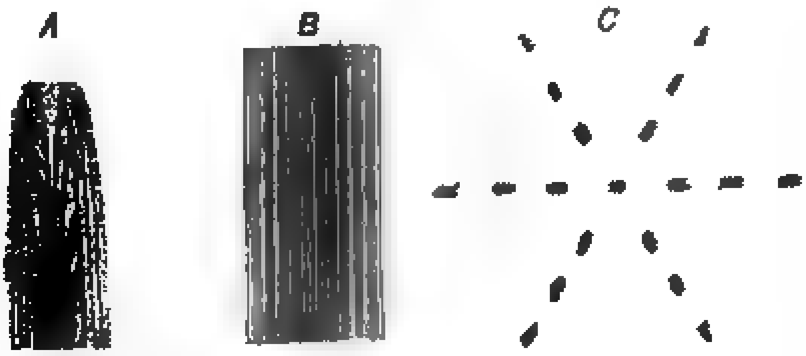


FIG. 579.

935. Miscellane[illegible] Phenomena.—In figure 579 are shown at *A* the diffraction bands in the shadow of the pointed end of a needle. It will be observed that there is a central bright band, *broadest* near the very point, while where the needle

[redacted]

is thicker many fine interference bands are seen in the shadow. This is shown in B which is the shadow of a somewhat thicker wire showing the many fine bands due to the interference of the waves coming around the two sides of the wire. At C is shown the diffraction pattern which may be seen by looking through the cloth of a silk umbrella toward an electric arc lamp.

A small round obstacle gives rise to a series of diffraction rings, and where the rings due to a great number of fine particles are all of the same size and are superposed the effect may be very intense. This is the explanation of the *coronas* seen so often around the moon. They are brightest when the light from the moon comes through a region full of minute water particles *nearly uniform in size.* These coronal rings are larger the smaller the particles that cause them, and the average diameter of the water drops can be immediately calculated from the angular radius of the rings.

Beautiful coronas may be seen on looking at an electric light or gas flame through a piece of glass coated with *lycopodium* powder, which is made up of minute discs of nearly uniform size. First breathe upon the glass, then pour some of the powder upon it and shake off the loose dust.

936. Diffraction in Case of a Lens.—In our study of lenses we saw that a lens transforms a wave of light coming from a distant point into a con-

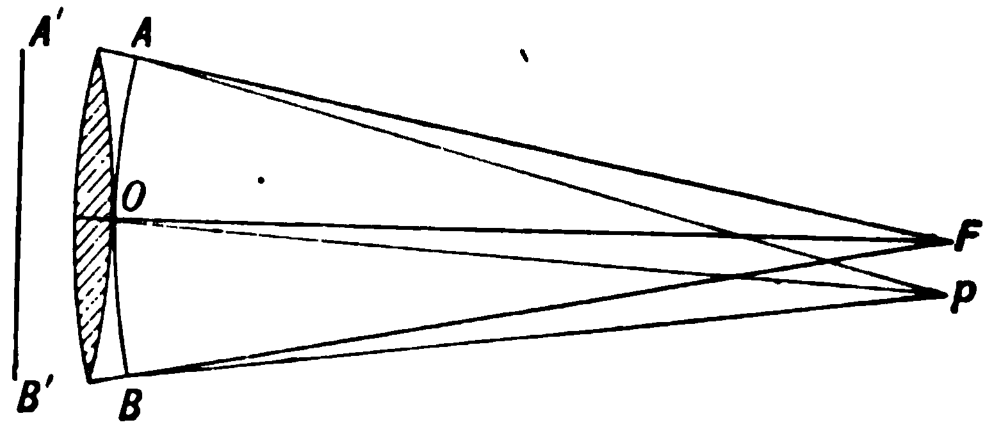


Fig. 580.

cave spherical wave which has its center at the focus toward which it converges. Thus the wave $A'B'$ becomes concave, as at AB, and *if the latter is perfectly spherical the lens is perfect.* The geometrical theory of optics would lead us to infer that in that case the light would all converge rigorously to the point F, but the wave theory shows that this cannot be so.

To determine the effect at F of the wave AOB we must again have recourse to Huygens' principle and consider the resultant effect as due to wavelets having their centers in the concave surface AOB. Clearly all will reach F in the same phase, since it is equidistant from all, hence F must be a point of maximum brightness. A little below F there must be some point p which is

ı the average a half-wave length farther off from the upper half of the surce AOB than it is from the lower half. At that point waves from one half the surface will interfere with those from the other half and produce cometе darkness. But between p and F the interference is only partial and ınsequently the light intensity must shade off from F to p. Since the light symmetrical about OF, there must be a ;tle *spot* of light formed at the focus, havg the distance Fp as its radius. The ırve in figure 581 shows by its height how ıe intensity of the light in the focal spot aries from F to p.

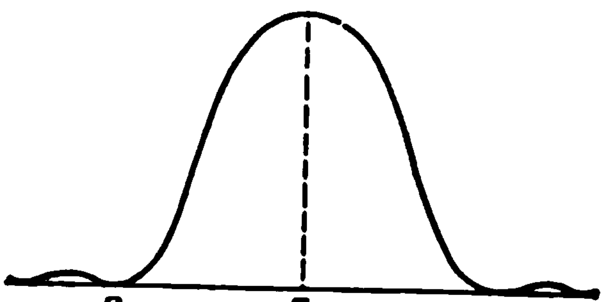


FIG. 581.

937. Resolving Power of Optical ıstruments.—The fact that the focal ɔot has an appreciable size has a most imɔrtant bearing on the *resolving power* of optical instruments, for when a ns forms an image of any object each *point* in the object is represented by little *spot* in the image, and the sharpness of definition in the image depends n the *smallness* of the focal spots.

Now, it may be proved that the *effective diameter* of the focal spot, or *iffraction image of a point*, as it is called, is equal to $\frac{\lambda F}{D}$, where λ is the wave ıngth of light, D is the diameter of the lens, and F is its focal length. Hence ɔr a given focal length the focal spot will be smaller the larger the lens.

The *angular diameter* of the focal spot is $\frac{\lambda}{D}$ which is equal to 4.5″ of arc hen $D = 1$ inch. Therefore a telescope having a perfect object-glass one ıch in diameter will be just capable of resolving a double star whose components are 4.5″ apart. For in that case the star images formed in the tele:ope will be two spots of light just touching each other. If the object-glass ı 2 in. in diameter it may then be capable of resolving stars only 2.3″ part. Evidently no magnification by the eye-piece will increase the resolvıg power as it will simply show two larger spots of light touching each other ıstead of two smaller ones. Helmholtz has shown that in consequence of he size of the focal spot it is impossible to have a microscope that will enable he eye to distinguish separate lines which are less than $1/35{,}000$ of an inch part, and even this limit can be reached only by oil-immersion lenses.

938. Diffraction Grating.—One of the most useful instruıents for the formation of spectra and for the measurement of he length of light waves is the *diffraction grating*, so called beause the first gratings made by Fraunhofer were veritable ratings made of fine parallel wires spaced at equal intervals. Iore accurate gratings are made by ruling with a diamond on lass or on a polished mirror surface of speculum metal an imıense number of parallel equidistant straight lines, and copi

or replicas of these ruled gratings are made by photography or by direct impression on a plate of celluloid.

The effect of such a grating, of the transparent sort, is shown in the figure opposite. At S is placed a narrow slit upon which is concentrated a beam of sunlight, the slit is supposed perpendicular to the plane of the paper so that its section is shown at S. In front of the slit is placed a lens L which forms a sharply defined image of the slit on the distant screen at O. If the grating is now interposed as shown, with its bars or rulings parallel with the slit, there are seen upon the screen several spectra on each side of the central image, which are said to be of the first,

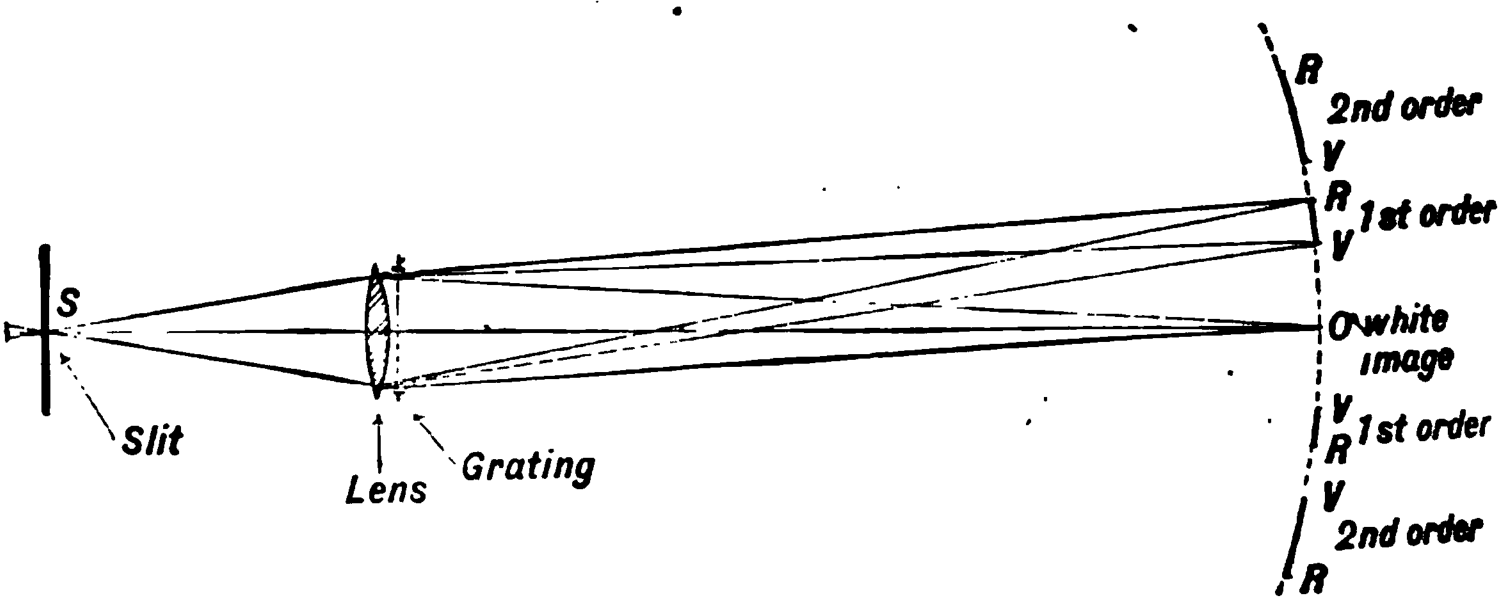


FIG. 582.—Spectra formed by diffraction grating.

second, or third order, etc., according to their distances from the center. These spectra have their violet ends toward the center and their lengths are nearly proportional to the numbers expressing their orders.

If the grating has only two or three bars to the millimeter the spectra will be very narrow, forming a group of bright bands on each side of the central image. But as the rulings are made closer together the spectra are longer and more spread out.

Very perfect gratings were made by Professor Rowland, of Baltimore, on a ruling engine devised by him. In many of these gratings 14,438 lines are ruled to the inch, or about 568 lines per millimeter.

939. How Gratings Produce Spectra.—Let the grating consist of a set of opaque bars, which are represented in cross-section, greatly magnified, by the heavy lines in figure 583.

When a series of flat waves comes from the left, as shown by

ıe arrows, the ether particles in the openings *a b c*, etc., are simulıneously set in vibration, and by Huygens' principle each par:le is a center from which wavelets spread out in all directions to the region beyond.

Now, if a convergent lens is placed in front of the grating as .own at *L*, a flat wave parallel with the grating will be converted ; the lens into a concave wave converging upon its principal cus at *O*, the lens retarding the middle portion of the wave ore than the edges, so that all parts reach *O* at the same instant. herefore wavelets starting simultaneously from all the grating

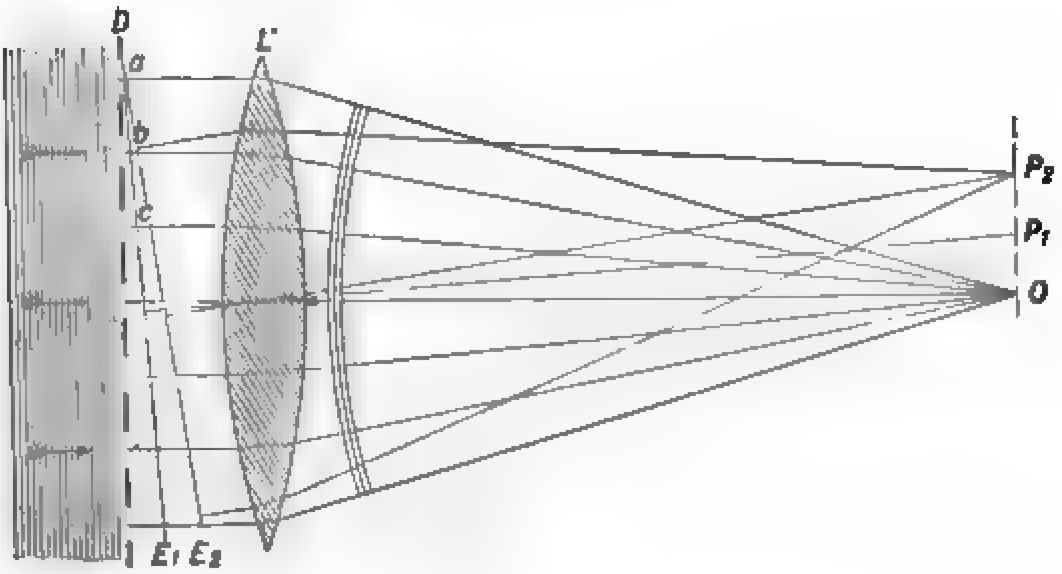


Fig. 583.

ɔenings will by the effect of the lens reach *O* at the same time ɔd in the same phase. **The point *O* will therefore be bright hatever may be the wave length of the light.**

In the same way by the effect of the lens an oblique wave ırallel to DE_1 is brought to focus at P_1 on the line through the ɛnter of the lens and perpendicular to DE_1. Therefore, in ɔnsequence of the lens it takes light equally long to reach P_1 om any point whatever on DE_1, and consequently wavelets om the grating that agree in phase on reaching DE_1, will also gree in phase at P_1.

Suppose that DE_1 is drawn through the edge of one grating ɔace in such a direction that it is distant exactly one wave ɛngth from the corresponding edge of the next grating space, as ıown in figure 584 where *fe* is supposed just equal to a wave ɛngth. Then a wavelet starting from a point in the opening *a* ill reach DE_1 at the same instant as the wavelet which started

from the corresponding point in b just *one complete period before*, and the two wavelets will therefore reach DE_1 in the same phase. So also the wavelet reaching DE_1 from the corresponding point in c will agree in phase with those from a and b, and thus as wavelets from all the openings reach DE_1 in the same phase they

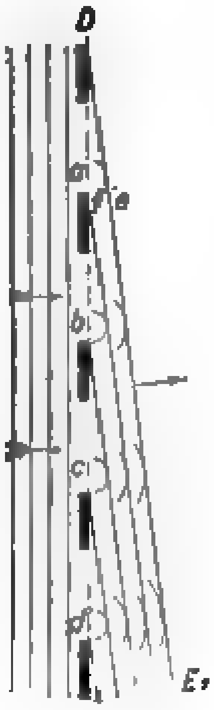


Fig. 584.

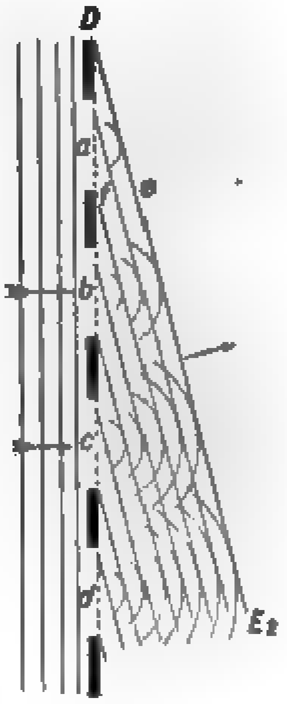


Fig. 585.

will agree in phase at P_1, which will therefore be bright, and may be called the *first order* bright spot.

Let us now consider a line DE_2 (Fig. 585) so oblique that fe is equal to two whole wave lengths. Then again wavelets from corresponding points in all the openings will agree in phase on reaching DE_2 and consequently the point P_2 to which they are

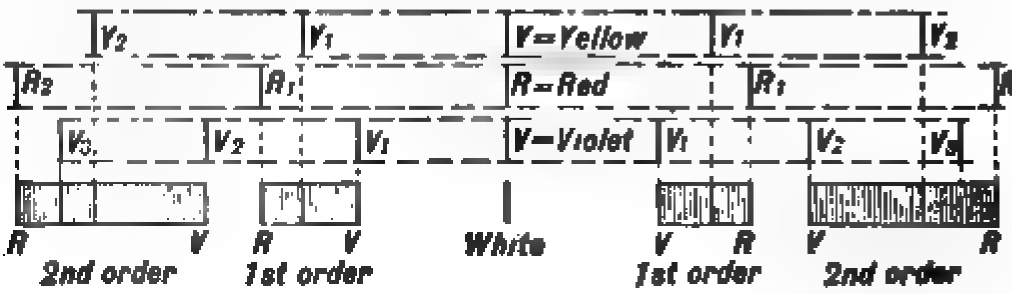


Fig. 586.—Formation of diffraction spectra.

converged by the lens is a bright point. Thus on each side of the central spot at O there will be bright spots of the first, second, etc., orders.

If the light were homogeneous or all of one wave length there would be only these bright spots, all of the same color; for in-

stance, if the grating is illuminated with sodium light there will appear a central yellow band with narrow yellow bands on each side somewhat as shown in the upper row in figure 586. With a source giving out longer waves, as of red light, the bands would be farther apart; for the distance *fe* in figure 584 would be greater and hence the line DE_1 would be more inclined, making the point P_1 farther from the center. While if the waves were shorter the bands would be closer together, as shown at V_1V_2 in the third row of figure 586. Consequently when *white* light shines upon the grating, having all wave lengths present, each wave length produces a bright band at the appropriate distance from the center, and therefore there results the spectra of the different orders represented in the lower part of the figure, the violet end of each order being toward the center, showing that wave lengths increase from the violet toward the red end of the spectrum.

940. Effect of Removing Grating.—If the grating is removed the side spectra all vanish, leaving only the central image at *O*. For the wavelets which were cut out by the bars of the grating now interfere with the wavelets which formed the side spectra. This is easily seen by a consideration of figure 587. Let *ac* be drawn from the edge of one grating space so that *bc*, its distance from the corresponding edge of the next space, is one wave length, then *bc* will be the direction in which the first order spectrum is formed. Imagine the grating bar between *a* and *b* removed so that light may now proceed from all points between *a* and *b*. Let *e* be a point halfway between *a* and *b*, then *ef* is one-half a wave length, and corresponding to any point *h* between *a* and *e* there is a point *g* between *e* and *b* which is just a half wave length farther from *ac*. Waves therefore which start in the same phase from *h* and *g* must reach *ac* in opposite phases, and consequently light going out in the direction *bc* from points between *e* and *b* will be exactly interfered with and neutralized by light from points between *e* and *a*, and *there will therefore be no first order spectrum.* In a similar way it may be shown that if the grating bars are removed there will be no side spectra of any order.

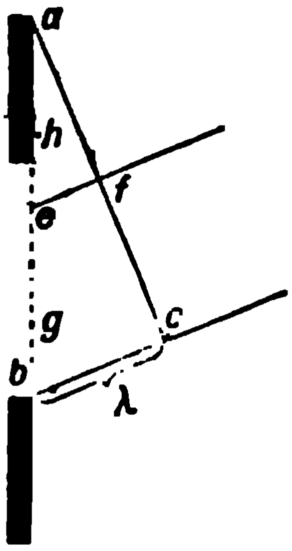


FIG. 587.

941. Resolving Power of Grating.—A grating should have a large number of bars and spaces for two reasons. First, the brightness of the diffraction spectra will be greater the larger the number of grating spaces. And, second, the power of a grating to give a sharply defined spectrum is proportional to the total number of grating spaces, other things being equal. For let AB, figure 588, represent a grating of 1000 spaces, and let AD be so drawn that its distance from the first grating space next to A is one wave length, from the second space its distance is 2 wave lengths, etc., from the 500th space at C its distance is 500 wave lengths represented by CE, and from the 1000th space at B its distance BD is 1000 wave lengths. Wavelets from all the openings of the grating therefore reach AD in the same phase and therefore conspire to form the bright first order spectrum in the direction EF. But now suppose the direction of AD to be slightly changed so that BD = 1001 wave lengths, then CE will equal 500½ wave lengths, and light from B and C will therefore reach AD in *opposite* phases; so also light from the next opening above B will reach AD in opposite phase to that from the next above C, and so on, light from the openings between B and C opposing that from the corresponding openings between C and A. There will therefore be no light of the given wave length in that direction.

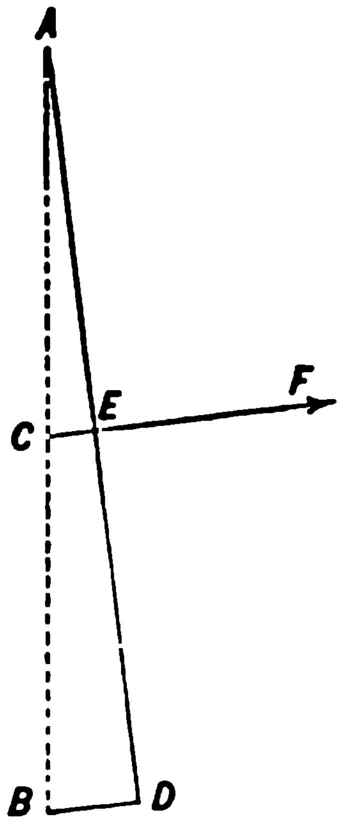


Fig. 588.

It thus appears that when BD is 1000 wave lengths there is a bright band of the first order in the direction EF, but this bright band must be exceedingly narrow, for so slight a change in direction of EF as will change BD to 1001 or to 999 wave lengths will take us beyond its limits. *Hence the more lines there are in the grating the narrower will be the bright image due to any one wave length and the closer together two spectrum lines may be and yet be separately distinguishable.*

942. Measurement of Wave Length of Light.—Diffraction gratings afford one of the most convenient means of measuring the wave length of light. The grating may be mounted on a *spectrometer*, as shown in figure 589, so that light from the slit S passes through the lens of the collimator and falls upon the grating at G in plane waves. The observer adjusts the telescope T so that the image of some line in the first order spectrum falls on the cross-hairs in the telescope. The telescope may then be moved into the position T' shown by the dotted lines, so that the central bright image (Fig. 582) comes on the cross-hairs, then the angle between these two positions of the telescope, which is read from the graduated circle, is the angle dce or x in the small

ı and this is equal to the angle *bac*. But the triangle *acb* ·angled at *b*, and *bc* is equal to the wave length λ which is .etermined, while *ac* is known from the measurement of ting and is called the grating space. Representing *ac* by ve $cb = ac.\sin x$, or

$$\lambda = s \sin x.$$

. this formula the wave length may be determined when

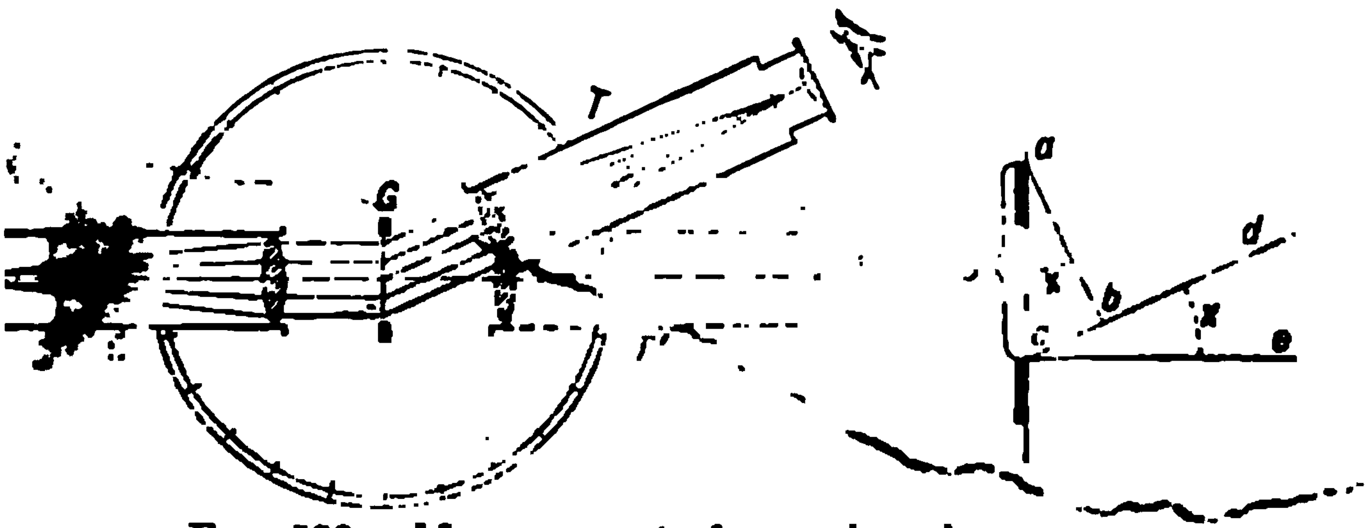


FIG. 589.—Measurement of wave lengths.

een measured as above described; since s is known from tion $s = \frac{1}{n}$, where n is the number of lines per millimeter rating.

Wave Lengths of Some Spectrum Lines.—

Table of Wave Lengths

Lines in the solar spectrum			Wave lengths in millionths of a millimeter
limit of visible red.	Fraunhofer's	A.....	759.4
.	Fraunhofer's	B.....	686.7
rogen line.	Fraunhofer's	C.....	656.3
ines.	Fraunhofer's	D_1....	589.6
		D_2....	589.0
rogen line.	Fraunhofer's	F.....	486.1
mit of visible violet rays.	[illegible]ofer's	H.....	396.9
		K.....	393.4

Concave Gratings.—It [illegible] discovered by Rowland that grating is ruled on a polished *concave* mirror surface of *on a flat* one very perfect diffraction spectra may be

[redacted]

953 - 956

spectrum. By sliding A toward S the first order spectrum may be brought in front of the eye-piece.

An important advantage of the spectrum photographs made with this apparatus is that the distances between spectrum lines are proportional to the differences in their wave lengths, so that a scale of equal parts may be made, which when applied to the photograph will give the wave length of every line on the plate.

Problems

1. Two flat pieces of glass touching at one edge and separated at the other by a thin piece of tinfoil show 30 bright interference bands when examined in sodium light reflected perpendicularly from the thin air film. What is the thickness of the tinfoil?
2. A narrow slit illuminated by light of wave length $600\mu\mu$ gives rise to diffraction bands on a screen 2 meters behind the slit. The two dark bands, one on each side of the central bright band and nearest to it, are just 1 cm. apart. Find the width of the slit.
3. A glass transmission diffraction grating has 50 lines to the millimeter. How far will the first order spectra of sodium light be from the central line when the screen is 6 meters distant?
4. What orders of diffraction spectra will be absent in the spectra produced by a transmission grating in which the bars are exactly equal in width to the spaces between them? See §940.

Polarized Light

945. Polarization by Tourmaline.—If two plates of tourmaline, cut parallel to the axis of the crystal and of suitable thickness, are placed one upon the other with their axes parallel, light will be transmitted through both plates; but if one is gradually turned on the other the transmitted beam will become fainter until when the two are crossed at right angles there is complete extinction. It is thus seen that light after coming through the first plate of tourmaline is different from ordinary light; for the second plate must have its axis in a particular direction in order to transmit the beam, while in case of ordinary light the transmitted beam is equally intense whatever may be the direction of the crystal axis.

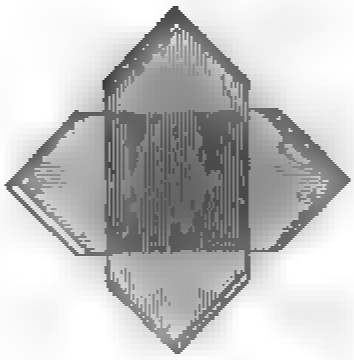


Fig. 591.—Crossed tourmalines.

A beam of light having this characteristic is said to be *polarized* and the first plate of tourmaline, which impresses this peculiarity on the light is called the **polarizer.** The second plate of tourmaline, which reveals the fact that the beam is polarized, is known as the **analyzer.**

946. Direction of Vibrations.—If the vibrations in waves of light, like those in sound waves, were perpendicular to the wave front or *in the ray direction,* rotating the tourmaline plate about the ray as an axis would not change its relation to the direction of vibration and consequently the vibrations could not be extinguished in that way, but would pass through both polarizer and analyzer even when they were crossed. We must therefore conclude that **in light waves the vibrations are wholly at right angles to the ray direction.** The French physicist Fresnel was the first to draw this conclusion.

947. Nature of Polarized Light.—In homogeneous light, or light of one wave length, the vibrations must be in circles or in some form of ellipse or straight line, but in white light they are doubtless very complicated and irregular. But however complicated they may be, each may be conceived as the resultant of two rectilinear vibrations at right angles to each other. For instance, let the path of an ether particle during a short interval be represented by the convoluted line in the diagram, the beam of light being perpendicular to the paper.

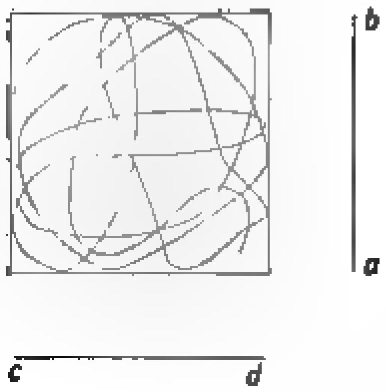


FIG. 592.

It is clear that its actual motion at any instant may be considered as made up of an up and down motion in the direction of the line *ab* combined with a sidewise motion in the direction of *cd.* Any circumstance which would cause one of these component vibrations to be suppressed without affecting the other would leave the particle oscillating along a straight line. It is precisely this which is believed to be effected by the tourmaline plate. Suppose that it absorbs all vibrations at right angles to its axis, while it transmits those which are in the direction of the axis, then the light transmitted through the first tourmaline will all be vibrating in one direction, *and if* the axis of the second tourmaline is at right angles to the *first,* no light will get through.

A beam of plane polarized light is therefore believed to be one in which the vibrations all take place in some one direction perpendicular to the ray.

948. Mechanical Illustration.—How it is possible for tourmaline to absorb one component of vibration and transmit the other may be seen from the following mechanical illustration. Let a weight of a pound or so be hung as a pendulum from the end of a light strut of wood which reaches out, say, 4 ft. from the wall and is stayed in position by cords *a*, *b*, and *c*, as shown in figure 593. The cords *b* and *c* are somewhat slack and tied into a loop of cord 3 or 4 in. long, which can slip across the end of the strut and is kept in position by three small nails one above, one below, and one passing through it and limiting the amount of sidewise slip.

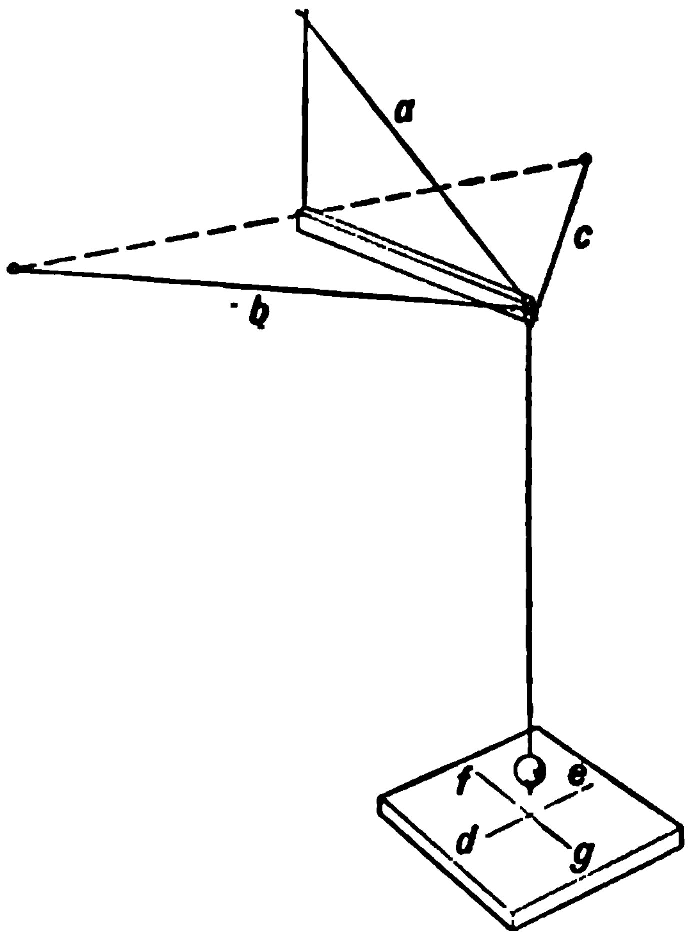


FIG. 593.

When properly adjusted if the weight is set swinging in the direction *de*, it gives up motion to the strut and soon comes to rest, while if it swings in the direction *fg* the strut is not disturbed and the motion of the pendulum persists. If we now set the weight swinging diagonally or around in a circle, the sidewise component of the vibration is soon suppressed and the weight is left swinging in the direction *fg*.

Just so, if there is any frictional resistance to the light vibrations in one direction in the tourmaline, the energy of vibrations taking place in that direction will be dissipated in heat and they will be absorbed, while those components of vibration at right angles to that direction may be freely transmitted.

949. Polarization by Reflection and Refraction.—When a beam of light falls obliquely on a piece of flat unsilvered glass so that the angle of incidence is about 57½°, the reflected beam is polarized, as may be ascertained by examining the light with a tourmaline plate. It is found that when the axis of the tour-

[redacted]

maline is parallel to the plane of incidence the beam is absorbed by the tourmaline, while if the axis of the tourmaline is perpendicular to the plane of incidence the reflected beam is largely transmitted. And so in general, when light is reflected at the surface of any transparent substance it is found that **for a certain angle of incidence the reflected beam is almost completely polarized. This angle is known as the polarizing angle.**

In this case it is found that **the refracted beam is also polarized in a direction at right angles to that of the reflected beam.** For a crystal of tourmaline having its axis parallel to the plane of incidence absorbs the reflected rays, while it must be held with its axis at right angles to the plane of incidence to absorb most completely the refracted beam.

If the reflected beam contained all those component vibrations of the incident light which are *at right angles* to the plane of incidence while the refracted beam contained all the vibrations *parallel* to the plane of incidence, each beam would be *completely polarized* and they would be equally intense, each having half the energy of the incident beam. But the reflected beam is usually much less intense than the refracted one, and consequently the refracted beam cannot be *completely* polarized, but must contain some of both components of vibration.

In case of glass the beam reflected at the polarizing angle contains only about 9 per cent. of the energy of the incident beam. To increase the effect it is common to make use of a pile of thin plates of glass instead of a single reflecting surface. By this device the reflected beam is brighter and the light refracted through the plates is more completely polarized.

Light is polarized in this way also at the surface of opaque substances, such as black glass, which absorb the refracted beam and do not have metallic luster.

Metals and substances having metallic luster reflect both components of vibration and cannot be used to polarize by reflection. Consequently light cannot be polarized by reflection from an ordinary silvered mirror.

950. Brewster's Law. It was discovered by Sir David Brewster that **the polarizing angle for any substance is that angle of incidence at which the reflected and refracted rays are at right**

angles to each other. This is known as Brewster's law of the polarizing angle; it leads at once to the relation

$$\tan p = n$$

where p is the polarizing angle and n is the index of refraction of the substance. For

$$n = \frac{\sin p}{\sin r} \qquad \text{(see figure 594)}$$

but if the angle between the reflected and refracted rays is 90°, p and r must be complementary and $\sin r = \cos p$. Therefore

$$n = \frac{\sin p}{\cos p} = \tan p.$$

By the use of this relation the index of refraction of opaque substances, such as dense black glass, may be approximately determined from a measurement of the polarizing angle.

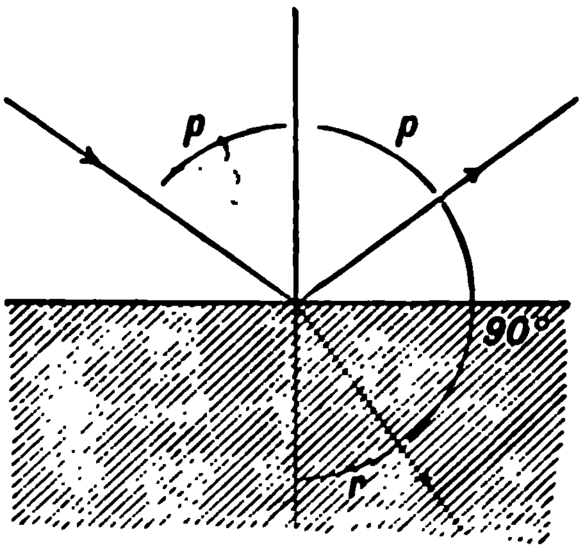


Fig. 594.—Polarizing angle.

951. Plane of Polarization.—Light polarized in the manner that has been described is said to be *plane polarized* to distinguish it from circularly and elliptically polarized light, which will be discussed later.

By common consent a beam of light polarized by reflection is said to be polarized in the plane of incidence or its plane of polarization is said to be parallel to the plane of incidence, while the plane of polarization of the refracted beam is at right angles to the plane of incidence. It is to be understood that this is simply a convention.

To find the plane of polarization of any beam of plane polarized light it is only necessary to let it fall on a plate of glass at the polarizing angle and then turn the reflecting plate about the incident beam as an axis until the reflected ray has maximum brightness. The plane of incidence is then the plane of polarization of the incident beam. In this way it may be found that the plane of polarization of a beam of light transmitted through tourmaline is at right angles to the axis of the tourmaline.

The direction of vibration in plane polarized light is believ

[redacted]

to be at right angles to its plane of polarization. This may be inferred from the following case of polarization.

952. Polarization by Fine Particles.—When a beam of light shines through a cloud of fine particles it is scattered or diffused to some extent and it is found that the light sent out at right angles to the direction of the original beam is plane polarized. This is easily shown by reflecting a beam of sunlight down into a tall glass jar filled with water made slightly soapy so that it shows a delicate bluish tint. Light is scattered sidewise in all directions so that the path of the beam appears bright, and by means of a tourmaline plate it is found that light coming out in such a direction as CD (Fig. 595) is plane polarized, *its plane of polarization being the vertical plane through* CD and AB.

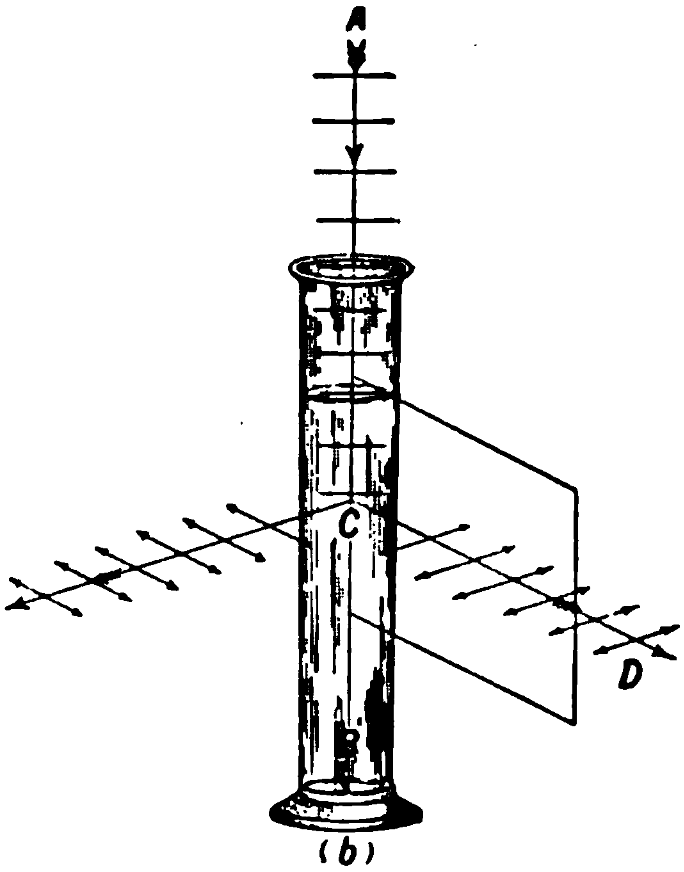


Fig. 595.

In this case it seems easy to see that on whatever side of AB the light may come out, the vibrations in the scattered light must be in a *horizontal direction.* For in the incident beam AB, the wave fronts are horizontal and consequently all the vibrations are in horizontal planes, and therefore *the vibrations in the scattered light may also be expected to be horizontal,* for the particles which scatter the light are too small to cause an actual turning of the wave front such as takes place in ordinary reflection from an oblique surface.

The above experiment therefore points to the conclusion that the direction of vibration in a plane polarized beam is at right angles to the plane of polarization, as stated in the previous paragraph.

953. Color of the Sky.—When the particles are small compared with the wave length of light the shorter waves are most strongly scattered, so that the diffused light is bluish, while the transmitted beam has a larger proportion of the long wave lengths, and therefore appears yellowish or even red.

Thus when we look toward the sun through a thick layer of air filled with fine particles, as at sunset, we see the familiar red and yellow tints; but looking at right angles to the direction of the sun, the diffused light from the sky is bluish and is also found to be polarized.

954. Polarization by Double Refraction.—When a crystal of Iceland spar is laid on a printed page, the letters are all seen *double*. A single black dot on the paper appears as two, and if the crystal while lying on the paper is slowly rotated about a vertical axis, one image of the dot is seen to revolve about the other. The paths of the rays of light in this case are shown in figure 596. Light from the black dot at *P* passes to the eye at *E* in two beams, one of which *PAE* is refracted at the surface according to the ordinary law, while the other is bent in an unusual way at *B*. The first is known as the *ordinary* ray and the other as the *extraordinary*. These two beams are found to be *oppositely polarized.* This may be shown by means of a tourmaline placed on the Iceland spar. If the axis of the tourmaline is in the direction of the line joining the two images *P* and *P′*, the extraordinary ray *PBE* is transmitted while *PAE* is extinguished, while the reverse is true if the axis of the tourmaline is at right angles to the line joining *A* and *B*.

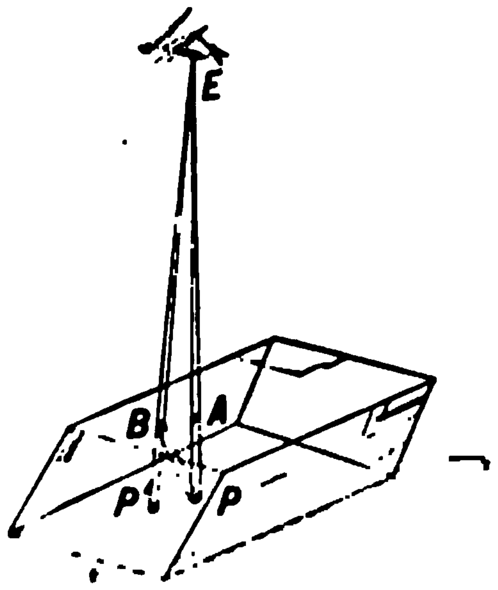


Fig. 596.—Double refraction.

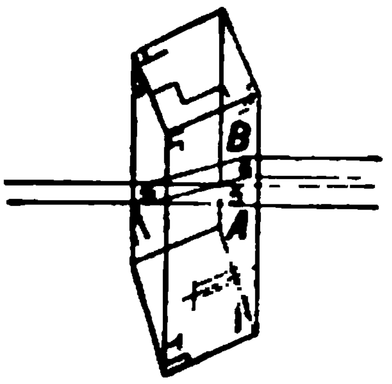


Fig. 597.—Double refraction of a narrow pencil of light.

When a narrow beam of sunlight falls perpendicularly on one face of a crystal of Iceland spar it is divided into two beams, one of which, the *ordinary* beam, passes straight through, while the other is refracted in an oblique direction in the crystal but emerges parallel to the first at the second face of the crystal, as shown in figure 597. If the incident beam is sufficiently narrow the emergent beams will be separate, otherwise they will overlap. On testing the two beams with the tourmaline plate they are found to be oppositely polarized. The polarization in this case is complete,

[redacted]

each beam transmitting only one component of vibration, so that if the incident light is unpolarized, each of the two beams will have just one-half the intensity of the original beam.

955. The Double-image Prism of Fresnel.—In order to separate a beam of ordinary light of the full size of the crystal plate into two oppositely polarized beams Fresnel cut the second face of the crystal obliquely, forming a prism, from which the two beams emerged in slightly divergent directions, as shown in the upper part of figure 598. By placing a suitable prism of glass in the reverse position against the prism of spar, both beams may be bent upward enough to restore one of them to its original direction, as shown in the lower diagram. At a little distance from the prism the two beams become quite separate in consequence of their divergence. On looking through such a prism all objects are seen double. This is one of the best means of obtaining polarized light where it is desired to transmit both of the two component beams.

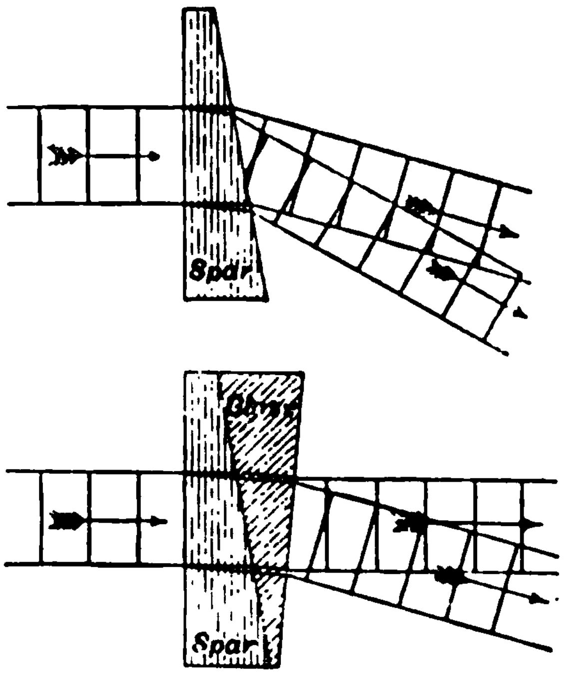


FIG. 598.—Double-image prism.

956. Nicol's Prism.—When we wish to obtain only one beam of polarized light, a Nicol's prism may be used. To make such a prism a long crystal of spar is taken having the form shown in

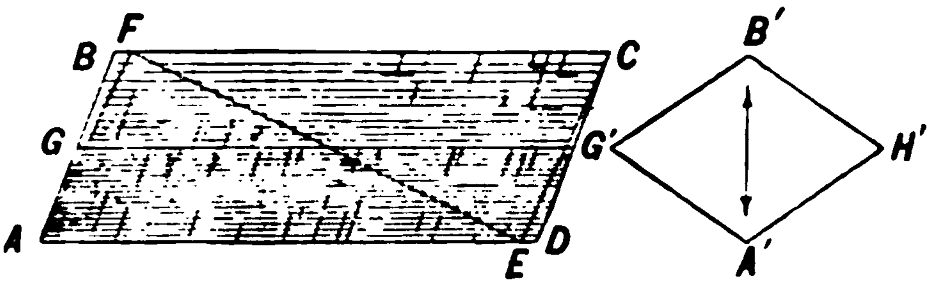


FIG. 599.

figure 599, where *ABCD* represents a side view and *A'G'B'H'* an end view. New end faces *AF* and *EC* are cut, inclined about 3° more than the natural faces, and the crystal is then divided by an oblique cut *FE* which is perpendicular to the plane *ABCD* and also perpendicular to the new end faces. The two surfaces of the cut *FE* are ground and polished and cemented together with Canada balsam, and the prism is then mounted in a protecting case which permits light to pass through it endwise.

The incident beam I on entering such a prism is doubly refracted, the ordinary ray in the crystal travels with less velocity than in Canada balsam, and meeting the surface FE at an angle greater than the critical angle, is *totally reflected* (§843) off to one side, as shown at O. But the extraordinary ray travels in the crystal with a greater velocity than in Canada balsam and therefore cannot be totally reflected and so passes through the prism

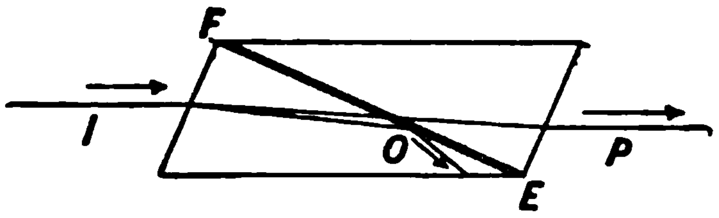


FIG. 600.

and emerges as a plane polarized beam in which the direction of vibration is perpendicular to $G'H'$, the longer axis of the rhombus which forms the end of the prism.

A Nicol's prism, or Nicol as it is often called, appears perfectly transparent like clear glass, but the transmitted beam has only half the intensity of the incident one when the latter is not polarized.

957. Double-image Prism as Analyzer.—Let a lens L (Fig. 601) be placed in front of an opening at O, through which a beam of

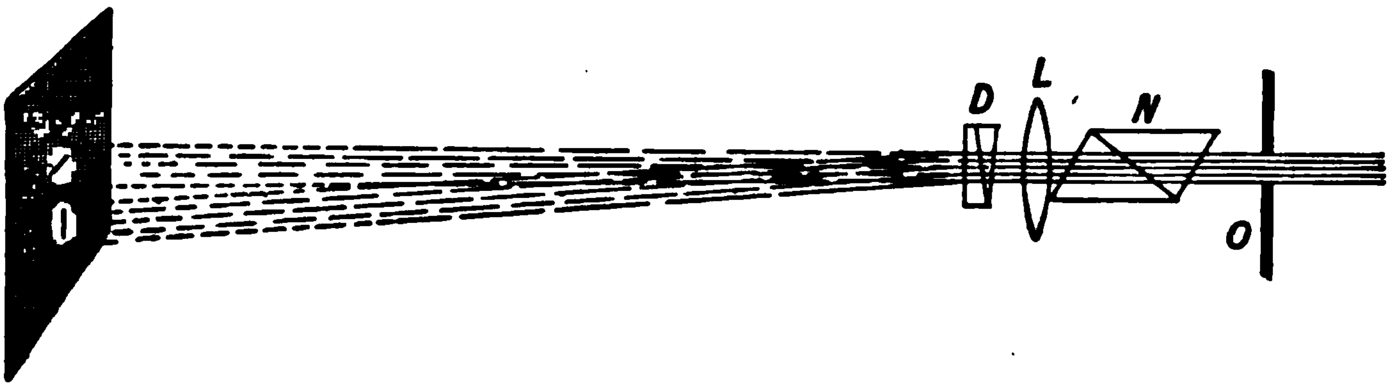


FIG. 601.

unlight passes, so as to form a bright image of the opening on a creen at S. On interposing a double-image prism D two images S and S' are formed. Let us suppose the vibrations in the lower beam to be in the direction of the line SS' joining the centers of he two spots, while in the ot right angles to that direction.

Now interpose a Ni s polarized before eaching D. Ther tion shown in the

[redacted]

cut, in which it transmits only vibrations in the direction SS', there will be seen on the screen only the spot S'. If the Nicol is slowly rotated about the beam as an axis, the spot S appears, at first faint but growing brighter, while S' grows dimmer, until, when the Nicol has been turned through 90°, S' has vanished and S receives all the light.

To understand these changes let the student in looking at the diagram (Fig. 602) imagine himself looking along the beam of light from O toward the screen, and let N represent the direction of the vibrations transmitted by the Nicol. The double-image prism resolves the vibrations into the two components S and S' which have different velocities in the crystal and are therefore separated, one going to the spot S and the other to S'.

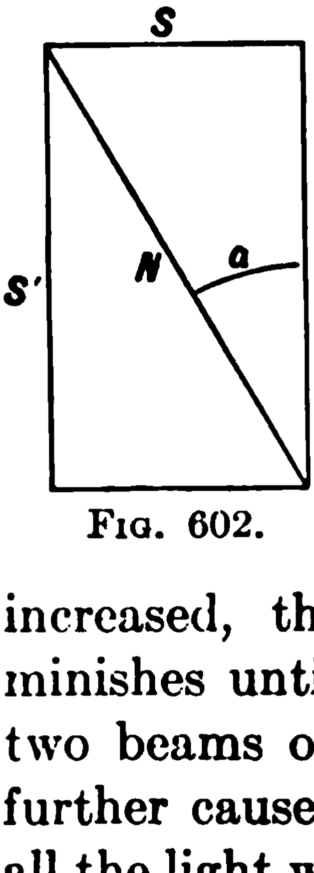


Fig. 602.

The lines S and S' represent by their lengths the amplitudes of the vibrations in the two beams; and it is evident that as the angle a is increased, the amplitude of S increases and that of S' diminishes until when a is 45°, S and S' will be equal and the two beams of light will be equally bright. Turning the Nicol further causes S to become brighter than S' and when a is 90°, all the light will be transmitted in S, and S' will have vanished.

958. The Wave Surface in Iceland Spar.—The Dutch physicist Huygens as early as 1690, to explain the double refraction of Iceland spar, advanced the very ingenious idea that a wave of light in such a crystal, instead of spreading out from a center as a spherical wave, divides into two waves, one of which advances as a spherical wave, just as in glass or water, and gives rise to the ordinary ray, while the other wave spreads out as an ellipsoid of revolution and gives rise to the extraordinary ray. He showed that this assumption explained the double refraction of Iceland spar, but he could not explain the polarization of the two beams. This was accomplished by Fresnel, who, in 1821, not only showed that polarization may be explained by the assumption that the vibrations in light waves are *transverse*, but also explained how to account for the separation of the wave in Iceland spar into the spherical and ellipsoidal surfaces conceived by Huygens.

cording to Fresnel, the cause of this separation is the fact that the ity of light in a crystal depends on the direction of the vibrations in the front. In a crystal there is a certain direction called the optic axis, and eland spar, waves in which the vibrations are at right angles to the optic advance through the crystal with less velocity than waves in which the ations are parallel to the axis, while if the vibrations are neither parallel at right angles to the axis the velocity is intermediate.

in the wave surface shown in figure 603 AB is the direction of the optic is. Vibrations on the surface of the spherical wave sheet are everywhere the direction of *parallels of latitude* about A and B as poles, and are there-re everywhere at right angles to the di-ection AB. This wave therefore advances rith the same velocity in all directions and nust be spherical in form. On the el-lipsoidal surface the vibrations are in the direction of the *meridians*, consequently at D and at all points on what may be called the equatorial belt of the ellipsoid the vi-brations are parallel to AB, at C they are inclined to the axis, while at A they are perpendicular to it, hence the velocity is greatest in directions such as OD, less in the direction OC, and in the direction OA it is the same as for the spherical sheet, for the vibrations in both are perpendicular to the axis.

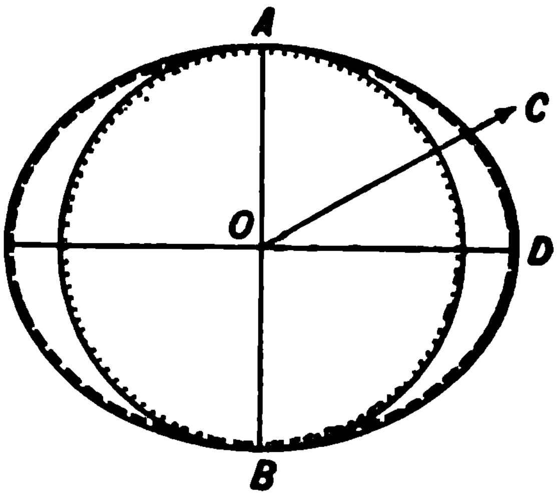


FIG. 603.—Huygen's wave surface.

959. Explanation of Double Refraction.—Let a beam of light fall perpendicularly upon the surface of a crystal of Iceland spar, meeting the surface at AB (Fig. 604), which shows a vertical section through the spar and beam), and let Ax and Bx' be in the direction of the optic axis of the crystal. Then when a wave meets the surface at AB it sets up vibrations at A and B and all intermediate points, which spread out as wavelets in the crystal, each having the form of the wave surface described in the last paragraph. Those components of vibration which are parallel to the line AB go to form the ellipsoidal sheets, while those which are perpendicular to the plane of the diagram form the spherical sheets. The original wave front will thus be separated into two, one of which, CD, is the resultant of the spherical waves and contains vibrations at right angles to the diagram (indicated by dots in the figure), while the other, EF, is the resultant of the ellipsoidal wavelets and has its vibrations parallel to AB (indicated by dashes in the figure). The former is the ordinary ray and the latter the extraordinary. It will be observed that the extraordinary wave EF has greater velocity in the crystal than the ordinary ray, and it moves obliquely, for it must be the envelope of all the ellipsoidal wavelets from A and B and intermediate points.

If the second surface of the crystal is parallel to the wave fronts CD and EF, both beams will emerge perpendicular to the surface, for all points in the wave front EF reach the refracting surface at the same instant, and giving rise to spherical wavelets in the outer medium must advance perpendicular to the surface.

960. Most Crystals Double Refracting.—All crystals except those belonging to the so-called regular or cubical system are more or less double refracting. Crystals of the hexagonal and tetrahedral systems have a single optic axis, as in case of Iceland spar and quartz, and are said to be *uniaxial*. While those of the three remaining crystal systems have two optic axes and are called *biaxial*.

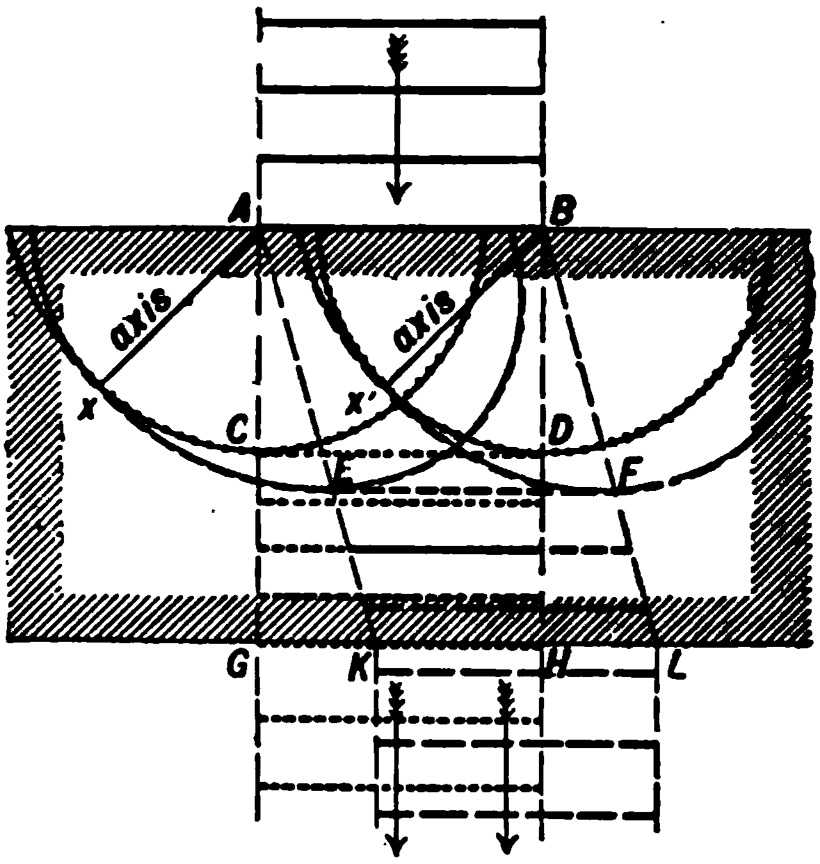


Fig. 604.—Double refraction.

961. Rotation of Plane of Polarization.—When a beam of plane polarized light is sent through a crystal of quartz in the direction of its optic axis the plane of polarization is rotated through an angle which depends on the thickness of the quartz and the wave length of the light. Suppose the diagram (Fig. 605) represents the cross-section of a crystal of quartz through which light is coming up toward the observer. Let AC be the direction of vibration in the incident beam, then red light may be rotated through the angle AOR and come out vibrating along the direction RR' and violet light being still more strongly rotated may emerge vibrating along VV', with intermediate wave lengths between. If the incident beam is of white light then the emergent beam is also white, as all the light is transmitted, but if an analyser such as a Nicol's prism, is used, in such a position as to transmit

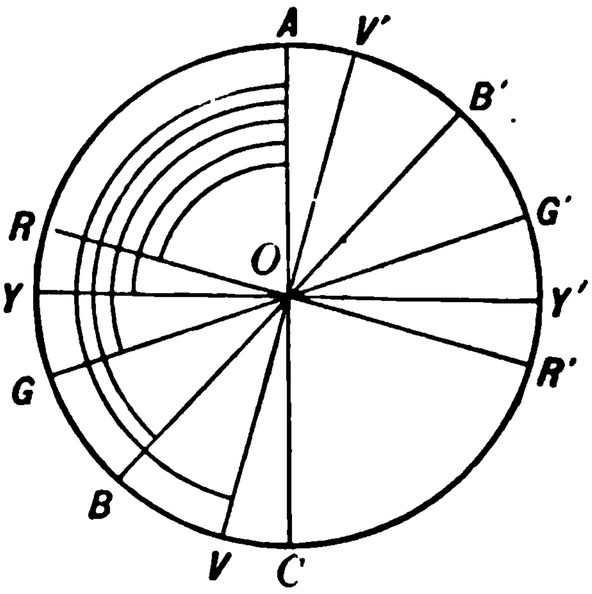


Fig. 605.

vibrations in the direction RR', the transmitted light will be red, for vibrations at right angles to RR' will be completely cut out, and those in other intermediate directions only partially transmitted. As the analyzer is rotated the tint of the light changes becoming bluish or violet when vibrations in the direction VV' are transmitted while those in the direction RR' are extinguished.

If a double-image prism (§955) is used as analyzer the two beams of oppositely polarized light coming from the prism will be of complementary colors when the original incident beam is white, for one will transmit all the component vibrations which are excluded from the other.

Some crystals of quartz rotate the plane of polarization to the right and some to the left, the form of the crystal itself showing to which class a given specimen belongs.

962. Rotation by Liquids.—Still more remarkable is the rotation of the plane of polarization by certain liquids, among which

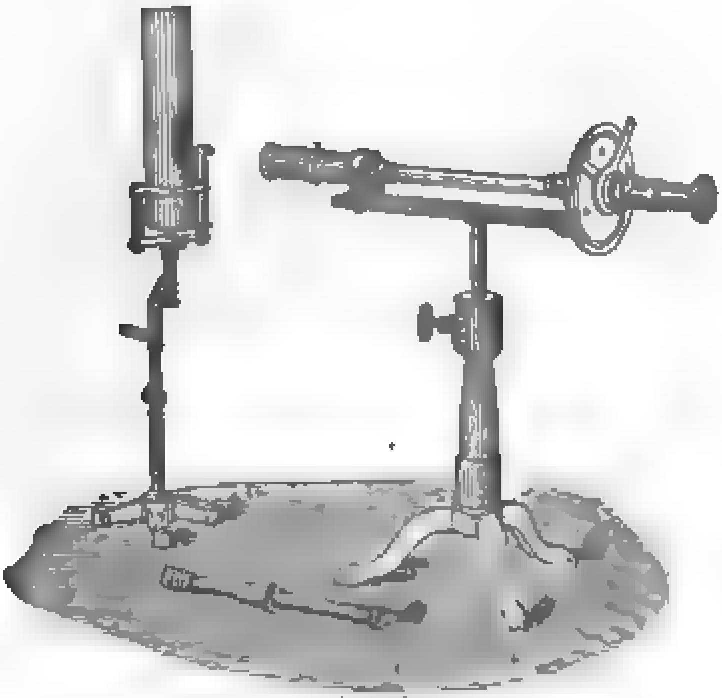


FIG. 606.—Saccharimeter.

may be mentioned turpentine, and solutions in water of tartaric acid, malic acid, and sugar. Both right and left varieties of tartaric and malic acids are known, and a solution containing equal amounts of the two varieties is neutral.

Cane sugar, or sucrose, rotates to the right, but by treatment with acid it may be broken up into a mixture of dextrose which rotates to the right, and of levulose which rotates to the left.

The proportion of cane sugar in a mixture of cane sugar and glucose may be determined by measuring the rotation of the solution both before and after the acid treatment, since the rotation due to glucose is not altered by the process.

An apparatus designed for the exact measurement of the rotation of the plane of polarization by sugar solutions is known as a saccharimeter.

963. Colors from Crystal Plates in Polarized Light.—When polarizer and analyzer are *crossed*, or so placed that the analyzer transmits only vibrations at right angles to those coming from the polarizer, no light will pass through the combination. But if a thin plate of mica or other crystal of suitable thickness is interposed between polarizer and analyzer, as at C in figure 607,

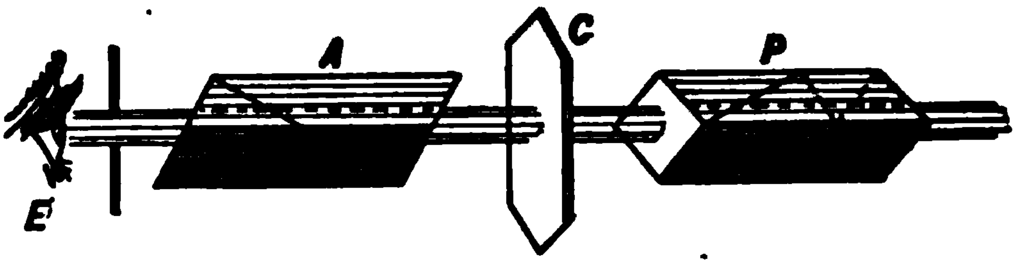


Fig. 607.

it may appear vividly colored as seen through the analyzer if the incident light is white. When the crystal plate is slowly rotated, keeping its plane perpendicular to the beam of light, the color is seen to be most intense when the optic axis in the crystal plate makes an angle of 45° with the plane of polarization of the beam, and fades out into darkness when the optic axis of the crystal is either parallel or perpendicular to that plane. The color depends on the thickness of the crystal, and a variety of beautiful colors may often be observed in mica plates in which some parts are thicker than others.

If an ordinary plate of glass is interposed between polarizer and analyzer no effect is observed, the light remains entirely cut off by the analyzer. But if the glass is *in a state of strain*, it acts like a crystal plate and appears bright to the eye at E. For instance, if a rod of plate glass is held across the beam at C so that its length makes an angle of 45° with the plane of polarization of the incident beam, on slightly bending the rod, it appears *bright* along the edges but dark in the center, for one edge is

[redacted]

ıed and the other compressed by the bending, but the center ıs unstrained.

when a piece of glass is heated in a flame and examined en the crossed Nicols, bright regions are seen, due to the s resulting from unequal heating, a the heat gradually diffuses gh the plate it loses its double ting power and becomes dark.

ces of glass that have been heated suddenly cooled remain in a ed or tempered state, and when ned with polarized light in the manner show characteristic pat- as in figure 608.

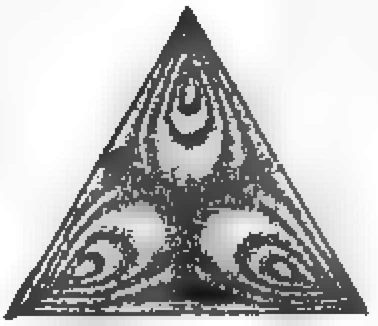


Fig. 608.—Strain figure in a triangle of tempered glass by polarized light.

his way it may be determined whether the glass for a tele- lens has been thoroughly annealed.

. **Circular and Elliptical Polarized Light.**—To understand oduction of colors in the case just discussed it will be necessary to r first what happens when a beam of plane polarized light of *one ngth* passes through a crystal plate. In figure 609 the beam of polarized supposed to be coming up toward the eye of the reader. To avoid on, the crystal plate and analyzer, instead of being shown superposed

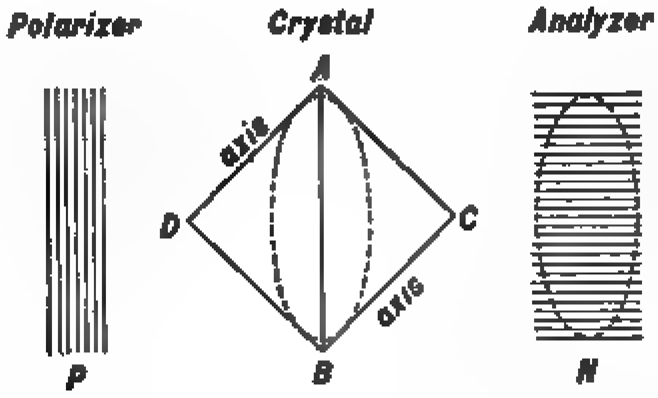


Fig. 609.

polarizer as they would actually appear to one looking along the beam, resented as shifted to one side so that each may be seen separately. cident light is supposed to be vibrating in the direction shown by the , P, with simple harmonic motion, since it is supposed homogeneous. ting the crystal it sets up vibrations in the same direction in the face it enters as represented by the line AB.

let us suppose that the crystal plate is placed with its axis in the direc- A or BC, at 45° to the direction of vibration in the incident beam.

wave length, and the emergent beam will be circularly polarized and so half transmitted by the analyzer.

On the whole, therefore, the crystal in this case would appear red or orange through the analyzer. But from a crystal plate of twice the thickness both the red and violet waves would emerge vibrating as in the incident beam and would be suppressed by the analyzer, while some intermediate wave length would be completely transmitted and the crystal would appear green.

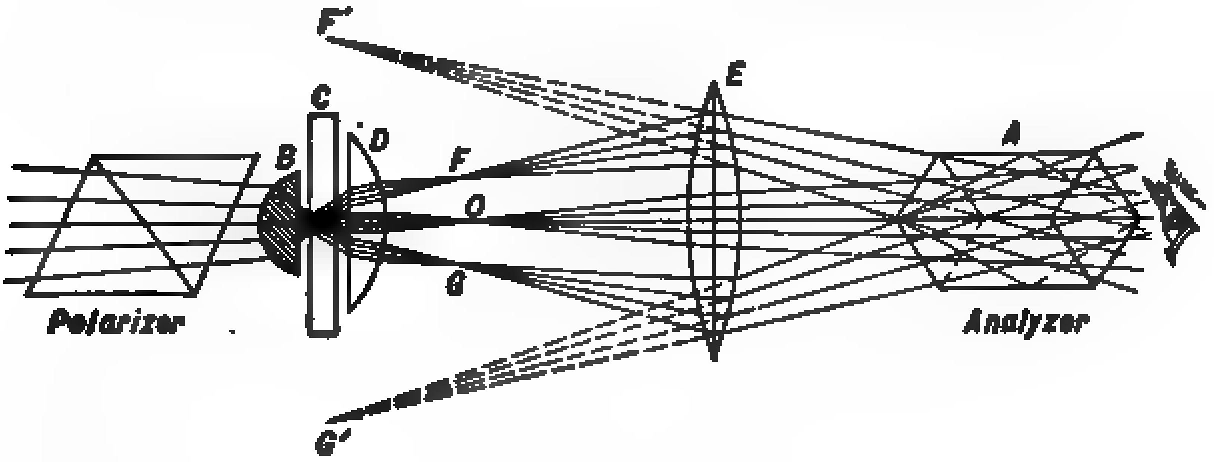


Fig. 611.—Optical system of polariscope.

966. Polarization Figures with Convergent Light.—When a thin plate of crystal is examined in an instrument called a polariscope, using a strongly convergent beam of polarized light, a polarization figure is obtained

Fig. 612.—Polarization figure for uniaxial crystal, perpendicular to axis, Nicols crossed.

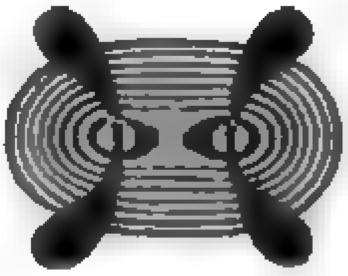


Fig. 613.—Polarization figure of biaxial crystal.

which is of great use to the mineralogist in revealing the optical properties of the crystal.

The optical system of the polariscope is shown in figure 611. The beam of light coming from the polarizer is converged on the crystal by the lens *B*. On the opposite side of the crystal plate *C* is a second short-focus lens *D*, beyond which is the eye lens *E* and analyzer *A*.

If the crystal is a plate cut from a uniaxial crystal perpendicular to its axis and if the analyzer and polarizer are crossed, a figure consisting of colored rings intersected by a black cross, as shown in figure 612, is seen at *F′O′G′* by the observer.

It is clear that rays coming to O in the center of the figure are those that have passed perpendicularly through the crystal section, while rays coming to other points of the figure have passed more or less obliquely through the crystal. Now, the more oblique the rays the greater the thickness of crystal traversed, hence the color seen at any point in the figure depends on the distance of that point from the center at O.

All points on a ring equidistant from O show the same color, because the rays at these points have traversed equal thickness of crystal at an equal inclination to the optic axis.

Rays passing through the crystal in certain directions, however, have their vibrations in such relation to the optic axis of the crystal that they are transmitted without any change in their polarization. All such are cut out by the analyzer and form the black cross.

In figure 613 is shown a more complicated polarization figure produced by a biaxial crystal, such as mica.

Reference on Polarization

EDWIN EDSER: *Light for Students.*

ELECTRICITY AND LIGHT

967. Magnetic Rotation of Light.—When a transparent substance is in a powerful magnetic field a beam of plane polarized light sent through it in the direction of the lines of force, has its plane of polarization rotated. This discovery was made by Faraday in 1845 and was the first evidence of a relation between light and electricity and magnetism.

The rotation is usually in the direction of the magnetizing current, or clockwise looking in the direction of the lines of force, though it is opposite in a solution of ferric chloride in water. The amount of the rotation is greatest in substances having a large index of refraction, and in a given substance is proportional to the length of the column and to the strength of the magnetic field.

If the light is reflected back again through the tube the rotation is doubled; that is, the rotation of the plane of polarization produced in this way is the same whether the light passes through the field in the positive direction of the lines of force or the reverse.

This last fact can be explained only by supposing *an actual rotatory motion of some sort* taking place in the magnetic field.

In this respect the magnetic rotation of the plane of polarization is different from that produced by quartz.

968. The Kerr Effect.—[illegible] covered by Faraday [illegible] beam of polarized [illegible] magnet the plane of [illegible]

969. Maxwell's Electromagnetic Theory of Light.—[illegible] year 1862 Maxwell [illegible] very short electromagnetic waves. [illegible] for the theory [illegible]

1. The [illegible] as that of light waves. [illegible] medium in [illegible] turbance. It is [illegible] waves are the same [illegible] municated by the same [illegible]

2. The [illegible] cases been found to be [illegible] those media.

3. Maxwell showed [illegible] through [illegible] *would also be* [illegible] case of metals [illegible] and also the most [illegible]

4. The [illegible] transmitting [illegible] right angles [illegible] in light waves [illegible]

970. Zeeman Effect.—[illegible] electricity and light [illegible] Maxwell's theory [illegible] covered by Zeeman [illegible] a sodium flame [illegible] wave lengths [illegible] powerful magnet [illegible] each single line [illegible] group of lines. [illegible] plained by the Dutch [illegible] sumption that light waves are electromagnetic waves originating in little vibrating *negatively charged electrons having the same mass and charge as the electrons in cathode rays.*

For a fuller discussion of the Zeeman effect see *Modern Theory of Physical Phenomena* by Righi.

971. Pressure of Light.—It was shown by Maxwell in 1873 that if light waves are electromagnetic they must exert a pressure against any surface on which they fall; and that the pressure against a reflecting surface must be twice as great as against an absorbing one. But the amount of this force is so small that for many years no one succeeded in proving its existence; for in full sunlight, according to Maxwell's theory, the pressure against a reflecting mirror one meter square is only one dyne, or less than the weight of one milligram.

But in 1900 Lebedew in Russia, and in 1901 Nichols and Hull in this country, were able to show that there is such a pressure, and in later experiments to prove that its amount is just what Maxwell's theory indicates.

The pressure of light has been shown by Fitzgerald and Arrhenius to be the probable cause of comet's tails, and Arrhenius has also proposed a very interesting explanation of the Aurora Borealis which depends in part on this same pressure.

APPENDIX 1

...T's THEOREM AND THE ABSOLUTE SCALE OF TEMPERATURE

...ork of Compression or Expansion.—When steam is ad... to a cylinder C and the piston P is pushed back through ...ance x, the force exerted by the steam against the piston ..., where p represents the pressure of the steam and A ... area of the piston head, and the work done is pAx, or

$$W = pAx.$$

...s the piston moves through a distance x the volume of the ...ker is enlarged by the amount xA. ...at representing the increase in ...e by v, we have

$$W = pv.$$

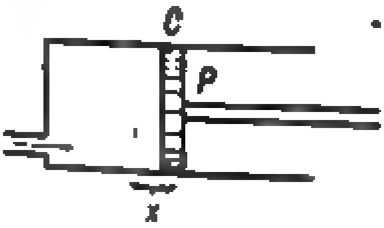


FIG. 614.

...is, *when any substance while ex... a pressure p expands in volume by an amount* [illegible] *...nt of work equal to the product pv.*

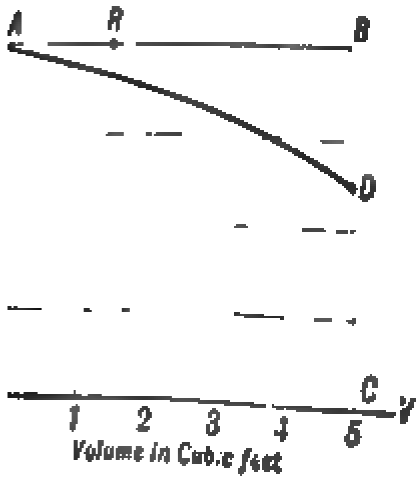


FIG. 615.

2. Work on the Pressure-volume Diagram.—[illegible] compression [illegible] veniently [illegible] on the [illegible] For [illegible]

...sured upward from [illegible] ... at the side; [illegible] ...ay 40 lbs. to the [illegible] the space from zero [illegible] ...epresented by the [illegible] ...es from A to B (Fig. 615 [illegible] The work [illegible]

[redacted]

which we have seen to be equal to the product of the pressure p by the increase in volume v is represented in the diagram by the area $ABCO$, or the area between the line AB and the base line, measured in appropriate units. If pressures are measured in pounds per square foot, and volumes in cubic feet, the work pv will be given in foot-pounds. In the above case the work is $5 \times 40 \times 144 = 28{,}800$ ft.-lbs., and the area of each small rectangle in the diagram represents 1440 ft. lbs. of work.

In case the pressure drops off as the volume increases, as indicated by the line AD in the diagram (Fig. 615), the work of expansion is represented by the area $ADCO$ between the curve AD

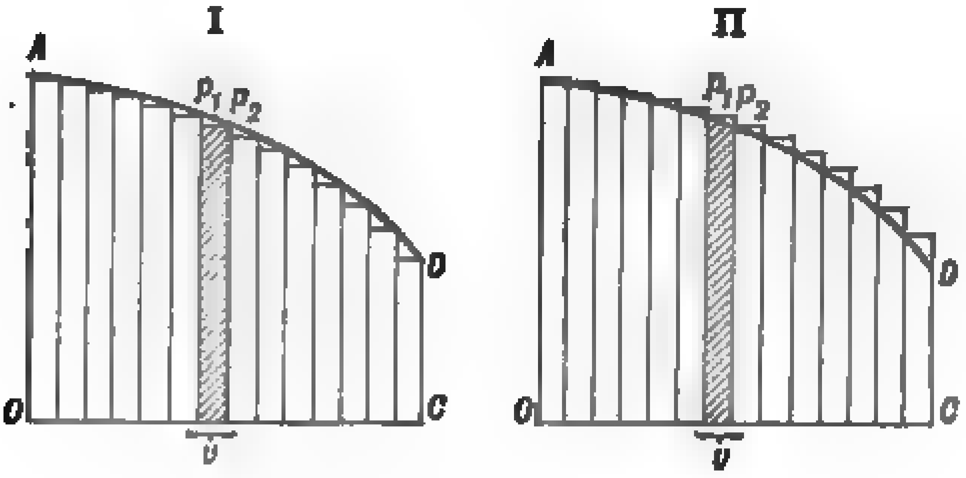


Fig. 616.

and the base line OC. For imagine the increase in volume to be made by a succession of small steps, one of which is represented by v in figure 616. Then, if during the expansion v the pressure changes from p_1 to p_2, the work done will have a value between p_1v and p_2v, and will be greater than the shaded strip in diagram *I* and less than the corresponding strip in diagram *II*. The total work done in the expansion from A to D will be greater than the sum of all the little shaded strips in diagram *I* and still be less than those strips in diagram *II*. But the greater the number of strips in the expansion, the smaller will be the width of the strips and the more nearly will the sum of the areas in both cases approach the area $ADCO$ as a limit; that area, therefore, must represent the actual work in the expansion from A to D.

Hence in general **the work of an expansion represented by any line AD in the pressure-volume diagram, is equal to the area under that line down to the base line, or line of zero pressure.**

3. Carnot's Cycle.—In the year 1824 a French engineer, Sadi Carnot, then only 28 years of age, published a work of extraordinary originality on "the motive power of heat," in which he arrived at results of fundamental importance through the consideration of the properties of an ideal heat engine, in each complete stroke of which the working substance was conceived to be put through a special series of four changes known as a Carnot's cycle, by which it was brought again to its original condition.

The working substance S is supposed to be enclosed in a cylinder having side walls and piston absolutely non-conducting, as indicated by heavy lines in the diagram, while the bottom is a perfect conductor of heat. Three stands are provided upon

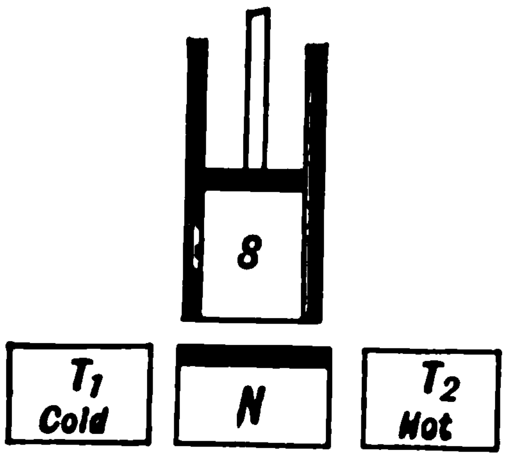


Fig. 617.—Carnot's engine.

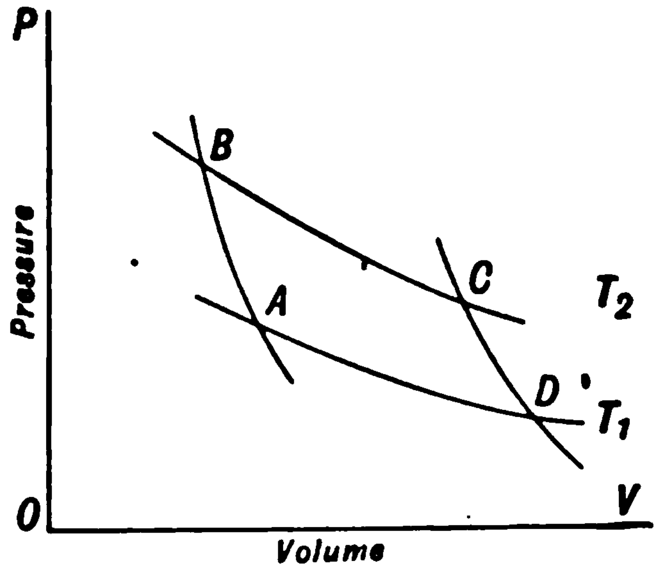


Fig. 618.

which the cylinder may be placed, a perfectly conducting hot stand at temperature T_2, a perfectly conducting cold stand at temperature T_1, and a perfectly non-conducting stand N.

Suppose the cylinder has been standing on the cold stand, and the working substance S, which may be supposed to be steam or air or any substance whatever, has come to the temperature T_1 and has a pressure and volume represented by the point A on the diagram (Fig. 618).

1. The cylinder is transferred to the non-conducting stand N and the substance is compressed until its temperature rises to T_2 in consequence of the work expended in its compression, and comes to the volume and pressure represented by B. In this operation no heat can flow into or out of the substance, it is therefore called an *adiabatic* operation or change.

2. The cylinder is shifted to the hot stand and the substance allowed to expand. As it expands it does work and would be

cooled except that heat freely flows in from the hot stand keeping its temperature constant at T_2. Let it expand in this way by some convenient amount until its volume and pressure may be represented by C on the diagram. The change from B to C is called *isothermal* because the temperature has remained constant.

3. Now place the cylinder again on the non-conducting stand, and let S expand still further. It will cool in consequence of the work done in the expansion since no heat is supplied, and may be cooled in this way to the temperature T_1 of the cold stand. This change is *adiabatic* and represented by the line CD on the diagram.

4. Placing the cylinder on the cold stand the piston is pushed down compressing the substance from D to its original volume at A. During this operation the heat of compression is taken up by the cold stand as fast as it is developed and the temperature of the substance is thus kept constant at T_1. This change is *isothermal*, and while it is taking place heat flows out of the working substance into the cold stand.

This series of operations in which the working substance is subjected to two isothermal and two adiabatic changes and brought again to its original pressure volume and temperature, is known as a Carnot's Cycle; its peculiarity is that no heat transfer takes place except at two definite temperatures.

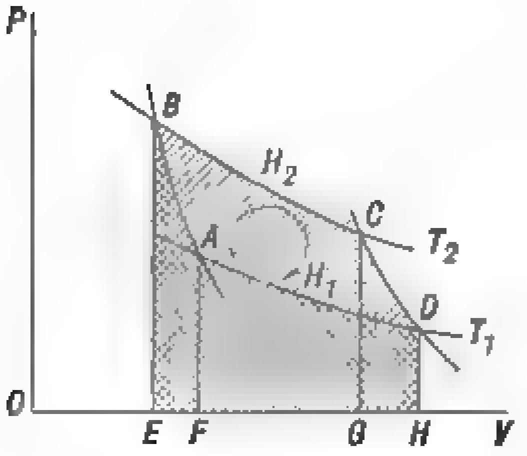


Fig. 619.

4. Work in a Carnot's Cycle.—The work done by the working substance in expanding from B to C is represented by the area $EBCG$ (Fig. 619), and during the adiabatic expansion from C to D it does work represented by the area $GCDH$, while the work done *upon* the substance as it is compressed first from D to A and then from A to B is measured by the double crossed area $EBADH$, below the line DAB. The net amount of work obtained from the engine in a complete cycle is then represented by the excess *of the* work done *by* the substance in expanding, over the work

›ne *upon* it in the compression, and is given by the area $ABCD$. During the isothermal expansion BC at the higher tempera-ıre T_2 heat is taken in from the hot stand, and during the iso-.ermal compression DA heat is given out to the cold stand, hile no heat at all is transferred during the two adiabatic ıanges. But the engine has done an amount of work during ıe cycle equal to the area $ABCD$, and since the working sub-ance is in exactly the same state at the end as it was in the be-nning, the energy expended in work cannot have come from the ıbstance, but must have come from the energy supplied in the ›rm of heat, otherwise the law of the conservation of energy ould be violated. Experiment confirms this conclusion and we ıd that **the heat H_2 taken in by the substance in expanding om B to C is more than the heat H_1 given out in the compres-on from D to A, the difference between the two being mechan-ally equivalent to the work of the cycle** $ABCD$; so that if work ıd heat are measured in the same units, which can be done nce both are forms of energy, we have

$$W = H_2 - H_1$$

here W is the work represented by the area $ABCD$.

5. **First Law of Thermodynamics.**—The conclusion just reached based on what is known as *the first law of thermodynamics*, hich is simply the law of the conservation of energy as applied › the relation of heat to work. It asserts that *wherever work ı obtained by any heat process an equivalent amount of heat issappears, and vice versa.*

6. **Efficiency of an Engine.**—The efficiency of a heat engine is he ratio of the work which the engine does to the energy that as to be supplied to it from the hot stand or boiler. In the ase just discussed the efficiency E may be expressed by the ›rmulas

$$E = \frac{W}{H_2} \quad \text{or,} \quad E = \frac{H_2 - H_1}{H_2}$$

7. **Reversible and Irreversible Operations.**—Some operations re in their energy relations reversible and some are irreversible. 'or instance when a weight is raised energy is expended, and when ; is lowered again an equal amount is given back. This is a eversible operation; but when work is done against friction

energy is transformed into heat, and when the motion is reversed energy is not recovered but still more is spent in heat, and the operation is not reversible. So the conduction of heat from hotter to colder bodies is an irreversible operation, while the adiabatic heating or cooling of a substance as its volume is changed without any conduction of heat taking place, is a reversible change.

A reversible engine is one in which the operations are all reversible. If an engine working in a Carnot's cycle were to be made so that during the isothermal expansion in which heat is taken in from the hot source there were absolutely no difference of temperature between the working substance and the source, and if there were a corresponding transfer of heat with no difference of temperature during the isothermal compression while heat is given out, the operations would then all be reversible.

If such an engine were driven backward the working substance would be put through the cycle of operations represented in figure 618 in the reverse order. Expanding from A to D it would take in the same quantity of heat H_1 at the cold temperature T_1 as it had given out at that temperature when direct acting; and in the compression from C to B it would give out the same quantity of heat H_2 to the hot body as it had taken in when direct acting. And in this reversed action the heat given out would be more than that taken in, by the work of the cycle W, which would in this case have to be supplied from outside, for more work would be done upon the working substance in the compression than would be done by it in the expansion.

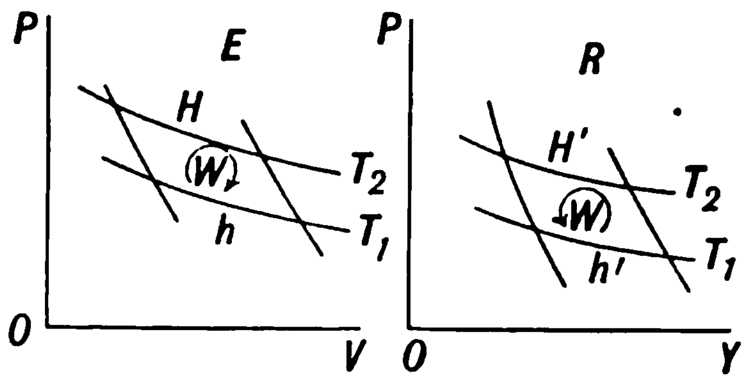


Fig. 620.

It is evident that no actual engine is ever reversible, nevertheless the results obtained by considering the properties of such engines are of very great importance.

8. Carnot's Theorem. — *No engine can be more efficient than a reversible engine working between the same limits of temperature.* For, if possible, let some engine E be more efficient than the reversible engine R working between the same limits of temperature T_1 and T_2, and suppose the strokes of the engines are so

adjusted that the work done per cycle is the same by one as by the other. Then let the more efficient engine E be coupled to the reversible engine R so as to drive it backward or in the reversed direction. This it will be able to do for the work required to drive the reversible engine backward is exactly what it would have performed if direct acting. In every cycle the engine E takes in heat H at the upper temperature and gives out heat h at the lower temperature and the work W which it does is equal to $H-h$ as we have seen (§4). The reversible engine, on the other hand, takes in heat h' at the lower temperature and gives out heat H' at the higher temperature and in this case also $H'-h'=W$; and so since W is the same for one as for the other, we have $H-h=H'-h'$. But by hypothesis the efficiency of E is greater than the efficiency of R, that is

$$\frac{W}{H} > \frac{W}{H'}, \text{ therefore } H < H' \text{ and } h < h'$$

That is, the heat H' returned to the boiler is more than the heat H taken from it, and the heat h' taken out of the cold body is more than the heat h which is given to it. There results, therefore, a steady transfer of heat from the cold to the hot body, the cold body growing colder and the hot body hotter through the agency of the combined engines working continuously without any outside assistance. This result is believed to be impossible, for it contradicts all experience. This conviction when formulated is called **the second law of thermodynamics** and may be stated thus: **It is impossible by any continuous self-sustaining process for heat to be transferred from a colder to a hotter body.**

We conclude then that *no engine can be more efficient than a reversible engine working between the same limits of temperature,* and consequently, *all reversible engines working between the same limits of temperature are equally efficient,* whatever working substance is used in the engine, whether air, steam, gas, or substance of any sort.

9. Absolute Temperature Scale.—Let us suppose that the diagrams in figure 621 are for two different gases or vapors between the same limits of temperature T_1 and T_2, then according to the theorem just given a reversible engine working with one substance in the cycle $ABCD$ will be exactly as efficient as another

reversible engine using the other substance and working around the cycle $A'B'C'D'$. Consequently

$$\frac{W}{H} = \frac{W'}{H'}$$

where W and W' are the work areas of the two cycles and H and H' are the quantities of heat taken in at the temperature T_2.

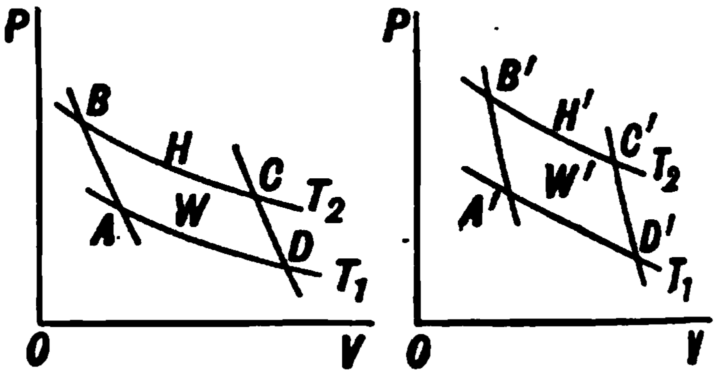


Fig. 621.

Since the ratio $^W/_H$ depends only on the temperatures T_1 and T_2 and not at all on the kind of substance used, Lord Kelvin proposed making it the basis of a truly absolute scale of temperature which should be quite independent of the individual peculiarities of different substances. This scale may be easily understood by the aid of figure 622, which is the pressure volume diagram of some substance, for which AB and CD are two adiabatic curves. Suppose T_1 and T_2 are two standard temperatures used to fix the size of the degree of the scale. For instance T_2 may be the standard boiling temperature of water, and T_1 the freezing point of water, and we may decide upon a scale in which, as in the centigrade scale, there shall be 100° between the two temperatures. Then if isothermal lines for the substance are drawn between T_1 and T_2 so as to divide the whole area W into 100 equal parts, these will correspond to successive degrees of temperature of the Kelvin scale, and the area between any two isothermal lines which are 1° apart will be one one-hundredth of W, or a.

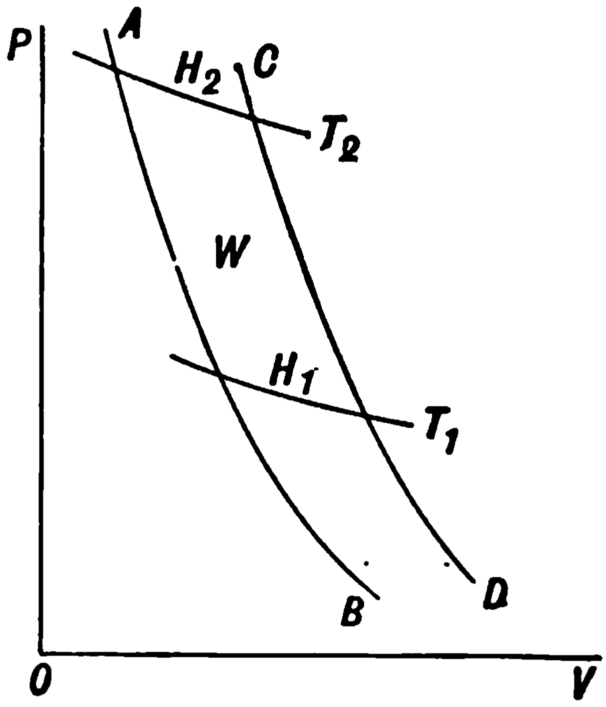


Fig. 622.

But in passing from the adiabatic line AB to the adiabatic CD at the temperature T_1 a quantity of heat energy H_1 is absorbed by the substance which is less than H_2, the amount absorbed at T_2, a temperature 100° higher, by an amount equal to W, or $100a$, and at 1° below T_1 the heat taken in in passing from one adiabatic

the other will be less than H_1 by a; at 2° below T_1 it will be less y $2a$; and so on. But if we take as many degrees below T_1 as is contained in H_1, we shall reach a temperature at which no ɜat at all would be taken in or given out in the isothermal ıange from one adiabatic to the other. Now, if this temperaıre is taken as the lower one in a Carnot's cycle, and any other ɜmperature be taken as the higher one, a reversible engine workıg in the cycle will have unit efficiency, that is, all the heat taken at the upper temperature will be transformed into mechanical ork, and none will be given out at the lower temperature.

No higher efficiency than this can be possible and the lower ɜmperature thus defined is taken as the absolute zero.

Experiment shows that H_1 is nearly 273 times a, so that the eezing point of water is about 273° above the absolute zero of ıe Kelvin scale, or as we may write it 273°.K., and the boiling ɔint being 100° higher will be 373°.K.

If 180° had been taken between the freezing and boiling points of water, as the ordinary Fahrenheit scale, the freezing temperature of water would ıve been found 491° and the boiling point 671° above the absolute zero.

It is interesting to note that temperatures measured on the ɔsolute scale of Lord Kelvin agree very closely with temperaıres measured from the so-called absolute zero of the air therometer (§394); but whereas the zero of the air thermometer as based on the behavior of a particular class of substances, ıe gases, the zero of the Kelvin scale is independent of the ·operties of any particular substance.

10. Efficiency of a Heat Engine on Kelvin's Scale.—From the ɔove explanation of the Kelvin scale of temperature it will be en that *the heat taken in by a substance in passing from one given liabatic to another given adiabatic at any temperature is proporɔnal to the number of degrees on the Kelvin scale which that temɪrature is above the absolute zero; that is*

$$\frac{H_1}{T_1} = \frac{H_2}{T_2} = a$$

$$H_1 = aT_1 \qquad \text{and} \qquad H_2 = aT_2$$

But we have seen (§6) that the efficiency E of a reversible ıgine may be expressed in terms of the heat H_2 taken in at tl

higher temperature and the heat H_1 given out at the lower temperature by the relation

$$E = \frac{H_2 - H_1}{H_2}.$$

So that substituting the values just given for H_1 and H_2 we have

$$\left.\begin{array}{l}\text{Efficiency of a reversible engine acting}\\ \text{between the temperatures } T_1 \text{ and } T_2\end{array}\right\} = \frac{T_2 - T_1}{T_2}$$

where the temperatures are measured on the Kelvin absolute scale.

But no engine is more efficient than a reversible engine working between the same limits of temperature; hence the expression gives an upper limit to the efficiency of any heat engine whatever. It will be noted that to secure high efficiency in a steam engine there must be great difference in temperature between the steam as it enters and as it escapes, and the high efficiency of gas engines is due in part to the high initial temperature of the exploding mixture.

APPENDIX II

Proof of Newton's Wave Formula

The following proof of Newton's law that in case of a compressional wave $V = \sqrt{\frac{E}{d}}$ is due to the distinguished English engineer and physicist, W. M. Rankine. Let a sound wave be moving forward in a medium in the direction and with velocity indicated by the arrow V, and let A and B be two parallel planes, perpendicular to V, which move forward with the same velocity as the wave, and therefore are always the same distance apart.

Let u represent the velocity of a particle due to its oscillation relative to the medium,

p the pressure in the medium,
d the density of the medium,
v the volume of unit mass, $v = \frac{1}{d}$;

and let the values of these quantities at the plane A be represented by $u_1p_1d_1v_1$ and at plane B by $u_2p_2d_2v_2$.

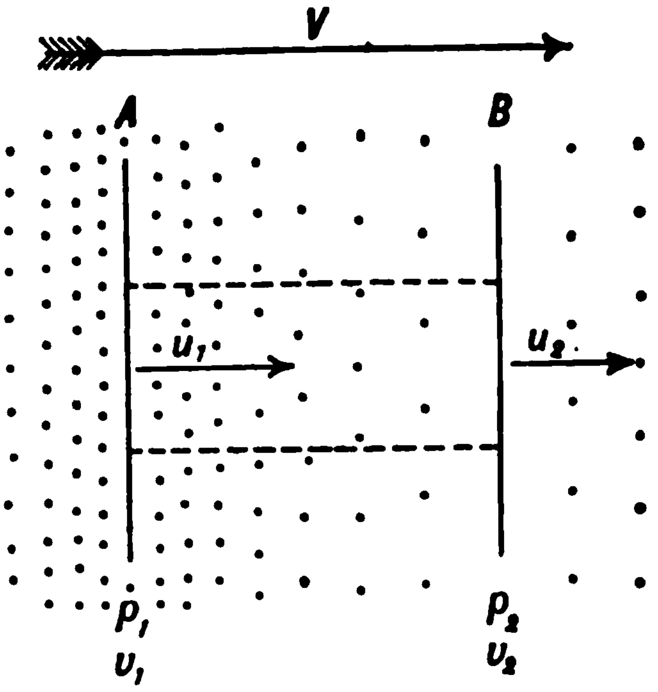


Fig. 623.

Now suppose for definiteness that the plane A is at the point of maximum compression of the wave, then u_1 will be maximum and in a forward direction as indicated in the figure, p_1 and d_1 will both have their maximum values, and these values remain constant as the plane moves on, for it moves *with the wave*. So also the conditions at plane B, at same other point in the wave, though different from those at A remain constant as the plane moves on.

To an observer moving with the planes A and B the medium will be seen to stream through the planes from right to left. At B its velocity relative to the plane will be $V-u_2$ and this then will be the number of cubic centimeters of the medium that pass through each square centimeter of B in one second. Let M be the *mass* of this volume, and we have

$$Mv_2 = V - u_2 \tag{1}$$

The mass M passing B per second must be the same as the mass passing A in the same time otherwise there will be a change in the amount of matter between A and B as the wave moves on, therefore

$$Mv_1 = V - u_1 \tag{2}$$

At a point where the velocity of the particles is zero the medium has its normal density d, and normal volume per unit mass v, hence at that point

$$Mv = V \tag{3}$$

Now consider a column of the medium reaching from A to B and having 1 sq. cm. cross section, the mass M entering this region through B in one second has momentum Mu_2, and the same mass leaving at A has momentum Mu_1 which we have supposed greater. The column thus experiences a *loss of momentum per sec.* equal to $Mu_1 - Mu_2$. But the conditions in the column *do not change* as the wave moves along, so the loss must be balanced by an equal gain in momentum. This is supplied by the difference between the pressures p_1 and p_2 on the two ends of the column, p_1 urging it toward the right and p_2 toward the left. The column must, therefore, by Newton's Second Law of Motion, experience a gain in momentum per second from left to right, equal to $p_1 - p_2$; and this momentum must be equal to that which the column loses in the same time.

We have therefore $$M(u_1 - u_2) = p_1 - p_2 \tag{4}$$

But subtracting (2) from (1)

$$(u_1 - u_2) = M(v_2 - v_1)$$

whence $$M^2(v_2 - v_1) = p_1 - p_2$$

And by equation (3)

$$\frac{V^2}{v^2} = \frac{p_1 - p_2}{v_2 - v_1} \quad \text{or} \quad V^2 = v\left(v\,\frac{p_1 - p_2}{v_2 - v_1}\right).$$

But by §§ 240 and 242, $E = v\dfrac{p_1 - p_2}{v_2 - v_1}$ therefore, $V^2 = vE$.

And since $v = \dfrac{1}{d}$ we have finally $V = \sqrt{\dfrac{E}{d}}$.

TABLES OF CONSTANTS

Figures refer to pages

INDEX

Figures refer to pages.

[redacted]